

Utleddninger

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March 4, 2022

Chapter 1

CHPT

1.1 Leading order Lagrangian

1.1.1 EM contribution only

Subs $\pi_a/f \rightarrow \pi_a$,

$$\Sigma = \exp \{i\pi_a \tau_a\} = 1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \quad (1.1)$$

$$Q = \frac{1}{6} + \frac{1}{2}\tau_3 \quad (1.2)$$

$$Q\Sigma = \frac{1}{2} \left[\frac{1}{3} \left(1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \right) + \tau_3 \left(1 + i\pi_a \tau_a - \frac{1}{2}\pi_a \pi_a \right) \right] \quad (1.3)$$

$$= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{6}\pi_a \pi_a + i\pi_a \tau_3 \tau_a + \frac{i}{3}\pi_a \tau_a + \tau_3 - \frac{1}{2}\pi_a \pi_a \tau_3 \right] \quad (1.4)$$

$$Q\Sigma^\dagger = \frac{1}{2} \left[\frac{1}{3} - \frac{1}{6}\pi_a \pi_a - i\pi_a \tau_3 \tau_a - \frac{i}{3}\pi_a \tau_a + \tau_3 - \frac{1}{2}\pi_a \pi_a \tau_3 \right] \quad (1.5)$$

Using $\text{Tr} \{ \tau_a \tau_b \tau_c \tau_d \} = 2(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd}\delta_{ad}\delta_{cb})$, and defining $\delta_{ab}^i = \delta_{ai}\delta_{bi}$,

$$\text{Tr} \{ Q\Sigma Q\Sigma^\dagger \} = \frac{1}{2^2} \text{Tr} \left\{ \frac{1}{9} - 2\frac{1}{2 \cdot 3^2}\pi_a \pi_a + \pi_a \pi_a \tau_3 \tau_a \tau_3 \tau_a + \frac{1}{8}\pi_a \pi_a + 1 - \pi_a \pi_a \right\} \quad (1.6)$$

$$= \frac{1}{2} \left(\frac{1}{9} + 1 - \frac{1}{3^2}\pi_a \pi_a - \pi_a \pi_a + \frac{1}{9}\pi_a \pi_a + \pi_a \pi_a (2\delta_{ab}^3 - \delta_{ab}) \right) \quad (1.7)$$

$$= \frac{5}{9} - \pi_1^2 - \pi_2^2. \quad (1.8)$$

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1.1.2 Free energy EM contribution

$$\mathcal{F}/u_0 = - \left(\cos \alpha + \frac{1}{2} \frac{\mu_I^2}{\bar{m}^2} \sin^2 \alpha + \frac{1}{2} \Delta \cos^2 \alpha \right) \quad (1.9)$$

Introduce $y^2 = \mu_I^2/\bar{m}^2 = x^{-2}$.

$$-\frac{1}{u_0} \frac{d\mathcal{F}}{d\alpha} = (1 + [y^2 - \Delta] \cos \alpha) \sin \alpha = 0. \quad (1.10)$$

New phase at $y = 1 + \Delta$, where

$$\cos \alpha = \frac{1}{y^2 - \Delta} \implies \sin^2 \alpha = 1 - \frac{1}{(y^2 - \Delta)^2} \quad (1.11)$$

Pressure

$$\tilde{p}' = -\mathcal{F}/u_0 = \frac{1}{y^2 - \Delta} + \frac{1}{2}y^2 \left(1 - \frac{1}{(y^2 - \Delta)^2}\right) + \frac{1}{2}\Delta \frac{1}{(y^2 - \Delta)^2} \quad (1.12)$$

$$= \frac{1}{2}y^2 + \frac{1}{y^2 - \Delta} - \frac{1}{2} \frac{y^2 + \Delta}{(y^2 - \Delta)^2} \quad (1.13)$$

$$\frac{1}{2} \left(y^2 + \frac{1}{y^2 - \Delta} \right). \quad (1.14)$$

Normalize

$$p = p' - p'|_{y^2=(1-\Delta)} = \frac{1}{2} \left(y^2 + \frac{1}{y^2 + \Delta} - 2 - \Delta \right). \quad (1.15)$$

