

```
In [1]: %%display latex
load("three_flavor.sage")
```

```
In [2]: v = v_mu
n = 2
S, Sd, dS, dSd = get_S(n, l2, indx="mu")
```

```
In [3]: indxs = ["mu", "nu", "rho"]

v = []
COM = []
COMd = []
for i in indxs:
    d = var("d"+i, latex_name="\\delta_\\ "+i, domain="real")
    v = d*mu
    COM.append(-(v*S - S*v)) # [v_mu, Sigma]
    COMd.append(v*Sd - Sd*v) # [v_nu, Sigma^\\dagger]
```

```
In [4]: dS = []
dSd=[]
for i in indxs:
    _, _, dSi, dSdi = get_S(n, l2, indx=i)
    dS.append(dSi)
    dSd.append(dSdi)
```

```
In [5]: kin = []
for i, ind in enumerate(indxs):
    term1 = mat_prep(dS[i]*(dSd[i]), n=n)
    term2 = -I*mat_prep(dS[i]*COMd[i] - COM[i]*dS[i], n=n)
    term3 = mat_prep(COM[i]*COMd[i], n=n)
    kin.append(term1 + term2 + term3)
```

```
In [6]: L1 = (kin[0].trace()**2).series(e, n+1).truncate().trig_reduce().full_simplify()
print_coeff2(L1)
```

1 :

$$4 \delta_{\mu}^4 \mu_I^4 \sin(\alpha)^4$$

e :

$$16 \delta_{\mu}^4 \mu_I^4 \varphi_2 \cos(\alpha) \sin(\alpha)^3$$

e^2 :

$$\begin{aligned} & -8 \delta_{\mu}^4 \mu_I^4 \varphi_1^2 \sin(\alpha)^4 - 32 \delta_{\mu}^4 \mu_I^4 \varphi_2^2 \sin(\alpha)^4 - 8 \delta_{\mu}^4 \mu_I^4 \varphi_3^2 \sin(\alpha)^4 - 4 \delta_{\mu}^4 \mu_I^4 \varphi_4^2 \sin(\alpha)^4 - 4 \delta_{\mu}^4 \mu_I^4 \varphi_5^2 \sin(\alpha)^4 - 4 \delta_{\mu}^4 \mu_I^4 \varphi_6^2 \sin(\alpha)^4 \\ & - 4 \delta_{\mu}^4 \mu_I^4 \varphi_7^2 \sin(\alpha)^4 + 8 \delta_{\mu}^4 \mu_I^3 \mu_S \varphi_4^2 \cos(\alpha) \sin(\alpha)^2 + 8 \delta_{\mu}^4 \mu_I^3 \mu_S \varphi_5^2 \cos(\alpha) \sin(\alpha)^2 - 8 \delta_{\mu}^4 \mu_I^3 \mu_S \varphi_6^2 \cos(\alpha) \sin(\alpha)^2 - 8 \delta_{\mu}^4 \mu_I^3 \mu_S \varphi_7^2 \cos(\alpha) \sin(\alpha)^2 \\ & + 8 \delta_{\mu}^4 \mu_I^4 \varphi_1^2 \sin(\alpha)^2 + 24 \delta_{\mu}^4 \mu_I^4 \varphi_2^2 \sin(\alpha)^2 + 2 \delta_{\mu}^4 \mu_I^4 \varphi_4^2 \sin(\alpha)^2 + 8 \delta_{\mu}^4 \mu_I^2 \mu_S^2 \varphi_4^2 \sin(\alpha)^2 + 2 \delta_{\mu}^4 \mu_I^4 \varphi_5^2 \sin(\alpha)^2 + 8 \delta_{\mu}^4 \mu_I^2 \mu_S^2 \varphi_5^2 \sin(\alpha)^2 \\ & + 2 \delta_{\mu}^4 \mu_I^4 \varphi_6^2 \sin(\alpha)^2 + 8 \delta_{\mu}^4 \mu_I^2 \mu_S^2 \varphi_6^2 \sin(\alpha)^2 + 2 \delta_{\mu}^4 \mu_I^4 \varphi_7^2 \sin(\alpha)^2 + 8 \delta_{\mu}^4 \mu_I^2 \mu_S^2 \varphi_7^2 \sin(\alpha)^2 + \frac{16}{3} i \sqrt{3} \delta_{\mu}^3 \partial_{\mu} \varphi_1 \mu_I^3 \varphi_8 \sin(\alpha)^3 \\ & + 4 \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^3 \varphi_4 \cos(\alpha) \sin(\alpha)^2 - 8 \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^2 \mu_S \varphi_4 \cos(\alpha) \sin(\alpha)^2 - 4 \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^3 \varphi_5 \cos(\alpha) \sin(\alpha)^2 + 8 \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^2 \mu_S \varphi_5 \cos(\alpha) \sin(\alpha)^2 \\ & - 4 \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^3 \varphi_6 \cos(\alpha) \sin(\alpha)^2 - 8 \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^2 \mu_S \varphi_6 \cos(\alpha) \sin(\alpha)^2 + 4 \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^3 \varphi_7 \cos(\alpha) \sin(\alpha)^2 + 8 \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^2 \mu_S \varphi_7 \cos(\alpha) \sin(\alpha)^2 \\ & + 4 i \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^3 \varphi_4 \sin(\alpha)^3 - 8 i \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^2 \mu_S \varphi_4 \sin(\alpha)^3 + 4 i \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^3 \varphi_5 \sin(\alpha)^3 - 8 i \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^2 \mu_S \varphi_5 \sin(\alpha)^3 \\ & + 4 i \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^3 \varphi_6 \sin(\alpha)^3 + 8 i \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^2 \mu_S \varphi_6 \sin(\alpha)^3 + 4 i \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^3 \varphi_7 \sin(\alpha)^3 + 8 i \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^2 \mu_S \varphi_7 \sin(\alpha)^3 \\ & - 4 \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^3 \varphi_4 \sin(\alpha)^2 + 8 \delta_{\mu}^3 \partial_{\mu} \varphi_5 \mu_I^2 \mu_S \varphi_4 \sin(\alpha)^2 + 4 \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^3 \varphi_5 \sin(\alpha)^2 - 8 \delta_{\mu}^3 \partial_{\mu} \varphi_4 \mu_I^2 \mu_S \varphi_5 \sin(\alpha)^2 \\ & + 4 \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^3 \varphi_6 \sin(\alpha)^2 + 8 \delta_{\mu}^3 \partial_{\mu} \varphi_7 \mu_I^2 \mu_S \varphi_6 \sin(\alpha)^2 - 4 \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^3 \varphi_7 \sin(\alpha)^2 - 8 \delta_{\mu}^3 \partial_{\mu} \varphi_6 \mu_I^2 \mu_S \varphi_7 \sin(\alpha)^2 \\ & + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_1^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_2^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_3^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_4^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_5^2 \mu_I^2 \sin(\alpha)^2 \\ & + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_6^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_7^2 \mu_I^2 \sin(\alpha)^2 + 8 \delta_{\mu}^2 \partial_{\mu} \varphi_8^2 \mu_I^2 \sin(\alpha)^2 \end{aligned}$$

