Exercise 10 - TFY4345 Classical Mechanics

2020

1 Velocity addition and Lorentz transformation matrices

Suppose three inertial systems S, S' and S'' are moving with collinear motion along their respective x_1-axes. Let the velocity of S' relative S be v_1, and the velocity of S'' relative S' be v_2. Write down the Lorentz transformation matrices L and L' corresponding to the transformations $S \to S'$ and $S' \to S''$. Use these to derive Einstein's addition rule for velocities based on the matrix elements of the transformation matrix L'' corresponding to $S \to S''$.

[See also Exam 2018 (December), problem 3, where S' moves in z-direction and S'' moves in the x' direction.]

2 Light from a fluorescent tube

[Exam 2016]

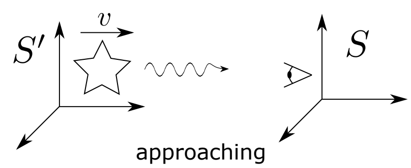
A fluorescent tube lamp is stationary in a reference frame S, parallel to the z axis. The tube lights up simultaneously (in S) along its entire length L_0 at the time t. The tube has one end at $z = 0$, and the other at $z = L_0$. Consider an observer in a reference system S' moving with a velocity v in the z-axis.

a) We now consider two spacetime events in S, the lighting up of the tube in position z at time t, and in position $z + \Delta z$ also at time t. Use the Lorentz transformation to calculate the spacetime coordinates of these two events in the S' frame, (z' at time t') and ($z' + \Delta z'$ at time $t' + \Delta t'$).

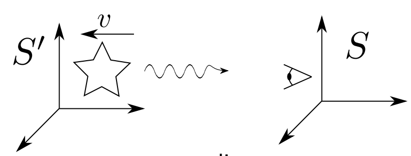
b) For the observer in S' the light does not appear to turn on simultaneously along the tube. Show that the observer in S' the lighting up of the tube propagates with an apparent velocity $u = c^2/v$

3 Relativistic Doppler effect

Consider a light source (system S') and receiver (system S) approaching one another with a relative speed v. The length of the wave train in S is $L = c\Delta t - v\Delta t$, and it contains n wave lengths.



approaching



receding

a) Derive the associated wave length λ and frequency f.

b) Consider the situation in terms of the proper time of the moving source (S') and the corresponding frequency f_0. Derive the relation

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0.$$

c) Consider a receding source. What does the relation between the frequencies look like now?