DigiPen
INSTITUTE OF TECHNOLOGY
EUROPE - BILBAO

MAT300 Curves and Surfaces
Spring, 2020

Prerequisites:

MAT250, MAT258.

It will be assumed that the student has knowledge in linear algebra, at least up to the level acquired in MAT250. Topics with which students should be familiar include ***vector spaces, polynomial vector spaces, basis and dimensions, linear combinations, vectors of coordinates, linear independence, change of basis, linear transformations, and a first approach to interpolation.***

It will also be assumed that the student has some basic knowledge in MAT140, in particular in ***linear and affine transformations, barycentric coordinates, and invariance of barycentric coordinates with respect to an affine basis under affine transformations.***

Topics that are also assumed from MAT258 are the ***logical proofs of statements, the Pascal-Tartaglia triangle, combinatory numbers and binomial coefficients.***

Student are encouraged recommended to revisit the above topics before entering in deeper math.

Moreover, it is desirable and very convenient that students that followed this course before, do a review of their notes, exercises, programming projects, exams and the teacher's feedback to know their weak points. It is also convenient to schedule appointments to discuss these weak points in private with the teacher, and determine how to improve the knowledge and grades with respect to previous evaluations.

General Information:

Class Schedule:	*Mo, We 16.00 – 17.20*
Class room:	*Sophie Germain*
Professor:	*Julia Sánchez Sanz*
Contact:	*Ext. 1015, julia.sanchez@digipen.edu*
Class web page:	*Moodle MAT300 at distance.digipen.edu*
Office Hours:	*Meetings scheduled by appointment (Monday to Thursday)*

Description:

This course is an introduction to parameterized polynomial curves and surfaces with a view toward applications in computer graphics. It discusses both the algebraic and constructive aspects of these topics. Algebraic aspects include vector spaces of functions, special polynomial and piecewise polynomial bases, polynomial interpolation, and polar forms. Constructive aspects include the de Casteljau algorithm and the de Boor algorithm. Other topics may include an introduction to parametric surfaces and multivariate splines.

Course Objectives and Learning Outcomes:

- *To become familiar with basic concepts of nonlinear geometry: parametric curves and surfaces.*
- *To implement the acquired knowledge in linear algebra (MAT140 and MAT250) in the definition of polynomial curves and surfaces using linear techniques: vector spaces, basis and linear and affine transformations.*
- *To implement the acquired knowledge in discrete mathematics (MAT258) in the construction and coding of the Bernstein polynomials in the Bezier curves.*
- *To learn the geometrical properties of the different types of polynomial curves: taking into account which type of object I want to model, determine which modeling technique is better.*
- *To learn two established algorithms in geometric modeling: the De Casteljau algorithm and the De Boor algorithm.*
- *To learn the advantages and disadvantages of the different interpolation techniques, taking into account speed, accuracy, or the selection of meshes.*
- *To be able to present mathematical conclusions and to answer theoretical questions coherently, completely and accurately.*

The learning process will consist of theoretical lectures in which the main theorems and techniques will be presented, some of them proved analytically, in the whiteboard. These lectures will be combined with some practical examples and Octave visualizations (code and plots).

The main investment of time and effort considering practice should be carried out by the student, doing exercises, participating in class and asking for appointments to discuss problems and doubts.

Students will demonstrate their assimilation of the contents through homework, programming assignments, a midterm and a final exam.

