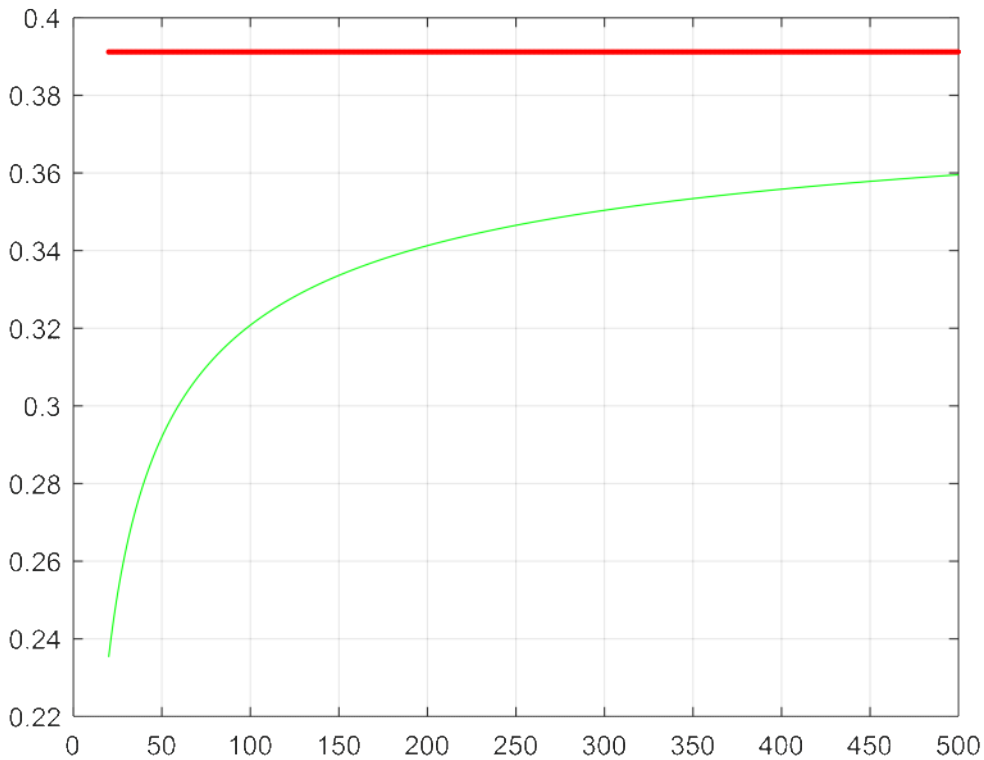
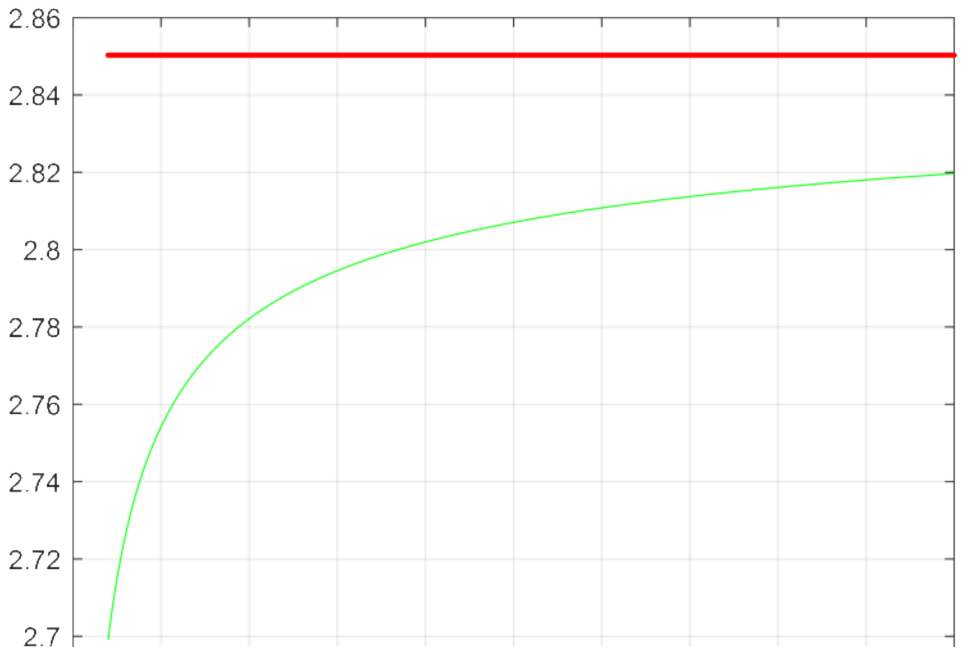
Tasks (deadline: 10.12.2020)

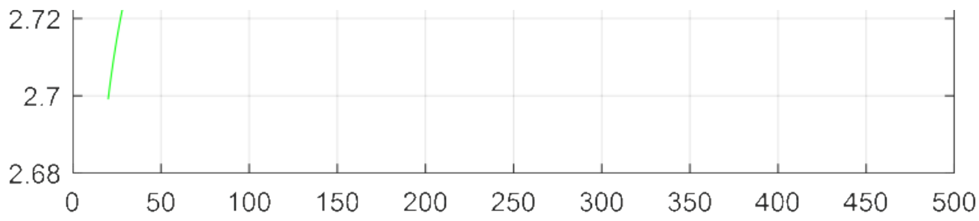
- **3.1:** Implement binomial tree pricing of lookback options with floating strike using Cheuk and Vorst method.
- 3.2*: Implement binomial tree pricing of lookback options with fixed strike using Cheuk and Vorst method (task is not obligatory, but an extra point will be given for completing it; the formulas can be found in the Cheuk and Vorst article).

