Digital Signal Processing Fundamentals [5ESC0]

Lab5ESC2

'Answer form'

Assignment 1 to 10

Group number: 13

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Assignment 1: Convolution

To evaluate the convolution by hand, either the graphical or the mathematical method can be applied. It is easier to explain the process using mathematics:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} (\delta[k+3] + \delta[k+1] + \delta[k-1] + \delta[k-3])\left(-\frac{1}{2}\delta[n-k+1] + \delta[n-k] - \frac{1}{2}\delta[n-k-1]\right)$$

Multiplication of the delta terms gives the opportunity to evaluate which delta pulse is timed at which k, giving a final result:

$$y[n] = -\frac{1}{2}\delta[n+4] + \delta[n+3] - \delta[n+2] + \delta[n+1] - \delta[n] + \delta[n-1] - \delta[n-2] + \delta[n-3] - \frac{1}{2}\delta[n-4]$$

This results corresponds to the Matlab plot:

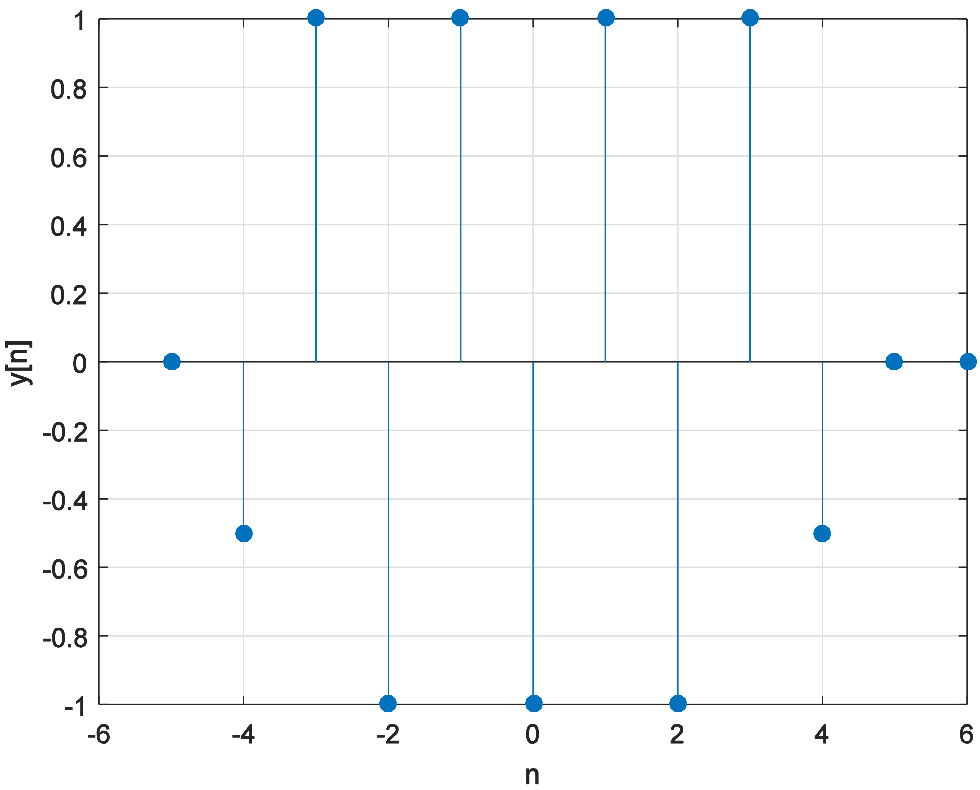

Figure 1: Convolution result of assignment 1

Assignment 2: Fade-in and fade-out of convolution

a) What is the length of the output and the length of the fade-in and fade-out phenomenon, as a function of N and M?

Length of a convolution equals: $\mathrm{length(x[n]) + length(h[n]) - 1 = N + M - 1}$

The number of fade-in and fade-out samples is the same and equals: $M - 1$

b) How many, and which, output samples have no fade-in and/or fade-out?

