

Exercise 2 - TFY4345 Classical Mechanics

2020

1 Damped oscillator

A particle with mass m moves with a low velocity \dot{x} . The particle is a damped oscillator with center in the origin and spring constant k . The frictional force is

$$F_f = -\frac{\partial \mathcal{F}}{\partial \dot{x}},$$

where \mathcal{F} is Rayleigh's dissipation function.

(a) Show that $\mathcal{F} \propto \dot{x}^2$, and that the viscous energy loss per minute \dot{W}_f can be written as $\dot{W}_f = 2\mathcal{F}$.

(b) Assume then that $\mathcal{F} = 3\pi\mu a\dot{x}^2$, where μ is the dynamic viscosity and a the particle radius. Start from Lagrange's equation, and show that the equation for motion can be written as

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = 0.$$

Express λ and ω_0 in terms of the constants above.

(c) Solve the equation $x(t)$ assuming $x(0) = x_0$ and $\dot{x}(0) = 0$ when $\lambda/\omega \ll 1$, and show that one approximately has

$$\overline{\dot{W}_f} = m\lambda(\omega_0 x_0)^2 e^{-2\lambda t},$$

where the bar denotes time average.

2 Operator identities

Use the Levi-Civita tensor to prove the following vector-operator relation:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}.$$

(Note: There will be no such assignment in the exam where one has to play with this tensor.)

3 Shortest path in polar coordinates

Show using polar coordinates and the Euler equation that the shortest distance between two points is a straight line, $r = b/\sin(\phi)$, $b = \text{const.}$. In this case, the Euler equation is

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0.$$

(Hint: here, you have to choose which of the variables r, ϕ are the parametre. (x in the above equation). Choose r .)

4 Forces of constraint

Consider a mathematical pendulum in two dimensions. Evaluate the equations of motion, and find the tension force within the pendulum string by using the Euler equations and an undetermined multiplier λ . Interpret the resulting force of constraint.

(Hint: Set the constraint such that the wire length ℓ is constant.)