Nuclear $\gamma\gamma$ Angular Correlations

January 31, 2024

Information for the lab report: Answer the questions posed starting from section 7. Comment on, or discuss your results when appropriate. Your lab report alone should be sufficient for someone else to perform the experiment and reproduce your data analysis.

1 Aim of this Experiment

1.1 Physics

Electromagnetic multipole radiation has a non-isotropic angular distribution. Considering a gamma decay of a nucleus, the orientation of the emission distribution arises from the spin orientation of the parent nucleus.
In thermal equilibrium, the net orientation is zero as all spin states are populated with an equal density. All spin configurations are equally probable and in this case, the gamma ray distribution of an ensemble of nuclei is isotropic.

A non-equilibrium population of the spin quantum states can be achieved in a cascaded gamma decay of a nucleus.

By detecting the first photon at a defined position, information about the quantum state of that specific mother nucleus is gained. In particular, probabilities for the possible values of the quantum number m can be calculated which turn out to be non-equal.

Therefore, a non isotropic angular distribution of the second photon can be measured if the first one was detected in a specific direction.

The aim of the experiment is to prepare the experiment and to measure the angular correlation of the $\gamma - \gamma$ cascade of ^{60}Co.

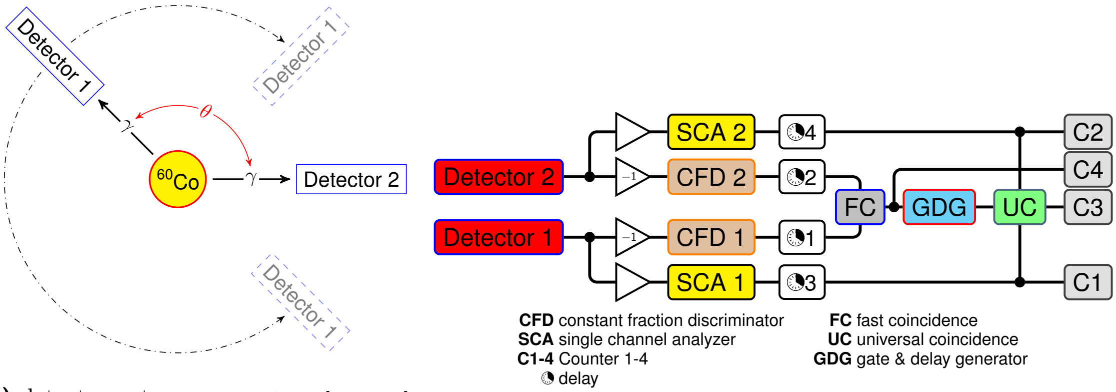
1.2 Lab proficiency

Another heavy focus of this lab course is to enhance the students lab proficiency. Important skills include:

- Do measurements time efficient:
  - Focus on the main goals: Measure all data needed, but not more than that
  - Don't waste time on measuring unimportant data or putting too much precision in single aspects that won't help with the precision of the final result
  - Be well prepared: Make a check list of things to do for each task.
- Be sure to have measured properly:
  - Do cross checks all the time, realize mistakes as quickly as possible
  - verify that the measured data contains all required information
  - Plot all data immediately! This way, mistakes can become apparent and can be fixed. e. g. a typo when writing down a measurement. If there is the slightest doubt: re-measure a data point. Don't ever change data once you left the lab, even if the mistake seems obvious.
- Do all of this in the very limited time you have access to the lab.

2 Experimental Setup

The setup consists of two NaI(Tl) scintillation detectors, which are mounted on a table. Both detectors are located on a circle which has the radioactive source in the center. One of the detectors is movable, thus the angle between the detectors can be varied (see Fig. 1a).



(a) detector setup – taken from [4, p. 1] (edited)

(b) electronic setup – taken from [4, p. 7]

Figure 1: Experimental Setup

Electronic modules are provided to build up a fast-slow coincidence. The setup is shown in Fig. 1b. Counter modules are available to measure the coincidence count rate and the individual count rates of each detector. The software tells a rotating engine to given angle θ. It also starts and stops measurements after given time.