Total length: $\boldsymbol{K = N + M - 1 = 8 + 3 - 1 = 10}$

Non-faded samples: $K - 2(M - 1) = 6$

Assignment 3: Causality

a) Value for L in order to make the impulse response causal:
The first sample (n = -5) needs to be at the origin (n = 0) to be causal. The delta pulse therefore needs an L of 5, resulting in: $\delta[n-5]$

b) $h_{causal}[n] = \begin{cases} 1 & \text{for n} = 0, 1, 2 \\ 0 & \text{elsewhere} \end{cases}$

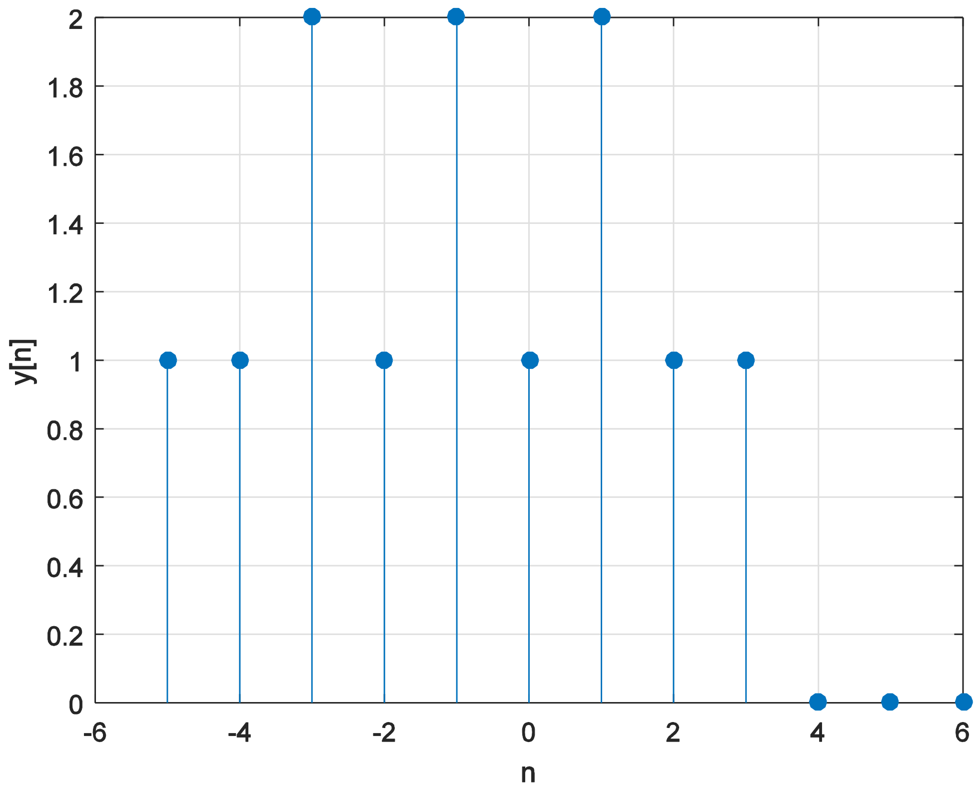

Figure 2: Convolution result of assignment 3 with non-causal filter

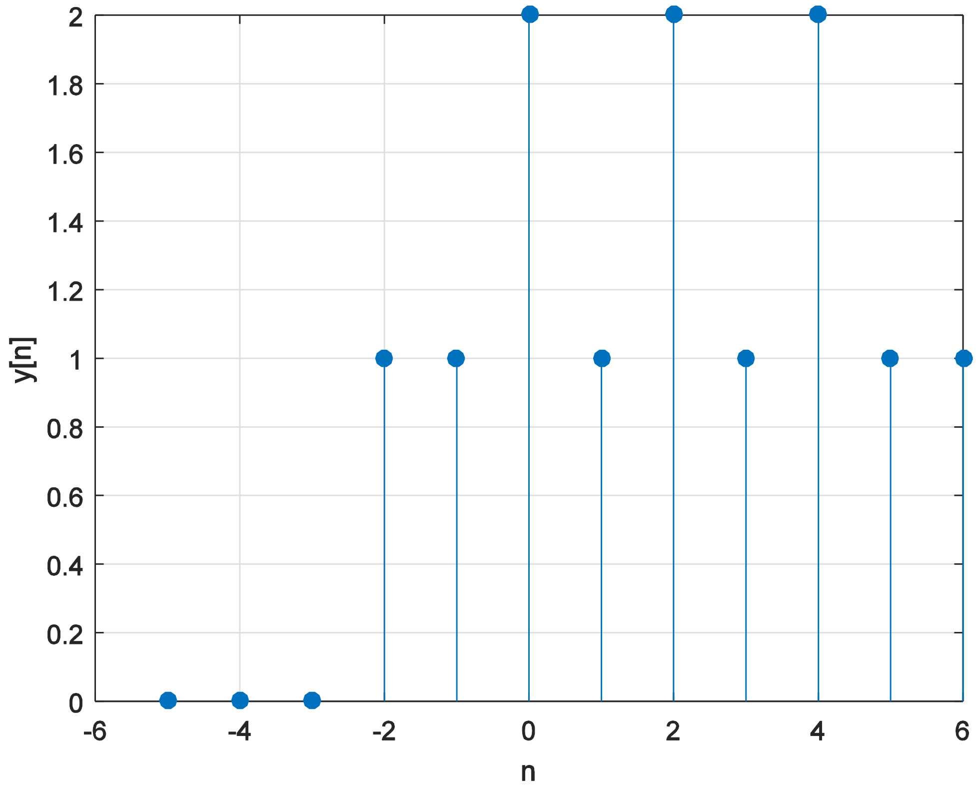

Figure 3: Convolution result of assignment 3 with causal filter

It is clear that the non-causal filter produces output before the input has even started. This is not the case anymore with the causal filter.

Assignment 4: Frequency response

