QCD Lagrangian

Martin Johnsrud

The QCD Lagrangian is, in compact notation

$$\mathcal{L} = \sum_f \bar{q}_f \left(i \not{D} - m_f \right) q_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} \tag{1}$$

Including all indices gives

$$\begin{aligned} \mathrm{QCD}^{f' c' j}_{f\, c\, j} &= \mathbb{1}_{f f'} \gamma^\mu_{j j'} \left[\mathbb{1}_{c c'} \partial_\mu - i g \lambda^a_{c c'} A_{a\mu} \right] - m_{f f'} \mathbb{1}_{c c'} \mathbb{1}_{j j'} \\ \mathcal{L}_1 &= \bar{q}_{f\, c\, j}\, \mathrm{QCD}^{f' c' j}_{f\, c\, j}\; q_{f' c' j'} \end{aligned}$$

Here, $f \in \{u, d, s, c, t, b\}$ are flavors, $c \in \{r, g, b\}$ are colors, $j \in \{0, 1, 2, 3\}$ are spinor indices, $\mu \in \{0, 1, 2, 3\}$ are space-time indices, $a \in \{1, ...8\}$ are indices for the $\mathfrak{su}\,(3)$ color algebra.

The gluon field strength tensor is given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$

where A^a_μ is the gluon field potential, g the field coupling strength and f^{abc} is the structure constants of the $\mathrm{SU}\,(3)$ gauge group of the gluon potential. The covariant derivative is given by

$$D_\mu = \partial_\mu - i g \lambda^a A^a_\mu,$$

which ensures the Lagrangians is invariant under the gauge transformation

$$q(x) \to U(x) q(x), \quad U(x) = \exp(i \lambda^a \chi^a(x)) \in \mathrm{SU}\,(3)\,. \tag{2}$$

Global symmetires

The light quarks are quarks with the flavors the subset $l \in \{u, d, s\}$. Approximating $m_{ll'} = m_u \delta_{ll'}$ gives the Lagrangian a global $\mathrm{U}\,(3)$ symmetry. In the chiral limit $m_u \to 0$, (HVORFOR KAN EN GJØRE DETTE) the Lagrangian splits into two parts, left- and right-handed, which both have the global flavor symmetry. Thus, the classical global symmetry becomes $\mathrm{U}\,(3)_L \times \mathrm{U}\,(3)_R$