Classical Mechanics TFY 4345 – Exercise 4

1. Mathematical pendulum. The pendulum is lifted with a constant acceleration *a*. Derive the Hamiltonian and Hamilton's equations of motion. Does the relation $H = E$ hold and is H a constant of motion?

2. Spherically symmetric potential. A moving particle experience a potential $U(r) = -k/r$. Determine the Hamiltonian and Hamilton's equations of motion using spherical coordinates.

3. Earth's orbit. Assume the Earth's orbit to be circular and that the sun's mass suddenly decreases to one half. What orbit will Earth then have? Will it escape from the solar system? Use the equation for eccentricity.

4. Einstein's correction. A single particle with mass *m* moves in a central force field and experiences a force

$$f(r) = -\frac{k}{r^2} + \frac{\beta}{r^3}$$

where β is a constant. This is an extension of the original Kepler problem to include Einstein's correction from the general theory of relativity. Show that the resulting trajectory can be written in the form

$$\frac{p}{r} = 1 + \varepsilon\cos(\gamma\theta)$$

which gives an ellipse for $\gamma = 1$ and precession of the orbit otherwise. For Mercury, the value of β is 43 arc seconds / century. Find the expressions for ε, *p*, and γ.

Hint 1: Start from the related integral in the original Kepler problem, lecture notes & course book.
Hint 2: Identify that $\gamma^2 = 1 + \frac{\beta m}{\ell^2}$ before integration, use it later in the definitions of ε and *p* (note that $\gamma = 1$ in the original Kepler problem).