a) Frequency response $H(e^{j\theta})$ of non-causal impulse response:

$$h[n] = \delta[n] + \frac{1}{2}\delta[n+1] + \frac{1}{2}\delta[n-1]$$

Transform to the frequency domain:

$$H(e^{j\theta}) = 1 + \frac{1}{2}e^{-j\theta} + \frac{1}{2}e^{j\theta}$$

Can be rewritten using Euler's expressions:

$$H(e^{j\theta}) = 1 + \cos\theta$$

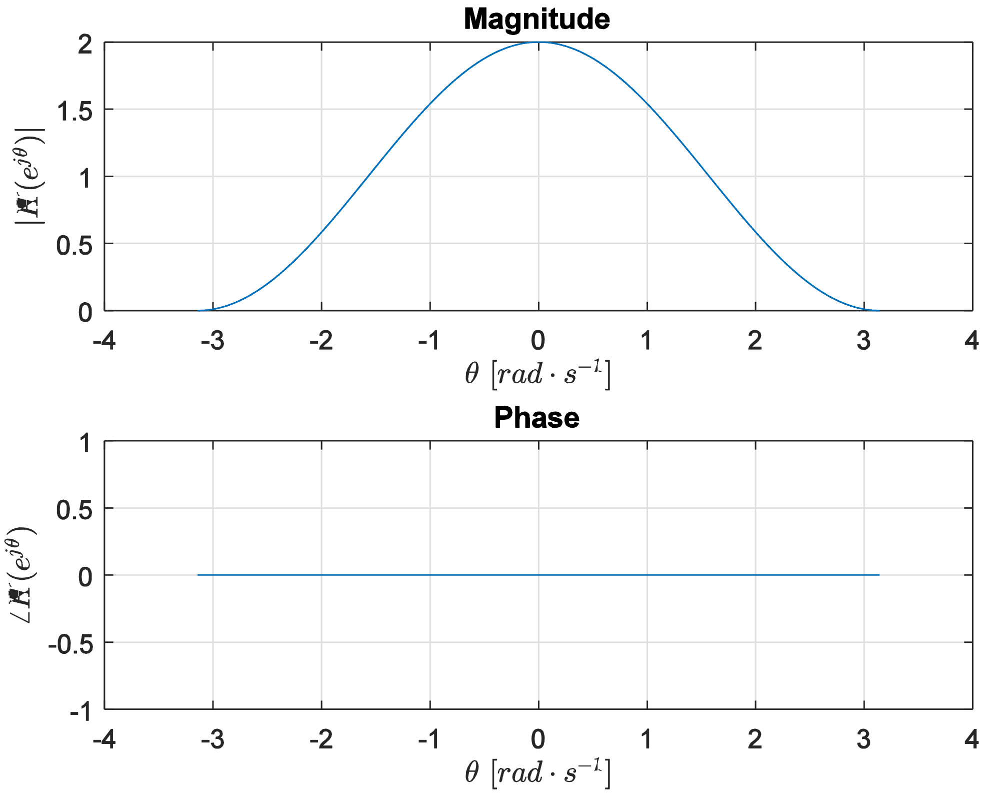

Figure 4: Magnitude and phase of $H(e^{j\theta})$

b) What is the main character of this filter (low-pass, high-pass or band-pass)? Give a short explanation:

By looking at the magnitude plots, it is evident that this filter can be used as a low pass filter. Only the lower frequencies will be passed. Band-pass filters have two cosine shaped bumps in their magnitude plot and a high-pass filter is a 'mirrored version' of the lpf's magnitude plot.