3 Knowledge

- Definition: $\gamma\gamma$ cascade + angular correlation; example: $0 \longrightarrow 1 \longrightarrow 0$ $\gamma\gamma$ cascade
- naïve theory of $\gamma\gamma$ directional correlation
- How can electric and magnetic fields disturb the correlation? Is it relevant for ^{60}Co? Why (not)?
- What does a scintillation detector do?
- Decay-scheme of ^{60}Co (Fig. 2). How does the spectrum look like when measured with the scintillators? What additional features appear?
- What is a fast-slow coincidence? Which components are required to build it up? What is the purpose of each component? What adjustment / configuration / calibration has to be done, before the fast-slow coincidence can be used?
- What is the effect of the detectors' solid angles on the measured correlation? How can it be compensated? What are random coincidences (+ how to compensate)? How does misalignment show in the data (+ how to compensate)?
- Combining results from count rate measurements. How to add values, times, how to calculate errors.

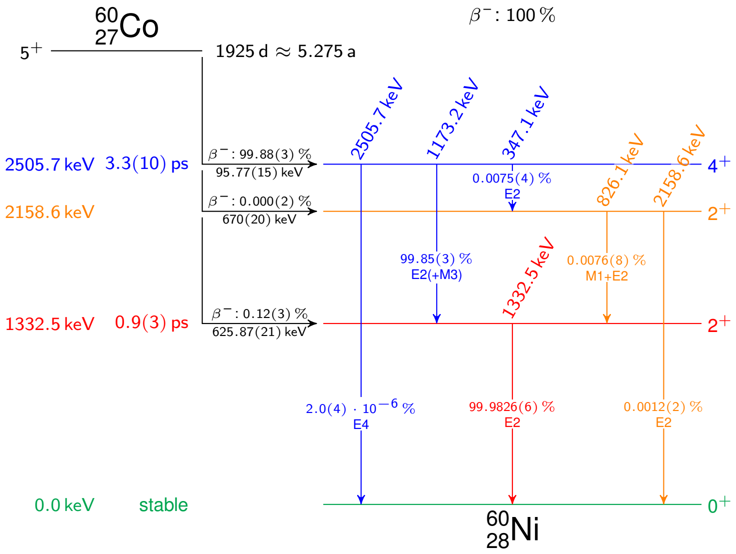


Figure 2: Decay scheme of ^{60}Co – taken from [4, p. 2] (edited), based on data from [5, 6]

4 Literature

4.1 Theory

K. Siegbahn: Vol. 2: α-, β- and γ-Ray Spectroscopy

- Pages 997 – 1000: Introduction to angular correlation of nuclear radiation
- Pages 1005 – 1007: Naïve theory
- Pages 1029 – 1033: Theoretical Prediction
- Pages 1190 – 1195: Solid angle correction factors
- Pages 1197 – 1198: F-Coefficients
- Pages 1698 – 1699: Table: Solid angle correction factors

4.2 Experimental Methods

W. R. Leo: *Techniques for Nuclear and Particle Physics Experiments*, Springer, 2$^{\text{nd}}$ Rev. edition (1994)

- Optional: 7. Scintillation Detectors, 8. Photomultipliers
- 12.3 NIM Logic Signals
- 14.8 Delay lines, 14.9 Discriminators, 14.10 Single-Channel Analyzers, 14.15 Scalers, 14.17 Coincidence Units, 14.21 Gate and Delay Generators
- 15. Pulse height Selection and Coincidence Technique until including 15.2.1 SCA Calibration and Energy Spectrum Measurement, 15.4 Basic Coincidence Technique (including all subsections)
- 15.5 Combining Pulse Height Selection and Coincidence Determination. The Fast-Slow Circuit
- 17.2 Time-Pickoff Methods – including 17.2.3

4.3 Additional / Optional

- *Siegbahn Vol. 2: α-, β- and γ-Ray Spectroscopy*, pp. 1029 – 1035, 1101 – 1104, 1190 – 1195, 1695
- Melissinos: *Experiments in Modern Physics*, pp. 412 – 429, 461 – 476
- Riezler/Kopitzki: *Kernphysikalisches Praktikum*
- Schatz/Weidinger: *Nuclear Condensed Matter Physics: Nuclear Methods and Applications*, Wiley, 1st edition (1996), pp. 14 – 20, 63 – 68, 80 – 85

5 Tasks for preparation

This experiment requires performing a few tasks before the experiment day, in addition to the preparation of the theory. These tasks are listed in the following.

