Classical Mechanics TFY 4345 – Exercise 7

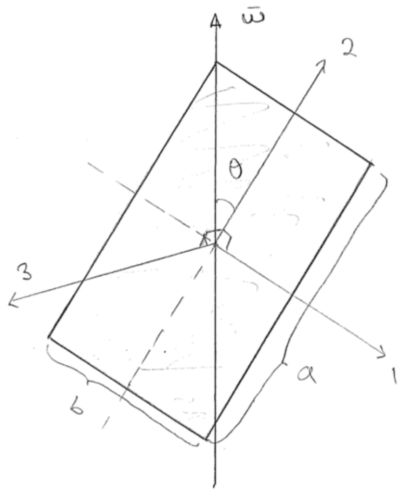
1. Inertia tensor.

A very thin rectangular slab has been placed in the *xyz*-coordinate system such that the origin is in one of the slab corners and the sides are along the *x*- and *y*-axes. The corresponding side lengths are *a* and *b*. Since the slab is very thin, we can assume that $z = 0$ throughout.

(a) Evaluate the individual elements of the inertia tensor.
(b) Set $a = b$ and solve the principal moments of inertia and the corresponding principal axes.

2. Rotating tilted slab (Exam Aug. 2018)

A very thin rectangular slab (see figure) rotates with an angular velocity $\boldsymbol{\omega}$ around its diagonal. The side lengths of the slab are *a* and *b* and the mass is *m*. The principal moments of inertia are $ma^2/12$, $mb^2/12$ and $m(a^2+b^2)/12$; the principal axes 1 and 2 go along the same directions as slab edges and 3 is perpendicular to the slab plane and goes through the slab center.



(a) Derive the angular momentum vector $\boldsymbol{L}$ of the slab.
(b) What is the angle between $\boldsymbol{L}$ and $\boldsymbol{\omega}$?
(c) What is the rotational kinetic energy T_{rot}?

3. Cone rolling on a plane (Exam Dec. 2016).

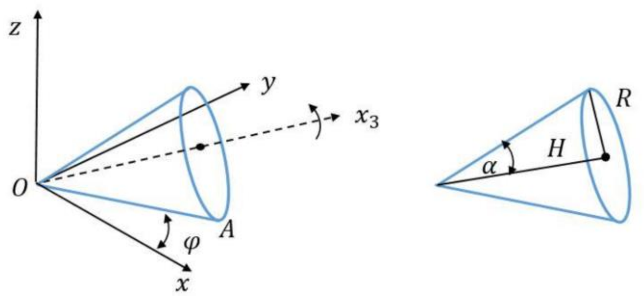


Figure: A solid cone rolls without slipping on a flat horizontal plane (*xy*-plane). The momentary line of contact with the plane is OA. The angle between the x-axis and the line OA is φ. The rolling induces rotation of the cone around the principal symmetry axis of the cone (x_3- axis in the figure). Dimensions of the cone: height (*H*), radius (*R*) and cone half-angle (α). Notice that these are related by: $\frac{R}{H}\tan(\alpha)$.

We shall consider the motion of a solid cone that is rolling on surface (*xy*-plane) without slipping.

a) The center of mass of the cone is situated on the symmetry axis x_3, at a distance ℓ from the origin O. Calculate the velocity of the center of mass, V_{cm}, as a function of ℓ, α and $\dot{\varphi}$. Here $\dot{\varphi}$ is the time derivative of φ.

b) Explain why the angular velocity vector $\vec{\omega}$ of the rolling cone is directed along the line OA. Show that $|\vec{\omega}| = \frac{\cos(\alpha)}{\sin(\alpha)}\dot{\varphi}$.

c) Find the components of $\vec{\omega}$ along the principal axes (symmetry axes) of the cone.

d) The principal moment of inertia of the cone (for rotation around the point O) is: $I_1 = I_2 = \frac{3}{20}m(R^2 + 4H^2)$ and $I_3 = \frac{3}{10}mR^2$, where m is the mass of the cone. Calculate the kinetic energy of the cone as a function of m, H, α and $\dot{\varphi}$.