Assignment 5: Frequency response of causal filter

a) Expression for the frequency response $H_{causal}(e^{j\theta})$:

$$h_{causal}[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-2] + \delta[n-1]$$

Transform to the frequency domain:

$$H_{causal}(e^{j\theta}) = \frac{1}{2} + \frac{1}{2}e^{-j2\theta} + e^{-j\theta}$$

b)

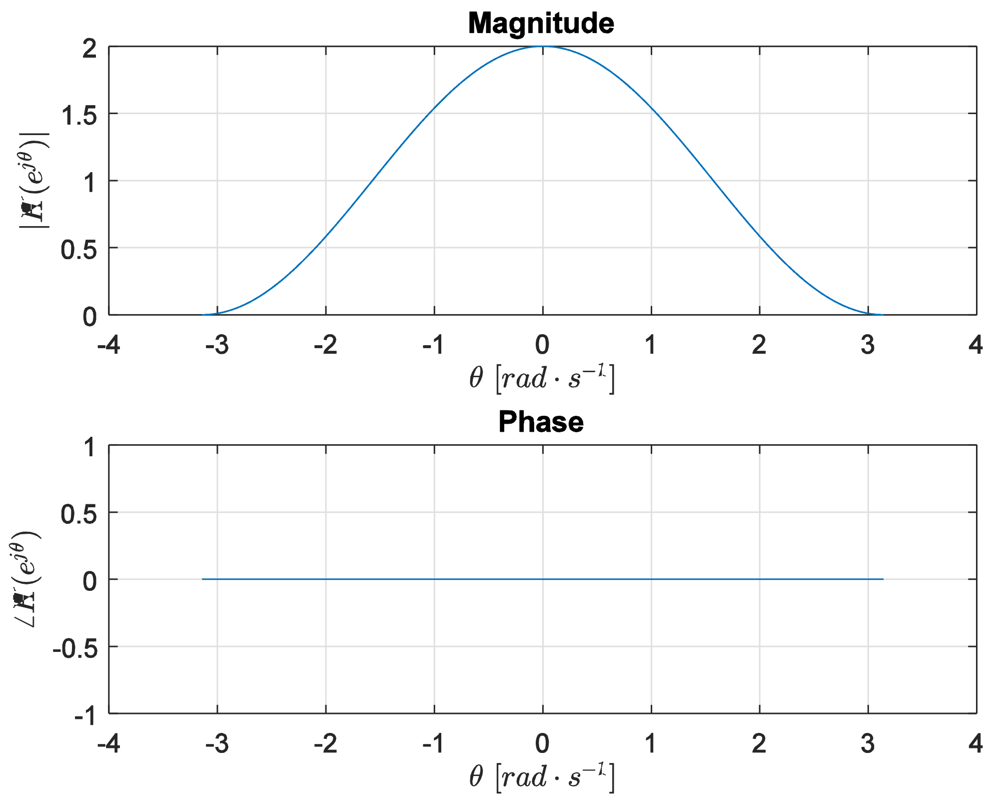

Figure 5: Magnitude and phase of $H_{causal}(e^{j\theta})$

c) Write $H_{causal}(e^{j\theta})$ as the product $H(e^{j\theta})\cdot D(e^{j\theta})$ and explain the difference between $H(e^{j\theta})$ and $H_{causal}(e^{j\theta})$:

$$H(e^{j\theta}) = 1 + cos\theta = 1 + \frac{1}{2}e^{-j\theta} + \frac{1}{2}e^{j\theta}$$

$$D(e^{j\theta}) = e^{-jL\theta} \text{ for } L = 1, D(e^{j\theta}) = e^{-j\theta}$$

$$H(e^{j\theta})D(e^{j\theta}) = \left(1 + \frac{1}{2}e^{-j\theta} + \frac{1}{2}e^{j\theta}\right)e^{-j\theta} = e^{-j\theta} + \frac{1}{2}e^{-j2\theta} + \frac{1}{2}$$

The difference between $H_{causal}(e^{j\theta})$ and $H(e^{j\theta})$ is the delay of $e^{j\theta}$, which equals the delay of $\delta[n-1]$ in the time domain.

