

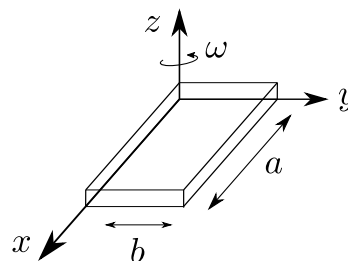
Exercise 7 - TFY4345 Classical Mechanics

2020

1 Inertia tensor

A very thin uniform rectangular slab of mass M has been placed in the xyz -coordinates system, such that the origin is in one of the slab corners, and the sides are along the x - and y -axes. The corresponding side lengths are a and b . Since the slab is very thin, we can assume that $z = 0$ throughout.

- Evaluate the individual elements of the inertia tensor.
- Set $a = b$, and solve for the principal moments of inertia, and the corresponding principal axes.



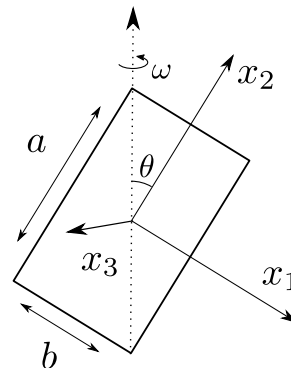
2 Rotated tilted slab

(Exam Aug. 2018)

Consider the same thin slab. However, now it rotates around an axis parallel to its diagonal, with an angular velocity vector ω . As the axis of rotation has change, so has its principal axis. The principal axes x_1 and x_2 are parallel to the slab edges, as indicated in the figure, while the axis x_3 is perpendicular to the slab, and goes through its center. The side lengths of the slab are a and b , with the principal moments of inertia

$$I = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} = M \begin{pmatrix} \frac{1}{12}a^2 & 0 & 0 \\ 0 & \frac{1}{12}b^2 & 0 \\ 0 & 0 & \frac{1}{12}(a^2 + b^2) \end{pmatrix}$$

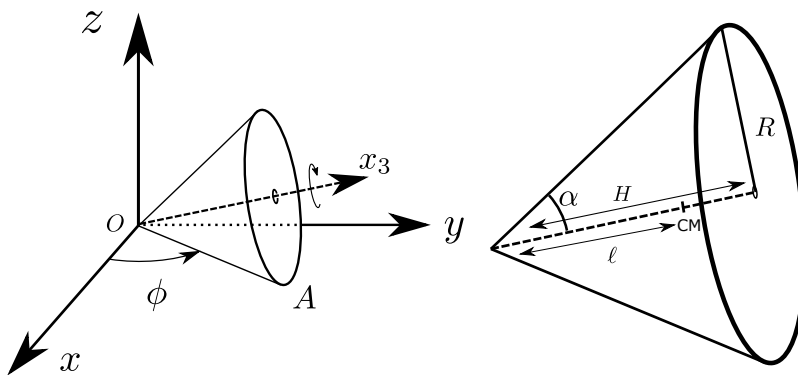
- Derive the angular momentum vector \mathbf{L} of the slab, in terms of a, b, M and ω .
- What is the angle α between \mathbf{L} and ω ?
- What is the rotational kinetic energy T ?



3 Cone rolling on a plane

(Exam Dec. 2016)

We shall consider the motion of a solid cone that is rolling on the surface (xy -plane), without slipping. The center of mass of the cone is situated on the symmetry axis x_3 , which goes through the center of the bottom of the cone. It is at a distance ℓ from the origin O . The height of the cone is H , and the radius of the bottom of the cone is R , and the cone half angle is α . These are related by $R \cos(\alpha) = \sin(\alpha)H$. The momentary line of contact between the cone and the xy -plane is the line OA , which is at an angle ϕ relative to the x -axis.



- (a) Calculate the velocity of the center of mass, V_{CM} as a function of ℓ, α and $\frac{d}{dt}\phi = \dot{\phi}$.
- (b) Explain why the angular velocity vector $\boldsymbol{\omega}$ of the rolling cone is directed along the line OA , the line of contact between the cone and the xy -plane. Show that

$$\omega = |\boldsymbol{\omega}| = \frac{1}{\tan(\alpha)} \dot{\phi}$$

- (c) x_3 is one of the principal axis of rotations, what are the other? Remember that the coordinate transformation that takes xyz into the principal axis $x_1x_2x_3$ is a rotation. Due to the symmetry of the cone, we have some freedom in choosing the other two axis. Choose x_1 to be in the plane spanned by $\boldsymbol{\omega}$ and x_3 , and find the components of $\boldsymbol{\omega}$ along the principal axes.
- (d) The principal moment of inertia of the cone, for rotation around the point O is

$$I = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} = \frac{3}{20} M \begin{pmatrix} R^2 + 4H^2 & 0 & 0 \\ 0 & R^2 + 4H^2 & 0 \\ 0 & 0 & 2R^2 \end{pmatrix},$$

where M is the mass of the cone. Calculate the kinetic energy of the cone as a function of M, H, α and $\dot{\phi}$.