Exercise x - TFY4345 Classical Mechanics

2020

1 Binding energy of the deuteron

[This is a short one]
Deuteron can be split by gamma rays in a nuclear experiment in the reaction $\gamma + {}^2\mathrm{H} \rightarrow p + n$. Calculate the energy required for the gamma rays for this process to occur, in electron volts. The quantities needed are

- Mass of a proton (${}^1\mathrm{H} = p$): 1.007825u
- Mass of a neutron (n): 1.008665u
- Mass of deuteron (${}^2\mathrm{H}$): 2.014102
- 1 u = 931.5 MeVc^{-2}

2 Frequency shift on a rotating disk

(FIGUR)
A radioactive ${}^{57}\mathrm{Co}$ element is situated on the periphery of a rotating disk. The peripheral velocity is u. The radiation from the cobalt is received by an observer located in the center of the disk. Let f_0 be the eigenfrequency of the radiation in the the inertial system where the element is momentarily at rest. Find the frequency f of the radiation observed by the observer in the center.

3 Fast moving particle in two inertial frames

[Exam Dec. 2019]
We shall consider a particle with rest mass m seen from two different inertial reference systems. In the reference system s, the particle has velocity

$$\mathbf{u} = (u_x, u_y, u_z).$$

The reference system S' is moving along the z axis with a velocity v relative to the system S. In this frame, the particle has the velocity

$$\mathbf{u}' = (u'_z, u'_y, u'_z).$$

Find the explicit relationship between $\mathbf{u}$ and $\mathbf{u}'$, i.e. derive the transformation that give $\mathbf{u}'$ from the components of $\mathbf{u}$. [Hint] Start with the relationship between the two frames, the Lorentz transformation. What is the definition of the components u_i, u'_j?

4 Lorentz transformation of energy and momentum

We shall consider a particle with res mass m seen from two inertial reference systems, as in the last exercise. The systems S and S' has a relative velocity v along the z-axis, and the particle has the constant velocity $\mathbf{u} = (u_x, u_y, u_z)$ and $\mathbf{u'} = (u_x, u_y, u_z)$ in S and S' respectively.

a) Einstein's velocity addition formulas are

$$\begin{aligned} u'_x &= \frac{u_x}{\gamma(1 - vu_z/c^2)} \\ u'_y &= \frac{u_x}{\gamma(1 - vu_y/c^2)} \\ u'_z &= \frac{u_z - v}{1 - vu_z/c^2} \end{aligned}$$

(The difference between these, and the ones derived in the last exercise, is that they do not assume the third reference system e.g. the particle moves along the z-axis). Use these to show that

$$\frac{1}{\sqrt{1 - u'^2/c}} = \gamma \frac{1 - vu_z/c^2}{\sqrt{1 - u^2/c^2}}.$$

Here, $\gamma = 1/\sqrt{1 - v/c^2}$, $u^2 = |\mathbf{u}| = u_x^2 + u_y^2 + u_z^2$ and $u'^2 = |\mathbf{u'}| = u'^2_x + u'^2_y + u'^2_z$.

b) The energy E and 3-momentum $\mathbf{p}$ of the particle in the reference system S is

$$E = \frac{mc^2}{\sqrt{1 - (u/c)^2}}, \quad \mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1 - (u/c)^2}},$$

while in the S' system it is

$$E' = \frac{mc^2}{\sqrt{1 - u'/c^2}}, \quad \mathbf{p}' = \frac{m\mathbf{u'}}{\sqrt{1 - u'/c^2}}.$$

Use the equation derived in a) to to find the transformation rule of energy of momentum, i.e. express E' and $\mathbf{p}'$ in terms of E and $\mathbf{p}$.