In [7]:

$e^2 :$

$$\begin{aligned}
& -4\delta_\mu^4\mu_I^4\varphi_1^2\sin(\alpha)^4 - 16\delta_\mu^4\mu_I^4\varphi_2^2\sin(\alpha)^4 - 4\delta_\mu^4\mu_I^4\varphi_3^2\sin(\alpha)^4 - 2\delta_\mu^4\mu_I^4\varphi_4^2\sin(\alpha)^4 - 2\delta_\mu^4\mu_I^4\varphi_5^2\sin(\alpha)^4 - 2\delta_\mu^4\mu_I^4\varphi_6^2\sin(\alpha)^4 \\
& - 2\delta_\mu^4\mu_I^4\varphi_7^2\sin(\alpha)^4 + 4\delta_\mu^4\mu_I^3\mu_S\varphi_4^2\cos(\alpha)\sin(\alpha)^2 + 4\delta_\mu^4\mu_I^3\mu_S\varphi_5^2\cos(\alpha)\sin(\alpha)^2 - 4\delta_\mu^4\mu_I^3\mu_S\varphi_6^2\cos(\alpha)\sin(\alpha)^2 - 4\delta_\mu^4\mu_I^3\mu_S\varphi_7^2\cos(\alpha)\sin(\alpha)^2 \\
& + 4\delta_\mu^4\mu_I^4\varphi_1^2\sin(\alpha)^2 + 12\delta_\mu^4\mu_I^4\varphi_2^2\sin(\alpha)^2 + \delta_\mu^4\mu_I^4\varphi_4^2\sin(\alpha)^2 + 4\delta_\mu^4\mu_I^2\mu_S^2\varphi_4^2\sin(\alpha)^2 + \delta_\mu^4\mu_I^4\varphi_5^2\sin(\alpha)^2 + 4\delta_\mu^4\mu_I^2\mu_S^2\varphi_5^2\sin(\alpha)^2 \\
& + \delta_\mu^4\mu_I^4\varphi_6^2\sin(\alpha)^2 + 4\delta_\mu^4\mu_I^2\mu_S^2\varphi_6^2\sin(\alpha)^2 + \delta_\mu^4\mu_I^4\varphi_7^2\sin(\alpha)^2 + 4\delta_\mu^4\mu_I^2\mu_S^2\varphi_7^2\sin(\alpha)^2 + \frac{8}{3}i\sqrt{3}\delta_\mu^3\partial_\mu\varphi_1\mu_I^3\varphi_8\sin(\alpha)^3 \\
& + 2\delta_\mu^3\partial_\mu\varphi_5\mu_I^3\varphi_4\cos(\alpha)\sin(\alpha)^2 - 4\delta_\mu^3\partial_\mu\varphi_5\mu_I^2\mu_S\varphi_4\cos(\alpha)\sin(\alpha)^2 - 2\delta_\mu^3\partial_\mu\varphi_4\mu_I^3\varphi_5\cos(\alpha)\sin(\alpha)^2 + 4\delta_\mu^3\partial_\mu\varphi_4\mu_I^2\mu_S\varphi_5\cos(\alpha)\sin(\alpha)^2 \\
& - 2\delta_\mu^3\partial_\mu\varphi_7\mu_I^3\varphi_6\cos(\alpha)\sin(\alpha)^2 - 4\delta_\mu^3\partial_\mu\varphi_7\mu_I^2\mu_S\varphi_6\cos(\alpha)\sin(\alpha)^2 + 2\delta_\mu^3\partial_\mu\varphi_6\mu_I^3\varphi_7\cos(\alpha)\sin(\alpha)^2 + 4\delta_\mu^3\partial_\mu\varphi_6\mu_I^2\mu_S\varphi_7\cos(\alpha)\sin(\alpha)^2 \\
& + 2i\delta_\mu^3\partial_\mu\varphi_6\mu_I^3\varphi_4\sin(\alpha)^3 - 4i\delta_\mu^3\partial_\mu\varphi_6\mu_I^2\mu_S\varphi_4\sin(\alpha)^3 + 2i\delta_\mu^3\partial_\mu\varphi_4\mu_I^3\varphi_6\sin(\alpha)^3 + 4i\delta_\mu^3\partial_\mu\varphi_4\mu_I^2\mu_S\varphi_6\sin(\alpha)^3 + 2i\delta_\mu^3\partial_\mu\varphi_5\mu_I^3\varphi_7\sin(\alpha)^3 \\
& + 4i\delta_\mu^3\partial_\mu\varphi_5\mu_I^2\mu_S\varphi_7\sin(\alpha)^3 - \frac{16}{3}i\sqrt{3}\delta_\mu^2\partial_\mu\varphi_2\partial_\mu\varphi_8\mu_I^2\cos(\alpha)\sin(\alpha)^3 - 2\delta_\mu^3\partial_\mu\varphi_5\mu_I^3\varphi_4\sin(\alpha)^2 + 4\delta_\mu^3\partial_\mu\varphi_5\mu_I^2\mu_S\varphi_4\sin(\alpha)^2 \\
& + 2\delta_\mu^3\partial_\mu\varphi_5\mu_I^2\mu_S\varphi_4\sin(\alpha)^2 + 2\delta_\mu^3\partial_\mu\varphi_4\mu_I^3\varphi_5\sin(\alpha)^2 - 4\delta_\mu^3\partial_\mu\varphi_4\mu_I^2\mu_S\varphi_5\sin(\alpha)^2 + 2\delta_\mu^3\partial_\mu\varphi_7\mu_I^3\varphi_6\sin(\alpha)^2 + 4\delta_\mu^3\partial_\mu\varphi_7\mu_I^2\mu_S\varphi_6\sin(\alpha)^2 \\
& - 2\delta_\mu^3\partial_\mu\varphi_6\mu_I^3\varphi_7\sin(\alpha)^2 - 4\delta_\mu^3\partial_\mu\varphi_6\mu_I^2\mu_S\varphi_7\sin(\alpha)^2 + 8\delta_\mu^2\partial_\mu\varphi_2^2\mu_I^2\sin(\alpha)^4 + \frac{8}{3}\delta_\mu^2\partial_\mu\varphi_8^2\mu_I^2\sin(\alpha)^4 - 2\delta_\mu^2\partial_\mu\varphi_4^2\mu_I^2\cos(\alpha)\sin(\alpha)^2 \\
& - 2\delta_\mu^2\partial_\mu\varphi_5^2\mu_I^2\cos(\alpha)\sin(\alpha)^2 - 2\delta_\mu^2\partial_\mu\varphi_6^2\mu_I^2\cos(\alpha)\sin(\alpha)^2 - 2\delta_\mu^2\partial_\mu\varphi_7^2\mu_I^2\cos(\alpha)\sin(\alpha)^2 - 4i\delta_\mu^2\partial_\mu\varphi_5\partial_\mu\varphi_6\mu_I^2\sin(\alpha)^3 \\
& + 4i\delta_\mu^2\partial_\mu\varphi_4\partial_\mu\varphi_7\mu_I^2\sin(\alpha)^3 + 4\delta_\mu^2\partial_\mu\varphi_1^2\mu_I^2\sin(\alpha)^2 - 4\delta_\mu^2\partial_\mu\varphi_2^2\mu_I^2\sin(\alpha)^2 - 4\delta_\mu^2\partial_\mu\varphi_3^2\mu_I^2\sin(\alpha)^2 + 2\delta_\mu^2\partial_\mu\varphi_4^2\mu_I^2\sin(\alpha)^2 \\
& + 2\delta_\mu^2\partial_\mu\varphi_5^2\mu_I^2\sin(\alpha)^2 + 2\delta_\mu^2\partial_\mu\varphi_6^2\mu_I^2\sin(\alpha)^2 + 2\delta_\mu^2\partial_\mu\varphi_7^2\mu_I^2\sin(\alpha)^2 + \frac{4}{3}\delta_\mu^2\partial_\mu\varphi_8^2\mu_I^2\sin(\alpha)^2
\end{aligned}$$

