Coalitions and Group Decisions

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Games with more than 2 players

Games with > 2 players are more complex:

- players can form *coalitions*: groups that cooperate to optimize their utility.
- players need to agree on joint decisions: social choice.

Cooperative Game

- Agents A, B and C represent servers; they can choose to not work (n) or work (w) at cost=5.
- A client is willing to pay 12 for a regression model and 20 for a regression model with causal analysis.
- One server alone cannot meet the deadline (payoff 0), two servers can produce the regression model, three servers can also produce causal analysis but extra revenue goes to agent A for license fees.

		BC			
		nn	nw	wn	ww
A	n	(0,0,0)	(0,0,-5)	(0,-5,0)	(0,1,1)
	w	(-5,0,0)	(1,0,1)	(1,1,0)	(7,-1,-1)

Highest (combined) payoff: (w,w,w) $\Rightarrow$ 5

But not a Nash equilibrium!

Coalitions without utility transfer

Possible coalitions in this game:

- AB, BC, AC: utility $= 2$ (when third agent is excluded).
- grand coalition ABC: utility $= 5$

Coalitions AB, BC, AC are *stable*: no agent has an incentive to leave the coalition.
Coalition ABC is not stable: agents B and C can get higher payoff by leaving the coalition!

Coalitions with utility transfer

Side contract: in grand coalition, A pays 1.5 each to B and C:

		BC			
		nn	nw	wn	ww
A	n	(0,0,0)	(0,0,-5)	(0,-5,0)	(0,1,1)
	w	(-5,0,0)	(1,0,1)	(1,1,0)	(4,0.5,0.5)

$\Rightarrow$ Grand coalition is a Nash equilibrium.
Coalitional game theory:

- *coalition formation*: which group gets the highest combined revenue?
- *payoff distribution*: how are the rewards distributed?

Stability of coalitions

		BC			
		nn	nw	wn	ww
A	n	(0,0,0)	(0,0,-5)	(0,-5,0)	(0,1,1)
	w	(-5,0,0)	(1,0,1)	(1,1,0)	(4,0.5,0.5)

- B and C are better forming their own coalition: each gets 1 instead of 0.5!
- Definition: a coalition N is stable if no subset $S \subset N$ gives higher utility for all agents in S than they get in N.
- When utility can be redistributed, sufficient that S as a whole gets higher utility than S gets in N.

Stability and the core

- Question: is the grand coalition (all agents) stable?
- Rephrased: for what payoff distributions is the GC stable?
- This set of payoff distributions is called the *core* of the game.

In the example game, the core is given by:

$$\begin{aligned} payoff(A) &\geq 6 \\ payoff(B) &\geq 6 \\ payoff(C) &\geq 6 \end{aligned}$$

However, the core may often be empty!

Determining the core

- Let the *characteristic function* v(S) be the value that can be achieved by a coalition S; N is the coalition of all agents.
- Condition(Bondereva-Shapley): Core is nonempty iff.

$$v(N) \geq \sum_{S \subseteq N} \lambda(S) v(S)$$

for every function λ $(2^{|N|} \rightarrow [0,1])$ that is balanced:

$$\forall i \in N, \sum_{S : i \in S} \lambda(S) = 1$$

- However, exponentially many $S \Rightarrow$ checking requires exponential time.

Games with nonempty core

- Superadditive game:

$$\forall S, T \subset N, if S \cap T = \phi, v(S \cup T) \geq v(S) + v(T)$$

- Convex game (implies superadditive):

$$\forall S, T \subset N, v(S \cup T) \geq v(S) + v(T) - v(S \cap T)$$

  Example game is convex

- Theorem: all convex games have a nonempty core!
- Stable payoff distribution is given by Shapley value.

Determining the right payoffs

- Shapley value = vector $(\phi_1, \phi_2, ..., \phi_n)$ giving the expected distribution of returns of the game.
- Shapley value should satisfy certain conditions $\Rightarrow$ axioms.
- For convex games, Shapley value should be in the core.

Conditions for a unique Shapley value

A *carrier* of a game is a minimal coalition of agents such that the result of the game is always completely decided by these agents.

1. an agent who is not member of any carrier has value $\phi_i = 0$
2. a permutation of agents gives the same permutation of Shapley values.
3. when the agents play two games I and J in parallel, the Shapley value of the combined game is the sum of the Shapley values for the individual games I and J.

$\Rightarrow$ there is a unique Shapley value!

$\Rightarrow$ for convex games, the Shapley value is in the core!

