Classical Mechanics TFY 4345 – Exercise 6

1. Elastic scattering in laboratory coordinates.

Consider the relation between the laboratory and centre-of-mass (CM) scattering angles

$$cos\vartheta = \frac{cos\Theta + \rho}{\sqrt{1 + 2\rho cos\Theta + \rho^2}}$$

and the resulting expression for cross sections

$$\sigma'(\vartheta) = \sigma(\Theta)\frac{(1 + 2\rho cos\Theta + \rho^2)^{3/2}}{1 + \rho cos\Theta}$$

Let us assume that we have two particles with **equal masses m_1 and m_2** which scatter elastically ($\rho=m_1/m_2=1$). The first particle is the incoming particle while the latter is initially at rest. Derive the connection between the **scattering angles** and **cross sections** based on the equations above. Furthermore, what is the **ratio for the kinetic energy** for particle 1 before and after scattering (E_1/E_0)?

Hint: You will need this general trigonometric relation with the scattering angles:

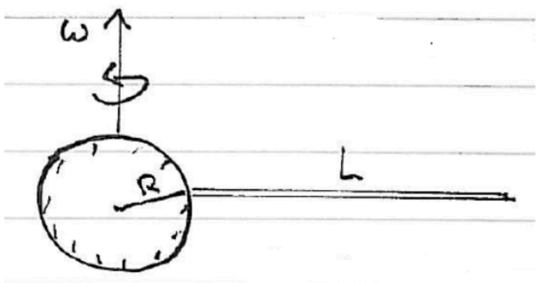
$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + cos\alpha}{2}}$$

2. Rotating system in cylindrical coordinates.

Write down Lagrange's and Hamilton's equations in cylindrical coordinates $q_i = q_i(r,\theta,z)$ for a particle with mass m in a potential $V = V(q_i)$. This is an easy one.

3. Centrifugal force and gravitation.

A rigid strong rod with uniform mass density hangs in a stationary position from a fixed point at the equator, directed upwards, without being suspended at any point (the rod thus follows the rotation of Earth). Here, the integrated gravitational effect equals to the centrifugal term. What is the length L of the rod? The radius R of Earth can be set to 6400 km.



Hint: You will need to consider two integrals. You can either calculate the angular velocity of Earth yourself (think!) or look at the lecture notes.

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4. Coriolis effect on a falling particle.

Find the horizontal deflection from the plumb line caused by the Coriolis force acting on a particle falling freely in the Earth's gravitational field from a height *h* above the Earth's surface. What is the direction?

Hint: You can safely neglect the x- and y- components of the velocity while building up the determinant for the cross-product. Otherwise the problem becomes too complicated (→ numerical solution).

Historical note: The direction of the deflection was already predicted by Isaac Newton (1679) and confirmed experimentally by Robert Hooke and several others. The most careful measurements were performed by F. Reich (1831), who dropped pellets down a mine shaft 188 meters deep and observed a mean deflection of 28 mm. This value is smaller than the prediction by theory (your result) due to air resistance effects.

Physical note: If one shoots the particle upwards initially so that it reaches the altitude *h* before falling down, the final deflection on the ground will be in the reverse direction (Exam December 2017). Think about it!