In [17]:

```
chiS = chi*Sd + S*chi.T
L6 = (chiS.trace())^2
L6 = L6.series(e, n+1).truncate().trig_reduce().full_simplify()
print_coeff2(L6)
```

1 :

$$16\bar{m}^4\cos(\alpha)^2 + 16m_S^2\bar{m}^2\cos(\alpha) + 4m_S^4$$

e :

$$-32\bar{m}^4\varphi_2\cos(\alpha)\sin(\alpha) - 16m_S^2\bar{m}^2\varphi_2\sin(\alpha)$$

e^2 :

$$\begin{aligned}
& -16\bar{m}^4\varphi_1^2\cos(\alpha)^2 - 32\bar{m}^4\varphi_2^2\cos(\alpha)^2 - 16\bar{m}^4\varphi_3^2\cos(\alpha)^2 - 8\bar{m}^4\varphi_4^2\cos(\alpha)^2 - 8\bar{m}^4\varphi_5^2\cos(\alpha)^2 - 8\bar{m}^4\varphi_6^2\cos(\alpha)^2 - 8\bar{m}^4\varphi_7^2\cos(\alpha)^2 \\
& - \frac{16}{3}\bar{m}^4\varphi_8^2\cos(\alpha)^2 + \frac{32}{3}\sqrt{3}\Delta m^2\bar{m}^2\varphi_3\varphi_8\cos(\alpha) - 8m_S^2\bar{m}^2\varphi_1^2\cos(\alpha) - 8m_S^2\bar{m}^2\varphi_2^2\cos(\alpha) - 8m_S^2\bar{m}^2\varphi_3^2\cos(\alpha) + 8\Delta m^2\bar{m}^2\varphi_4^2\cos(\alpha) \\
& - 12m_S^2\bar{m}^2\varphi_4^2\cos(\alpha) + 8\Delta m^2\bar{m}^2\varphi_5^2\cos(\alpha) - 12m_S^2\bar{m}^2\varphi_5^2\cos(\alpha) - 8\Delta m^2\bar{m}^2\varphi_6^2\cos(\alpha) - 12m_S^2\bar{m}^2\varphi_6^2\cos(\alpha) - 8\Delta m^2\bar{m}^2\varphi_7^2\cos(\alpha) \\
& - 12m_S^2\bar{m}^2\varphi_7^2\cos(\alpha) - \frac{40}{3}m_S^2\bar{m}^2\varphi_8^2\cos(\alpha) + \frac{16}{3}\sqrt{3}\Delta m^2m_S^2\varphi_3\varphi_8 + 16\bar{m}^4\varphi_2^2 + 4\Delta m^2m_S^2\varphi_4^2 - 4m_S^4\varphi_4^2 + 4\Delta m^2m_S^2\varphi_5^2 - 4m_S^4\varphi_5^2 - 4\Delta m^2m_S^2\varphi_6^2 - 4m_S^4\varphi_6^2 - 4\Delta m^2m_S^2\varphi_7^2 - 4m_S^4\varphi_7^2 - \frac{16}{3}m_S^4\varphi_8^2
\end{aligned}$$

In [10]:

```
L7 = mat_prep(chi*Sd - S*chi.T, n=n)
L7 = (L7.trace())^2
L7 = L7.series(e, n+1).truncate().trig_reduce().full_simplify()
print_coeff(L7)
```

e^2 :

$$-16\Delta m^4\varphi_3^2 + \frac{32}{3}\left(\sqrt{3}\Delta m^2\bar{m}^2\cos(\alpha) - \sqrt{3}\Delta m^2m_S^2\right)\varphi_3\varphi_8 - \frac{16}{3}\left(\bar{m}^4\cos(\alpha)^2 - 2m_S^2\bar{m}^2\cos(\alpha) + m_S^4\right)\varphi_8^2$$