The text requires knowledge about the experiment, so it makes sense to read the theory and the description of the experiment first.

5.1 Theoretical Considerations

5.1.1 Which distances to choose?

The angular correlation function is given for point like detectors. A real detector covers a certain solid angle.
A larger detector helps to gather more events and therefore get a better statistical precision. However, a bigger solid angle will also smear out the angular correlation. Correction factors (see [2]) can be used to compensate this effect.
You should ponder both effects to find a good compromise.

The following hints are supposed to help you to make a decision, based on (semi) quantitative considerations.

For a very small solid angle, the correction factor will be close to 1, i.e. no correction. An increased solid angle will significantly increase the statistical precision, while the correction factor remains close to 1 if the solid angle is still small enough.
At some point, the correction will become significant and eventually increase so quickly that it outweighs any gain in statistical precision. When does that happen?

Use the following hints to examine this problem quantitatively:

- How does the count rate of one detector change with the distance?
- How does the coincidence rate change with the distance? Give a formula!
- How does the asymmetry seen in the experiment differ from the theoretical prediction? Again, give a formula.
- How do the correction factors influence the error of the corrected correlation coefficients?
- Using the values provided in Siegbahn for 2 ” × 2 ” crystals: which distance is likely to yield most precise results?
- If you cannot get an answer with mathematical certainty, give an argument what seems reasonable.

Hint: If you get stuck, use the assumption that the errors on the coefficients in the formula are directly proportional to the error bars of the data points.

5.1.2 Which angles to pick?

Among many compromises that have to be taken in this experiment, one is between measurement time per angle on the one side and number of angles on the other side. The total time available is limited, so you can either increase the statistical precision for few measurements or make many measurements which have a larger statistical error.

Deciding what is the optimal compromise requires a careful definition of the goal of the measurement.

Without any theoretical assumptions, measurements over the whole angular range are necessary, performed in small angular steps, in order to get a full picture of the angular correlation.

However, the picture changes if one assumes a certain angular distribution that is predicted by theory. Data points at certain angles give a stronger constraint on the fit parameters than those at other angles.

When measuring a linear dependency, the data points should be as far apart as possible to get a good precision.

As the function describing the angular correlation is more complex, also it is more involved to find the optimal points to perform measurements.

To get into this, consider the theoretical prediction for the angular correlation of the 4-2-0-cascade which is given by

$$f(\theta) = A \cdot \left(1 + B \cdot \cos^2(\theta) + C \cdot \cos^4(\theta)\right) \tag{1}$$

A is a scaling factor which is proportional to measurement time and intensity of the source. The information about the angular correlation is contained in parameters B and C, defining the intensity of the two contributions.
This can also be expressed differently by defining other coefficients:

$$\alpha = B + C$$
$$\beta = B - C.$$

Assuming that errors can be propagated in the Gaussian way, determining α and β precisely yields precise values for B and C.

To find out which spots are best suited to determine α and β, do the following steps:

- Express the angular correlation using the coefficients A, α, and β.
- Plot the function using the predicted values for α and β. Use $A = 1$.
- Plot the function again, but once with slightly varied α and once with slightly varied β
- How does the function change? At which points do you see the largest change?

5.1.3 How to correct for de-adjustment?

In a practical setup, the alignment of the source can only be done to a certain precision. While it is relatively easy to move the detector on a circle, it is more difficult to have the source located exactly in the center of this circle. **How will this misalignment manifest in the data? How can you correct it?** Give formulas for the the count rate in each detector $f_{\text{movable}}(\theta)$ and $f_{\text{fixed}}(\theta)$ as well as the coincidence rate $f_{\text{coinc}}(\theta)$.
To simplify the calculation, you can assume that there is no angular correlation (i.e. $B = C = 0$) and that there are no random coincidences.

5.2 Preparation of the Experiment Tasks

5.2.1 Analysis tools

You are provided with old data. Plot all data sets and apply fits.
Use these tools / scripts / ... to plot your own measurements directly, during the lab course.
When these measurements were done, at least one mistake was made in each. Try to spot them.

