

Gravgård

Martin Kjøllesdal Johnsrud

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Alt som ikke kom med...

0.1 Tree level pion star

We introduce the new dimensionless variable $1 + x^2 = \mu_I^2/\bar{m}^2$. This is reminiscent of the dimensionless Fermi momentum $x_f = p_f/m$ in {section: cold fermi star}. By an argument using a right triangle, we can verify that $\cos a = b$ implies $\sin a = \sqrt{1 - b^2}$. Substituting the dimensionless variable into the free energy density, we get

$$\mathcal{F} = -\frac{u_0}{2} \left(1 + x^2 + \frac{1}{1 + x^2} \right). \quad (1)$$

We have introduced the characteristic energy density $u_0 = \bar{m}^2 f^2$. As we found in {section: cold fermi star}, pressure is given by negative the free energy density, normalized to $\mu_I = \bar{m}$, or $x = 0$. We choose $p_0 = u_0$, so the dimensionless pressure can be written

$$\tilde{p} = -\frac{1}{u_0}(\mathcal{F} - \mathcal{F}_{x=0}) = \frac{1}{2} \frac{x^4}{1 + x^2}. \quad (2)$$

The charge density corresponding to a chemical potential is given by minus the derivative of the free energy with respect to that chemical potential. We must, however, not assume any dependence of α on μ_I . The isospin density therefore is

$$n_I = -\frac{\partial \mathcal{F}}{\partial \mu_I} = f^2 \mu_I \sin^2 \alpha = \frac{u_0}{\mu_I} \frac{2x^2 + x^4}{1 + x^2}, \quad (3)$$

With this, the dimensionless energy density

$$\tilde{u} = -\tilde{p} + \frac{1}{u_0} n_I \mu_I = \frac{1}{2} \frac{4x^2 + x^4}{1 + x^4} \quad (4)$$

0.2 Feil analyse av three-flavor vakum

$$\frac{1}{4} \text{Tr} \{ \nabla_\mu \Sigma_\alpha \nabla^\mu \Sigma_\alpha^\dagger \} = -\frac{1}{2} \sin^2 \alpha \left\{ \mu_I^2 \left[n_1^2 + n_2^2 + \frac{1}{4}(n_6^2 + n_7^2) \right] + \frac{1}{4} [\mu_I^2 + 3\mu_8^2] [n_4^2 + n_5^2] \right\} \quad (5)$$

$$\frac{1}{4} \text{Tr} \{ \chi \Sigma^\dagger + \Sigma \chi^\dagger \} = M_1^2 \cos \alpha \quad (6)$$

$$\mathcal{H} = -\frac{1}{8} \sin^2 \alpha \left[\mu_I^2 (4a^2 + b^2 + c^2) + 3\mu_8^2 (b^2 + c^2) + 2\sqrt{3}\mu_8 \mu_I (b^2 - c^2) \right] + M_1^2 \cos \alpha \quad (7)$$

$$= -\frac{1}{8} \sin^2 \alpha \left[4\mu_I^2 a^2 + (\mu_I + \sqrt{3}\mu_8)^2 b^2 + (\mu_I - \sqrt{3}\mu_8)^2 c^2 \right] \quad (8)$$

Choose, without loss of generality, $n_1 = n_4 = 0$, which leaves $n_2 = \cos \beta$, $n_5 = \sin \beta$, and thus

$$\Sigma = \exp \{ i\alpha(\cos \beta \lambda_2 + \sin \beta \lambda_5) \} = \cos \alpha + i(\lambda_2 \cos \beta + \lambda_5 \sin \beta) \sin \alpha. \quad (9)$$

0.3 Feil three-flavor chpt beregning

We need to parametrize the ground state, as we did in subsection: parametrization, and define Let

$$\Sigma_\alpha = \exp \{ i\alpha n_a \lambda_a \} = \cos \alpha + i n_a \lambda_a \sin \alpha, \quad \alpha = \frac{1}{f} \sqrt{\pi_a^0 \pi_a^0}, \quad n_a = \frac{\pi_a^0}{\sqrt{\pi_b^0 \pi_b^0}}. \quad (10)$$

The relevant terms are then

$$\frac{1}{4}\text{Tr}\{\nabla_\mu\Sigma_\alpha\nabla^\mu\Sigma_\alpha^\dagger\} = \frac{1}{2}\sin^2\alpha\left[\mu_I^2(n_1^2+n_2^2) + \frac{1}{4}(\mu_I+2\mu_s)^2(n_4^2+n_5^2) + \frac{1}{4}(\mu_I-2\mu_s)^2(n_6^2+n_7^2)\right] \quad (11)$$

$$\frac{1}{4}\text{Tr}\{\chi\Sigma^\dagger + \Sigma\chi^\dagger\} = M_1^2\cos\alpha \quad (12)$$

We notice that both terms are independent of μ_B . With this, the static Hamiltonian is

$$\mathcal{H}_0 = -\frac{1}{2}f^2\sin^2\alpha\left[\mu_I^2a^2 + \mu_{K^\pm}^2b^2 + \mu_{K^0}^2c^2\right] - f^2M_1^2\cos\alpha \quad (13)$$