Assignment 6: Impulse response of low pass filter

a) Expression for the impulse response:

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\theta})e^{jn\theta}\,d\theta = \frac{1}{2\pi}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1e^{jn\theta}\,d\theta = \frac{1}{j2\pi n}\left[e^{jn\theta}\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

Solves to:

$$h[n] = \frac{1}{j2\pi n}\left(e^{jn\left(\frac{\pi}{3}\right)} - e^{-jn\left(\frac{\pi}{3}\right)}\right) = \frac{\sin\left(n\frac{\pi}{3}\right)}{\pi n}$$

b)

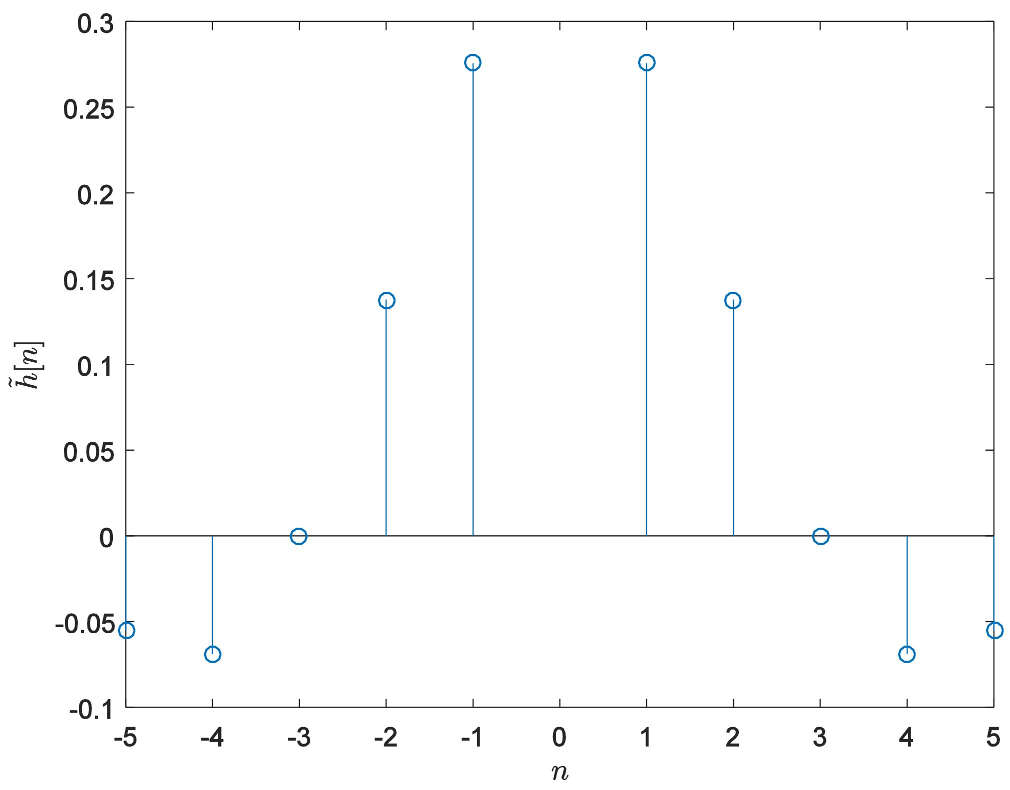

Figure 6: impulse response for N = 11

c) Play x[n] and explain what you hear:

Your hear a lower tone, where all the higer frecensies are filter out of the signal. The result is a low bass tone with a noticable “click” sound and a “drop” like sound at the end of the signal (due to the fade out)

Assignment 7: Calculation of convolution via FTD and IFTD

Calculation:

