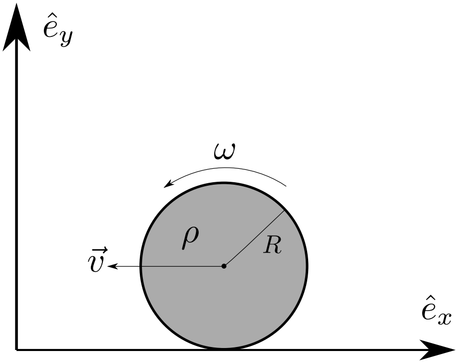
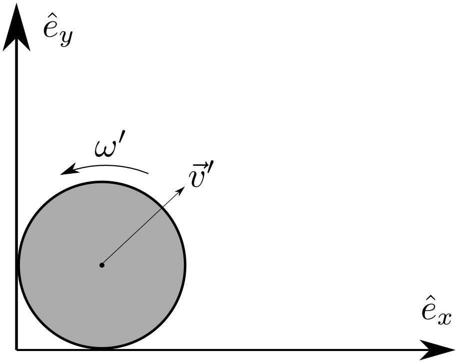
Question

- Sphere of density ρ, radius R with velocity $\vec{v} = -v_0\,\hat{e}_x$.
- Hit wall, bounces elastically and without slipping
- What is velocity $\vec{v}'$ after collision, and how does it depend on v_0, ρ and R? What happens for small R?
- What if sphere is hollow?



Quantities

- Angular velocity: ω
- Mass: $m = \frac{4\pi}{3}\rho R^3$.
- Moment of inertia: $I = \frac{2}{5}mR^2$.
- After: ω', $\vec{v}' = v_x\hat{e}_x + v_y\hat{e}_y$.



Conditions

- No slip at ground: $\omega = \frac{v_0}{R}$.
- No slip at collision: $\omega' = \frac{v_y}{R}$.
- Energy:

$$E = \frac{1}{2} m |\vec{v}|^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{7}{5} m v_0^2,$$

$$E' = \frac{1}{2} m |\vec{v}'|^2 + \frac{1}{2} \frac{I}{R^2} \omega'^2 = \frac{1}{2} m \left(v_x^2 + \frac{7}{5} v_y^2 \right)$$

- Elasticity:

$$E = E' \implies v_0^2 = \frac{5}{7} v_x^2 + v_y^2$$

Conservation of angular momentum

- Angular momentum around point of collision: No arm, no torque, conservation of angular momentum.
- Before collision: No angular momentum from center-of-mass motion, only spin angular momentum: $|\vec{L}| = I\omega = \frac{2}{5}mv_0R$.
- After, additional term from y component of velocity: $|\vec{L}'| = I\omega' + mv_yR = \frac{7}{5}mv_yR$.
- Conservation of angular momentum, $\vec{L} = \vec{L}'$, $\implies v_y = \frac{2}{7}v_0$.

Solution

- Combining gives

$$v_0^2 = \frac{5}{7} v_x^2 + \left(\frac{2}{7}\right)^2 v_0^2 \implies v_x^2 = \frac{9}{7} v_0^2$$

- Finally

$$\vec{v}' = v_0 \left(\frac{3}{\sqrt{7}} \hat{e}_x + \frac{2}{7} \hat{e}_y \right) \approx v_0 (1.13 \hat{e}_x + 0.29 \hat{e}_y)$$

- Independent of ρ, R — no other constants of same dimension, scale free.

Hollow sphere

- Only difference: new moment of inertia $I = \frac{2}{3}mr^2$
- $v_0^2 = \frac{3}{5}v_x^2 + v_y^2$, $v_y = \frac{2}{5}v_0$,

$$\implies \vec{v}' = v_0 \left(\sqrt{\frac{7}{5}}\hat{e}_x + \frac{2}{5}\hat{e}_y \right) \approx v_0(1.18\hat{e}_x + 0.4\hat{e}_y)$$