# 2020 Machine Learning

# **First Assignment**

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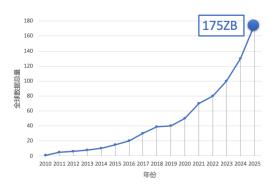
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## 1 åijŢèĺĂ

## 1.1 å;ŞäżŁçŽĎæŮűäżčèČŇæŹŕâĂŤâĂŤåďğæŢřæŊő

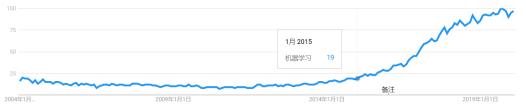
œŮľåIJĺ20äÿŰçžłæIJń, 䞊eĄŤç¡Śä¿£åůšçżŔåśŢçŐråĞžäžĘåŁ£äÿŊåŔŕæŇąçŽĎèűŃåŁ£, éŽŔçİÄ䞊eĄŤç¡ŚçŽĎès åďgæŢřæŊőæĹŘäÿžäžĘè£Źäÿłä£ąæĄŕæŮűäżčçŽĎæăĞå£ŮèŕŊ. åőČçŽĎåŔŚåśŢè£ĚéçåĹř獎èĞşåŔŕèČ¡æĹŚäżňè£ðäřsåůšçżŔåIJĺ獧æťżçŽĎåŘĎäÿłæŰźéľċåŔŮåĹřåőČçŽĎå¡śåŞŊäžĘ. IBMæŔŘåĞžåďgæŢřæŊőæIJĽ5äÿłçĽźçĆź, åĹĘåĹńæŸŕVolumeãĂĄVelocityãĂĄVarietyãĂĄValueãĂĄVeracity, åĹĘåĹńäżčèąĺçİĂåďgéĞŔãĂĄéńŸéçãĂĄåďŽæäťåŻ¿1æŔŔè£řäžĘè£Śè£ĞåŐżåŊĄåźťäżěåŔŁæIJłæľĕ䞍åźſçŽĎæŢřæŊőéĞŔåćđeŢ£èűŃåŁ£, åŔŕäżěå¿ĹæŸŐ柿çŽĎçè£Źäź§æĎŔåŚşçİĂæŢřæŊőéĞŔçŽĎåcđeŢ£äijŽçŁźåęĆæŇĞæŢřçĹĘçĆÿäÿĂèĹňåŊĄåĹĘè£ĚçŇŻ, åŻäæŋď, åęĆä¡ŢæIJĽæŢĹåIJřåĹľçŤĺåęĆæŋďæţůéĞŔçŽĎæŢřæŊőä¿£æŸŕå¡ŞåŁąäźŃæĂě.



åŻ; 1: Rapid growth of data.

獜äžŐåŕźäžŐæŢřæŊóådĎçŘĘçŽĎéIJĂæśĆ, äÿĂ䞯æŁĂæIJŕåęĆâĂIJæŢřæŊőæŇŰæŐŸâĂİ,ãĂĄâĂIJæIJžåŹĺån åőČäżňæŮĺåIJĺè£ŘçŤĺçŐřäżčåijžåďgçŽĎeőąçőŮæIJžåŠŇäijŸçgĂçŽĎçĺŃåžŔäÿŐçőŮæşŢ. åŕźå¡ŞäżŁæţůéĞŔçŽĎæŢřæ äżŐèĂŇæĽ¿åĹřäyĂ䞯æIJĽäżůåĂijçŽĎä£ąæĄŕ. éŽŔçİĂæŢřæŊőéĞŔçŽĎåďgåźĚåćďåďg, åŊŢåŔřèőąçőŮæIJžåůšçżŔżèĂŇäżěHadoopäÿžäżčèąĺçŽĎåijÄæžŘåĹĘåÿČåijŔèőąçőŮæďűæďĎåĹŹæŔŘä¿ŻäžĘåĹĘåÿČåijŔèőąçőŮçŽĎæŁĂæIJŕæ′åŘŇæŮű, éŽŔçİĂ Caffe åŠŇ TensorFlow çŋĽéńŸæŢĹçŐĞçŽĎæůśåžęåŋęäźăæąĘæďűèćńåijĂæžŘ, èőÿåďŽåřŔåďŃåĚňåŔ䞊èĄŤç¡ŚåIJĺäÿŊæŰŋåŔŚåśŢ, æŢřæŊőçŽĎ獧æĹŘäź§äÿŊäijŽåĄIJäÿŃèĎŽæŋě.

åęĆåŻ; 1 æĽĂçďž, 2020åźťåĚĺçŘČæŢřåŋŮåőĞåőŹåřĘäijŽåůšçżŔèűĚè£Ğ50ZB, èŹ¡çĎűæĹŚäżňçŻőåĽŊ裟äÿŊæjäÿŊè£ĞåŔŕäżèèĆŕåőŽçŽĎæŸŕ, æŢřæŊőäijŽæĹŘäÿžäÿĂéąźéĞŊèęĄçŽĎèţĎæžŘ, åIJĺåďgæŢřæŊőæŮűäżč, 缟åĹńæŸŕæIJłæİĕçŽĎæŹžèČ¡åŇŰæŮűäżč, åęĆæIJžåŹĺåŋęäźäè£ŹçğŊåŕźäžŐæŢřæŊőçŽĎåďĎçŘĘæŁĂæIJŕäÿĂåőŽäæŋčåęĆéľň䞌æĽĂèĺĂ, äžžçśżçď¿äijŽçŽĎæIJłæİĕäÿĂåőŽäijŽè£ŻåĚěæŢřæŊőåďĎçŘĘæŁĂæIJŕ( Data Technology, DT )æŮűäżč.



