Numerical Computing II

Homework 15: Zero Finders

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Exercise 15.4

```
% determine "close" starting x
x = 2;

% define f(x)
function f = f(z)
    f = erf(z);
end

% define f'(x)
function fp = fp(z)
    fp = (2/sqrt(pi)) * e^(-z/2);
end

% begin Newton's Method
while abs(f(x)) > 1e-14
    x = x - f(x)/fp(x);
end
```

We define the functions $f(x)$ and $f'(x)$ as the error function and the error function's derivative. This code produces a very fast approximation for "nice" initial values of x. The initial values of x are of crucial importance because the error function $f(x)$ returns 1 for all values of x greater than 3.3, which causes trouble for Newton's Method, as Newton's Method produces recursive values of x_k for when $f(x_{k+1}) = f(x_k)$, which happens frequently in this function.

For this function, we receive a zero after 6 iterations of the while loop with initial iterate of 2.

Exersize 15.5

We set $a \in \mathbb{R}$. Then to find $a^{1/3}$ using Newton's Method, we can solve $x = a^{1/3}$ or, equivalently, $x^3 - a = 0$. So, we simply use

$$f(x) = a - x^3 \tag{1}$$
$$f'(x) = -3x^2 \tag{2}$$

To use Newton's Method, we must specify an initial iterate, b, such that $x_1 = b$ and $x_2 = x_1 + f(b)/f'(b)$. We need to choose b such that:

$$x_2 \neq x_1$$
$$x_2 \neq b.$$

That is, we want to choose a b which will not recursively return itself when given to Newton's Method:

$$x_2 \neq b - f(b)/f'(b)$$
$$x_2 \neq b - a - b^3/-3b^2$$
$$b \neq \sqrt[3]{a}.$$

The other conditions are simply that $b \neq 0$, as that would cause a division by zero to occur in our first iteration.

Our method can use a few termination conditions. First, we can't allow a division by zero, so stop our function when $x_i = 0$. Next, we realize that $x_i = \sqrt[3]{a}$ would cause our method to recurse, so we must stop at that condition.

Exersize 15.6

We need to find x such that

$$\sigma x^3 = (1 + x)^2. \tag{3}$$

That is, solve the function

$$f(x) = (1 + x)^2 - \sigma x^3. \tag{4}$$

```
r = 6.9707e8; % radius of sun's orbit
v = 11.7;     % velocity of sun's orbit
T = 4332.1;   % period of rotation
M = 1.989e30; % mass of sun
G = 6.67e-11; % gravitational constant

% define a few constants
theta = G*M / (r*v^2);
x     = 0;

% function we want to zero
function f = f(z,theta)
    f = (1 + z)^2 - theta * z^3;
end

% derivative function
function fp = fp(z,theta)
    fp = 2*(1 + z) - 3 * theta * z^2;
end

% use Newton's Method
while abs(f(x,theta)) > 1e-16
    x = x -f(x,theta) / fp(x,theta);
end
```

This is quite easy to do with Newton's Method, and we receive the zero value of x after 22 iterations of $x = 8.9651 * 10^{-04}$. This is equivalent to saying that 3 reaches equality when the ratio of $m/M = x = 8.9651 * 10^{-04}$. Then, using this ratio, we can solve for m, the mass of Jupitor by using:

$$
\begin{aligned}
& m/M = x \\
\implies & m = x * M \\
& m = 8.9651 * 10^{-04} * 1.989 * 10^{30} \\
& \quad = 1.7832 * 10^{27},
\end{aligned}
$$

which is a very close approximation of the actual mass of Jupiter, which is $1.8987 * 10^{27}$ kilograms.