Exercise 2 solutions - TFY4345 Classical Mechanics

2020

1 Damped oscillator

(a) The frictional force is

$$F_f = -\frac{\partial \mathcal{F}}{\partial v}.$$

The work done by friction is force times distance, so the work per unit time is

$$\dot{W}_f = -F_f v = \frac{\partial \mathcal{F}}{\partial v} v \implies \mathcal{F} = Cv^2.$$

(As $\mathcal{F}$ is a (velocity) potential, we can dismiss any constants, just as with regular potentials.) This means

$$\dot{W}_f = 2Cv^2 = 2\mathcal{F}.$$

(b) The Lagrangian with a velocity-dependent potential is

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + \frac{\partial \mathcal{F}}{\partial \dot{x}} = 0.$$

Inserting the Lagrangian for a harmonic oscillator,

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2,$$

and the given velocity potential $\mathcal{F} = 3\pi\mu a\dot{x}^2$, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} = m\ddot{x},\ \frac{\partial L}{\partial x} = -kx,\ \frac{\partial \mathcal{F}}{\partial \dot{x}} = 6\pi\mu a\dot{x},$$
$$\implies m\ddot{x} + 6\pi\mu a + kx = 0,$$

or

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = 0, \quad \lambda = \frac{3\pi\mu a}{m},\ \omega_0 = \sqrt{\frac{k}{m}}.$$

(c)

2 Operator identities

3 Shortest path in polar coordinates

4 Forces of constraint