Exercise 7 solutions - TFY4345 Classical Mechanics

2020

1 Inertia tensor

(SIDEREFFERANSE)
The inertia tensor of a solid object V with the mass density $\rho(\vec{r})$ is defined as

$$I_{ij} = \int_V \rho(\vec{r})\big(\delta_{i,j} r^2 - x_i x_j\big)\mathrm{d}V. \tag{1}$$

We assume the slab is so thin that the z-direction can be neglected, and that it has a constant mass density $\rho = M/ab$. The integral then becomes.

$$I_{ij} = \frac{M}{ab}\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y r^2 - x_i x_j). \tag{2}$$

The tensor is symmetric, so the integrals we need to evaluate are

$$\begin{cases} I_{11} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \\ I_{12} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \\ I_{13} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \\ I_{22} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \\ I_{13} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \\ I_{13} = \dfrac{M}{ab}\displaystyle\int_0^a \mathrm{d}x \int_0^b \mathrm{d}y \end{cases} \tag{3}$$