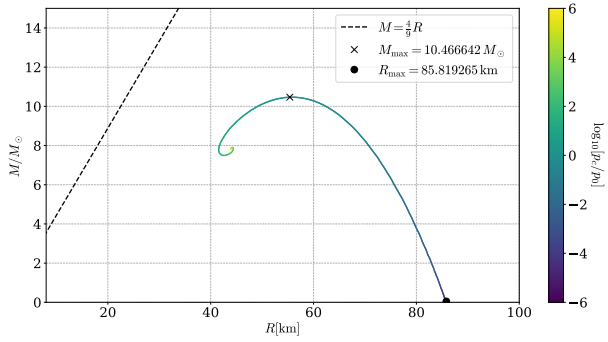
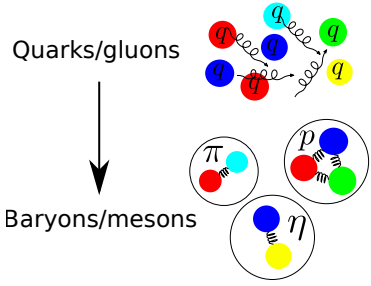
Pion stars



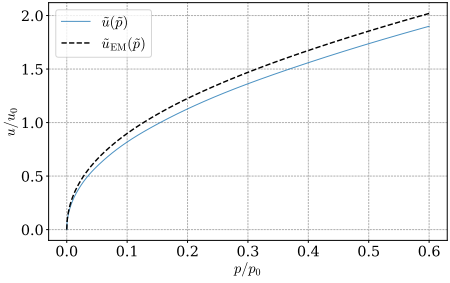
- New proposal: stars made of pions
- Microscopic part: what are the thermodynamics of pions?
- Macroscopic part: hydrodynamics of astrophysical objects
- Questions: What are the mass-radius relations of pion stars? How do EM-interactions/including three quarks/loop corrections influence the star?

Thermodynamics of quarks

- Effective theory for QCD: Chiral perturbation theory
  $\mathcal{L} = \sum_f \bar{q}_f(\gamma^\mu[\partial_\mu - iq\lambda^a A^a_\mu] + m_f)q_f + G^a_{\mu\nu}G^{\mu\nu}_a$
  $\longrightarrow \frac{1}{4}f^2\,\mathrm{Tr}\{\nabla_\mu\Sigma(\nabla^\mu\Sigma)^\dagger\} + \frac{1}{4}f^2\,\mathrm{Tr}\{\chi^\dagger\Sigma + \Sigma^\dagger\chi\}$
  $+\frac{1}{3}l_1\,\mathrm{Tr}\{\nabla_\mu\Sigma(\nabla^\mu\Sigma)^\dagger\}^2 + \ldots, \quad \Sigma \in \mathrm{SU}(3).$

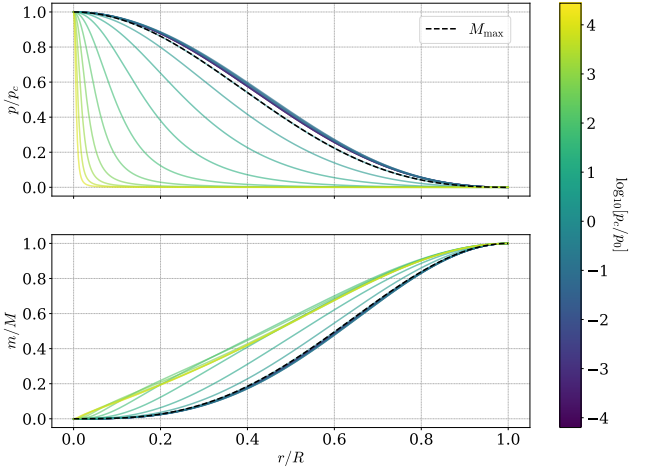


- $\Sigma = e^{i\alpha\lambda_2/2}\exp\{i\pi_a\lambda_a/f\}\,e^{i\alpha\lambda_2/2}$
- Free energy $F = -T\ln\left[\int \mathcal{D}\pi e^{-S}\right]$
- Include isospin and strangeness chemical potential, EM-interactions, loops...
- Phase transitions: pion condensation
- Calculate equation of state



