Semester assigmen ElMag - Simulation of aurora

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Auroras come from charged particles, launch into outer space by sun storms, beeing guided by the magnetic field of the earth down in the atmosphere. (...)

Parametres

This problem has many parametres, q, M, R, m, μ_0, which is respectivley the particle charge and mass, the earths mass and magnetic dipole moment and the magnetic peremability of space. These can be reduced to 3, namely characteristic time, length and magnetic field,

$$t_0 = \frac{MR^3}{qm\mu_o}, \quad x_0 = R, \quad B_0 = \frac{m\mu_0}{R^3}.$$

Real world values for these are

$$t_0 = \frac{1.6 \cdot 10^{-27}}{1.6 \cdot 10^{-19}} \frac{(6.47 \cdot 10^6)^3}{8 \cdot 10^{22} 1.26 \cdot 10^{-6}} = 270 \cdot 10^{-6}\mathrm{s}$$

$$x_0 = 6.47 \cdot 10^6 \mathrm{m}, \quad B_0 = \frac{8 \cdot 10^{22} 1.26 \cdot 10^{-6}}{(6.47 \cdot 10^6)^3} = 0.0037 \cdot 10^{-2} \mathrm{kg\,S^{-2}\,A^{-1}}.$$

These will be used as units through the exercise, however we see that $x_0/t_0 \sim 240c$, so we choose to slow down the simulation by a factor $5 \cdot 10^5$, i.e. using $t0 \cdot 5 \cdot 10^5 \sim 135$s as the time unit. Then, a typical coronal ejection has speed unity.

Magnetic field

We can model the earths magnetic field as a diploe. Let the x-axis point from the centre of the earth towards the sun, the y-axis roughly paralel to the earths orbit, and then the z-axis northwards. Let

$$\hat{m} = \hat{e}_i \hat{m}_i$$

be the normailzed direction of earths magnetic dipole-moment. The earths magnetic field is then given by

$$B_j = \frac{1}{4\pi} \frac{3\hat{m}_i \hat{x}_i \hat{x}_j - \hat{m}_j}{r^2}, \tag{1}$$

where

$$r = \sqrt{x_i x_i}, \quad \hat{x}_i = x_i / r.$$

Figure shows the magnetic field as seen from the eraths orbit, and on the other side than the sun.

Charged particles

Let $\hat{e}_i B_i(x_j)$ be the earths magnetic field. The magnetic force on a particle with charge q, position $\hat{e}_i x_i(t)$ and veloxity $\hat{e}_i \dot{x}_i(t)$ is then

$$F_i = \epsilon_{ijk} \dot{x}_j B_k. \tag{2}$$

By newtons second law, we get the equation of motion for the particle,

$$\ddot{x}_i = \epsilon_{ijk} \dot{x}_j B_k \tag{3}$$

Rewriting to a first order set of equations, we get

$$\frac{\mathrm{d}}{\mathrm{d}t} y = \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_i \\ \dot{x}_i \end{pmatrix} = \begin{pmatrix} \dot{x}_i \\ \epsilon_{ijk} \dot{x}_j B_k \end{pmatrix} = f(y).$$

This can then be solved numerically by a runge kutta method.