Textbooks:

During the course I will follow contents from different books and lecture notes. Main recommended literature:

- ***Elementary linear algebra.*** *H. Anton & C. Rorres. 9th Edition, Applications version 2005. John Willey & Sons, Inc. ISBN: 0-471-66959-8 (Chapters 5, 7 and 8, in the library).*
- ***Lecture notes by Prof. Matt Klassen.*** *MAT 300 Supplementary notes Spring 2013. (In Moodle).*
- ***Practical Linear Algebra. A geometry toolbox.*** *G. Farin & D. Hansford.A K Peters, Ltd. 2005. ISBN: 1-56881-234-5. (Chapter 18, in Science department).*
- ***Bezier and B-Spline Techniques.*** *H. Prautzsch, W. Boehm and M. Paluszny. Mathematics and Visualization series, Springer-Verlag 2002. ISBN: 3-540-43761-4. (Chapters 2, 4, 5, 6 and 9, in the library).*
- ***Numerical Methods. J. D. Faires & R. Burden.*** *3Rd Edition, 2003. Brooks/Cole Cenage Learning. ISBN: 978-0-534-40761-2, 0-534-40761-7. (Chapter 3 and 8.4, in the library).*
- *https://pomax.github.io/bezierinfo*

Course Materials Required:

Students will be allowed to use the TI-89 or TI-Nspire calculator (or any other calculator with equivalent features) during assignments and exams. Additional material, if necessary, will be provided via Moodle.

Outline and Tentative Dates:

Week	Topic
1	*Introduction to parametrized curves and surfaces. Review of polynomial vector spaces.*
2	*Basis and dimension. Isomorphism with Euclidean vector spaces, vectors of coordinates change of basis.*
3	*The interpolation Problem.* ***Project I overview.***

4	*Polynomial interpolation of curves using Lagrange and Newton bases. Introduction to interpolation of surfaces.*
5	*Interpolation with Derivatives. Osculating curves. Piecewise Polynomials.*
6	*Cubic Splines.* ***Project II overview.***
7	*Vector spaces of splines.* ***Midterm exam.***
8	*Bernstein Polynomials. Bezier Curves.*
9	*De Casteljau algorithm. Subdivision of curves.*
10	*Polar forms and Nested linear interpolation.* ***Project III overview.***
11	*Derivatives of Bezier curves.* Splines with different order of continuity.
12	*Knot sequences. B-Splines.*
13	*B-Spline curves. De Boor Algorithm.* ***Project IV Overview.***
14	***Easter break.***
15	*Review (only if the contents are covered)*
16	**Final exam**

This entire syllabus, particularly the timeline, may be adjusted or changed at any time by the instructor. Test Weeks are tentative.

Grading Policy:

A total score will be calculated from homework, programming projects, midterm and final exam as follows:

- *15% Homework,*
- *25% Programming Projects,*
- *25% Midterm Exam,*
- *35% Final Exam.*

The minimum average ***grade to pass the subject is 70%****. A minimum* ***average of 60% among the midterm and the final*** *is also required. Moreover, a minimum of* ***50% in the final*** *is mandatory to pass the course. Your final letter grade will be computed as follows:*

x%	Grade
$x>93$	A
$90<x<93$	A-
$87<x<90$	B+
$83<x<87$	B
$80<x<83$	B-
$77<x<80$	C+
$73<x<77$	C
$70<x<73$	C-
$60<x<70$	D
$x<60$	F

Homework submissions and grading policy:

- *Before submitting an assignment your grade is a 0, not a 100. This means that you obtain points for doing steps right, you do not loose points and* ***I do not subtract points from a non existing 100.***

- *Exercises should be clear, precise and tidy.*

- *Please, submit homework in pen (blue or black),* ***I will not accept homework submitted in pencil.***

- *If I do not understand what you are doing in a particular exercise you will get a zero in that exercise.*

- *If the process that you follow to complete an exercise is totally incorrect you will get a zero in that exercise.*

- *If the process that you follow is correct but you arrive to a conclusion that is totally inconsistent with the theory learned in class, you will get a zero in that exercise.*

- *If the results that you obtain are derived through a not enough precise or partially incorrect process, or if your reasoning is not consistent at some point with the theory learned in class you will have a penalty of 10-50% (depending on the importance) in that exercise.*

- *If an error is accumulating along an exercise, that will not penalize the rest of the exercise unless this means inconsistency with the theory learned in class. In case of inconsistency we will follow the above procedure.*