We have defined the chemical potentials $\mu_{K^\pm} = \frac{1}{2}(\mu_I+2\mu_S) = \mu_u - \mu_s$ and $\mu_{K^0} = \frac{1}{2}(\mu_I-2\mu_S) = -\mu_d + \mu_s$, and

$$a^2 = n_1^2 + n_2^2, \quad b^2 = n_4^2 + n_5^2, \quad c^2 = n_6^2 + n_7^2, \quad a^2 + b^2 + c^2 = 1 - n_3^2 - n_8^2. \quad (14)$$

All the terms with a square chemical potential factors are positive definite, which means that the Hamiltonian will always be minimized by $n_3 = n_8 = 0$. Furthermore, we can without loss of generality chose $n_1 = n_4 = n_6 = 0$. This corresponds to changing basis of $\mathfrak{su}(3)$. Depending on the signs of μ_I and μ_S , we must have either $b = 0$ or $c = 0$. If $\text{sgn}(\mu_I) = \text{sgn}(\mu_S)$, then $\mu_{K^\pm} > \mu_{K^0}$, and $c = 0$. Likewise, if $\text{sgn}(\mu_I) = -\text{sgn}(\mu_S)$, then $\mu_{K^\pm} < \mu_{K^0}$, and $b = 0$. To begin with, we assume the former. Define $a^2 = \cos^2\beta$, which implies $b^2 = \sin^2\beta$. The Hamiltonian density is then

$$\mathcal{H}_0 = -\frac{1}{2}f^2\left[\mu_I^2\cos^2\beta + \mu_{K^\pm}^2\sin^2\beta\right]\sin^2\alpha - f^2M_1^2\cos\alpha. \quad (15)$$

The β parameter is set, as α , by minimizing \mathcal{H} . We have

$$\frac{\partial\mathcal{H}}{\partial\beta} = \frac{1}{2}(\mu_I^2 - \mu_{K^\pm}^2)f^2\sin^2\alpha\cos 2\beta, \quad \frac{\partial^2\mathcal{H}}{\partial\beta^2} = f^2(\mu_I^2 - \mu_{K^\pm}^2)\sin^2\alpha\sin 2\beta. \quad (16)$$

We see that, if we are in the pion condensate phase where $\alpha \neq 0$, the stationary points for β are 0 and $\pi/2$. However, which one these that is a minimum depends on the sign of $\mu_I^2 - \mu_{K^\pm}^2$, as this determines the sign of the second derivative. For $\mu_I^2 > \mu_{K^\pm}^2$, $\beta = 0$, while for $\mu_I^2 < \mu_{K^\pm}^2$ we have $\beta = \pi/2$. The analysis for $\text{sgn}(\mu_I) = -\text{sgn}(\mu_S)$ is the same, only with μ_{K^\pm} changed to μ_{K^0} . The different ground states are then

$$\Sigma_0 = \mathbb{1}, \quad \Sigma_\pi = \exp\{i\alpha\lambda_2\}, \quad \Sigma_{K^\pm} = \exp\{i\alpha\lambda_5\}, \quad \Sigma_{K^0} = \exp\{i\alpha\lambda_7\}. \quad (17)$$

As we found for two flavors, this corresponds to a transformation of the vacuum to a new ground state by, $\Sigma_0 \rightarrow A_\alpha\Sigma_0A_\alpha$. We must therefore transform the excitations around ground state in the same way. However, now the transformation depend on which phase we are in. We therefore parametrize the fields as

$$\Sigma = A_\alpha^i[U(x)\Sigma_0U(x)]A_\alpha^i, \quad U(x) = \exp\left\{i\frac{\pi_a\lambda_a}{2f}\right\}, \quad A_\alpha^i = \exp\{i\alpha\lambda_i\}. \quad (18)$$

Here, there is no sum over i . Rather, $i = 2, 5$, or 7 , dependent on if we are in the pion condensate, the charged kaon condensate or neutral kaon condensate.

0.3.1 failed Leading order

We work in the pion condensate, with $e = 0$. The relevant terms are then

$$\begin{aligned} \frac{f^2}{8B_0}\text{Tr}\{\chi\Sigma^\dagger + \Sigma\chi^\dagger\} = & -\frac{1}{4}(m_u + m_d)\cos\alpha(\pi_1^2 + \pi_2^2 + \pi_3^2) - \frac{1}{4}\left[(m_u + m_s)\cos^2\frac{\alpha}{2} - m_d\sin^2\frac{\alpha}{2}\right](\pi_4^2 + \pi_5^2) \\ & - \left[(m_d + m_s)\cos^2\frac{\alpha}{2} - m_u\sin^2\frac{\alpha}{2}\right](\pi_6^2 + \pi_7^2) + \frac{1}{12}[(m_u + m_d + 2m_s)\cos\alpha + 2m_s]\pi_8^2 \\ & - \frac{1}{2\sqrt{3}}(m_u - m_d)\pi_3\pi_8 - \frac{1}{2}(m_u + m_d)\sin\alpha\pi_2 + \frac{1}{2}(m_u + m_d)\cos\alpha + \frac{1}{4}m_s(\cos\alpha + 1) \end{aligned} \quad (19)$$