In [11]:

```
L8 = mat_prep((chi*Sd)^2 + (S*chi.T)^2, n=n)
L8 = L8.trace().series(e, n+1).truncate().trig_reduce()
print_coeff2(L8)
```

1 :

$$4\left(2\cos(\alpha)^2 - 1\right)\bar{m}^4 + 4\Delta m^4 + 2m_S^4$$

e :

$$-16 \bar{m}^4 \varphi_2 \cos(\alpha) \sin(\alpha)$$

e^2 :

$$\begin{aligned} & \frac{2}{9} \sqrt{3} \left(12 \left(\sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 \varphi_1^2 + 12 \left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 \varphi_2^2 + 48 \Delta m^2 \bar{m}^2 \varphi_3 \varphi_8 \cos(\alpha) + 12 \right. \\ & \quad \left(\left(\sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 \right) \varphi_3^2 + 3 \\ & \quad \left(\left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 + 2 \sqrt{3} \Delta m^2 m_S^2 - \sqrt{3} m_S^4 + 2 \left(\sqrt{3} \Delta m^2 \cos(\alpha) - \sqrt{3} m_S^2 \cos(\alpha) \right) \bar{m}^2 \right) \varphi_4^2 + 3 \\ & \quad \left(\left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 + 2 \sqrt{3} \Delta m^2 m_S^2 - \sqrt{3} m_S^4 + 2 \left(\sqrt{3} \Delta m^2 \cos(\alpha) - \sqrt{3} m_S^2 \cos(\alpha) \right) \bar{m}^2 \right) \varphi_5^2 + 3 \\ & \quad \left(\left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 - 2 \sqrt{3} \Delta m^2 m_S^2 - \sqrt{3} m_S^4 - 2 \left(\sqrt{3} \Delta m^2 \cos(\alpha) + \sqrt{3} m_S^2 \cos(\alpha) \right) \bar{m}^2 \right) \varphi_6^2 + 3 \\ & \quad \left(\left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 - 2 \sqrt{3} \Delta m^2 m_S^2 - \sqrt{3} m_S^4 - 2 \left(\sqrt{3} \Delta m^2 \cos(\alpha) + \sqrt{3} m_S^2 \cos(\alpha) \right) \bar{m}^2 \right) \varphi_7^2 + 4 \\ & \quad \left. \left(\left(2 \sqrt{3} \sin(\alpha)^2 - \sqrt{3} \right) \bar{m}^4 - \sqrt{3} \Delta m^4 - 2 \sqrt{3} m_S^4 \right) \varphi_8^2 \right) \end{aligned}$$

In [16]:

```
L4 = kin[0].trace()*chiS.trace()
L4 = L4.series(e, n+1).truncate().trig_reduce().full_simplify()
print_coeff(L4)
```

1 :

$$-8 \delta_\mu^2 \bar{m}^2 \mu_I^2 \cos(\alpha)^3 - 4 \delta_\mu^2 m_S^2 \mu_I^2 \cos(\alpha)^2 + 8 \delta_\mu^2 \bar{m}^2 \mu_I^2 \cos(\alpha) + 4 \delta_\mu^2 m_S^2 \mu_I^2$$

e :

$$24 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_2 \cos(\alpha)^2 \sin(\alpha) + 8 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_2 \cos(\alpha) \sin(\alpha) - 8 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_2 \sin(\alpha)$$

e^2 :

