

# Utleddninger

Martin Kjøllestad Johnsrud

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# Chapter 1

## CHPT

### 1.1 Leading order Lagrangian

#### 1.1.1 EM contribution only

Subs  $\pi_a/f \rightarrow \pi_a$ ,

$$\Sigma = \exp \{i\pi_a \tau_a\} = 1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \quad (1.1)$$

$$Q = \frac{1}{6} + \frac{1}{2}\tau_3 \quad (1.2)$$

$$Q\Sigma = \frac{1}{2} \left[ \frac{1}{3} \left( 1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \right) + \tau_3 \left( 1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \right) \right] \quad (1.3)$$

$$= \frac{1}{2} \left[ \frac{1}{3} - \frac{1}{6}\pi_a \pi_a + i\pi_a \tau_3 \tau_a + \frac{i}{3}\pi_a \tau_a + \tau_3 - \frac{1}{2}\pi_a \pi_a \tau_3 \right] \quad (1.4)$$

$$Q\Sigma^\dagger = \frac{1}{2} \left[ \frac{1}{3} - \frac{1}{6}\pi_a \pi_a - i\pi_a \tau_3 \tau_a - \frac{i}{3}\pi_a \tau_a + \tau_3 - \frac{1}{2}\pi_a \pi_a \tau_3 \right] \quad (1.5)$$

Using  $\text{Tr} \{ \tau_a \tau_b \tau_c \tau_d \} = 2(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd}\delta_{ad}\delta_{cb})$ , and defining  $\delta_{ab}^i = \delta_{ai}\delta_{bi}$ ,

$$\text{Tr} \{ Q\Sigma Q\Sigma^\dagger \} = \frac{1}{2^2} \text{Tr} \left\{ \frac{1}{9} - 2\frac{1}{2 \cdot 3^2}\pi_a \pi_a + \pi_a \pi_a \tau_3 \tau_a \tau_3 \tau_a + \frac{1}{8}\pi_a \pi_a + 1 - \pi_a \pi_a \right\} \quad (1.6)$$

$$= \frac{1}{2} \left( \frac{1}{9} + 1 - \frac{1}{3^2}\pi_a \pi_a - \pi_a \pi_a + \frac{1}{9}\pi_a \pi_a + \pi_a \pi_a (2\delta_{ab}^3 - \delta_{ab}) \right) \quad (1.7)$$

$$= \frac{5}{9} - \pi_1^2 - \pi_2^2. \quad (1.8)$$

,

#### 1.1.2 Free energy EM contribution

$$\mathcal{F}/u_0 = - \left( \cos \alpha + \frac{1}{2} \frac{\mu_I^2}{\bar{m}^2} \sin^2 \alpha + \frac{1}{2} \Delta \cos^2 \alpha \right) \quad (1.9)$$

Introduce  $y^2 = \mu_I^2/\bar{m}^2 = x^{-2}$ .

$$-\frac{1}{u_0} \frac{d\mathcal{F}}{d\alpha} = (1 + [y^2 - \Delta] \cos \alpha) \sin \alpha = 0. \quad (1.10)$$

New phase at  $y = 1 + \Delta$ , where

$$\cos \alpha = \frac{1}{y^2 - \Delta} \implies \sin^2 \alpha = 1 - \frac{1}{(y^2 - \Delta)^2} \quad (1.11)$$

Pressure

$$\tilde{p}' = -\mathcal{F}/u_0 = \frac{1}{y^2 - \Delta} + \frac{1}{2}y^2 \left(1 - \frac{1}{(y^2 - \Delta)^2}\right) + \frac{1}{2}\Delta \frac{1}{(y^2 - \Delta)^2} \quad (1.12)$$

$$= \frac{1}{2}y^2 + \frac{1}{y^2 - \Delta} - \frac{1}{2} \frac{y^2 + \Delta}{(y^2 - \Delta)^2} \quad (1.13)$$

$$\frac{1}{2} \left( y^2 + \frac{1}{y^2 - \Delta} \right). \quad (1.14)$$

Normalize

$$p = p' - p'|_{y^2=(1-\Delta)} = \frac{1}{2} \left( y^2 + \frac{1}{y^2 + \Delta} - 2 - \Delta \right). \quad (1.15)$$

Isospin density

$$\frac{\mu_I}{u_0} n_I = -\frac{\mu_I}{u_0} \frac{d\mathcal{F}}{d\mu_I} = y^2 \sin^2 \alpha. \quad (1.16)$$

Energy density

$$\tilde{u} = -\tilde{p} + \frac{1}{u_0} \mu_I n_I = \frac{1}{2} \left( y^2 + \frac{1}{y^2 - \Delta} - 2 - \Delta + 2y^2 \left[ 1 - \frac{1}{(y^2 - \Delta)^2} \right] \right) \quad (1.17)$$

$$= \frac{1}{2} \left( y^2 - \frac{3y^2 - \Delta}{(y^2 - \Delta)^2} + 2 + \Delta \right). \quad (1.18)$$

