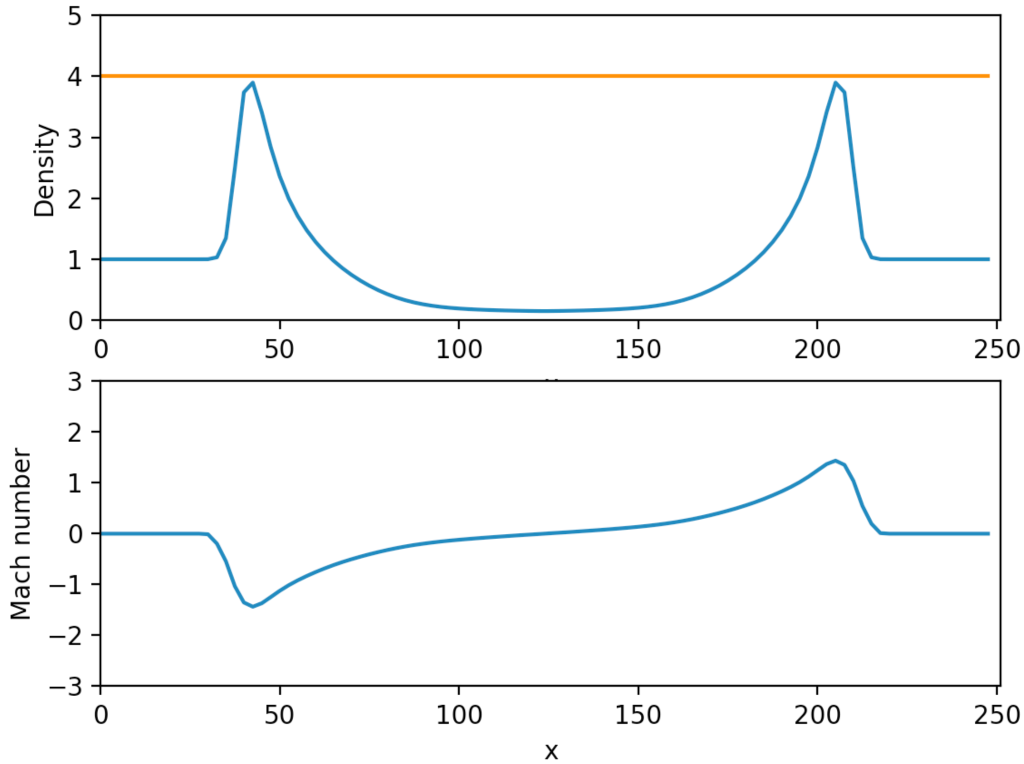
Question # 2

Part 1) In class we derive that the post shock solution for a adiabatic strong shock is

$$\rho_2/\rho_1 = \frac{\gamma+1}{\gamma-1}$$

Given that we start at $\rho_1 = 1 \Rightarrow \rho_2/\rho_1 = \rho_2/1 = \rho_2$ travelling peaks should be at $\frac{\gamma+1}{\gamma-1} = \frac{5/3+1}{5/3-1} = 4$



Part 2)

From lecture 15.2 we know that the width of the shock is given by shock width $\sim \frac{\nu}{u}$

