## Gravgård

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Alt som ikke kom med...

## 0.1 Tree level pion star

We introduce the new dimensionless variable  $1 + x^2 = \mu_I^2/\bar{m}^2$ . This is reminicent of the dimensionless Fermi momentum  $x_f = p_f/m$  in {section: cold fermi star}. By an argument using a right triangle, we can vertify that  $\cos a = b$  implies  $\sin a = \sqrt{1 - b^2}$ . Substituting the dimensionless variable into the free energy density, we get

$$\mathcal{F} = -\frac{u_0}{2} \left( 1 + x^2 + \frac{1}{1 + x^2} \right). \tag{1}$$

We have introduced the characteristic energy density  $u_0 = \bar{m}^2 f^2$ . As we found in {section: cold fermi star}, pressure is given by negative the free energy density, normalized to  $\mu_I = \bar{m}$ , or x = 0. We choose  $p_0 = u_0$ , so the dimensionless pressure can be written

$$\tilde{p} = -\frac{1}{u_0} (\mathcal{F} - \mathcal{F}_{x=0}) = \frac{1}{2} \frac{x^4}{1+x^2}.$$
 (2)

The charge density corresponding to a chemical potential is given by minus the derivative of the free energy with respect to that chemical potential. We must, however, not assume any dependence of  $\alpha$  on  $\mu_I$ . The isospin density therefore is

$$n_I = -\frac{\partial \mathcal{F}}{\partial \mu_I} = f^2 \mu_I \sin^2 \alpha = \frac{u_0}{\mu_I} \frac{2x^2 + x^4}{1 + x^2}.,$$
 (3)

With this, the dimensionless energy density

$$\tilde{u} = -\tilde{p} + \frac{1}{u_0} n_I \mu_I = \frac{1}{2} \frac{4x^2 + x^4}{1 + x^4} \tag{4}$$

## 0.2 Feil analyse av three-flavor vakum

$$\frac{1}{4} \text{Tr} \left\{ \nabla_{\mu} \Sigma_{\alpha} \nabla^{\mu} \Sigma_{\alpha}^{\dagger} \right\} = -\frac{1}{2} \sin^{2} \alpha \left\{ \mu_{I}^{2} \left[ n_{1}^{2} + n_{2}^{2} + \frac{1}{4} (n_{6}^{2} + n_{7}^{2}) \right] + \frac{1}{4} [\mu_{I}^{2} + 3\mu_{8}^{2}] [n_{4}^{2} + n_{5}^{2}] \right\}$$
 (5)

$$\frac{1}{4}\operatorname{Tr}\left\{\chi\Sigma^{\dagger} + \Sigma\chi^{\dagger}\right\} = M_1^2 \cos\alpha \tag{6}$$

$$\mathcal{H} = -\frac{1}{8}\sin^2\alpha \left[\mu_I^2 \left(4a^2 + b^2 + c^2\right) + 3\mu_8^2 (b^2 + c^2) + 2\sqrt{3}\mu_8\mu_I (b^2 - c^2)\right] + M_1^2\cos\alpha \tag{7}$$

$$= -\frac{1}{8}\sin^2\alpha \left[4\mu_I^2 a^2 + (\mu_I + \sqrt{3}\mu_8)^2 b^2 + (\mu_I - \sqrt{3}\mu_8)^2 c^2\right]$$
 (8)

Choose, without loss of generality,  $n_1 = n_4 = 0$ , which leaves  $n_2 = \cos \beta$ ,  $n_5 = \sin \beta$ , and thus

$$\Sigma = \exp\left\{i\alpha(\cos\beta\lambda_2 + \sin\beta\lambda_5)\right\} = \cos\alpha + i(\lambda_2\cos\beta + \lambda_5\sin\beta)\sin\alpha. \tag{9}$$