$$\begin{aligned} & 12 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_1^2 \cos(\alpha)^3 + 36 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_2^2 \cos(\alpha)^3 + 12 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_3^2 \cos(\alpha)^3 + 6 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_4^2 \cos(\alpha)^3 + 6 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_5^2 \cos(\alpha)^3 \\ & \quad + 6 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_6^2 \cos(\alpha)^3 + 6 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_7^2 \cos(\alpha)^3 + \frac{4}{3} \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_8^2 \cos(\alpha)^3 - \frac{8}{3} \sqrt{3} \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_3 \varphi_8 \cos(\alpha)^2 + 4 \\ & \quad \delta_\mu^2 m_S^2 \mu_I^2 \varphi_1^2 \cos(\alpha)^2 + 8 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_2^2 \cos(\alpha)^2 + 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_3^2 \cos(\alpha)^2 - 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_4^2 \cos(\alpha)^2 + 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_4^2 \cos(\alpha)^2 \\ & \quad + 8 \delta_\mu^2 \bar{m}^2 \mu_I \mu_S \varphi_4^2 \cos(\alpha)^2 - 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_5^2 \cos(\alpha)^2 + 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_5^2 \cos(\alpha)^2 + 8 \delta_\mu^2 \bar{m}^2 \mu_I \mu_S \varphi_5^2 \cos(\alpha)^2 + 2 \\ & \quad \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_6^2 \cos(\alpha)^2 + 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_6^2 \cos(\alpha)^2 - 8 \delta_\mu^2 \bar{m}^2 \mu_I \mu_S \varphi_6^2 \cos(\alpha)^2 + 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_7^2 \cos(\alpha)^2 + 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_7^2 \cos \\ & \quad (\alpha)^2 - 8 \delta_\mu^2 \bar{m}^2 \mu_I \mu_S \varphi_7^2 \cos(\alpha)^2 + \frac{8}{3} \delta_\mu^2 m_S^2 \mu_I^2 \varphi_8^2 \cos(\alpha)^2 - 4 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_1^2 \cos(\alpha) - 28 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_2^2 \cos(\alpha) - 12 \\ & \quad \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_3^2 \cos(\alpha) - 4 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_4^2 \cos(\alpha) + 4 \delta_\mu^2 m_S^2 \mu_I \mu_S \varphi_4^2 \cos(\alpha) + 8 \delta_\mu^2 \bar{m}^2 \mu_S^2 \varphi_4^2 \cos(\alpha) - 4 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_5^2 \cos(\alpha) + 4 \\ & \quad \delta_\mu^2 m_S^2 \mu_I \mu_S \varphi_5^2 \cos(\alpha) + 8 \delta_\mu^2 \bar{m}^2 \mu_S^2 \varphi_5^2 \cos(\alpha) - 4 \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_6^2 \cos(\alpha) - 4 \delta_\mu^2 m_S^2 \mu_I \mu_S \varphi_6^2 \cos(\alpha) + 8 \delta_\mu^2 \bar{m}^2 \mu_S^2 \varphi_6^2 \cos(\alpha) - 4 \\ & \quad \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_7^2 \cos(\alpha) - 4 \delta_\mu^2 m_S^2 \mu_I \mu_S \varphi_7^2 \cos(\alpha) + 8 \delta_\mu^2 \bar{m}^2 \mu_S^2 \varphi_7^2 \cos(\alpha) - \frac{4}{3} \delta_\mu^2 \bar{m}^2 \mu_I^2 \varphi_8^2 \cos(\alpha) + \frac{8}{3} \sqrt{3} \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_3 \varphi_8 + \frac{16}{3} i \\ & \quad \sqrt{3} \delta_\mu \partial_\mu \varphi_1 \bar{m}^2 \mu_I \varphi_8 \cos(\alpha) \sin(\alpha) - 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_2^2 - 4 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_3^2 + 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_4^2 - 3 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_4^2 + 4 \delta_\mu^2 m_S^2 \mu_S^2 \varphi_4^2 + 2 \\ & \quad \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_5^2 - 3 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_5^2 + 4 \delta_\mu^2 m_S^2 \mu_S^2 \varphi_5^2 - 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_6^2 - 3 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_6^2 + 4 \delta_\mu^2 m_S^2 \mu_S^2 \varphi_6^2 - 2 \Delta m^2 \delta_\mu^2 \mu_I^2 \varphi_7^2 \\ & \quad - 3 \delta_\mu^2 m_S^2 \mu_I^2 \varphi_7^2 + 4 \delta_\mu^2 m_S^2 \mu_S^2 \varphi_7^2 - \frac{8}{3} \delta_\mu^2 m_S^2 \mu_I^2 \varphi_8^2 + 4 \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_4 \cos(\alpha)^2 - 8 \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_S \varphi_4 \cos(\alpha)^2 - 4 \\ & \quad \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_5 \cos(\alpha)^2 + 8 \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_S \varphi_5 \cos(\alpha)^2 - 4 \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_6 \cos(\alpha)^2 - 8 \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_S \varphi_6 \cos(\alpha)^2 + 4 \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_7 \cos \\ & \quad (\alpha)^2 + 8 \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_S \varphi_7 \cos(\alpha)^2 + 4 i \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_4 \cos(\alpha) \sin(\alpha) - 8 i \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_S \varphi_4 \cos(\alpha) \sin(\alpha) + 4 i \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_5 \cos \\ & \quad (\alpha) \sin(\alpha) - 8 i \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_S \varphi_5 \cos(\alpha) \sin(\alpha) + 4 i \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_6 \cos(\alpha) \sin(\alpha) + 8 i \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_S \varphi_6 \cos(\alpha) \sin(\alpha) + 4 i \\ & \quad \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_7 \cos(\alpha) \sin(\alpha) + 8 i \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_S \varphi_7 \cos(\alpha) \sin(\alpha) + \frac{8}{3} i \sqrt{3} \delta_\mu \partial_\mu \varphi_1 m_S^2 \mu_I \varphi_8 \sin(\alpha) + 2 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_I \varphi_4 \cos(\alpha) - 4 \\ & \quad \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_4 \cos(\alpha) - 4 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_4 \cos(\alpha) + 8 \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_S \varphi_4 \cos(\alpha) - 2 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_I \varphi_5 \cos(\alpha) + 4 \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_5 \cos(\alpha) \\ & \quad + 4 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_5 \cos(\alpha) - 8 \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_S \varphi_5 \cos(\alpha) - 2 \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_I \varphi_6 \cos(\alpha) + 4 \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_6 \cos(\alpha) - 4 \\ & \quad \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_6 \cos(\alpha) + 8 \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_S \varphi_6 \cos(\alpha) + 2 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_I \varphi_7 \cos(\alpha) - 4 \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_7 \cos(\alpha) + 4 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_7 \cos \\ & \quad (\alpha) - 8 \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_S \varphi_7 \cos(\alpha) + 2 i \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_I \varphi_4 \sin(\alpha) - 4 i \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_4 \sin(\alpha) + 2 i \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_I \varphi_5 \sin(\alpha) - 4 i \\ & \quad \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_5 \sin(\alpha) + 2 i \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_I \varphi_6 \sin(\alpha) + 4 i \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_6 \sin(\alpha) + 2 i \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_I \varphi_7 \sin(\alpha) + 4 i \\ & \quad \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_7 \sin(\alpha) - 2 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_I \varphi_4 + 4 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_4 + 2 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_I \varphi_5 - 4 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_5 + 2 \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_I \varphi_6 + 4 \\ & \quad \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_6 - 2 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_I \varphi_7 - 4 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_7 + 8 \delta_\mu \varphi_1^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_2^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_3^2 \bar{m}^2 \cos(\alpha) + 8 \\ & \quad \delta_\mu \varphi_4^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_5^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_6^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_7^2 \bar{m}^2 \cos(\alpha) + 8 \delta_\mu \varphi_8^2 \bar{m}^2 \cos(\alpha) + 4 \partial_\mu \varphi_1^2 m_S^2 + 4 \partial_\mu \varphi_2^2 m_S^2 + 4 \\ & \quad \partial_\mu \varphi_3^2 m_S^2 + 4 \partial_\mu \varphi_4^2 m_S^2 + 4 \partial_\mu \varphi_5^2 m_S^2 + 4 \partial_\mu \varphi_6^2 m_S^2 + 4 \partial_\mu \varphi_7^2 m_S^2 + 4 \partial_\mu \varphi_8^2 m_S^2 \end{aligned}$$

```
In [13]: L5 = (kin[0]*chiS).trace()
L5 = L5.series(e, n+1).truncate().trig_reduce().full_simplify()
print_coeff2(L5)
```