5.2.2 List of Measurements

This script only describes very briefly measurements that have to be performed. References are provided where measurements are explained in more detail.

To be able to successfully do all measurements you should **prepare a list of all measurements that have to be made, including one schematic for each.**

6 Experimental Procedure

The angular correlation of the two gamma photons from ^{60}Co is determined by measuring the rate of decays for different angles.

Two detectors are placed on a circle around the ^{60}Co sample. One of the detectors is movable, which allows to measure the angular dependence of event rate.

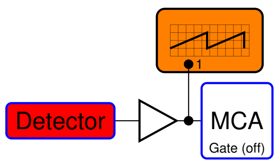
Both detectors have to register an event A) with the right energy each, B) at the same time. A Fast-Slow-Coincidence (see Fig. 1b) is used to ensure both. A counter is needed at the output of the global coincidence to measure the coincidence event rate. Two more counters measure the rate of detected photons in each detector. These values are needed to compensate for imperfections of the setup. A fourth counter measures the fast-coincidence event rate.

7 Preparation and Calibration of the Setup

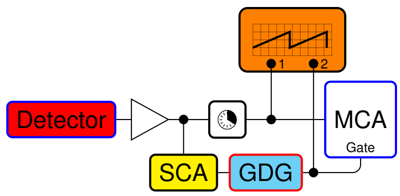
7.1 Set the gain of the main amplifiers

Goal: The signal amplitude on the output of the amplifiers should be as high as possible. A little headroom to the maximum signal amplitude of the electronics ($V_{\text{max}} = 10\,\text{V}$) has to be maintained.

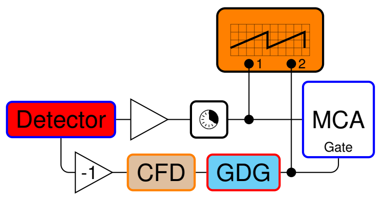
- Use an oscilloscope and the MCA (see Fig. 3a). The tutor will help you to set it up that the photopeaks are visible on the screen.
- The photopeaks should have an amplitude of $8\,\text{V} - 9\,\text{V}$.
- Take screen shots for the lab report!



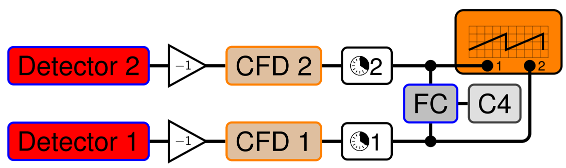
(a) circuit diagram to measure spectrum – taken from [4, p. 25] (edited)



(b) circuit diagram to setup SCA window – taken from [4, p. 25]



(c) circuit diagram to setup CFD threshold – taken from [4, p. 25]



(d) circuit diagram to setup the delay in the Fast Branch – taken from [4, p. 25]

Figure 3: circuit diagrams – all taken from [4, p. 25]

7.2 Calibrate the SCAs

Goal: set the SCAs such, that they select pulses corresponding to the photo peaks of the spectrum.

- Built up Fig. 3b and set MCA gate to coincidence
- set up delay of GDG such that photopheaks are in middle of SCA output
- **Option 1:** Configure the SCAs that the energy window of each module covers both photo-peaks
- **Option 2:** Set the thresholds, that each SCA only covers one of the photons.
- Acquire the MCA spectrum once with internal gate[1], once with SCA gate.

7.3 Find a good threshold for the CFDs

Goal: Find a proper setting for the threshold of each CFD.
Background: In addition to what is described in 17.2.3 of [3], CFDs have also a leading edge discriminator which enables the zero-crossing detector. Setting it to a too low threshold will result in the discriminator triggering on electronic noise. A too high threshold will also suppress the signals corresponding to true events.

- Build up Fig. 3c and keep MCA gate to coincidence
- Pick a threshold that removes the major part of the noise, but still allows to detect all photons
- Acquire the MCA spectrum once with internal gate[1], once with gate generated using the CFD.

7.4 Adjust the delay in the Fast Branch

Goal: Find the optimal setting for the delay in the fast branch. Get data to determine characteristics of the coincidence.
The output signals of both CFDs should arrive at the same time at the fast coincidence module.

