Exercise 2 - TFY4345 Classical Mechanics

2020

1 Damped oscillator

A particle with mass m moves with a low velocity $\dot{x}$. The particle is a damped oscillator with center in the origin and spring constant k. The frictional force is

$$F_f = -\frac{\partial \mathcal{F}}{\partial \dot{x}},$$

where $\mathcal{F}$ is Rayleigh's dissipation function.

(a) Show that $\mathcal{F} \propto \dot{x}^2$, and that the viscous energy loss per unit time $\dot{W}_f$ can be written as $\dot{W}_f = 2\mathcal{F}$.

(b) Assume then that $\mathcal{F} = 3\pi\mu a\dot{x}^2$, where μ is the dynamic viscosity and a the particle radius. Start from Lagrange's equation, and show that the equation for motion can be written as

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = 0.$$

Express λ and ω_0 in terms of the constants k, m, μ and a.

(c) Solve the equation for $x(t)$ when $\lambda/\omega_0 \ll 1$, assuming $x(0) = x_0$ and $\dot{x}(0) = 0$, and show that one approximately has

$$\overline{\dot{W}_f} = m\lambda\left(\omega_0 x_0\right)^2 e^{-2\lambda t},$$

where the bar denotes time average.

2 Operator identities

Use the Levi-Civita tensor to prove the following vector-operator relation:

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla\left(\nabla \cdot \mathbf{A}\right) - \nabla^2\mathbf{A}.$$

(Note: There will be no such assignment in the exam where one has to play with this tensor.)

3 Shortest path in polar coordinates

Show using polar coordinates and the Euler equation that the shortest distance between two points is a straight line, $r = b/\sin(\phi)$, $b = \text{const}$. In this case, the Euler equation is

$$\frac{\partial f}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'} = 0.$$

(Hint: here, you have to choose which of the variables r, ϕ are the parameter, x in the above equation or time in physical problems, and which is the free variable, as y is in the equation above. Choose r to be the parameter.)

4 Forces of constraint

Consider a mathematical pendulum in two dimensions. Evaluate the equations of motion, and find the tension force within the pendulum string by using the Euler equations and an undetermined multiplier λ. Interpret the resulting force of constraint.

(Hint: Set the constraint such that the wire length ℓ is constant.)