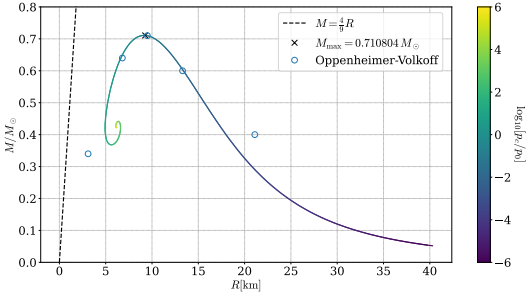
Hydrostatic equilibrium

- TOV equation govern pressure of perfect fluids in hydrostatic equilibrium
  $\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G}{r^2}\frac{(u+P)(m+4\pi r^3 P)}{\left(1-\frac{2Gm}{r}\right)}$,
  $\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 u$
- Numerical integration gives P, u and thus M, R.

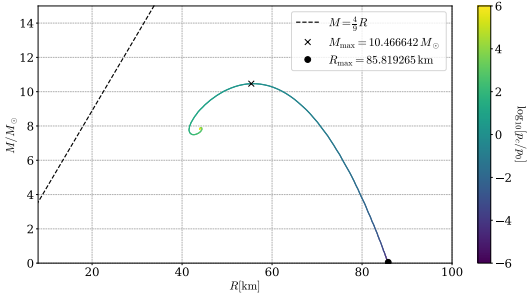


Spot the difference

Neutron star

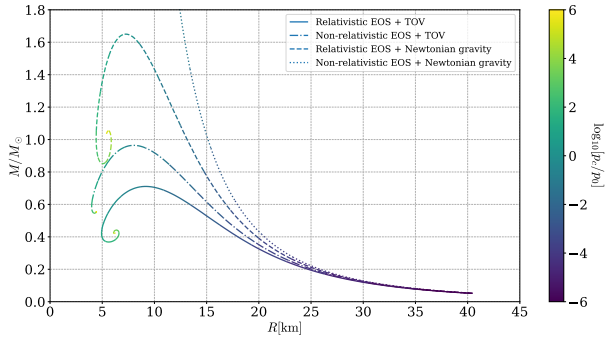


Pion star



Why does pion star have a maximum radius? As far as we can tell, no one has commented or given an explanation.