## 1.2 Newtonian stars

$$d\left(\frac{u}{n}\right) = -p d\left(\frac{1}{n}\right). \quad (1.19)$$

Polytrope:  $p = Ku^\gamma$ , internal energy (assuming  $\gamma \neq 1$ ):  $u' = u - mn$ , where  $m$  is particle mass,  $n$  particle number density,  $p = K(mn)^\gamma(1 + u'/(mn))^\gamma$ .

$$d\frac{u}{n} = d\frac{u'}{n} = \frac{1}{n} du' + u' d\frac{1}{n} = -k(mn)^\gamma \left(1 + \frac{u'}{mn}\right)^\gamma d\frac{1}{n} \quad (1.20)$$

$$\implies du' = \left( \frac{u'}{mn} + k(mn)^{\gamma-1} [1 + u'/(mn)]^\gamma \right) m dn. \quad (1.21)$$

non-relativistic limit,  $u' \ll mn$ , we get

$$u' = u - mn \sim \frac{k(mn)^\gamma}{\gamma - 1} \sim \frac{p}{\gamma - 1}, \quad (1.22)$$

as  $p \sim k(mn)^\gamma$ .

### 1.2.1 Energy

$$\Phi = -\frac{Gmu}{r}, \quad \frac{dp}{dr} = -\frac{Gmu}{r^2}. \quad (1.23)$$

Total kinetic energy is  $T$ , potential  $V$ .

$$T = 4\pi \int_0^R dr r^2 u' = \frac{4\pi}{\gamma-1} \int r^2 p, \quad (1.24)$$

$$V = -4\pi \int_0^R dr r^2 \frac{Gmu}{r} = \int dr r^3 \frac{dp}{dr} = -3 \cdot 4\pi \int dr r^2 p \quad (1.25)$$

$$\implies T = -3(\gamma-1)V. \quad (1.26)$$

With  $dm = 4\pi r^2 u dr$ , we get

$$I = 4\pi \int dr^2 r^2 p = \int dm \frac{p}{u} = - \int d\left(\frac{p}{u}\right) m, \quad (1.27)$$

where we integrated by parts and used  $m(0) = p(R)/u(R) = 0$ . (assum  $\gamma > 1$ ).

$$d\frac{p}{u} = \frac{\gamma-1}{\gamma} \frac{dp}{u} = -\frac{\gamma-1}{\gamma} \frac{Gm}{r^2} dr, \quad (1.28)$$

as  $\gamma p du = dp u$ . With this, we integrate by parts to obtain

$$I = \frac{\gamma-1}{\gamma} \int dr \frac{Gm^2}{r} = \frac{\gamma-1}{\gamma} \left[ - \int d\left(\frac{1}{r}\right) Gm^2 \right] = -\frac{\gamma-1}{\gamma} \left[ \frac{GM^2}{R} - 2 \int dm \frac{Gm}{r} \right] \quad (1.29)$$

using  $dm = 4\pi r^2 u$ , we get

$$\int dm \frac{Gm}{r} = 4\pi \int dr r Gmu = -3I, \implies I = \frac{5\gamma-6}{\gamma-1} \frac{GM^2}{R} \quad (1.30)$$

Combining  $E = T + V$ , we get

$$E = -\frac{3\gamma-4}{5\gamma-6} \frac{GM}{R^2} \quad (1.31)$$

## 1.3 Ward identities

(TODO: Gjør ward-identity utledningen med scalarfelt)

Consider, for defenders, a massless scalar field with the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi_i \partial^\mu \varphi_i - \mathcal{V}[\varphi]. \quad (1.32)$$

Assume this theory has a global  $O(N)$  symmetry,

$$\varphi_i \rightarrow \varphi'_i = M_{ij} \varphi_j = (\delta_{ij} + i\eta_\alpha T_{ij}^\alpha) \varphi_j. \quad (1.33)$$

so  $\mathcal{V}[\varphi'] = \mathcal{V}[\varphi]$ , where  $\varphi'$  is related to  $\varphi$  via a symmetry transformation. The system then has the conserved current

$$J_\alpha^\mu = (\partial^\mu \varphi_i) T_{ij}^\alpha \varphi_j. \quad (1.34)$$

The action, with scalar sources  $j$  and vector sources  $v_{i,\alpha}^\mu$  is then

$$S[\varphi, j, v] = \int d^4x \left( \frac{1}{2} \partial_\mu \varphi_i \partial^\mu \varphi_i - \mathcal{V}[\varphi] + j_i \varphi_i + v_\mu^\alpha J_\alpha^\mu \right). \quad (1.35)$$

As with the derivation of the Dyson-Schwinger equations, we now perform a *local*  $O(N)$  transformation but without setting the external sources to zero. The action then becomes

$$S[\varphi', j, v] = S[\varphi, j, v] + \int d^4x [j_i M_{ij} \varphi_j + J_\alpha, i^\mu()] \quad (1.36)$$

If a theory is globally invariant under some transformation,  $\varphi(x) \rightarrow \varphi(x) + i\epsilon V\varphi(x)$ , and assuming the measure is as well, then a *local* transformation  $\varphi(x) \rightarrow \varphi(x) + i\epsilon V(x)\varphi(x)$  will give...

$$aaa \quad (1.37)$$