Computing the Shapley value

- Characteristic function $v(S)$ = combined payoff that coalition S can achieve together.
- Let agents $\{a_1, .., a_n\}$ be ordered and form coalitions in that order:

$$C_1 = \{a_1\}, .., C_k = \{a_1, .., a_k\}, C_n = \{a_1, .., a_n\}$$

- Given this particular ordering, the value of $U(a_{k+1})$ to the coalition C_{k+1} is $v(C_{k+1}) - v(C_k)$.
- The Shapley value of an agent is the *average* value over all possible orderings of agents.

Example (1)

Characteristic function:

AB	BC	AC	ABC
12	12	12	20

Order	U(A)	U(B)	U(C)
ABC	0	12	8
ACB	0	8	12
BAC	12	0	8
BCA	8	0	12
CAB	12	8	0
CBA	8	12	0
average	6 2/3	6 2/3	6 2/3

Example (2)

If A contributes more than the others:

AB	BC	AC	ABC
16	12	16	20

Order	U(A)	U(B)	U(C)
ABC	0	16	4
ACB	0	4	16
BAC	16	0	4
BCA	8	0	12
CAB	16	4	0
CBA	8	12	0
average	8	6	6

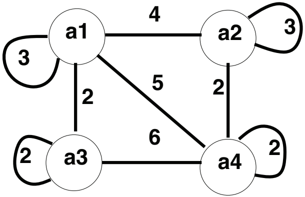
Computing the Shapley value efficiently

Explicitly computing all marginal contributions has exponential complexity. Are there classes of games where computation is efficient?

- weighted graph games: agents contribute to coalitions either individually or in pairs.
- marginal contribution nets: contribution can be in larger groups.
- weighted majority voting: Shapley value complex to compute.

Weighted graph games

Represent rewards of agents and pairs of agents as a graph:



Value of a coalition = sum of edge weights in the subgraph:

$$\begin{array}{rl} \{a1\} & value = 3 \\ \{a1, a2\} & value = 3 + 4 + 3 = 10 \\ \{a1, a2, a4\} & value = 3 + 4 + 3 + 2 + 5 + 2 = 19 \\ \{a1, a2, a3, a4\} & value = 29 \end{array}$$

Shapley value of a weighted graph game

$\Rightarrow$

$$
\begin{aligned}
Shapleyvalue(a_i) &= w((a_i, a_i)) \\
&+0.5 \sum_{\{e_i | e_i = (a_i, a_j), j \neq i\}} w(e_i)
\end{aligned}
$$

Example:

$$
\begin{aligned}
SV(a1) &= 3 + 0.5(4 + 5 + 2) = 8.5 \\
SV(a2) &= 3 + 0.5(4 + 2) = 6 \\
SV(a3) &= 2 + 0.5(2 + 6) = 6 \\
SV(a4) &= 2 + 0.5(5 + 2 + 6) = 8.5
\end{aligned}
$$

But not all games can be represented this way!

Marginal Contribution Nets

- Generalization of graphical games: also allow hyperedges.
- Computing the Shapley value: as in graphical games, but divide contributions by size of the edge (can be > 2).
- Generalize edges to conditions that could also exclude agents: can represent any game, but no easy way to compute Shapley value.

Coalition Structures

- In some cases, agents may have a negative effect on a coalition: consume more resources than they contribute.

$\Rightarrow$ the grand coalition does not achieve the best overall payoff.

$\Rightarrow$ search for optimal division into coalitions.

- Example: separate construction workers into several crews.
- Computationally very hard problem, but good approximate solutions.

Group decision making

- Social choice: group of agents to agree on one of n alternative decisions $d_1, .., d_n$.
- decision should reflect joint preferences; all agents carry equal weight.
- preferences are *ordinal*: only order is expressed, no preference strength/risk attitude.
- direct revelation voting protocol: agents express their preferences, *scoring rule* determines the outcome.
- categories: 2 or ≥ 3 choices.

Properties of voting protocols

- Pareto-optimality: if every agent prefers d_i over d_j, d_j cannot be preferred over d_i in the social choice.
- Monotonicity: if an agent raises its preference for the winning alternative, it remains the winner.
- Non-imposition: for each alternative d_i, there is some set of agent preference orders so that it is chosen as the winner (with monotonicity, implies Pareto-optimality).
- Independence of losing alternatives: if the social choice function prefers d_i over d_j, then this order does not change if another alternative d_l is introduced.
- Non-dictatorship: the protocol does not always choose the alternative preferred by the same agent.

Voting with 2 alternatives

- Every agent ranks alternative $d_1 \succ d_2$ or $d_2 \succ d_1$.
- Majority voting: among 2 alternatives, agents vote for the one they prefer.
- Rank $d_1 \succ d_2$ if at least half the agents vote for d_1.
- All votes count the same.

$\Rightarrow$ best agent strategy: vote for the preferred item.

- Satisfies all desirable properties.

