Classical Mechanics TFY 4345 – Exercise 10

1. Velocity addition rule and Lorentz transformation matrices.

Suppose three inertial frame system *S*, *S'* and *S''* which are in collinear motion along their respective x_1-axes. Let the velocity of *S'* with respect to *S* be v_1 and the velocity of *S''* relative *S'* to be v_2. Write out the corresponding Lorentz transformation matrices ***L*** and ***L'*** and derive the Einstein's velocity addition rule based on the resulting matrix elements by considering the direct transition from *S* to *S''* (corresponding to ***L''***).

[See also Exam 2018 (December), problem 3, where S' moves in z-direction and S´´ in x' direction.]

2. Light from a fluorescent tube. [Exam 2016]

A fluorescent tube lamp is stationary in a reference frame *S*. The tube is arranged such that it lights up simultaneously (in *S*) along its entire length L_0 at the time t = 0. Consider an observer in a reference system *S'* moving with a velocity *v* parallel to the orientation of the tube. Suppose the tube lies along the z axis, at rest in the reference frame *S*, with one end at z = 0, and the other at z = L_0.

a. We now consider two space time events in *S*: The lighting up of the tube in position z at time t, and in position $z + \Delta z$ also at time t. Use the Lorentz transformation to calculate the space-time coordinates of these two events in the *S'* frame (z' at time t'), ($z' + \Delta z'$ at time $t' + \Delta t'$).

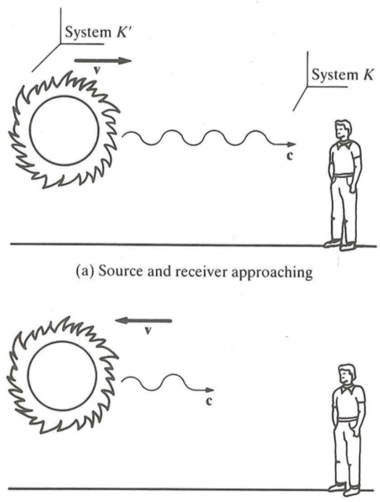
b. For the observer in *S'* the light does not appear to turn on simultaneously along the tube. Show that for the observer in *S'* the lighting up of the tube propagates with an apparent velocity: $|u| = c^2/v$.

3. Relativistic Doppler effect.

Consider a light source and receiver approaching one another with a relative speed *v* (inertial frames denoted as *K* and *K'*, could denote them as S and S´ alternatively). The length of the wave train is $L = c\Delta t - v\Delta t$ and it contains *n* wave lengths. First, derive the associated wave length λ and frequency ν. Second, consider the situation in terms of the proper time of the moving object (K´) and the corresponding frequency ν_0 and derive the relation

$$\upsilon = \frac{\sqrt{1+\beta^2}}{\sqrt{1-\beta^2}}\upsilon_0$$

Third, consider a receding object. How does the relation between the frequencies look like now?



(a) Source and receiver approaching

(b) Source and receiver receding