Classical Mechanics TFY 4345 – Exercise 2 (lecture hours 5-8)

1. Damped oscillator:

A particle with mass m moves with low velocity $\dot{x} = v$.

The frictional force is $F_f = -\partial\mathcal{F}/\partial v$, where $\mathcal{F}$ is Rayleigh's dissipation function. Show that if $\mathcal{F} \propto v^2$, the viscous energy loss $\dot{W}_f$ per unit time can be written as $\dot{W}_f = 2\mathcal{F}$. Assume that the particle is a damped oscillator with centre in the origin. The spring constant is k. Assume also that $\mathcal{F} = 3\pi\mu a v^2$ where μ is the dynamic viscosity and a is the particle radius. Start from Lagrange's equation and show that the equation of motion can be written as:

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = 0.$$

Express λ and ω_0 in terms of the above constants. Solve the equation for $x(t)$ assuming that $\dot{x}(0) = 0$ when $\lambda/\omega_0 \ll 1$ and show that one approximately has

$$\overline{\dot{W}_f} = m\lambda(\omega_0 x_0)^2 \mathrm{e}^{-2\lambda t}.$$

2. Use the Levi-Civita tensor to prove the following vector-relation:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

(Note: There will be no such assignment in the exam where one has to play with this tensor.)

3. Show by using polar coordinates (r, ϕ) and Euler equation for variational calculus that the shortest distance between two points is a straight line ($r = b/\sin\phi$, b = constant).

For reminder, the Euler equation is:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0.$$

4. Consider a mathematical pendulum in two dimensions. Evaluate the equations of motion and the tension force within the pendulum string by using the Lagrange equations and an undetermined multiplier λ. Interpret the resulting force of constraint. [Hint: Set the constraint such that the wire length (l) is a constant.]