Isospin density

$$\frac{\mu_I}{u_0} n_I = -\frac{\mu_I}{u_0} \frac{d\mathcal{F}}{d\mu_I} = y^2 \sin^2 \alpha. \quad (1.16)$$

Energy density

$$\tilde{u} = -\tilde{p} + \frac{1}{u_0} \mu_I n_I = \frac{1}{2} \left(y^2 + \frac{1}{y^2 - \Delta} - 2 - \Delta + 2y^2 \left[1 - \frac{1}{(y^2 - \Delta)^2} \right] \right) \quad (1.17)$$

$$= \frac{1}{2} \left(y^2 - \frac{3y^2 - \Delta}{(y^2 - \Delta)^2} + 2 + \Delta \right). \quad (1.18)$$

1.2 Newtonian stars

$$d\left(\frac{u}{n}\right) = -p d\left(\frac{1}{n}\right). \quad (1.19)$$

Polytrope: $p = Ku^\gamma$, internal energy (assuming $\gamma \neq 1$): $u' = u - mn$, where m is particle mass, n particle number density, $p = K(mn)^\gamma(1 + u'/(mn))^\gamma$.

$$d\frac{u}{n} = d\frac{u'}{n} = \frac{1}{n} du' + u' d\frac{1}{n} = -k(mn)^\gamma \left(1 + \frac{u'}{mn}\right)^\gamma d\frac{1}{n} \quad (1.20)$$

$$\implies du' = \left(\frac{u'}{mn} + k(mn)^{\gamma-1} [1 + u'/(mn)]^\gamma \right) m dn. \quad (1.21)$$

non-relativistic limit, $u' \ll mn$, we get

$$u' = u - mn \sim \frac{k(mn)^\gamma}{\gamma - 1} \sim \frac{p}{\gamma - 1}, \quad (1.22)$$

as $p \sim k(mn)^\gamma$.

1.2.1 Energy

$$\Phi = -\frac{Gmu}{r}, \quad \frac{dp}{dr} = -\frac{Gmu}{r^2}. \quad (1.23)$$

Total kinetic energy is T , potential V .

$$T = 4\pi \int_0^R dr r^2 u' = \frac{4\pi}{\gamma - 1} \int r^2 p, \quad (1.24)$$

$$V = -4\pi \int_0^R dr r^2 \frac{Gmu}{r} = \int dr r^3 \frac{dp}{dr} = -3 \cdot 4\pi \int dr r^2 p \quad (1.25)$$

$$\implies T = -3(\gamma - 1)V. \quad (1.26)$$

With $dm = 4\pi r^2 u dr$, we get

$$I = 4\pi \int dr^2 r^2 p = \int dm \frac{p}{u} = - \int d\left(\frac{p}{u}\right) m, \quad (1.27)$$

where we integrated by parts and used $m(0) = p(R)/u(R) = 0$. (assum $\gamma > 1$).

$$d\frac{p}{u} = \frac{\gamma - 1}{\gamma} \frac{dp}{u} = -\frac{\gamma - 1}{\gamma} \frac{Gm}{r^2} dr, \quad (1.28)$$

as $\gamma p du = dp u$. With this, we integrate by parts to obtain

$$I = \frac{\gamma - 1}{\gamma} \int dr \frac{Gm^2}{r} = \frac{\gamma - 1}{\gamma} \left[- \int d\left(\frac{1}{r}\right) Gm^2 \right] = -\frac{\gamma - 1}{\gamma} \left[\frac{GM^2}{R} - 2 \int dm \frac{Gm}{r} \right] \quad (1.29)$$

using $dm = 4\pi r^2 u$, we get

$$\int dm \frac{Gm}{r} = 4\pi \int dr r Gmu = -3I, \implies I = \frac{5\gamma - 6}{\gamma - 1} \frac{GM^2}{R} \quad (1.30)$$

Combining $E = T + V$, we get

$$E = -\frac{3\gamma - 4}{5\gamma - 6} \frac{GM}{R^2} \quad (1.31)$$