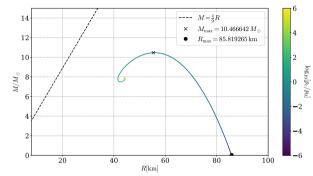
Pion stars

- ► New proposal: stars made of pions
- Microscopic part: what are the thermodynamics of pions?
- Macroscopic part: hydrodynamics of astrophysical objects

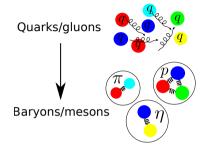


Questions: What is the mass-radius relations of pion stars? How do EM-interactions/including three quarks/loop corrections influence the star?

Chiral perturbation theory

- \blacktriangleright Quarks bind into baryon (protons/neutrons) and mesons (pions) at low temperatures (< $\sim 10^{12}$ K)
- The strong force is strong, so we can't do perturbation theory.

$$\mathcal{L} = \sum_{f} \bar{q}_{f} (\gamma^{\mu} [\partial_{\mu} - iq\lambda^{a} A_{\mu}^{a}] + m_{f}) q_{f} + G_{\mu\nu}^{a} G_{a}^{\mu\nu}$$



▶ Need an effective theory, The QCD vacuum spontaneously break a symmetry of the Lagrangian — Goldstone bosons (pions)

$$\mathcal{L}_{\mathsf{eff}} = \frac{f^2}{4} \operatorname{Tr} \left\{ \nabla_{\mu} \Sigma \nabla^{\mu} \Sigma^{\dagger} \right\} + \frac{f^2}{4} \operatorname{Tr} \left\{ \chi^{\dagger} \Sigma + \Sigma^{\dagger} \chi \right\} + l_1 \operatorname{Tr} \left\{ \nabla_{\mu} \Sigma (\nabla^{\mu} \Sigma)^{\dagger} \right\}^2 + \dots$$

 $\Sigma = \exp(i\pi_a \tau_a/f) \in SU(3)$, where SU(3) is the broken symmetry.

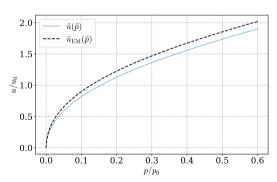


Thermodynamics

▶ With the effective Lagrangian, we can calculate free energy density

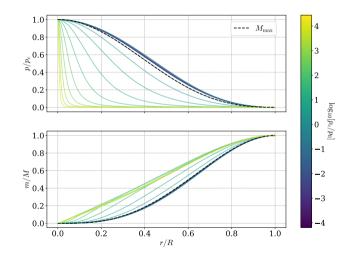
$$\triangleright \mathcal{F} = -\frac{i}{V\beta} \ln \left[\int \mathcal{D}\pi \, \exp \left(i \int d^4x \left[\mathcal{L}_{\mathsf{eff}} + \mu J \right] \right) \right]$$

- ► Include isospin and strangeness chemical potential, EM-interactions, loops...
- Phase transitions: pion condensation
- Calculate equation of state



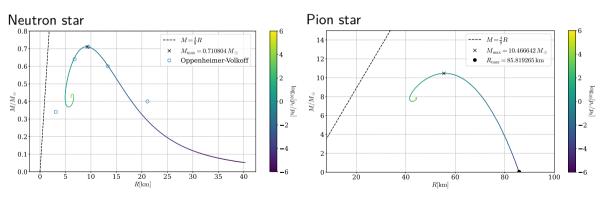
Hydrostatic equilibrium

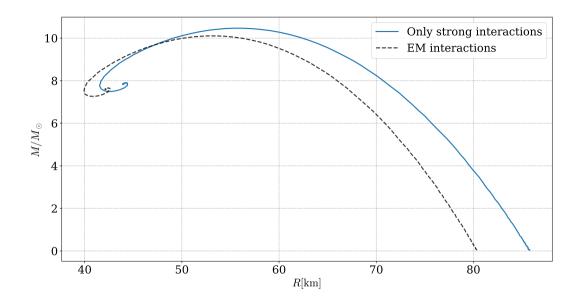
- TOV equation govern pressure of perfect fluids in hydrostatic equilibrium $\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G}{r^2}\frac{(u+P)\left(m+4\pi r^3P\right)}{\left(1-\frac{2Gm}{r}\right)},$ $\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 u$
- Numerical integration gives P, u and thus M, R.



Results

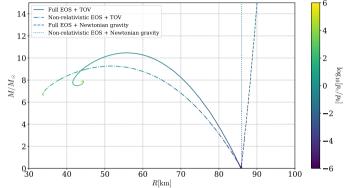
We get a family of results parametrized by central pressure.





Limit

- Newtonian and non-relativistic limit of EOS and TOV
- Used to derive Chandrasekhar limit
- Non-relativistic limit of pion equation of state is polytrope, $P = Ku^2$



▶ We can show that radius is independent of external pressure, and $R = \frac{\pi r^0}{\sqrt{12}}$.