- *If you do not finish an exercise because you do not want to waste time doing mathematical operations, you will have a 30% penalty in that particular exercise.*

- *Students should present their results in a coherent way following the theory learned in class and showing all the intermediate steps. If you present a result and you do not show how you obtained it you will get a penalty of 10% per intermediate step.*

- *If an exercise contains drawing tasks, the plots should be tidy, selfcontained, with different colors if necessary, and big enough to show all the necessary information. You can use a computer for plots if you want. If the plots are not enough clear you will get a penalty of 50-100% in that particular task.*

Programming projects submissions and grading policy:

Programming projects will be carried out in groups of three people.

The projects consist of coding routines in Octave for computing polynomial curves in 2D with different algorithms and meshes. With the routines, the students should attach a document explaining the theory behind the algorithms, the obtained results and some related questions. The specifications of each particular project will be provided via Moodle in the above tentative dates.

The grading policy for the programming assignments will be provided for each particular assignment, but it will follow the same structure as this one:

- *Codes: 60%*
    - *The programs work correctly 30%.*
    - *Codes are well structured, clean and Octave functions are implemented in order to be efficient 10%.*

- *The programs give the required outputs and no more information 5%.*
- *Routines are well commented, with headers and mathematical explanations with the correct terminology in each line 15%.*

- *Document: 40%*
  - *Instructions for running the code 3%.*
  - *Expected output 2%.*
  - *Mathematical explanation 25%.*
  - *Examples 5%.*
  - *Observations 5%.*

Mechanisms and Procedures:

There will be a making up for every assignment, project, midterm and/or final exam only if you have a compelling and well documented justification for missing it. Notice that ***make ups or changes of deadlines are only considered under relevant medical, familiar or administrative situations that cannot be postponed.***

Attendance Policy:

Class attendance at DigiPen Bilbao is mandatory. Missing more than 20% of the classes will result in an automatic failure for the course. Students have to arrive on time. 5 minutes after the class starts the door will be closed and ***I will not allow people entering during class time****. Please, use the breaks to go to the toilets. Students should notify the instructors in the event of a planned absence. If you have actually abandoned the course, it is still up to you to go through the withdrawal process.*

Late Policy:

Late homework and programming projects will be graded with a penalty of -10% per each day starting from the day of the submission. Students should notice that the teacher will not upload solutions of homework into Moodle before every student submits the assignment. ***If you submit***

an assignment late, your behavior has an impact in the rest of the students who will receive the solutions with delay. This impact can be critical before the midterm and final exams.

Deadlines can be changed only if the class as a group agrees on the change, the communication of the decision to the teacher is at least two days before the deadline, and the teacher agrees with the decision of the class. Exceptions to this policy are not considered except for making up procedures.*

(*) Students should consider as an agreement a common decision obtained after a discussion of the issue, taking into account the situation of all the students in the class, for instance which subjects and deadlines has each one. In this sense (and in this course) agreement is not the will of the majority.

Last Day to Withdraw:

In order to withdraw from a course it is not sufficient simply to stop attending class or to inform the instructor. In accordance with the policy, contact your advisor or the registrar to begin the withdrawal process. The last day for withdrawal from this course is on ***Monday March the 2nd 2020.***

Academic Integrity Policy:

Academic dishonesty in any form will not be tolerated in this course. Cheating, copying, plagiarizing, or any other form of academic dishonesty (including doing someone else's individual assignments) will result in, at the extreme minimum, a zero on the assignment in question, and could result in a failing grade in the course or even expulsion from DigiPen.

Disability Support Services:

If students have disabilities and will need formal accommodations in order to fully participate or effectively demonstrate learning in this class, they should contact the Administration Office at 94 6365163. The Administration Office welcomes the opportunity to meet with students to discuss how the accommodations will be implemented. Also, if you may need assistance in the event of an evacuation, please let the instructor know.