

# Semester assignment ElMag - Simulation of aurora

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Auroras come from charged particles, launch into outer space by sun storms, being guided by the magnetic field of the earth down in the atmosphere. (...)

## Parametres

This problem has many parametres,  $q, M, R_\odot, m, \mu_0$ , which is respectively the particle charge and mass, the earths mass and magnetic dipole moment and the magnetic permeability of space. These can be reduced to 4 characteristic sizes, namely length, magnetic field, time and energy

$$x_0 = R_\odot, \quad B_0 = \frac{\mu_0 m}{x_0^3}, \quad t_0 = \frac{M}{q} \frac{1}{B_0}, \quad E_0 = M \frac{x_0^2}{t_0^2}.$$

Real world values for these are

$$x_0 = 6.47 \cdot 10^6 \text{ m}, \quad B_0 = \frac{1.26 \cdot 10^{-6} \cdot 8 \cdot 10^{22}}{x_0^3} = 3.7 \cdot 10^{-5} \frac{\text{kg}}{\text{s}^2 \text{ A}}$$
$$t_0 = \frac{1.6 \cdot 10^{-27}}{1.6 \cdot 10^{-19}} \frac{1}{B_0} = 2.7 \cdot 10^{-4} \text{ s}, \quad E_0 = 1.6 \cdot 10^{-27} \frac{x_0^2}{t_0^2} = 9.2 \cdot 10^{-9} \frac{\text{kg m}^2}{\text{s}^2}$$

These will be used as units through the exercise. However was  $x_0/t_0 \sim 240c$ , so we choose to slow down the simulation by a factor  $5 \cdot 10^5$ , i.e. using  $t_0 \cdot 5 \cdot 10^5 \sim 135 \text{ s}$  as the time unit. Then, a typical coronal ejection has speed unity.

## Magnetic field

We can model the earths magnetic field as a dipole. Let the x-axis point from the centre of the earth towards the sun, the y-axis roughly parallel to the earths orbit, and then the z-axis northwards. Let

$$\hat{m} = \hat{e}_i \hat{m}_i$$

be the normalized direction of earths magnetic dipole-moment. The earths magnetic field is then given by

$$B_j = \frac{1}{4\pi} \frac{3\hat{m}_i \hat{x}_i \hat{x}_j - \hat{m}_j}{r^2}, \quad (1)$$

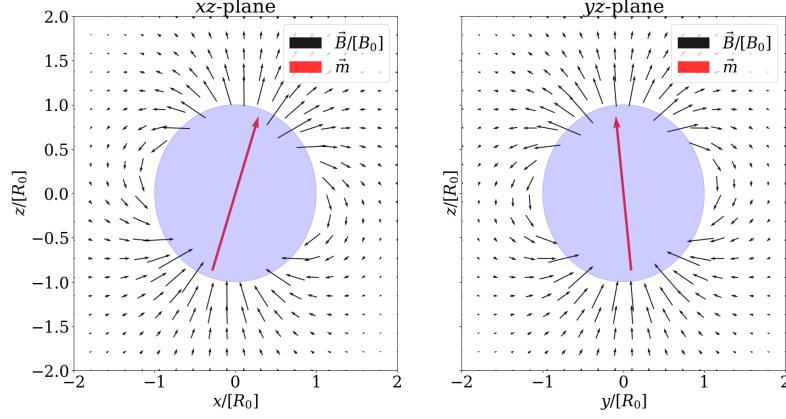


Figure 1: The earths magnetic field, in the  $xz$ - and  $yz$ -plane.

where

$$r = \sqrt{x_i x_i}, \quad \hat{x}_i = x_i / r.$$

Figure 1 shows the magnetic field as seen from the earths orbit, and on the other side than the sun.

## Charged particles

Let  $\hat{e}_i B_i(x_j)$  be the earths magnetic field. The magnetic force on a particle with charge  $q$ , position  $\hat{e}_i x_i(t)$  and velocity  $\hat{e}_i \dot{x}_i(t)$  is then

$$F_i = \epsilon_{ijk} \dot{x}_j B_k. \quad (2)$$

By newtons second law, we get the equation of motion for the particle,

$$\ddot{x}_i = \epsilon_{ijk} \dot{x}_j B_k \quad (3)$$

Rewriting to a first order set of equations, we get

$$\frac{d}{dt} y = \frac{d}{dt} \begin{pmatrix} x_i \\ \dot{x}_i \end{pmatrix} = \begin{pmatrix} \dot{x}_i \\ \epsilon_{ijk} \dot{x}_j B_k \end{pmatrix} = f(y).$$

This can then be solved numerically by a runge kutta method. This is shown in figure 2, where several particles with different starting positions are simulated as they approach the earth, and are deflected by the magnetic field of the earth. The only particles that are able to reach down to earths surface are particles approaching from higher up, explaining why we only see auroras near the poles.