European put lookback with floating strike (using CRR approximated tree)
S=100; T=0.5; r=0.05; sigma=0.02; Analytical price assuming that: Smin=S; Smax=S;



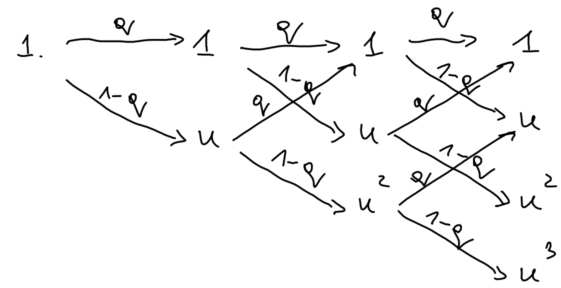
European call lookback with floating strike (using CRR approximated tree)
S=100; T=0.5; r=0.05; sigma=0.02. Analytical price assuming that: Smin=S; Smax=S;

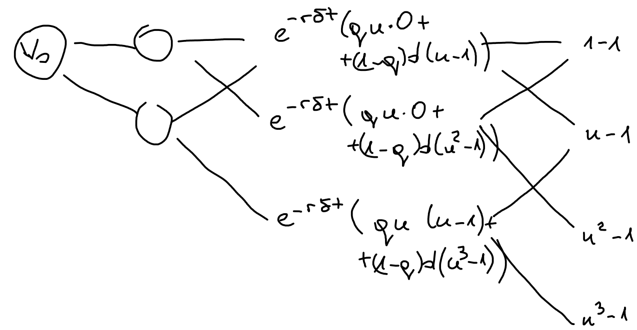




$$Y_t = \frac{\max_{0 \le \tau \le t} S_\tau}{S_t}$$

Y_t:





$$LP_{floating} = S \cdot V_0$$

$$\begin{cases} u = e^{\sigma\sqrt{\delta t}} \\ d = \frac{1}{u} \\ q = \frac{e^{r\delta t} - d}{u - d} \end{cases}$$

1 Calculate the payoff at T

$$Y_T - 1$$

2 Move backward on the tree

$$e^{-r\delta t}\left[q u V_t^{+} + (1-q) d V_t^{-} \right]$$

3 Multiply the value V_0 by S

Sketch of the algorithm for a <u>call</u> option

Let:

$$Z_t = \frac{S_t}{\min_{0 \le \tau \le t} S_\tau}$$ In analogy to the put option:

$$Z_t = 1 \quad , \quad Z_{t+\delta t} = \begin{cases} u & S_{t+\delta t} \nearrow \\ 1 & S_{t+\delta t} \searrow \end{cases}$$

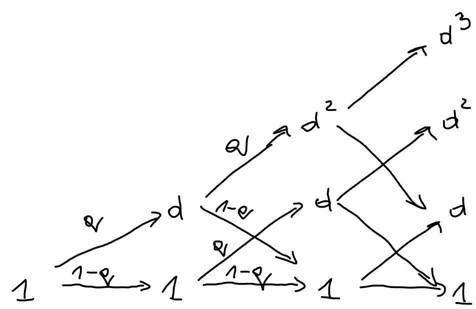
$$Z_t = u^m, \quad Z_{t+\delta t} = \begin{cases} u^{m+1} & S_{t+\delta t} \nearrow \\ u^{m-1} & S_{t+\delta t} \searrow \end{cases}$$

Define $Y_t = \frac{1}{Z_t}$. Recall that $\frac{1}{u} = d$

$$Y_t = 1, \quad Y_{t+\delta t} = \begin{cases} d & S_{t+\delta t} \nearrow & (q) \\ 1 & S_{t+\delta t} \searrow & (1-q) \end{cases}$$

$$Y_t = d^m, \quad Y_{t+\delta t} = \begin{cases} d^{m+1} & S_{t+\delta t} \nearrow & (q) \\ d^{m-1} & S_{t+\delta t} \searrow & (1-q) \end{cases}$$

We can build a tree for Y_t.



Note that the Lookback call payoff.

$$f_T = S_T(1 - Y_T)$$

Hence, we can find the price of call option using analogous algorithm as for put but with a modified tree for Y_t and $1 - Y_t$ payoff

1 Calculate the payoff at T:

$$1 - Y_T$$

2 Move backward on the tree

$$e^{-r\delta t}\left[q u V_t^{+} + (1-q) d V_t^{-} \right]$$

3 Multiply the value V_0 by S