Exercise 9 solutions - TFY4345 Classical Mechanics

2020

1 Coupled pendula

(FIGUR)
The kinetic energy of the masses are

$$T = \frac{1}{2}m\left[(b\dot{\theta}_1)^2 + (b\dot{\theta}_2)^2\right].$$

The displacement of the masses in the vertical direction is given by $b(1-\cos(\theta))$. As we are considering small oscillations, we only care about the stretching of the spring due to the horizontal movement of the pendula. This is given by $b(\sin(\theta_1) - \sin(\theta_2))$. Thus, the potential energy of the system is

$$U = mgb\left[(1-\cos(\theta_1)) + (1-\cos(\theta_2))\right] + \frac{1}{2}kb^2\left[\sin(\theta_1) - \sin(\theta_2)\right]^2.$$

Using the small angle approximation, we get $\sin(\theta) \approx \theta$, $1-\cos(\theta) \approx \frac{1}{2}\theta^2$, so the potential energy becomes

$$\frac{1}{2}mgb(\theta_1^2 + \theta_2^2) - \frac{1}{2}kb^2(\theta_1 - \theta_2).$$

2 Two coupled oscillators

(FIGUR)

3 Oscillating body with two attached pendula

(FIGUR)

4 Double pendulum

(FIGUR)