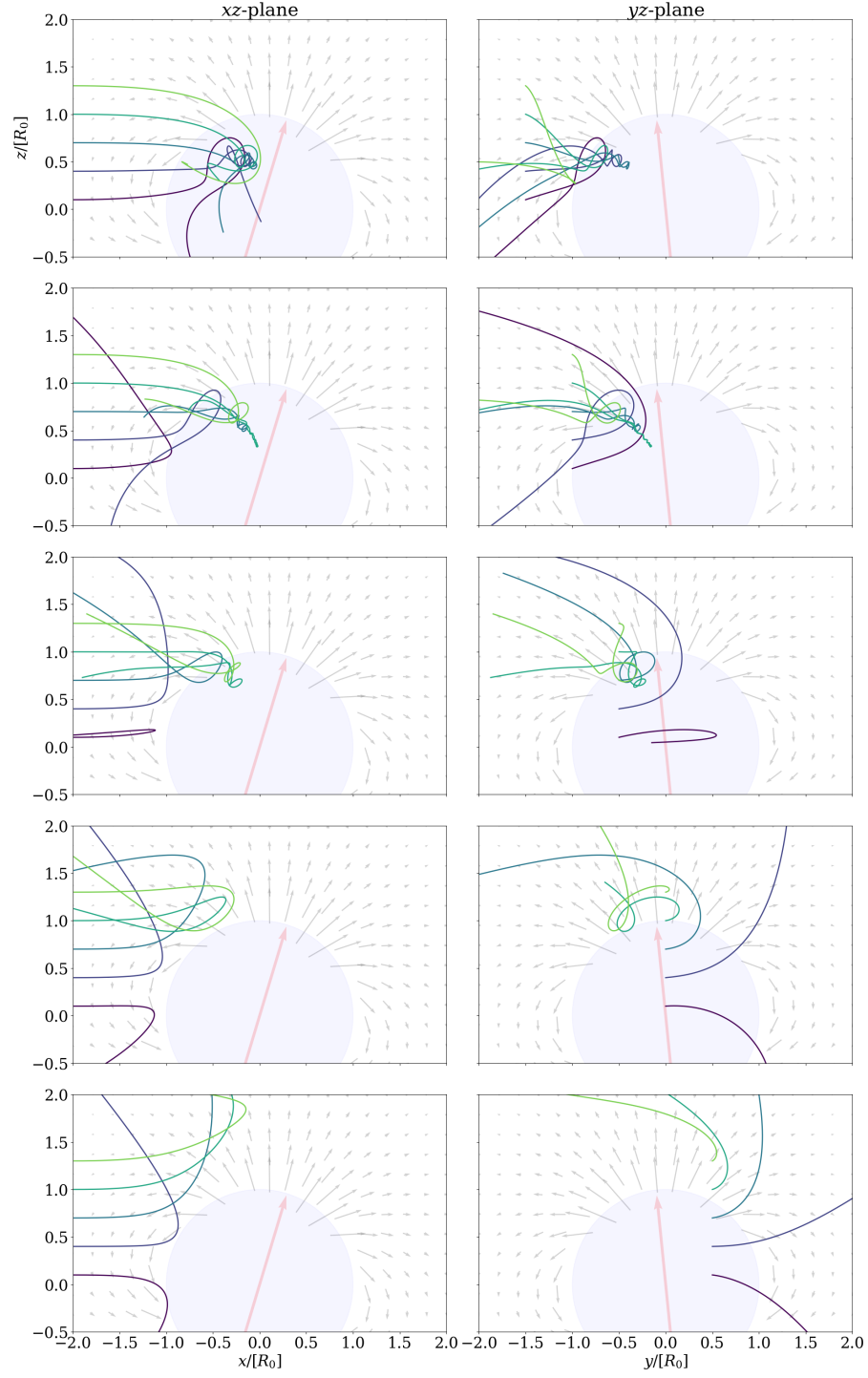


Figure 2: Charged particles approaching earth from different starting points. Even though the lines may overlap with the earth, it does not mean that they have hit the ground, as the graph only contains two dimensions.

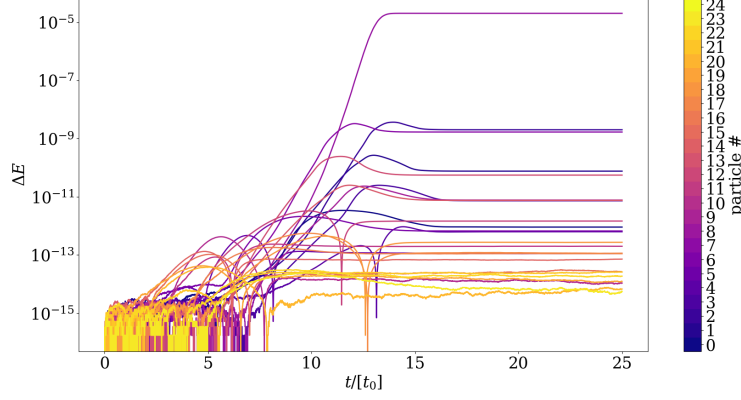


Figure 3: The relative shifts in energy, in comparison with the starting values. Particle 3 has the largest value,  $2.036 \cdot 10^{-5}$

## Accuracy

To test the accuracy of the numerical scheme, one can use the fact that magnetic forces never do any work on particles, as they always are perpendicular to the velocity of a particle. The potential energy of the particle should thus be conserved, as the simulation does not take into account friction, nor electromagnetic radiation. Kinetic energy is given by

$$E = \frac{1}{2} \dot{x}^2,$$

in units of  $E_0$ . Relative error,

$$\Delta E(t) = \frac{|E(t) - E(0)|}{E(0)},$$

is thus a useful quantity for evaluating the precision of the simulations. Figure shows this for all simulated paths.