FTD of $x[n]$ gives:

$$X(e^{j\theta}) = e^{j3\theta} + e^{j\theta} + e^{-j\theta} + e^{-j3\theta}$$

FTD of $h[n]$ gives:

$$H(e^{j\theta}) = -\frac{1}{2}e^{j\theta} - \frac{1}{2}e^{-j\theta} + 1$$

$Y(e^{j\theta})$ can be calculated by multiplying $X(e^{j\theta})$ and $H(e^{j\theta})$:

$$\begin{aligned} Y(e^{j\theta}) = {} & e^{j3\theta} + e^{j\theta} + e^{-j\theta} + e^{-j3\theta} \\ & -\frac{1}{2}\left(e^{j4\theta} + e^{j2\theta} + 1 + e^{-j2\theta} + e^{j2\theta} + 1 + e^{-j2\theta} + e^{-j4\theta}\right) \end{aligned}$$

Using the common transforms for IFTD $y[n]$ can be calculated:

$$\begin{aligned} y[n] = {} & -\frac{1}{2}\delta[n+4] + \delta[n+3] - \delta[n+2] + \delta[n+1] - \delta[n] + \delta[n-1] - \delta[n-2] \\ & + \delta[n-3] - \frac{1}{2}\delta[n-4] \end{aligned}$$

Comparing $y[n]$ to the plotted result of Assignment 1, it can be concluded that both methods produce the same result.

Assignment 8: Sampling a sinusoidal signal

a)

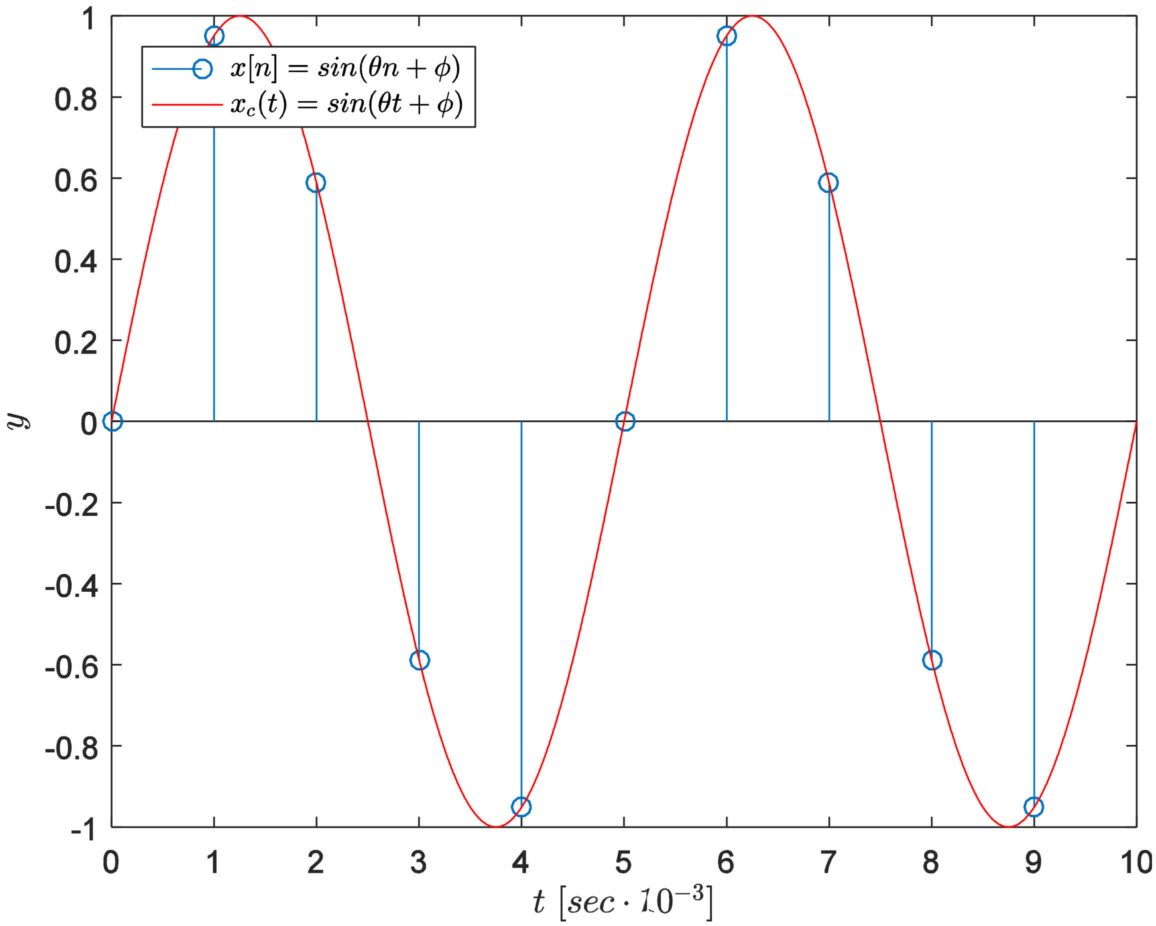

Figure 8: continuous signal $x_c(t)$ and samples x[n]

b) What do you notice when playing the sound?

