Classical Mechanics TFY 4345 – Solution set 11

1. Binding energy of a deuteron.

Deuteron can be split by gamma rays in a nuclear experiment $\gamma + {}^2H \rightarrow p + n$. Let us assume that the kinetic energy of the gamma ray is transferred *completely* to the rest masses of a proton and neutron.

Mass of a proton (${}^1H = p$): 1.007825 u

Mass of a neutron (n): 1.008665 u

Sum (p + n): 2.016490 u

Mass of deuteron (${}^2H$): 2.014102 u

Difference (Q): 0.002388 u

This difference in mass-energy is equal to hold the neutron and proton together as a deuteron.

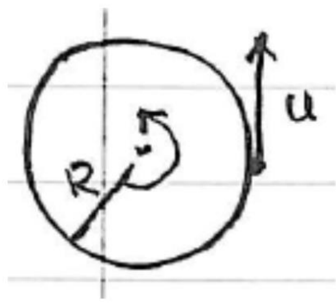
Conversion: 1 u = 931.5 MeV / c^2

Binding energy of a deuteron: 0.002388 x 931.e MeV = 2.22 MeV

Correspondingly, the energy of the initial gamma ray has to be *at least* 2.22 eV (threshold energy) to split a deuteron apart.

2. Frequency shift on rotating disk.

We have $\nu^0 = dN/dt^0$ where dN is the number of waves emitted in the time interval dt^0 in the instantaneous rest-system S'. We then have $dt^0 = d\tau$ (the eigentime of the system under consideration). If t is the time measured by the observer, we have $dt = \gamma d\tau$. The frequency measured in the center is then



$$\nu = dN/dt = dN/(\gamma d\tau) = \nu^0/\nu = \sqrt{1-\beta^2}\nu^0.$$

This is the transversal Doppler effect.

3. Fast moving particle in two inertial frames.

From the Lorentz transformations we can see immediately:

$$\begin{aligned} x' &= x && (27)\\ y' &= y && (28)\\ z' &= \gamma(z - vt) && (29)\\ t' &= \gamma(t - \frac{vz}{c^2}); \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} && (30) \end{aligned}$$

Let us differentiate:

$$u'_x = \frac{dx'}{dt'} = \frac{dx}{dt'} = \frac{dx}{\gamma(dt - \frac{vdz}{c^2})} = \frac{dx/dt}{\gamma(dt/dt - \frac{v(dz/dt)}{c^2})} = \underline{\frac{u_x}{\gamma(1 - \frac{vu_z}{c^2})}} \qquad (31)$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{dt'} = \ldots = \underline{\frac{u_y}{\gamma(1 - \frac{vu_z}{c^2})}} \qquad (32)$$

As one can see, also the x and y velocity components change.

What remains is the z-component

$$u'_z = \frac{dz'}{dt'} = \frac{\gamma(dz - vdt)}{\gamma(dt - \frac{vz}{c^2})} = \frac{dz/dt - v(dt/dt)}{dt/dt - \frac{v(dz/dt)}{c^2}} = \underline{\frac{u_z - v}{1 - \frac{vu_z}{c^2}}} \qquad (33)$$

This completes the velocity transformation. The results is know as the Einstein's velocity addition formula (general case). It ensures that the new velocity does not exceed the speed of light. In this case, the u_x and u_y components are affected although there is movement between S and S' along the z-axis only.

4. Lorentz transformation of energy and momentum.

a) Using Einstein velocity summation formulas (Brevik page 87) gives:

$$
\begin{aligned}
1-\frac{u'^2}{c^2} &= 1-\frac{u_x^2+u_y^2}{\gamma^2(1-\frac{vu_z}{c^2})^2c^2}-\frac{(u_z-v)^2}{(1-\frac{vu_z}{c^2})^2c^2} && (1)\\
&= \frac{1}{\gamma^2(1-\frac{vu_z}{c^2})^2}\left[\gamma^2(1-\frac{vu_z}{c^2})^2-\frac{u_x^2+u_y^2}{c^2}-\frac{\gamma^2(u_z-v)^2}{c^2}\right] && (2)\\
&= \frac{1}{\gamma^2(1-\frac{vu_z}{c^2})^2}\left[\gamma^2(1+\frac{v^2u_z^2}{c^2}-\frac{u_z^2}{c^2}-\frac{v^2}{c^2})-\frac{u_x^2+u_y^2}{c^2}\right] && (3)\\
&= \frac{1}{\gamma^2(1-\frac{vu_z}{c^2})^2}\left(1-\frac{u_x^2+u_y^2+u_z^2}{c^2}\right) && (4)\\
&= \frac{1}{\gamma^2(1-\frac{vu_z}{c^2})^2}\left(1-\frac{u^2}{c^2}\right) && (5)
\end{aligned}
$$

where we have used that $\gamma^2=\frac{1}{1-\frac{v^2}{c^2}}$

In conclusion we have shown that:

$$\frac{1}{\sqrt{1-u'^2/c^2}}=\gamma\frac{1-\frac{vu_z}{c^2}}{\sqrt{1-u^2/c^2}} \tag{6}$$

b)

Lorentz transformation of energy:

$$
\begin{aligned}
E' &= \frac{mc^2}{\sqrt{1-\frac{u'^2}{c^2}}} && (7)\\
&= \frac{mc^2\gamma(1-\frac{vu_z}{c^2})}{\sqrt{1-\frac{u^2}{c^2}}} && (8)\\
&= \gamma(E-vp_z) && (9)
\end{aligned}
$$

Similar calculation for momentum gives:

$$
\begin{aligned}
p'_x &= p_x && (10)\\
p'_y &= p_y && (11)\\
p'_z &= \gamma(p_z-\frac{v}{c^2}E) && (12)
\end{aligned}
$$