Majority voting with ≥ 3 alternatives

Generalize by voting for pairs of alternatives in sequence:

1. order alternatives $d_1, d_2, ..., d_n$.
2. let $x \leftarrow winner(d_1, d_2)$.
3. for $i \leftarrow 3$ to n $x \leftarrow winner(x, d_i)$
4. "surviving" x is the winner.

Vote organizer decides the order of alternatives.

Condorcet winners

- Condorcet winner:

  alternative that beats or ties all others in a pairwise majority vote.

- Depending on the preference structure, a Condorcet winner might not exist.
- Condorcet winner is Pareto-optimal, independent of loosing alternatives, satisfies monotonicity.
- Majority voting always selects the Condorcet winner.

Situation with no Condorcet winner

3 agents A_1, A_2 and A_3 choose between apples, pears and oranges:

$$\begin{aligned} A_1: &\quad a \succ p \succ o \\ A_2: &\quad p \succ o \succ a \\ A_3: &\quad o \succ a \succ p \end{aligned}$$

Thus:

a is preferred over p (A_1, A_3 over A_2)
p is preferred over o (A_1, A_2 over A_3)
o is preferred over a (A_2, A_3 over A_1)

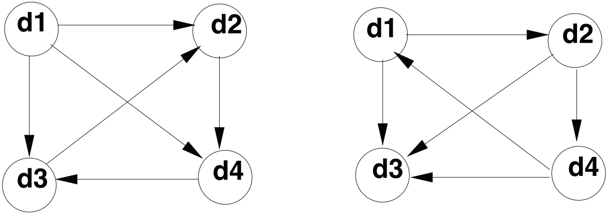
No choice is a Condorcet winner!

Manipulation in majority voting

1. order = a,p,o
   - a vs. p: a wins
   - a vs. o: **o** wins
2. order = o,p,a
   - o vs. p: p wins
   - p vs. a: **a** wins
3. order = o,a,p
   - o vs. a: o wins
   - o vs. p: **p** wins

Vote order determines outcome!

Majority graphs



- nodes = alternatives.
- directed arc from d_i to d_j: majority prefers d_i over d_j.
- Condorcet winner: node with only outgoing edges.
- left: $d1$ is a Condorcet winner (cycle does not matter).
- right: winning cycle of $d1, d2, d4$. $d3$ certainly not winner.

Manipulation of majority voting

- If there is a Condorcet winner, majority voting will select it.
- What if there is a cycle, i.e. no Condorcet winner?

$\Rightarrow$ outcome depends on sequence of votes!

- Winner is the alternative in the winning cycle that is introduced last.

$\Rightarrow$ vote organizer can always determine which is of these is chosen!

Other voting protocols

Some examples of voting protocols:

- Plurality voting: every agent votes for one alternative, order alternatives by number of votes.
- Plurality with elimination: proceed in $n-1$ rounds, at each round the least preferred alternative is eliminated and those that voted for it have to vote again for a remaining alternative.
- Approval voting: vote for every acceptable alternative; the one with the most votes wins.
- Borda count: give $n-1$ votes for most preferred, $n-2$ votes for second most preferred, ..., 0 vote for least preferred alternative. Alternative with most votes wins.
- Slater ranking: best approximation to majority graph.

Complexity considerations

- Voting with many alternatives can be a considerable burden: voter has to evaluate all alternatives and rank them!
- Protocols might require many rounds (majority voting) and heavy communication.
- Simpler alternative: only vote for most preferred alternative (plurality voting).

Problems with plurality voting

3 alternatives a,b,c:

499 agents:	$a \succ b \succ c$
3 agents:	$b \succ c \succ a$
498 agents:	$c \succ b \succ a$

b is the Condorcet winner, but:

- plurality would pick a
- plurality with elimination would eliminate b, then pick c (with 501 over 499 votes).

Weighting alternatives

- Plurality voting ignores preferences beyond the best one.

$\Rightarrow$ allow further expression.

- Borda count: give
  - $n-1$ votes to most preferred alternative
  - $n-2$ to second best,
  - ...
  - 0 votes to least preferred alternative.
- Agent could not give votes for alternatives that are very low.

Problems with Borda count (1)

3 alternatives a,b,c:

$a \succ c \succ b$	$b \succ a \succ c$	$c \succ b \succ a$
35 agents	33 agents	32 agents

Protocol	a	b	c
Borda	103	98	99
Plurality	35	33	32

without alternative c:

Protocol	a	b
Borda	35	65
Plurality	35	65

Removing c reverses choice from a to b!