- Use a fixed delay and a variable delay
- set Delay 1 (◔1 in Fig. 3d) to a fixed value, then variate Delay 2 (◔2) and measure counts of the fast-coincidence (FC) with Counter 4 (C4) for each value of Delay 2
- Select delay step size and measurement time for each setting such, that the whole measurement will take 5 – 10 min!

7.5 Adjust the delay in the Slow Branch

Goal: The signals of the SCAs and the output of the fast branch need to arrive at the three-fold coincidence module with proper alignment in time.

- Try both: Trigger on the output of the fast coincidence or on the output of one SCA. Which allows more easily to align the pulses and why?
- regulate the SCA delays (◔3 and ◔4) such that all the signals arrives at the same time – it may be necessary to change GDG delay
- change GDG width such that all logic peaks are of (nearly) the same length
- Take screen shots of the oscilloscope for the lab report!

[1]This corresponds to the Gate being set to "Off" in the spectrum window.

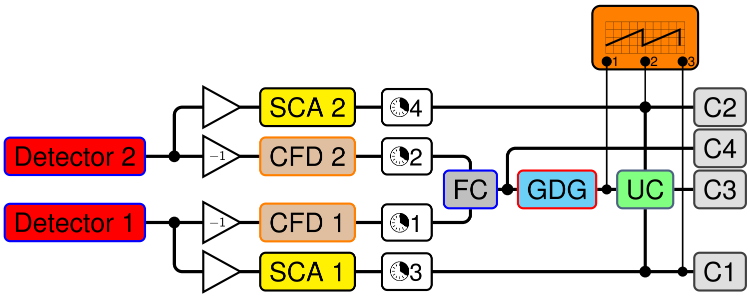


Figure 4: circuit diagram to adjust slow branch delays – taken from [4, p. 26] (edited)

8 Main measurement

8.1 Measurement of random coincidences

The measured coincidence rate is the sum of true and random coincidences. To get the true coincidence rate from the measurements, the random coincidence rate needs to be determined. In order to do so, set the variable delay in the fast circuit to such a value that true coincidences will never arrive within the resolving time. **Does an error arise from not changing the delay in the slow circuit? Argue under which conditions it is negligible. Are these conditions fulfilled?** Measure random coincidences until a sufficient statistical precision is achieved. This can be considered the case when the statistical error on the random coincidence rate does not significantly increase the error of the overall result.

Compare the measured value to the value that is expected from single detector count rate and resolving time of the coincidence unit.

8.2 Measurement of the angular correlation

Set both detectors to the measurement distance.

Before planing the measurement, an important property of the setup needs to be discussed: Its stability. In practice, no electronics are perfectly stable over time. For example, gain factors and thresholds change slowly.

Therefore you can let measure every point for several times. To reduce stability effects, it recommends to measure angles ascending from minimum to maximum angle, then descending from maximum to minimum (start again with ascending). This can be set easily with the given software! You just have to say how long you want to measure (for each point). Think about the measuring time for every point!

A slow drift of the detection efficiency will mostly cancel out, as data for both angles are affected in a similar way.

So after all, some statistical accuracy is traded in for a lower systematic error.

The data measured at optimized angles allows to pin down the correlation coefficients as precisely as possible using the assumption that the correlation can be described by the predicted distribution.

The intermediate positions allow to get an idea of the overall angular dependence.

As all measurements are repeated, it is also possible to determine the stability of the setup.

9 Analysis of the Data

9.1 Calibration Measurements

9.1.1 Setting the Main Amplifiers Gain

Show oscilloscope screenshots of both amplifiers output. Explain how you concluded that the shown signals correspond to the photopeaks, how the signal shape should be, and how it would look like if saturated.

9.1.2 Calibration of the SCAs

Plot the measured spectra. Indicate which thresholds were chosen for the final measurement. Calculate and plot the sensitivity of the SCA in dependence of the energy for each detector.

$$\eta_{\text{SCA,i}(E)} = \frac{N_{\text{SCA-gated}(E)}}{N_{\text{self-gated}(E)}} \tag{2}$$

The sensitivity of the SCA should be close to 100 % in the range of the photo peaks. Is it?

