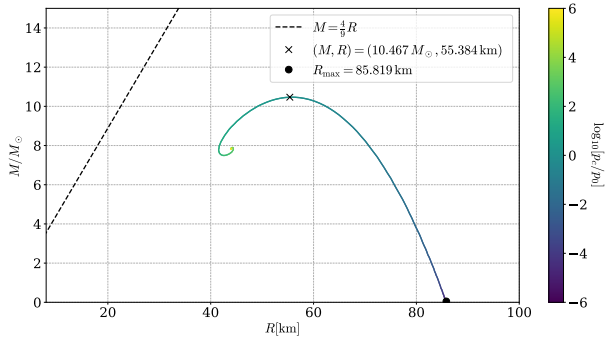
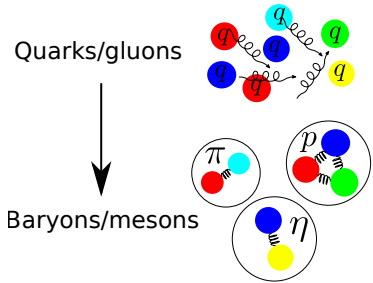
Pion stars



- New proposal: stars made of pions
- Microscopic part: what are the thermodynamics of pions?
- Macroscopic part: hydrodynamics of astrophysical objects
- Questions: What is the mass-radius relations of pion stars? How do EM-interactions/including three quarks/loop corrections influence the star?

Chiral perturbation theory

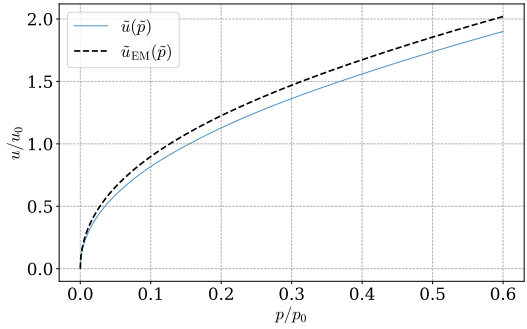
- Quarks bind into baryon (protons/neutrons) and mesons (pions) at low temperatures ($< \sim 10^{12}$ K)
- The strong force is strong, so we can't do perturbation theory.
  $\mathcal{L} = \sum_f \bar{q}_f(\gamma^\mu[\partial_\mu - iq\lambda^a A^a_\mu] + m_f)q_f + G^a_{\mu\nu}G^{\mu\nu}_a$



- Need an effective theory, The QCD vacuum spontaneously break a symmetry of the Lagrangian — Goldstone bosons (pions)
  $\mathcal{L}_{\text{eff}} = \frac{f^2}{4}\operatorname{Tr}\{\nabla_\mu \Sigma \nabla^\mu \Sigma^\dagger\} + \frac{f^2}{4}\operatorname{Tr}\{\chi^\dagger\Sigma + \Sigma^\dagger\chi\} + l_1 \operatorname{Tr}\{\nabla_\mu\Sigma(\nabla^\mu\Sigma)^\dagger\}^2 + \ldots$
- $\Sigma = \exp(i\pi_a\tau_a/f) \in \mathrm{SU}(3)$, where $\mathrm{SU}(3)$ is the broken symmetry.

Thermodynamics

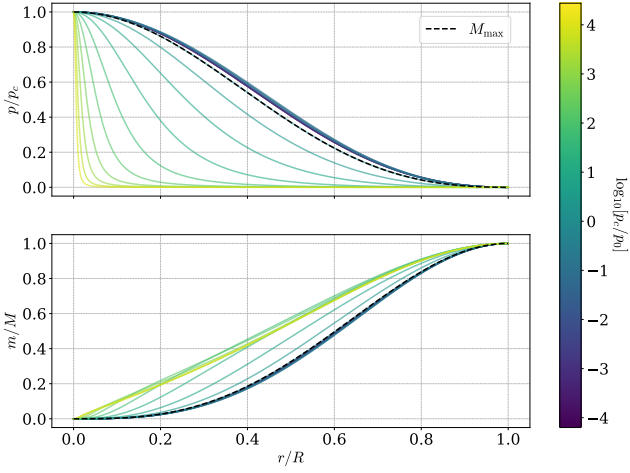
- With the effective Lagrangian, we can calculate free energy density
- $\mathcal{F} = -\frac{i}{V\beta} \ln \left[\int \mathcal{D}\pi \, \exp \left(i \int \mathrm{d}^4 x \left[\mathcal{L}_{\text{eff}} + \mu J \right] \right) \right]$
- Include isospin and strangeness chemical potential, EM-interactions, loops...
- Phase transitions: pion condensation
- Calculate equation of state



Hydrostatic equilibrium

- TOV equation govern pressure of perfect fluids in hydrostatic equilibrium
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G}{r^2}\frac{(u+P)\left(m+4\pi r^3 P\right)}{\left(1-\frac{2Gm}{r}\right)},$$
$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 u$$