0.4 Two-flavor electromagnetic effects

When including contribution from a dynamical photon field, the leading order Lagrangian is [eckerRoleResonancesChiral1995] **urechVirtualPhotonsChiral1995**]

$$\mathcal{L}_2^{\text{EM}} = \frac{1}{4}f^2\text{Tr}\{\nabla_\mu\Sigma\nabla^\mu\Sigma^\dagger\} + \frac{1}{4}f^2\text{Tr}\{\chi\Sigma^\dagger + \Sigma\chi^\dagger\} + e^2C\text{Tr}\{Q\Sigma Q\Sigma^\dagger\} \quad (20)$$

Q is the quark charge matrix, which for $N_f = 2$ is

$$Q = \frac{1}{3} \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\mathbb{1} + \frac{1}{6}\tau_3. \quad (21)$$

C and dimensionfull constant, and $\chi = 2B_0m$, where m is the quark mass matrix **??**. To find the electromagnetic effect on the pion mass, we assume $\mu_I = 0$. We use the parametrization $\Sigma = \exp\{i\pi_a\tau_a/f\}$, and the covariant derivative is in this case

$$\nabla_\mu\Sigma = \partial_\mu\Sigma - ie\mathcal{A}_\mu[Q, \Sigma]. \quad (22)$$

We expand to second order in π_a/f , which gives

$$\frac{1}{4}f^2\text{Tr}\{\nabla_\mu\Sigma\nabla^\mu\Sigma\} = \frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi_a + e\mathcal{A}^\mu(\pi_1\partial_\mu\pi_2 - \pi_2\partial_\mu\pi_1) + e^2\mathcal{A}^2(\pi_1^2 + \pi_2^2), \quad (23)$$

$$\frac{1}{4}f^2\text{Tr}\{\chi\Sigma^\dagger + \Sigma\chi^\dagger\} = \bar{m}^2\left(f^2 - \frac{1}{2}\pi_a\pi_a\right), \quad (24)$$

$$\text{Tr}\{Q\Sigma Q\Sigma^\dagger\} = \frac{5}{9} - \frac{\pi_1^2 + \pi_2^2}{f^2}. \quad (25)$$

Inserting this into Eq. (20), we get

$$\mathcal{L}_2^{\text{EM}} = \bar{m}^2f^2 + \frac{5}{9}e^2C + \frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi_a - \frac{1}{2}\bar{m}_\pm^2(\pi_1^2 + \pi_2^2) - \frac{1}{2}\bar{m}^2\pi_3^2 + e\mathcal{A}^\mu(\pi_1\partial_\mu\pi_2 - \pi_2\partial_\mu\pi_1) + e^2\mathcal{A}^2(\pi_1^2 + \pi_2^2). \quad (26)$$

where

$$\bar{m}_\pm^2 = \bar{m}^2 + 2\frac{e^2}{f^2}C. \quad (27)$$

This is the leading order electromagnetic contribution to the mass. It only affects the π_1, π_2 pions, as they are linear combinations of the charged pions π^\pm , while $\pi_3 = \pi^0$, the neutral pion. To leading order, $\bar{m} = m_\pi$, the neutral pion mass, and $\bar{m}_\pm = m_{\pi^\pm}$. From the values listed in **??**, we find

$$\Delta m_\pm := \frac{e}{f}\sqrt{2C} = \sqrt{m_{\pi^\pm}^2 - m_\pi^2} = 35.50 \text{ MeV}. \quad (28)$$

This corresponds to $C = 0.3771 u_0 = 5.824 \cdot 10^{-5} \text{ GeV}^4$. We now no longer assume $\mu_I = 0$. The zeroth-order expansion in π/f is

$$\Sigma = e^{i\alpha\tau_1} = \sin\alpha + i\tau_1\cos\alpha. \quad (29)$$

This gives the contributions

$$\text{Tr}\{\nabla_\mu\Sigma\nabla^\mu\Sigma^\dagger\} = 2\sin^2\alpha(\mu_I^2 + 2e\mu\mathcal{A}_0 + e^2\mathcal{A}^2), \quad (30)$$

$$\text{Tr}\{\chi\Sigma^\dagger + \Sigma\chi^\dagger\} = 4\bar{m}^2\cos\alpha, \quad (31)$$

$$\text{Tr}\{Q\Sigma Q\Sigma^\dagger\} = \cos^2\alpha - \frac{4}{9}. \quad (32)$$

We are interested in the static Lagrangian, the Lagrangian for $\pi_a = \mathcal{A}_\mu = 0$. Inserting these terms into Eq. (20), we get

$$\mathcal{L}_2^{\text{EM},0} = f^2\left[\frac{1}{2}\mu_I^2\sin^2\alpha + \bar{m}^2\cos\alpha + \frac{1}{2}\Delta m_\pm^2\left(\cos^2\alpha - \frac{4}{9}\right)\right]. \quad (33)$$

Can we include trace?

Compare/use Urec's results: $C = 61.1 \times 10^{-6} \text{ (GeV)}^4$