9.1.3 Finding the CFD threshold

Plot the measured data. Indicate the chosen threshold in the plot.
Calculate and plot the sensitivity of the CFD in dependence of the energy for each detector.

$$\eta_{\text{CFD,i}(E)} = \frac{N_{\text{CFD-gated}(E)}}{N_{\text{self-gated}(E)}} \tag{3}$$

The sensitivity of the CFD should be close to 100 % in the range of the photo peaks. Is it?

9.1.4 Adjusting the delay in the fast branch

Fit the function

$$f(t) = \frac{A}{2} \cdot \left(1 + \operatorname{erf}\left(\frac{t - \left(t_0 - \frac{w}{2}\right)}{\sigma}\right) \cdot \operatorname{erf}\left(\frac{\left(t_0 + \frac{w}{2}\right) - t}{\sigma}\right)\right) + A_0 \tag{4}$$

to the data. **Explain what each of the parameters represents.** Which property of the setup corresponds to the width of the function? Which property to the width of the slopes?

The resolving time should be as short as possible to reduce random coincidences. Also it should be as long as possible to ensure that true coincidences are certainly detected.

How will the shape of the prompt curve change if the resolving time is chosen too short or too long? Was the choice reasonable?

Plot the data and the fitted function. Add the chosen delays for main measurement and measurement of random coincidences respectively, as well as the resolving time determined by the fit.

9.1.5 Delay Adjustment in the slow branch

Show oscilloscope screen shots of the properly aligned pulses. Explain which features need to be aligned (center? edge?).

9.2 Main Measurement

1. Calculate the random coincidence rate from the direct measurement and from the resolving time and single detector count rates.

2. Investigate the stability of the setup over time. These quantities should stay constant:

    (a) Single detector count rate of the mobile detector at given angles (use the repeated measurements!)

    (b) Single detector count rate of the fixed detector

    (c) Coincidence count rate for a fixed angle (again, use the repeated measurements!)

    (d) fast coincidence count rate for a fixed angle

Can you see instability over time in the data? How well do the repeated measurements help?

3. You got several values for each angle. Calculate the sum of events and time to calculate the average event rate.
4. Subtract random coincidences from the measured rates. Can you use the same random coincidence rate for all angles? Alternatively, explain quantitatively why such a correction is not necessary.
5. Compensate the misalignment of the setup. Which function would you expect to see (based on misalignment) and how will it affect the coincidence rate? How can you remove the misalignment effect from coincidence rate?
6. Apply a least squares fit of the predicted function to the resulting data and apply solid angle corrections. Correction factors can be found in [2].
7. Try to fit angular correlation functions for cascades with different spins e.g. 0-1-0. To which degree can those be excluded?
8. Plot the angular distribution including data, fit, and prediction

References

[1] R. B. Firestone, Table of Isotopes 8th edition, (Wiley, New York, 1996)

[2] K. Siegbahn, ALPHA-, BETA-, AND GAMMA-RAY SPECTROSCOPY, Vol. 2, North Holland Publishing Company, Amsterdam (1965), pp. 1695

[3] W. R. Leo: *Techniques for Nuclear and Particle Physics Experiments*, Springer, 2nd Rev. edition (1994)

[4] P. Rosinsky: *Aufbau und Testung der Automatisierung winkelabhängiger γ-Messungen im Praktikum mit Hilfe einer motorisierten Messapparatur* (Bachelorarbeit) Bonn (2022)

[5] E. Browne, J. K. Tuli *NuDat 3.0 – ^{60}Co β^- decay (1925.28 d) – Nuclear Data Sheet 114, 1849* (2013) https://www.nndc.bnl.gov/ensnds/60/Ni/beta_decay_1925.28_d.pdf Literature Cutoff Date: 31-Dec-2012 – visited 16-Mar-2022

[6] E. Browne, J. K. Tuli *NuDat 3.0 – ^{60}Co decays – Nuclear Data Sheet 114, 1849* (2013) https://www.nndc.bnl.gov/nudat3/decaysearchdirect.jsp?nuc=60Co&unc=NDS Literature Cutoff Date: 31-Dec-2012 – visited 16-Mar-2022

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