Utledninger

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Chapter 1

CHPT

1.1 Leading order Lagrangian

1.1.1 EM contribution only

Subs $\pi_a/f \to \pi_a$,

$$\Sigma = \exp\{i\pi_a\tau_a\} = 1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a \tag{1.1}$$

$$Q = \frac{1}{6} + \frac{1}{2}\tau_3 \tag{1.2}$$

$$Q\Sigma = \frac{1}{2}\left[\frac{1}{3}\left(1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a\right) + \tau_3\left(1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a\right)\right] \tag{1.3}$$

$$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\pi_a\pi_a + i\pi_a\tau_3\tau_a + \frac{i}{3}\pi_a\tau_a + \tau_3 - \frac{1}{2}\pi_a\pi_a\tau_3\right] \tag{1.4}$$

$$Q\Sigma^\dagger = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\pi_a\pi_a - i\pi_a\tau_3\tau_a - \frac{i}{3}\pi_a\tau_a + \tau_3 - \frac{1}{2}\pi_a\pi_a\tau_3\right] \tag{1.5}$$

Using $\mathrm{Tr}\{\tau_a\tau_b\tau_c\tau_d\} = 2(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd}\delta_{ad}\delta_{cb})$, and defining $\delta^i_{ab} = \delta_{ai}\delta_{bi}$,

$$\mathrm{Tr}\{Q\Sigma Q\Sigma^\dagger\} = \frac{1}{2^2}\,\mathrm{Tr}\left\{\frac{1}{9} - 2\frac{1}{2\cdot 3^2}\pi_a\pi_a + \pi_a\pi_a\tau_3\tau_a\tau_3\tau_a + \frac{1}{8}\pi_a\pi_a + 1 - \pi_a\pi_a\right\} \tag{1.6}$$

$$= \frac{1}{2}\left(\frac{1}{9} + 1 - \frac{1}{3^2}\pi_a\pi_a - \pi_a\pi_a + \frac{1}{9}\pi_a\pi_a + \pi_a\pi_a(2\delta^2_{ab} - \delta_{ab})\right) \tag{1.7}$$

$$= \frac{5}{9} - \pi_1^2 - \pi_2^2. \tag{1.8}$$