Utledninger

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Chapter 1

CHPT

1.1 Leading order Lagrangian

1.1.1 EM contribution only

Subs $\pi_a/f \to \pi_a$,

$$\Sigma = \exp\{i\pi_a\tau_a\} = 1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a \tag{1.1}$$
$$Q = \frac{1}{6} + \frac{1}{2}\tau_3 \tag{1.2}$$

$$\begin{aligned} Q\Sigma &= \frac{1}{2}\left[\frac{1}{3}\left(1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a\right) + \tau_3\left(1 + i\pi_a\tau_a - \frac{1}{2}\pi_a\pi_a\right)\right] && (1.3)\\ &= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\pi_a\pi_a + i\pi_a\tau_3\tau_a + \frac{i}{3}\pi_a\tau_a + \tau_3 - \frac{1}{2}\pi_a\pi_a\tau_3\right] && (1.4)\\ Q\Sigma^\dagger &= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\pi_a\pi_a - i\pi_a\tau_3\tau_a - \frac{i}{3}\pi_a\tau_a + \tau_3 - \frac{1}{2}\pi_a\pi_a\tau_3\right] && (1.5)\end{aligned}$$

Using $\mathrm{Tr}\{\tau_a\tau_b\tau_c\tau_d\} = 2(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd}\delta_{ad}\delta_{cb})$, and defining $\delta^i_{ab} = \delta_{ai}\delta_{bi}$,

$$\begin{aligned} \mathrm{Tr}\{Q\Sigma Q\Sigma^\dagger\} &= \frac{1}{2^2}\,\mathrm{Tr}\left\{\frac{1}{9} - 2\frac{1}{2\cdot 3^2}\pi_a\pi_a + \pi_a\pi_a\tau_3\tau_a\tau_3\tau_a + \frac{1}{8}\pi_a\pi_a + 1 - \pi_a\pi_a\right\} && (1.6)\\ &= \frac{1}{2}\left(\frac{1}{9} + 1 - \frac{1}{3^2}\pi_a\pi_a - \pi_a\pi_a + \frac{1}{9}\pi_a\pi_a + \pi_a\pi_a(2\delta^3_{ab} - \delta_{ab})\right) && (1.7)\\ &= \frac{5}{9} - \pi_1^2 - \pi_2^2. && (1.8)\end{aligned}$$

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1.1.2 Free energy EM contribution

$$\mathcal{F}/u_0 = -\left(\cos\alpha + \frac{1}{2}\frac{\mu_I^2}{\bar{m}^2}\sin^2\alpha + \frac{1}{2}\Delta\cos^2\alpha\right) \tag{1.9}$$

Intrdouce $y^2 = \mu_I^2/\bar{m}^2 = x^{-2}$.

$$-\frac{1}{u_0}\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\alpha} = \big(1 + [y^2 - \Delta]\cos\alpha\big)\sin\alpha = 0. \tag{1.10}$$

New phase at $y = 1 + \Delta$, where

$$\cos\alpha = \frac{1}{y^2 - \Delta} \implies \sin^2\alpha = 1 - \frac{1}{(y^2 - \Delta)^2} \tag{1.11}$$

Pressure

$$\tilde{p}' = -\mathcal{F}/u_0 = \frac{1}{y^2 - \Delta} + \frac{1}{2}y^2\left(1 - \frac{1}{(y^2 - \Delta)^2}\right) + \frac{1}{2}\Delta\frac{1}{(y^2 - \Delta)^2} \tag{1.12}$$

$$= \frac{1}{2}y^2 + \frac{1}{y^2 - \Delta} - \frac{1}{2}\frac{y^2 + \Delta}{(y^2 - \Delta)^2} \tag{1.13}$$

$$\frac{1}{2}\left(y^2 + \frac{1}{y^2 - \Delta}\right). \tag{1.14}$$

Normalize

$$p = p' - p'|_{y^2=(1-\Delta)} = \frac{1}{2}\left(y^2 + \frac{1}{y^2 + \Delta} - 2 - \Delta\right). \tag{1.15}$$

Isospin density

$$\frac{\mu_I}{u_0}n_I = -\frac{\mu_I}{u_0}\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\mu_I} = y^2\sin^2\alpha. \tag{1.16}$$

Energy density

$$\tilde{u} = -\tilde{p} + \frac{1}{u_0}\mu_I n_I = \frac{1}{2}\left(y^2 + \frac{1}{y^2 - \Delta} - 2 - \Delta + 2y^2\left[1 - \frac{1}{(y^2 - \Delta)^2}\right]\right) \tag{1.17}$$

$$= \frac{1}{2}\left(y^2 - \frac{3y^2 - \Delta}{(y^2 - \Delta)^2} + 2 + \Delta\right). \tag{1.18}$$