1 :

$$-4\delta_\mu^2\bar{m}^2\mu_I^2\cos(\alpha)^3 + 4\delta_\mu^2\bar{m}^2\mu_I^2\cos(\alpha)$$

e :

$$12\delta_\mu^2\bar{m}^2\mu_I^2\varphi_2\cos(\alpha)^2\sin(\alpha) - \frac{8}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\cos(\alpha)^3 - 4\delta_\mu^2\bar{m}^2\mu_I^2\varphi_2\sin(\alpha) - 8i\Delta m^2\delta_\mu\partial_\mu\varphi_2\mu_I\cos(\alpha)^2\sin(\alpha) + \frac{8}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\cos(\alpha)$$

e^2 :

$$\begin{aligned} & 6\delta_\mu^2\bar{m}^2\mu^2\varphi_1^2\cos(\alpha)^3 + 18\delta_\mu^2\bar{m}^2\mu^2\varphi_2^2\cos(\alpha)^3 + 6\delta_\mu^2\bar{m}^2\mu^2\varphi_3^2\cos(\alpha)^3 + 3\delta_\mu^2\bar{m}^2\mu^2\varphi_4^2\cos(\alpha)^3 + 3\delta_\mu^2\bar{m}^2\mu^2\varphi_5^2\cos(\alpha)^3 + 3 \\ & \delta_\mu^2\bar{m}^2\mu^2\varphi_6^2\cos(\alpha)^3 + 3\delta_\mu^2\bar{m}^2\mu^2\varphi_7^2\cos(\alpha)^3 + \frac{2}{3}\delta_\mu^2\bar{m}^2\mu^2\varphi_8^2\cos(\alpha)^3 - \frac{4}{3}\sqrt{3}\Delta m^2\delta_\mu^2\mu^2\varphi_3\varphi_8\cos(\alpha)^2 - \Delta m^2\delta_\mu^2\mu^2\varphi_4^2\cos \\ & (\alpha)^2 + \delta_\mu^2m_S^2\mu^2\varphi_4^2\cos(\alpha)^2 + 4\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_4^2\cos(\alpha)^2 - \Delta m^2\delta_\mu^2\mu^2\varphi_5^2\cos(\alpha)^2 + \delta_\mu^2m_S^2\mu^2\varphi_5^2\cos(\alpha)^2 + 4 \\ & \delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_5^2\cos(\alpha)^2 + \Delta m^2\delta_\mu^2\mu^2\varphi_6^2\cos(\alpha)^2 + \delta_\mu^2m_S^2\mu^2\varphi_6^2\cos(\alpha)^2 - 4\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_6^2\cos(\alpha)^2 + \Delta m^2\delta_\mu^2\mu^2\varphi_7^2\cos(\alpha)^2 \\ & + \delta_\mu^2m_S^2\mu^2\varphi_7^2\cos(\alpha)^2 - 4\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_7^2\cos(\alpha)^2 + \frac{16}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\varphi_2\cos(\alpha)^2\sin(\alpha) + \frac{8}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_2\mu_I\varphi_8\cos \\ & (\alpha)^2\sin(\alpha) + \frac{8}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\cos(\alpha)^2\varphi_2(x)\sin(\alpha) + \frac{8}{3}\sqrt{3}\Delta m^2\delta_\mu\partial_\mu\varphi_2\mu_I\cos(\alpha)^2\varphi_8(x)\sin(\alpha) - 2\delta_\mu^2\bar{m}^2\mu^2\varphi_1^2\cos(\alpha) - 14 \\ & \delta_\mu^2\bar{m}^2\mu^2\varphi_2^2\cos(\alpha) - 6\delta_\mu^2\bar{m}^2\mu^2\varphi_3^2\cos(\alpha) - \frac{5}{2}\delta_\mu^2\bar{m}^2\mu^2\varphi_4^2\cos(\alpha) - 2\Delta m^2\delta_\mu^2\mu^2\mu_S\varphi_4^2\cos(\alpha) + 2\delta_\mu^2m_S^2\mu^2\mu_S\varphi_4^2\cos(\alpha) + 2 \\ & \delta_\mu^2\bar{m}^2\mu^2\varphi_5^2\cos(\alpha) - \frac{5}{2}\delta_\mu^2\bar{m}^2\mu^2\varphi_5^2\cos(\alpha) - 2\Delta m^2\delta_\mu^2\mu^2\mu_S\varphi_5^2\cos(\alpha) + 2\delta_\mu^2m_S^2\mu^2\mu_S\varphi_5^2\cos(\alpha) + 2\delta_\mu^2\bar{m}^2\mu^2\varphi_6^2\cos(\alpha) - \frac{5}{2} \\ & \delta_\mu^2\bar{m}^2\mu^2\varphi_6^2\cos(\alpha) - 2\Delta m^2\delta_\mu^2\mu^2\mu_S\varphi_6^2\cos(\alpha) - 2\delta_\mu^2m_S^2\mu^2\mu_S\varphi_6^2\cos(\alpha) + 2\delta_\mu^2\bar{m}^2\mu^2\varphi_7^2\cos(\alpha) - \frac{5}{2}\delta_\mu^2\bar{m}^2\mu^2\varphi_7^2\cos(\alpha) - 2 \\ & \Delta m^2\delta_\mu^2\mu^2\mu_S\varphi_7^2\cos(\alpha) - 2\delta_\mu^2m_S^2\mu^2\mu_S\varphi_7^2\cos(\alpha) + 2\delta_\mu^2\bar{m}^2\mu^2\varphi_8^2\cos(\alpha) - \frac{2}{3}\delta_\mu^2\bar{m}^2\mu^2\varphi_8^2\cos(\alpha) - 16i\Delta m^2\delta_\mu\partial_\mu\varphi_2\mu_I\varphi_2\cos \\ & (\alpha)^3 - \frac{8}{3}i\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\varphi_8\cos(\alpha)^3 - 8i\Delta m^2\delta_\mu\partial_\mu\varphi_1\mu_I\cos(\alpha)^3\varphi_1(x) - 8i\Delta m^2\delta_\mu\partial_\mu\varphi_2\mu_I\cos(\alpha)^3\varphi_2(x) - 8i\Delta m^2\delta_\mu\partial_\mu\varphi_3\mu_I\cos \\ & (\alpha)^3\varphi_3(x) - 4i\Delta m^2\delta_\mu\partial_\mu\varphi_4\mu_I\cos(\alpha)^3\varphi_4(x) - 4i\Delta m^2\delta_\mu\partial_\mu\varphi_5\mu_I\cos(\alpha)^3\varphi_5(x) - 4i\Delta m^2\delta_\mu\partial_\mu\varphi_6\mu_I\cos(\alpha)^3\varphi_6(x) - 4i \\ & \Delta m^2\delta_\mu\partial_\mu\varphi_7\mu_I\cos(\alpha)^3\varphi_7(x) - \frac{8}{3}i\Delta m^2\delta_\mu\partial_\mu\varphi_8\mu_I\cos(\alpha)^3\varphi_8(x) - 