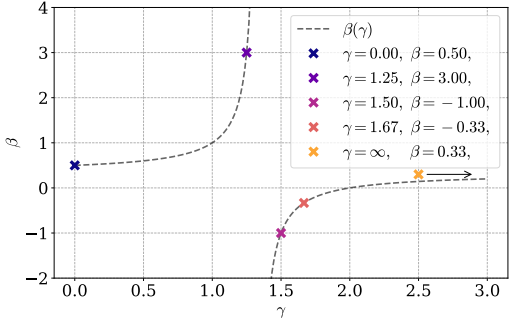
Non-relativistic approximation

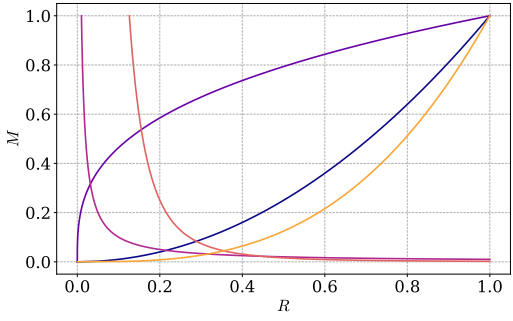


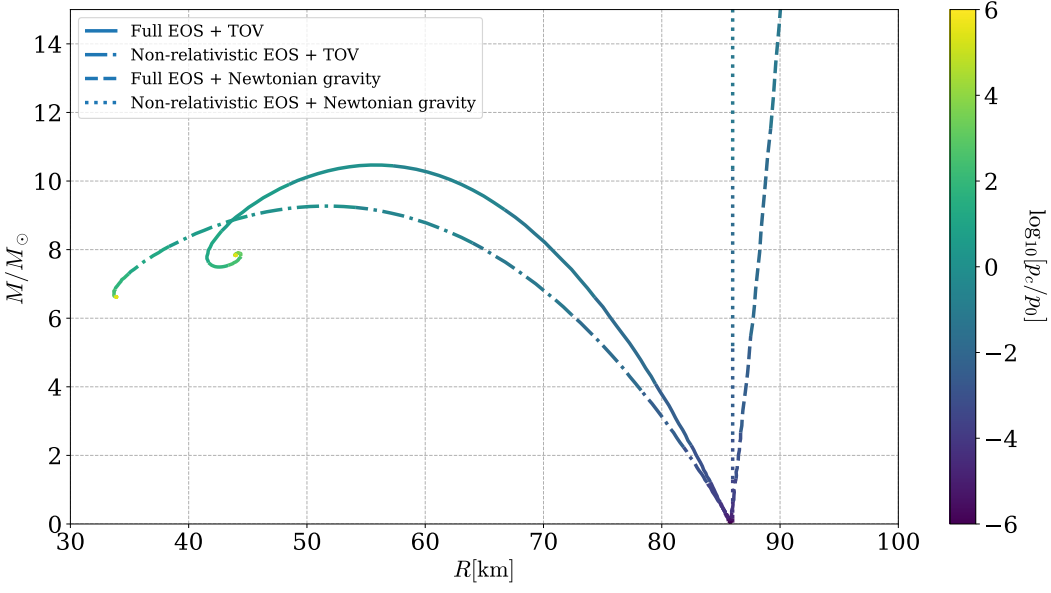
- Newtonian limit for TOV,
  $\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gmu}{r^2}$
  plus polytrope, $P = Ku^\gamma$,
  $\implies \xi^{-2}\frac{\mathrm{d}}{\mathrm{d}\xi}\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi} = -\theta^{1/(\gamma-1)}$,
  where $\xi \propto r$, $\theta \propto P^{1/(1-\gamma)}$
- Used to derive Chandrasekhar limit
- Non-relativistic limit of pion equation of state is polytrope, $P = Ku^2$
- We can show that radius is independent of external pressure, and $R = \frac{\pi r^0}{\sqrt{12}}$.

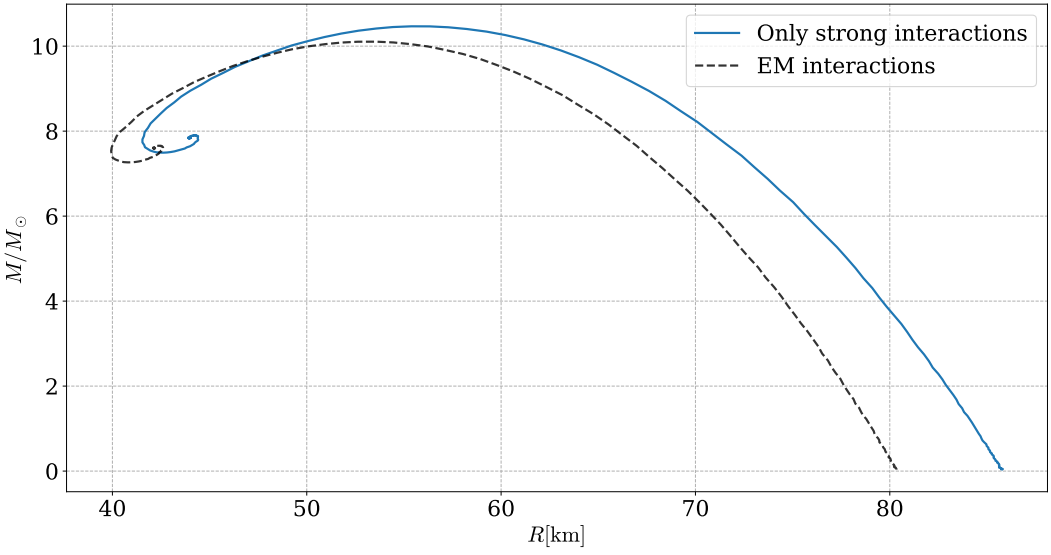
Polytrope mass-radius relation

$$M = R^{\beta}$$






Full EOS + TOV
Non-relativistic EOS + TOV
Full EOS + Newtonian gravity
Non-relativistic EOS + Newtonian gravity
$M/M_\odot$
R[km]
$\log_{10}[p_c/p_0]$
0
2
4
6
8
10
12
14
30
40
50
60
70
80
90
100
6
4
2
0
−2
−4
−6


Only strong interactions
EM interactions
$M/M_\odot$
R[km]
0
2
4
6
8
10
40
50
60
70
80