Question

- Sphere of density ρ, radius R with velocity $\vec{v} = -v_0 \hat{e}_x$.
- Hit wall, bounces elastically and without slipping
- What is velocity $\vec{v}'$ after collision, and how does it depend on v_0, ρ and R?
- What if sphere is hollow?

Quantities

- Angular velocity: ω
- Mass: $m = \frac{4\pi}{3}\rho R^3$.
- Moment of inertia: $I = \frac{2}{5} m R^2$.
- After: ω', $\vec{v}' = v_x \hat{e}_x + v_y \hat{e}_y$.

Conditions

- No slip at ground: $\omega = \frac{v_0}{R}$.
- No slip at collision: $\omega' = \frac{v_y}{R}$.
- Energy:

$$E = \frac{1}{2}m|\vec{v}|^2 + \frac{1}{2}\frac{I}{R^2}\omega^2 = \frac{1}{2}\frac{7}{5}mv_0^2,$$

$$E' = \frac{1}{2}m|\vec{v}'|^2 + \frac{1}{2}\frac{I}{R^2}\omega'^2 = \frac{1}{2}m\left(v_x^2 + \frac{7}{5}v_y^2\right)$$

- Elasticity:

$$E = E' \implies v_0^2 = \frac{5}{7}v_x^2 + v_y^2$$

Conservation of angular momentum

- Angular momentum around point of collision: No arm, no torque, conservation of angular momentum.
- No angular momentum from center-of-mass motion.
- Angular momentum by parallel axis theorem: $|\vec{L}| = (I + mR^2)\omega = \frac{7}{5} m v_0 R$.
- After, additional term from y component of velocity: $|\vec{L}'| = (I + 2mR^2)\omega' = \frac{12}{5} m v_y R$.
- Conservation of angular momentum, $\vec{L} = \vec{L}'$, $\implies v_y = \frac{7}{12} v_0$.

Solution

- Combining gives

$$v_0^2 = \frac{5}{7}v_x^2 + \left(\frac{12}{7}\right)^2 v_0^2 \implies v_x^2 = \frac{133}{144}v_0^2$$

- Finally

$$\vec{v}' = \frac{1}{12}v_0(\sqrt{133}\hat{e}_x + 7\hat{e}_y).$$