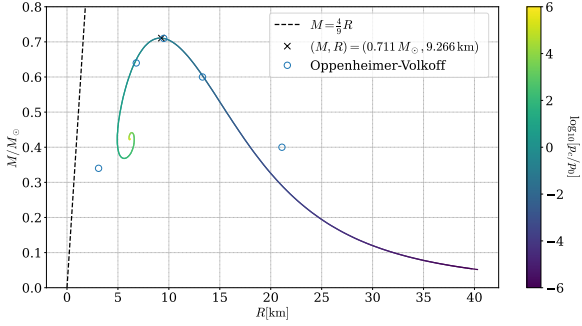
- Numerical integration gives P, u and thus M, R.



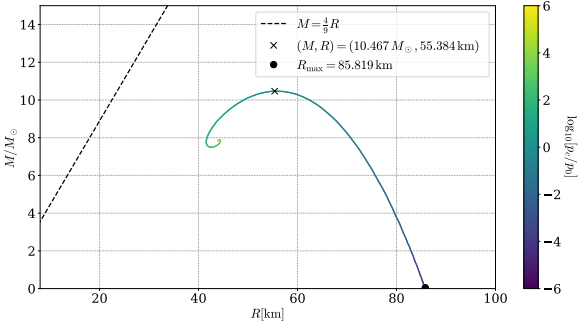
Results

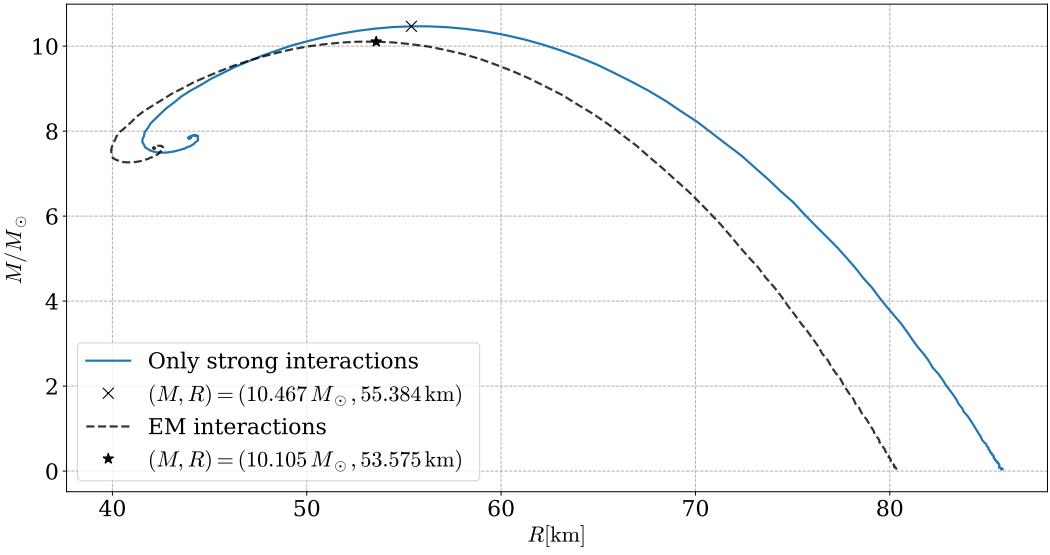
We get a family of results parametrized by central pressure.

Neutron star



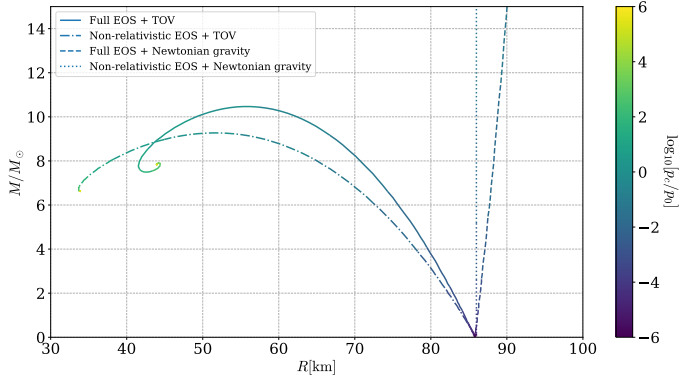
Pion star




10
8
6
4
2
0
$M/M_\odot$
40
50
60
70
80
R[km]
Only strong interactions
$(M, R) = (10.467\, M_\odot\,, 55.384\,\mathrm{km})$
EM interactions
$(M, R) = (10.105\, M_\odot\,, 53.575\,\mathrm{km})$

Limit

- Newtonian and non-relativistic limit of EOS and TOV
- Used to derive Chandrasekhar limit
- Non-relativistic limit of pion equation of state is polytrope, $P = Ku^2$



- We can show that radius is independent of external pressure, and $R = \frac{\pi r^0}{\sqrt{12}}$.