TFY 4345: Exercise 8: (lectures 41-46)

1. Derive the transformation equations for the type 4 generating function of canonical transformations

$$F = q_i p_i - Q_i P_i + F_4(p, P, t)$$

2. Using harmonic oscillator as an example, show that the Poisson bracket $[q, H]$ remains the same upon canonical transformation.

$$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$$

Canonical transformation:

$$q = \sqrt{\frac{2P}{m\omega}}\sin Q, \quad p = \sqrt{2Pm\omega}\cos Q, \quad H = \omega P$$

3. The transformation equations between two sets of coordinates are

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$$Q = \log(1 + \sqrt{q}\cos p)$$
$$P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p.$$

a) Show directly that Q, P are canonical variables if q and b are. Use the symplectic condition

$\bar{\bar{M}}^T \bar{\bar{J}} \bar{\bar{M}} = \bar{\bar{J}}$ where $\bar{\bar{M}}$ is the Jacobian matrix and $\bar{\bar{J}}$ is antisymmetric.

$$M_{ij} = \frac{\overset{\text{new}}{\zeta_i}}{\underset{\text{old}}{\eta_j}} \quad \text{and} \quad \bar{\bar{J}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The last stage is tedious and you may use tools such as Mathematica. (Solve $\bar{\bar{M}}$ in the first place)

b) Show that the function that generates this transformation is

$$F_3 = -(e^Q - 1)^2 \tan p$$

4. Solve the equations of motion of a free particle ($H = \frac{p^2}{2m}$) by using the Hamilton-Jacobi theory.