

Assignment 2

Martynas Lukosevicius, Alejo Perez Gomez, Shwetha Vandagadde Chandramouly

07/11/2020

Assignment 2

1.

Model:

$\hat{y} \sim N(w_0 + w^T X_i, \sigma^2)$ where $w \sim N(0, \frac{\sigma^2}{\lambda})$

- w - weights
- X - features
- λ - regularization penalty

2.

Scaling data:

```
library(readr)
parkinsons <- read_csv("parkinsons.csv")
cleaned <- parkinsons[c(-1:-4, -6)]
parkinsons.scaled <- scale(cleaned)
set.seed(12345)
n <- dim(parkinsons.scaled)[1]
id=sample(1:n, floor(n*0.6))
train=parkinsons.scaled[id,]
test=parkinsons.scaled[-id,]
```

3.

As we will be optimizing σ and w , likelihood and prior should contain all σ , even if data is scaled (so it means that $\sigma \sim 1$):

$$\log(\text{posterior}) = \log(\text{likelihood} * \text{prior}) = \log(\text{likelihood}) + \log(\text{prior})$$

a) loglikelihood:

$$\log(p(D|w)) = -\frac{n}{2} \log(2\pi\sigma^2) - \sum_{i=1}^n \frac{(y_i - w^T X_i)^2}{2\sigma^2}$$

```
loglikelihood <- function(w, sigma){
  n <- dim(train)[1]
  part1 <- -(n/ 2) * log(2 * pi*(sigma^2))
  sum <- 0
  for (i in 1:n) {
    y <- train[i, 1]
    x <- train[i, -1]
```

```

    temp <- (y - (t(w) %*% x))^2
    sum <- sum + as.vector(temp)
  }
  return(part1 - (sum/(2*(sigma)^2)))
}

```

b) Ridge part $\sim \log$ prior, where $\tau = \frac{\sigma^2}{\lambda}$:

$$\log(\text{prior}) = -\frac{1}{2} \log(2\pi\tau) - \frac{(w)^2}{2\tau}$$

function returns $-\log(\text{posterior})$

```

ridge <- function(x, lambda){
  w <- x[1:16]
  sigma <- x[17]
  tau <- sigma^2 / lambda
  part1 <- (-1/2) * log(2* pi * tau)
  part2 <- (w %*% w) / (2* tau)
  ridge <- part1 - part2
  return( - (loglikelihood(w,sigma) + ridge))
}

```

c) function to predict weights (w) and σ

```

ridgeOpt <- function(lambda){
  x <- rep(1,17)
  a <- optim(x ,ridge, method = "BFGS", lambda = lambda)
  w <- a$par[1:16]
  sigma <- a$par[17]
  return(a)
}

```

d) function to calculate degrees of freedom

```

DF <- function(lambda){
  m <- as.matrix(train[, -1])
  part1 <- t(m) %*% m + (lambda * diag(16))
  part2 <- m %*% solve(part1) %*% t(m)
  return(sum(diag(part2)))
}

```

4.

	MSE train	MSE test
lambda = 1	0.8872145	0.6342994
lambda = 100	0.8769148	0.6173668
lambda = 1000	0.9019595	0.6233459

$\lambda = 100$ is better than others because MSE for train set and for test set is lowest. MSE is good loss function because it comes from model's MLE.

5.

	AIC
lambda = 1	9610.013
lambda = 100	9562.313
lambda = 1000	9655.555

The optimal model is with lowest AIC score, in this case its a model with $\lambda = 100$. Hold out method requires to divide data into 3 parts, which wont allow to use all data for training, its not the case with AIC.