4\Delta m^2\delta_\mu\partial_\mu\varphi_7\mu_I\cos(\alpha)^2\varphi_4(x)\sin(\alpha) + 4\Delta m^2\delta_\mu\partial_\mu\varphi_6\mu_I\cos \\ & (\alpha)^2\varphi_5(x)\sin(\alpha) + 4\Delta m^2\delta_\mu\partial_\mu\varphi_5\mu_I\cos(\alpha)^2\varphi_6(x)\sin(\alpha) - 4\Delta m^2\delta_\mu\partial_\mu\varphi_4\mu_I\cos(\alpha)^2\varphi_7(x)\sin(\alpha) + \frac{4}{3}\sqrt{3}\Delta m^2\delta_\mu^2\mu^2\varphi_3\varphi_8 - \frac{8}{3}i \\ & \sqrt{3}\delta_\mu\partial_\mu\varphi_3\bar{m}^2\mu_I\varphi_8\cos(\alpha)^2 + \frac{8}{3}i\sqrt{3}\delta_\mu\partial_\mu\varphi_1\bar{m}^2\mu_I\varphi_8\cos(\alpha)\sin(\alpha) + \frac{1}{2}\Delta m^2\delta_\mu^2\mu^2\varphi_4^2 - \frac{1}{2}\delta_\mu^2m_S^2\mu^2\varphi_4^2 - 2\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_4^2 - 2 \\ & \Delta m^2\delta_\mu^2\mu^2\varphi_4^2 + 2\delta_\mu^2m_S^2\mu^2\varphi_4^2 + \frac{1}{2}\Delta m^2\delta_\mu^2\mu^2\varphi_5^2 - \frac{1}{2}\delta_\mu^2m_S^2\mu^2\varphi_5^2 - 2\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_5^2 - 2\Delta m^2\delta_\mu^2\mu^2\varphi_5^2 + 2 \\ & \delta_\mu^2m_S^2\mu^2\varphi_5^2 - \frac{1}{2}\Delta m^2\delta_\mu^2\mu^2\varphi_6^2 - \frac{1}{2}\delta_\mu^2m_S^2\mu^2\varphi_6^2 + 2\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_6^2 + 2\Delta m^2\delta_\mu^2\mu^2\varphi_6^2 + 2\delta_\mu^2m_S^2\mu^2\varphi_6^2 - \frac{1}{2} \\ & \Delta m^2\delta_\mu^2\mu^2\varphi_7^2 - \frac{1}{2}\delta_\mu^2m_S^2\mu^2\varphi_7^2 + 2\delta_\mu^2\bar{m}^2\mu^2\mu_S\varphi_7^2 + 2\Delta m^2\delta_\mu^2\mu^2\varphi_7^2 + 2\delta_\mu^2m_S^2\mu^2\varphi_7^2 - 3i\Delta m^2\delta_\mu\partial_\mu\varphi_4\mu_I\varphi_4\cos(\alpha)^2 \\ & + \Delta m^2\delta_\mu\partial_\mu\varphi_5\mu_I\varphi_4\cos(\alpha)^2 - i\delta_\mu\partial_\mu\varphi_4m_S^2\mu_I\varphi_4\cos(\alpha)^2 - \delta_\mu\partial_\mu\varphi_5m_S^2\mu_I\varphi_4\cos(\alpha)^2 + 2\delta_\mu\partial_\mu\varphi_5\bar{m}^2\mu_I\varphi_4\cos(\alpha)^2 + 2i \\ & \delta_\mu\partial_\mu\varphi_4\bar{m}^2\mu_S\varphi_4\cos(\alpha)^2 - 2\delta_\mu\partial_\mu\varphi_5\bar{m}^2\mu_S\varphi_4\cos(\alpha)^2 - \Delta m^2\delta_\mu\partial_\mu\varphi_4\mu_I\varphi_5\cos(\alpha)^2 - 3i\Delta m^2\delta_\mu\partial_\mu\varphi_5\mu_I\varphi_5\cos(\alpha)^2 \\ & + \delta_\mu\partial_\mu\varphi_4m_S^2\mu_I\varphi_5\cos(\alpha)^2 - i\delta_\mu\partial_\mu\varphi_5m_S^2\mu_I\varphi_5\cos(\alpha)^2 - 2\delta_\mu\partial_\mu\varphi_4\bar{m}^2\mu_I\varphi_5\cos(\alpha)^2 + 2\delta_\mu\partial_\mu\varphi_4\bar{m}^2\mu_S\varphi_5\cos(\alpha)^2 + 2i \\ & \delta_\mu\partial_\mu\varphi_5\bar{m}^2\mu_S\varphi_5\cos(\alpha)^2 - 3i\Delta m^2\delta_\mu\partial_\mu\varphi_6\mu_I\varphi_6\cos(\alpha)^2 + \Delta m^2\delta_\mu\partial_\mu\varphi_7\mu_I\varphi_6\cos(\alpha)^2 + i\delta_\mu\partial_\mu\varphi_6m_S^2\mu_I\varphi_6\cos(\alpha)^2 \\ & + \delta_\mu\partial_\mu\varphi_7m_S^2\mu_I\varphi_6\cos(\alpha)^2 - 2\delta_\mu\partial_\mu\varphi_7\bar{m}^2\mu_I\varphi_6\cos(\alpha)^2 + 2i\delta_\mu\partial_\mu\varphi_6\bar{m}^2\mu_S\varphi_6\cos(\alpha)^2 - 2\delta_\mu\partial_\mu\varphi_7\bar{m}^2\mu_S\varphi_6\cos(\alpha)^2 \\ & - \Delta m^2\delta_\mu\partial_\mu\varphi_6\mu_I\varphi_7\cos(\alpha)^2 - 3i\Delta m^2\delta_\mu\partial_\mu\varphi_7\mu_I\varphi_7\cos(\alpha)^2 - \delta_\mu\partial_\mu\varphi_6m_S^2\mu_I\varphi_7\cos(\alpha)^2 + i\delta_\mu\partial_\mu\varphi_7m_S^2\mu_I\varphi_7\cos(\alpha)^2 + 2 \\ & \delta_\mu\partial_\mu\varphi_6\bar{m}^2\mu_I\varphi_7\cos(\alpha)^2 + 2\delta_\mu\partial_\mu\varphi_6\bar{m}^2\mu_S\varphi_7\cos(\alpha)^2 + 2i\delta_\mu\partial_\mu\varphi_7\bar{m}^2\mu_S\varphi_7\cos(\alpha)^2 + i\Delta m^2\delta_\mu\partial_\mu\varphi_6\mu_I\varphi_4\cos(\alpha)\sin(\alpha) - 3 \\ & \Delta m^2\delta_\mu\partial_\mu\varphi_7\mu_I\varphi_4\cos(\alpha)\sin(\alpha) - i\delta_\mu\partial_\mu\varphi_6m_S^2\mu_I\varphi_4\cos(\alpha)\sin(\alpha) - \delta_\mu\partial_\mu\varphi_7m_S^2\mu_I\varphi_4\cos(\alpha)\sin(\alpha) + 2i\delta_\mu\partial_\mu\varphi_6\bar{m}^2\mu_I\varphi_4\cos(\alpha)\sin \\ & (\alpha) - 2i\delta_\mu\partial_\mu\varphi_6\bar{m}^2\mu_S\varphi_4\cos(\alpha$$