You hear nothing, this is because at $f = 4000\ Hz$ and a sample rate $fs = 4000\ Hz$ the sampled signal is being alliased and being mapped back to the fundamental freqency where it lies on the boundray of the fundamental freqency and is thus not being played.

Assignment 9: Visualization of 'aliasing' via time domain

Think of at least two different sinuses with different frequencies between 0Hz and 1Hz which cross the same sample points and plot these signals.

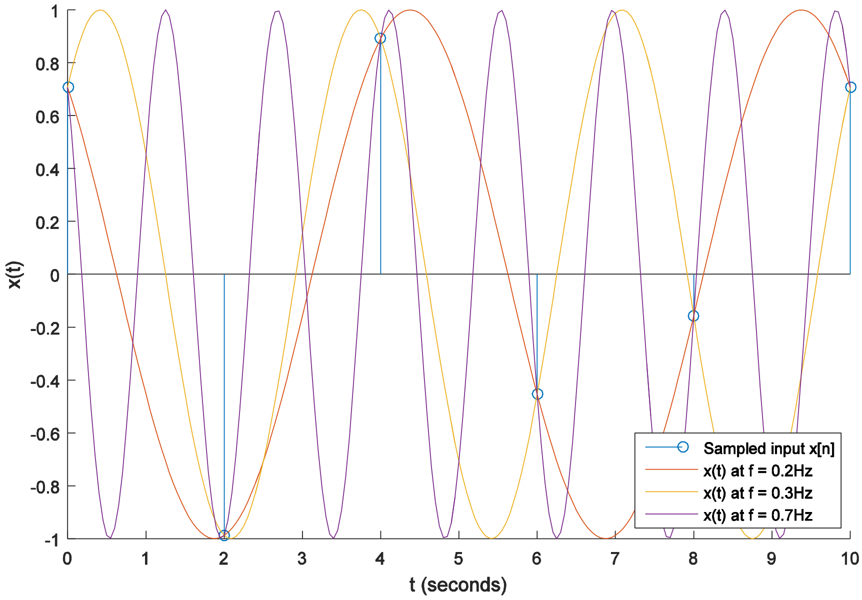

Figure 9: Plot of assignment 8

Explain your results:
Knowing that $x[n] = Acos(\theta n + \phi) \equiv Acos(\{\theta + 2\pi\}n + \phi) \equiv Acos(\{\theta - 2\pi\}n - \phi)$ it is evident that $\theta = 2\pi\left(\frac{f}{f_s}\right) \equiv 2\pi\left(\frac{f}{f_s} + 1\right)$
Solving these equations for f results in $f = [0.2, 0.3, 0.7]$. In the graph above can be seen that these sampled values correspond to their continuous signals, although the frequencies are actually different.

Assignment 10: Up- and down-sampling

a) Implement all 3 versions. Print Matlab code in appendix and upload Matlab code. Explain results:

The first output is obviously equal to the input, as long as D/A converter operates at the same sample rate.

The second output is distorted. Half of the original sampling data is lost, meaning that aliasing occurs. The latest 4 notes in the audio fragment are the result of aliasing: the frequencies have shifted to the lower part of the spectrum (they fell out of the fundamental interval due the limited sampling frequency).

The third output includes an LPF before sampling the signal down by K. This should eliminate all aliasing that occurred in the second output. It removes the high-frequency content that could get lost by removing samples.

The fact that the last couple of notes in the third output jump to lower frequencies, can be explained by the fact that the LPF does not remove all the high-frequency content, or that the down sample factor is too large that it still causes aliasing.

b) Implement all 3 versions. Print Matlab code in appendix and upload Matlab code. Explain results:

The first output is again equal to the input.

The second output contains a numerous amount of high-frequency content, caused by the 0-valued samples that have been added.

The third output should filter the high-frequency content and should therefore interpolate on the extra samples to smoothen the signal, which it does well, but not good enough (the first notes still contain some high-frequency content inside the signal).