DigiPen Institute of Technology, Bilbao

MAT300 Curves & Surfaces

Spring 2020. Homework 1: Deadline: 1-27-2020

---

1. (10%) Consider $x_0 = -3$, $x_1 = -1$, $x_2 = 0$, $x_3 = 2$ and $x_4 = 3$. Construct a polynomial $p : \mathbb{R} \to \mathbb{R}$ of minimum degree satisfying $p(x_0) = 100$, $p(x_1) = 14$, $p(x_2) = 1$, $p(x_3) = 65$ and $p(x_4) = 154$.

2. (10%) Is p (the result in exercise 1) the unique polynomial of degree at most 4 satisfying that conditions? justify your answer with algebraic statements (equivalent statements from MAT250). If p is not unique give a polynomial $q \neq p$ satisfying the conditions in exercise 1.

3. (10%) Is p (the result in exercise 1) the unique polynomial of degree at most 5 satisfying that conditions? justify your answer with algebraic statements. If p is not unique give a polynomial $q \neq p$ satisfying the conditions in exercise 1.

4. (5%) Construct the Vandermonde basis with constants x_0, x_1, x_2, x_3 and x_4 (those given in exercise 1). What is the space for which it is a basis?

5. (5%) Construct a shifted basis for the space in exercise 4 taking x_0 of exercise 1 as a constant.

6. (15%) Show that the Bernstein polynomials of degree n form a basis for P_n (for n arbitrary).

7. (5%) Construct the Bernstein basis for the space in exercise 4.

8. (10%) Construct a transformation corresponding to a change of basis from Bernstein to Vandermonde (using bases from the previous exercises).

9. (10%) Construct a transformation corresponding to a change of basis from Shifted to Vandermonde (using bases from the previous exercises).

10. (10%) Construct a transformation corresponding to a change of basis from the Bernstein to Shifted (using bases from the previous exercises).

11. (10%) Give the vector of coordinates of p (obtained in exercise 1) in the standard, the Vandermonde, the Shifted, and the Bernstein basis (using the above transformations).