$$\begin{aligned}
& + i \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_4 \sin(\alpha) + \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_4 \sin(\alpha) + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_S \varphi_4 \sin(\alpha) - 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_7 \mu_S \varphi_4 \sin(\alpha) - 2i \\
& \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_4 \sin(\alpha) - 2 \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_4 \sin(\alpha) - 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_I \varphi_5 \sin(\alpha) - \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_5 \sin(\alpha) + i \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_5 \sin(\alpha) \\
& + 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_S \varphi_5 \sin(\alpha) + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_7 \mu_S \varphi_5 \sin(\alpha) + 2 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_5 \sin(\alpha) - 2i \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_5 \sin(\alpha) - 2 \\
& \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_I \varphi_6 \sin(\alpha) + i \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_6 \sin(\alpha) + \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_6 \sin(\alpha) + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_S \varphi_6 \sin(\alpha) - 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_S \varphi_6 \sin \\
& (\alpha) + 2i \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_6 \sin(\alpha) + 2 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_6 \sin(\alpha) + 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_I \varphi_7 \sin(\alpha) - \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_7 \sin(\alpha) + i \\
& \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_7 \sin(\alpha) + 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_S \varphi_7 \sin(\alpha) + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_S \varphi_7 \sin(\alpha) - 2 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_7 \sin(\alpha) + 2i \\
& \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_7 \sin(\alpha) + \frac{8}{3} i \sqrt{3} \delta_\mu \partial_\mu \varphi_3 \bar{m}^2 \mu_I \varphi_8 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_I \varphi_4 + i \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_4 - \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_4 - 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_S \varphi_4 \\
& - 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_S \varphi_4 - 2i \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_4 + 2 \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_4 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_I \varphi_5 + \delta_\mu \partial_\mu \varphi_4 \bar{m}^2 \mu_I \varphi_5 + i \delta_\mu \partial_\mu \varphi_5 \bar{m}^2 \mu_I \varphi_5 + 2 \\
& \Delta m^2 \delta_\mu \partial_\mu \varphi_4 \mu_S \varphi_5 - 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_5 \mu_S \varphi_5 - 2 \delta_\mu \partial_\mu \varphi_4 m_S^2 \mu_S \varphi_5 - 2i \delta_\mu \partial_\mu \varphi_5 m_S^2 \mu_S \varphi_5 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_I \varphi_6 - i \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_6 \\
& + \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_6 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_S \varphi_6 + 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_7 \mu_S \varphi_6 - 2i \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_6 + 2 \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_6 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_7 \mu_I \varphi_7 \\
& - \delta_\mu \partial_\mu \varphi_6 \bar{m}^2 \mu_I \varphi_7 - i \delta_\mu \partial_\mu \varphi_7 \bar{m}^2 \mu_I \varphi_7 - 2 \Delta m^2 \delta_\mu \partial_\mu \varphi_6 \mu_S \varphi_7 + 2i \Delta m^2 \delta_\mu \partial_\mu \varphi_7 \mu_S \varphi_7 - 2 \delta_\mu \partial_\mu \varphi_6 m_S^2 \mu_S \varphi_7 - 2i \delta_\mu \partial_\mu \varphi_7 m_S^2 \mu_S \varphi_7 + 4 \\
& \partial_\mu \varphi_1^2 \bar{m}^2 \cos(\alpha) + 4 \partial_\mu \varphi_2^2 \bar{m}^2 \cos(\alpha) + 4 \partial_\mu \varphi_3^2 \bar{m}^2 \cos(\alpha) + 2 \partial_\mu \varphi_4^2 \bar{m}^2 \cos(\alpha) + 2 \partial_\mu \varphi_5^2 \bar{m}^2 \cos(\alpha) + 2 \partial_\mu \varphi_6^2 \bar{m}^2 \cos(\alpha) + 2 \\
& \partial_\mu \varphi_7^2 \bar{m}^2 \cos(\alpha) + \frac{4}{3} \partial_\mu \varphi_8^2 \bar{m}^2 \cos(\alpha) - \frac{8}{3} \sqrt{3} \Delta m^2 \partial_\mu \varphi_3 \partial_\mu \varphi_8 - 2 \Delta m^2 \partial_\mu \varphi_4^2 - 2 \Delta m^2 \partial_\mu \varphi_5^2 + 2 \Delta m^2 \partial_\mu \varphi_6^2 + 2 \Delta m^2 \partial_\mu \varphi_7^2 + 2 \\
& \partial_\mu \varphi_4^2 m_S^2 + 2 \partial_\mu \varphi_5^2 m_S^2 + 2 \partial_\mu \varphi_6^2 m_S^2 + 2 \partial_\mu \varphi_7^2 m_S^2 + \frac{8}{3} \partial_\mu \varphi_8^2 m_S^2
\end{aligned}$$

```
In [14]: H1 = (chi*chi.T).trace().expand()
          pprint(H1)
```

$$2 \Delta m^4 + m_S^4 + 2 \bar{m}^4$$

```
In [ ]:
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