Problems with Borda count (2)

4 alternatives a,b,c,d:

3 agents: $a \succ b \succ c \succ d$

2 agents: $b \succ c \succ d \succ a$

2 agents: $c \succ d \succ a \succ b$

Borda	a	b	c	d
Score	11	12	13	6

without alternative d:

Borda	a	b	c
Score	8	7	6

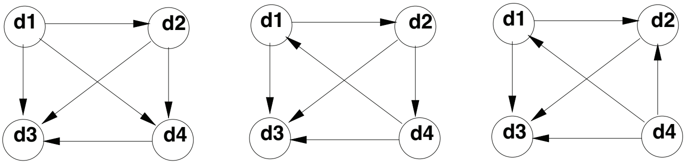
Removing d reverses order from $c \succ b \succ a$ to $a \succ b \succ c$!

Slater ranking

Combined ranking corresponds to a consistent majority graph: every alternative ranked higher beats a lower ranked one.
Slater ranking: among all possible rankings, choose the one that is closest to the agents' majority graph.
Algorithm:

- make agents vote between every pair of alternatives (or ask their preference order and simulate this vote).
- for each possible ordering, evaluate how many edges differ from the majority graph (possibly weighted by the strength of the majority).
- $\Rightarrow$ choose the one with the smallest discrepancy.
- combinatorial optimization problem: hard to solve!

Example: Slater ranking



d1>d2>d4>d3 **d4>d1>d2>d3**

2 of 24 possible orderings:

- left: edge $d_1 \rightarrow d_4$ is reversed.
- right: edge $d_2 \rightarrow d_4$ is reversed.

Kemeny Scores

- Ask agents to submit total orders of choices.
- For a candidate joint order, for each relation between subsequent choices d_i and d_{i+1}, count how many voters rank the two choices in the *opposite* way.
- Kemeny score of the joint order = sum of these counts.
- Winner = order with lowest Kemeny score.
- Search for joint order using branch-and-bound search.

Voting with Computers

- Computerized Voting Protocols allow more accurate decision making.
- Verification is complex: how to prove that chosen order is optimal?
- However, even simple voting protocols are hard to verify when votes are secret.

Manipulation

Voting may have anomalies, but can agents exploit them to their advantage?

Two forms:

- Manipulation of vote order by vote organizer (as in majority voting).
- Non-truthful voting: agent submits vote that does not correspond to its true preferences.

Manipulation by vote organizer

3 agents A_1, A_2 and A_3 choose between 3 alternatives a,b,c:

$$\begin{aligned} A_1: &\quad a \succ b \succ c \\ A_2: &\quad b \succ c \succ a \\ A_3: &\quad c \succ a \succ b \end{aligned}$$

- order a,b,c: **c** (a wins over b, c wins over a).
- order c,b,a: **a** (b wins over c, a wins over b).
- order c,a,b: **b** (c wins over a, b wins over c).

Options introduced later in the process have a higher chance!

The Gibbard-Satterthwaite Theorem

Every (deterministic) voting protocol for ≥ 3 alternatives must have one of these three properties:

- the protocol is dictatorial, i.e. one agent decides the outcome.
- there is some candidate who cannot win under any preference profile.
- there are situations where an agent has an interest to not vote according to its true preference, i.e. to manipulate the outcome by a non-truthful vote.

Example of non-truthful vote

3 alternatives a,b,c; plurality votes of other agents:

	a	b	c
Score	3	7	8

Agent X prefers $a \succ b \succ c$:

- votes for a (truthful): c wins
- votes for b (non-truthful): b might win

Non-truthful voting $\Rightarrow$ not clear what the outcome means!

Manipulability of voting

- For many voting protocols, determining if and how the outcome can be manipulated is NP-hard, but...
- This is only the worst case: the average case is likely to be easy.
- Example heuristics:
  - Plurality: vote for most preferred alternative that is within some ϵ of winning.
  - Sensitive rules (where all alternatives are ranked): rank desired outcome first, order all others in *opposite order of other agents' preference*.
- These heuristics will find almost all manipulations.

Randomized Voting

What if outcome could be chosen by a randomized process:

- Majority voting: *probability* of choosing outcome x = fraction of agents who voted for x.
- Voting for y instead of x increases $p(y)$ by $1/n$ and decreases $p(x)$ by the same amount: expected outcome less preferred!

$\Rightarrow$ no incentive to lie about preferences.

- However, random choice could be manipulated.

Better social choice protocols

Problems with voting:

- no consideration of strength of preference $\Rightarrow$ inconsistent situations.
- every voter counts the same in every decision.
- large potential for manipulation.

Better social choice protocols are based on maximizing social welfare $\Rightarrow$ mechanism design.

Conclusions

- Agents can often gain from cooperating in coalitions.
- Stability of coalitions: scale payoffs so that no agent has incentive to leave coalition (the core).
- Shapley value often falls in the core.
- Voting as social choice protocols.
- Majority voting finds Condorcet winners; but can be manipulated by choice of vote order.
- Anomalies of other voting protocols; incentives for non-truthful voting.