DigiPen Institute of Technology, Bilbao

MAT300 Curves & Surfaces

Spring 2020. Homework 2: Deadline: 5-2-2020

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1. (20%) Consider $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$ and $x_4 = 4$. Construct the Lagrange polynomials associated to that nodes and give their vectors of coordinates in the standard basis.

2. (15%) Show that the set of Lagrange polynomials obtained in exercise 1 is a basis for a certain polynomial vector space. Which space?

3. (10%) Consider a polynomial $p : \mathbb{R} \to \mathbb{R}$ satisfying $p(x_0) = 0$, $p(x_1) = 2$, $p(x_2) = 1$, $p(x_3) = 3$ and $p(x_4) = 3$. Give the vector of coordinates of p in the Lagrange basis.

4. (20%) Construct the divided differences for the nodes $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$ and $x_4 = 4$ being $f = p$ in exercise 3.

5. (15%) Construct the Newton basis associated to the nodes $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$ and $x_4 = 4$.

6. (10%) Show that the Newton basis in exercise 5 is indeed a basis of a certain polynomial vector space. Which space?

7. (10%) Give the vector of coordinates of p (the one in exercise 3) in the Newton basis (the one in exercise 5). **Hint:** you have computed the divided differences for that polynomial in exercise 4.