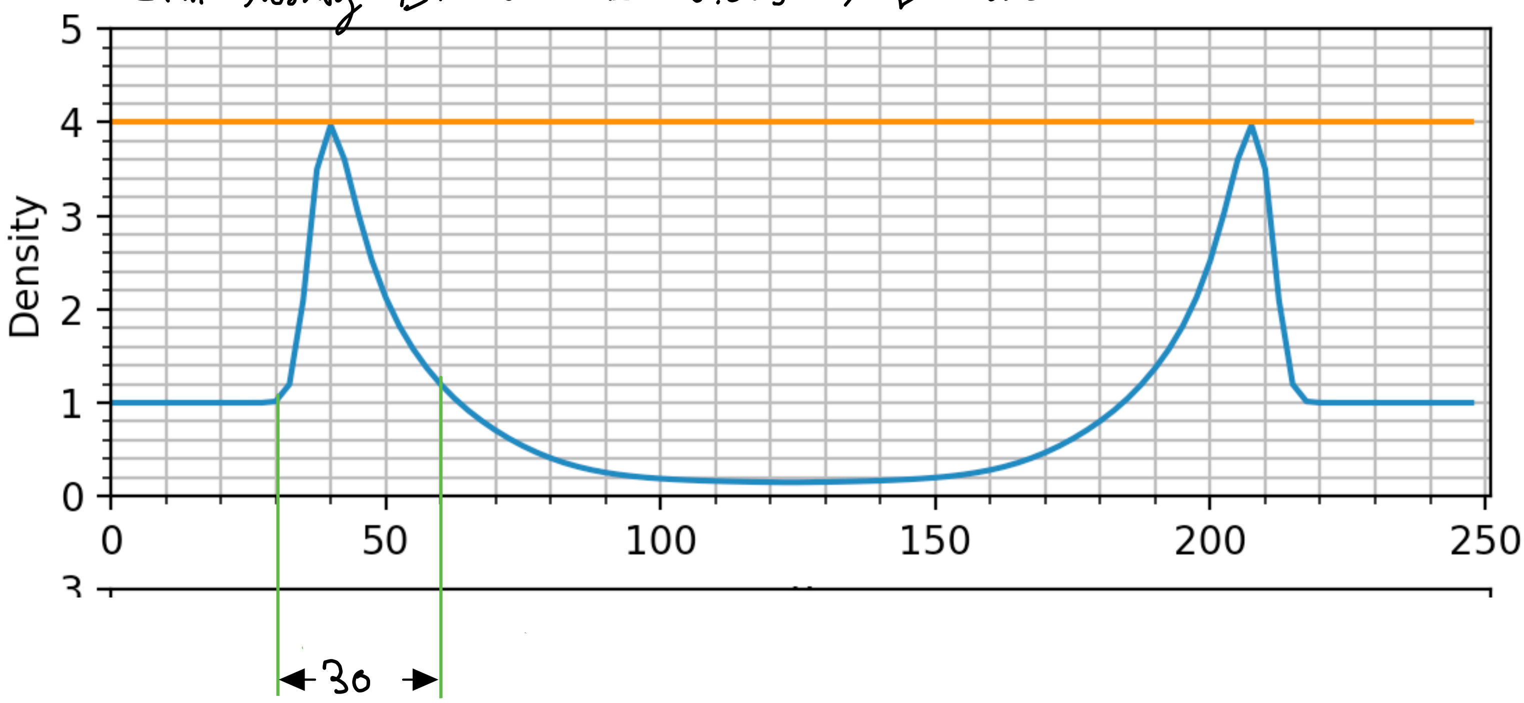
We know from the first set of notes that ↙ page 3
the numerical viscosity is given by

