DigiPen Institute of Technology Europe-Bilbao

MAT300 Curves & Surfaces

Spring 2020. **Homework 3**: **Deadline: 26-2-2020**

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1. (10%) Use a system of linear equations to find an interpolant polynomial in standard basis satisfying $p(1) = 6$, $p'(1) = 8$, $p''(1) = 42$, $p^{(3)}(1) = 168$, $p(2) = 91$, $p'(2) = 255$, and $p''(2) = 620$.

2. (20%) Compute the divided differences for a polynomial satisfying $p(1) = 6$, $p'(1) = 8$, $p''(1) = 42$, $p^{(3)}(1) = 168$, $p(2) = 91$, $p'(2) = 255$, and $p''(2) = 620$. Give the Newton basis and the vector of coordinates of the polynomial in that basis.

3. (10%) Obtain the change of basis transformation from Newton to Standard basis, and verify that the polynomials in exercises 1 and 2 are the same.

4. (30%) Given the polynomial

$$p(x) = \begin{cases} 1 + 2x + x^2, & x \in [-2, 0) \\ 1 + 2x - x^3, & x \in [0, 1) \\ 3 - x, & x \in [1, 3) \\ 12 - 7x + x^2, & x \in [3, 5] \end{cases}$$

   a) (5%) Determine to which polynomial vector space belongs p, **taking into account orders of continuity**.
   b) (10%) Construct a right shifted basis for that space.
   c) (15%) Give the vector of coordinates of p in that basis.

5. (30%) Consider a cubic spline such that $p(0) = 1$, $p(1) = 0$, $p(2) = -1$ and $p(5) = 1$.

   a) (15%) Give the vector of coordinates of such a spline in the right shifted basis.
   b) (15%) Give the piecewise expression.