Classical Mechanics TFY4345 - Exercise 1

1. Halley's comet - conservation of angular momentum

Halley's comet follows an elliptical orbit around the Sun, with a period of about 76 years. The Sun is a focal point of the ellipse. The closest distance between the comet and the Sun is 0.6 AU, and the farthest distance is 35AU. (1AU = mean distance between the Sun and the earth)

Use the sun as origin in your coordinate system.
(a) Explain why the net torque on Halley's comet is zero. This implies that the angular momentum is conserved.
(b) When the comet is closest to the Sun, it's velocity is measured to be 54km/s. Use conservation of angular momentum to calculate the velocity when the comet is farthest from the Sun.

2. Simple Pendulum

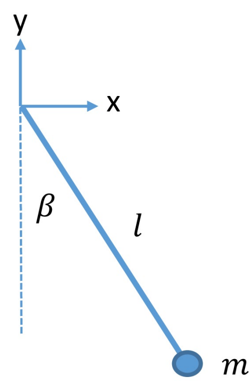

Figure 1: Simple pendulum: a mass m suspended in a massless cord. l is the distance between the pivot point and the center of the mass m. β is the angle between the y-axis and the cord.

Consider a simple pendulum subject to a uniform gravitational field $\vec{g} = -g\vec{e}_y$. Choose the pivot point as origin of your coordinate system. There are no friction forces.

(a) Show that the position vector of the mass m is: $\vec{R} = l\sin(\beta)\vec{e}_x - l\cos(\beta)\vec{e}_y$
(b) Find the potential energy of the mass, as a function of the angle β.
(c) Find the kinetic energy of the mass, as a function of β and $\dot{\beta} = \frac{d\beta}{dt}$
(d) The Lagrangian of the pendulum is $L = T - V$. Use Lagrange's equations to obtain the equation of motion for the pendulum.

3. Double Pendulum

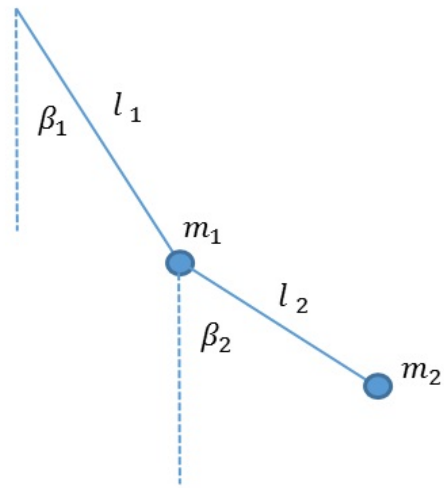

Figure 2: Double pendulum

(a)Find the Lagrangian $L = T - V$ for the coplanar[1] double pendulum in a uniform gravitational field. Choose the angles β_1 and β_2 as coordinates.
(b Obtain the equations of motion using Lagrange's equations.

For one more assignment, see the next page!

[1]coplanar objects = objects in the same plane

4. Show by direct substitution that

$$L'(q,\dot{q},t) = L(q,\dot{q},t) + \frac{dF(q,t)}{dt}$$

where *F* is an arbitrary function and leads to the same Lagrangian equation as $L(q,\dot{q},t)$.

Hint: Start from the Lagrange's equation and use the chain rule for partial derivatives for the function *F*.