Extending Hackett et al. (2024): A Lotka–Volterra–Type Model for Pollination Network Robustness

Abstract

Hackett et al. (2024) showed that landscapes subdivided into multiple habitat patches (triads) yield more even, complementary plant–pollinator networks and more consistent robustness to species loss than single-habitat (monad) landscapes. Here, we propose to replicate their sequential extinction simulations using a mechanistic mutualistic model with Holling-type II functional responses. We will embed n_P plant and n_V pollinator species across m patches into a coupled ODE system to (1) track continuous abundances rather than topology alone and (2) compare "topological" versus "dynamical" robustness metrics under rare-to-common species removal.

1 Introduction

Landscape heterogeneity can stabilize ecosystems by promoting species evenness and functional complementarity (Hackett et al., 2024). In a field study across 30 nine-hectare sites in southwest England and Wales, Hackett et al. demonstrated that *triad* landscapes (three 3 ha habitat patches) produced 30% more Class I fruits and had lower variance in network robustness than *monad* landscapes (one 9 ha patch) under sequential removal of plants from rarest to commonest (Hackett et al., 2024). Their extinction simulations used a purely structural approach—nodes deleted in rank order and pollinators allowed to rewire links probabilistically—without modeling continuous population dynamics.

Objective. We will extend their framework by embedding the same bipartite networks into a Lotka–Volterra–type mutualism model with Holling-type II saturation. Our goals are:

1. Replicate sequential "rare-to-common" removals by imposing mortality pulses on plant abundances.
2. Compare *topological robustness* (fraction of surviving species in the network approach) with *dynamical robustness* (fraction of species whose equilibrium abundances exceed a viability threshold).
3. Analyze how habitat number ($m = 1, 2, 3$ patches) and pollinator dispersal influence robustness, variability, and pollination service.

2 Model Framework

We consider n_P plant species and n_V pollinator species in m discrete patches. State variables:

$$P_{i,p}(t), \quad V_{j,p}(t), \quad i = 1, \ldots, n_P;\ j = 1, \ldots, n_V;\ p = 1, \ldots, m.$$

The dynamics are:

$$\frac{dP_{i,p}}{dt} = r_i\,P_{i,p}\Big(1 - \tfrac{P_{i,p}}{K_i}\Big) + \sum_{j=1}^{n_V} \frac{\gamma_{ij}\,P_{i,p}\,V_{j,p}}{1 + h_i\,\gamma_{ij}\,V_{j,p}}, \tag{1}$$

$$\frac{dV_{j,p}}{dt} = -\,d_j\,V_{j,p} + \sum_{i=1}^{n_P} \frac{\delta_{ji}\,V_{j,p}\,P_{i,p}}{1 + h_j\,\delta_{ji}\,P_{i,p}} + \sum_{\substack{q=1\\ q\neq p}}^{m} D_{j,pq}\,\big(V_{j,q} - V_{j,p}\big). \tag{2}$$

Parameter Definitions

- r_i, K_i: intrinsic growth rate and carrying capacity of plant i.
- d_j: per-capita mortality rate of pollinator j.
- γ_{ij}: mutualistic strength of pollinator j on plant i.
- δ_{ji}: mutualistic strength of plant i on pollinator j.
- h_i, h_j: handling-time parameters for plant i and pollinator j.
- $D_{j,pq}$: dispersal rate of pollinator j from patch q to patch p.

3 Parameterization

1. *Network extraction*: derive binary adjacency A_{ij} for each empirical site (monads, dyads, triads).
2. *Baseline rates*: set $r_i = 1$, $K_i = 1$, $d_j = 0.5$ uniformly.
3. *Mutualism strengths*: $\gamma_{ij} = \delta_{ji} = \gamma_0$ for all observed links; scan $\gamma_0 \in [0.01, 1]$.
4. *Handling times*: $h_i = h_j = h_0$; scan $h_0 \in [0.01, 1]$.
5. *Dispersal*: define $D_{j,pq} = D_0$ if patches p, q are adjacent in dyads/triads, else 0; scan $D_0 \in [0, 1]$.

4 Simulation Protocol

1. Integrate Eqs. (1)–(2) to steady state for each site without removals; record equilibrium (P^*, V^*) and verify feasibility.
2. Compute *dynamical stability* via Jacobian eigenvalues at (P^*, V^*).
3. **Species-loss experiment:**
   - Rank plants by empirical commonness C_i (rarest first, as in Hackett et al. 2024).
   - For step k, set $P_{k,p}(0) = 0\ \forall p$, re-integrate to T, record which $P_{i,p}, V_{j,p} < \varepsilon$.
4. *Metrics*:

- **Topological robustness:** fraction of species persisting in the original network approach.
- **Dynamical robustness:** fraction of species with $P_{i,p}(T), V_{j,p}(T) > \varepsilon$.
- **Pollination service:** $S_p = \sum_j \beta_j V_{j,p}$; compare mean and variance across $m = 1, 2, 3$.

5 Expected Outcomes and Analysis

- *Hypothesis 1*: Dynamical robustness and its variance will decrease (i.e. become more consistent) as m increases, mirroring topological results.
- *Hypothesis 2*: Non-additive gains in S_p for triads versus monads in the coupled LV model.
- Perform sensitivity analyses over (γ_0, h_0, D_0).

References

Hackett, T. D., Sauve, A., Maia, K. P., Montoya, D., Davies, N., Archer, R., Potts, S. G., Tylianakis, J. M., Vaughan, I. P., and Memmott, J. (2024). Multi-habitat landscapes are more diverse and stable with improved function. *Nature*, 633(September):115–119.