$$\nu = \frac{\Delta x^2}{2\Delta t}$$

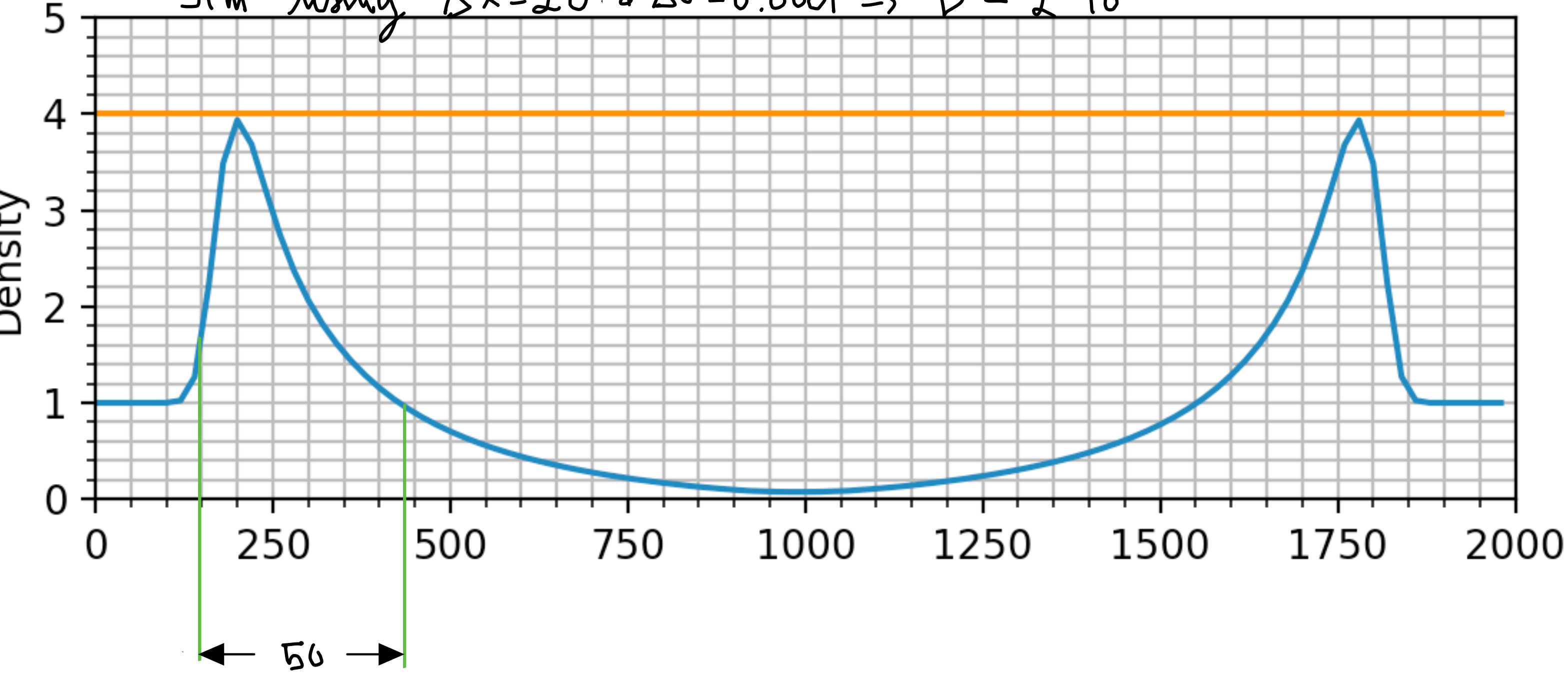
$$\Rightarrow \quad \boxed{\text{width} \sim \frac{\Delta x^2}{2\Delta t}\frac{1}{u} \propto \frac{\Delta x^2}{2\Delta t}\frac{1}{M}}$$

So the width is determine by our choice of the dx and dt accordingly with $\frac{\Delta x^2}{2\Delta t}$

Sim using $\Delta x = 2.5$ & $\Delta t = 0.005 \Rightarrow \nu = 625$



Sim using $\Delta x = 20$ & $\Delta t = 0.0001 \Rightarrow \nu = 2 \cdot 10^6$



As we can see increasing the nu increases the width

Important derivation for the simulation

Given $f_1 = \rho$, $f_2 = \rho u$, $f_3 = \rho e_{tot}$ we an expression for P & c_s in terms of f_1, f_2, f_3

First let's recall that

$$\rho e_{tot} = \frac{1}{2}\rho u^2 + \rho\left(\varepsilon + \frac{P}{\rho}\right)$$

From the lecture on shock we know that

$$\frac{\gamma P}{\rho} = (\gamma-1)(\epsilon + P/\rho) \Rightarrow \frac{\gamma P}{(\gamma-1)} = \rho\,(\epsilon + P/\rho)$$

$$\rho e_{tot} = \frac{1}{2}\rho u^2 + \frac{\gamma P}{(\gamma-1)}$$

$$\Rightarrow f_3 = \frac{1}{2} f_2^2 / f_1 + \frac{\gamma}{(\gamma-1)} P$$

$$\Rightarrow \boxed{P = \frac{\gamma-1}{\gamma}\left(f_3 - \frac{1}{2} f_2^2 / f_1\right)}$$

from this class we also know that

$$P = \rho \frac{c_s^2}{\gamma} \Rightarrow c_s^2 = \frac{\gamma}{\rho} P$$

$$\boxed{c_s^2 = \frac{\gamma}{f_1} P}$$