å**Ż**; 2: Google Trend about Machine Learning.

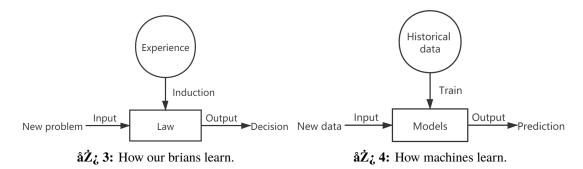
åŻ; 2 æŸŕâĂIJæIJžåŹĺåŋęäźăâĂİè£ŹäÿĂèfŊèŕŋåIJĺçăŤçľűéćĘå§§çŽĎèřůæŋŇæŘIJçťċçČŋåžęèűŃåŁ£[1],

åŔŕäżěçIJŃåĞž, èĞł2015åźťäżěæİĕ, æIJžåŹĺåŋęäźăçŽĎæŘIJçťcçČŋåžęä¿£æĂěè¡ň篝äÿŁ, è£ŹäÿĂ翪æđIJåŔŕèČ¡èęĄå¸AlphaGoæŸŕäÿĂæň¿éĂŽè£ĞæIJžåŹĺåŋęäźăæđĎåżžåĞžçŽĎåŻťæčŃåŕźåijĹçşżçż§, åĚűçňňäÿĂäżčçşżçż§äžŐ2015åźť3aåźűæIJĂçźĹäżě3:1çŽĎåďgæŕŤåĹĘåŔŰå¿ŮèČIJåĹľ. çżŔæŋďäÿĂå¡ź, AlphaGoäżěåŔŁåĚűèČŇåŘŐçŽĎæIJžåŹĺåŋęäźăè.

åÄijå¿ŮçŢŹæĎŔçŽĎæŸŕ, è£ŹéąźæŢřæŊőæŸŕäżŐ2004åźťåijĂåğŃçŽĎ, äź§åřśæŸŕèŕťGoogleåĚňåŔÿèĞşåřŚäżŐ20
åŘŇæăůçŽĎ, Facebookäź§åIJĺè£ŘçŤĺæIJžåŹĺåŋęäźäåŕźæŕŔäÿĂäÿłçŤĺæĹůçŽĎæŕŔäÿĂæİąæţŔèğĹèőřå¡Ţ,
æŕŔäÿĂæňąçĆźèţđè£ŻèąŇåĹĘæđŘ, äżŐèĂŇäÿžçŤĺæĹůæŐléĂĄæIJĂäÿžåŘĹéĂĆçŽĎåĘĚåőźäżěåŔŁåź£åŚŁ.
åŻ¡åĘĚåĚňåŔÿåŕźæIJžåŹĺåŋęäźääź§æIJĽåŊĄåĹĘæůśåĚěçŽĎåžŤçŤĺ, åęĆAlibabaçŽĎäžžèĎÿæŤŕäżŸ,

èČ¡åď§åIJÍåĞä䞣çŤÍæĹůäÿŋåďgèğĎæÍąæŋčåijŔäÿŁçž£äžžèĎÿæŤŕäżŸçşżçż§, èŕťæŸŐäžĘAlibabaåĚňåŔÿæIJňèžńåŕźäz èĂŇäżŐ2000åźťåijĂåğŃåřśæIJĽåŊşæŮűéĂŽèőŕäÿŽåŁąçŽĎTencentåĚňåŔÿäź§åřĘæIJžåŹÍåŋęäźäåžŤçŤÍèĞşå¿ĹåďŽäž åęĆQQäÿŋâĂIJåŔŕèČ¡èőďèŕĘçŽĎäžžâĂİ, æĹŰèĂĚé§şäźŘè¡ŕäżű, èğĘéćŚè¡ŕäżűéĞŇåŔŕèČ¡åŰIJæňćçŽĎèğĘéćŚæĹŰèŹ éĞŇéĬćéČ¡æIJĽæIJžåŹÍåŋęäźăçŽĎå¡śåŋŘ.

çśżæŕŤäžŐäžžèĎŚçŽĎåŋęäźäè£ĞçÍŃ, æIJžåŹĺçŽĎåŋęäźääijijäźŐåĚůæIJĿçŻÿäijijçŽĎæĂğètĺ, äžžèĎŚçŽĎåŋęäźäè )åŔŕäżěæŁ¡èśąäÿžäżŐçżŔéłŇäźŃäÿŋæĂżçżŞåĞžèğĎå¿Ń, éĄĞåĹřæŰřéŮőéćŸæŮűå§žäžŐè£ĞåŐżæĂżçżŞèĂŇæİĕ玼 èĂŇæIJžåŹĺåŋęäźäçŽĎåŋęäźäè£ĞçĺŃ(**åŻ; 4**)åĹŹåŔŕäżěæŁ¡èśąäÿžèőąçőŮæIJžäżŐäžžäÿžè¿ŞåĚĕçŽĎåŐĘåŔšæŢřæŊő åŕźäžŐæŰřåĞžçŐřçŽĎçŽĎæŢřæŊő, 獜åůšçżŔèőŋçżČåě¡çŽĎæĺąåđŃ翏åĞžéćĎæţŃ.



ä¡ĘæIJžåŹĺåŋęäźăæIJňèžńåŊťåźűäÿŊæŸŕäyĂäÿłåŊĄåĹĘåźťè¡żçŽĎäžŃ缾, åőČæIJĂæŮľçŽĎåžŤçŤĺåŔŕäżĕ裡æž SamuelåĘŹåĞžçŽĎäÿĂäÿłèůşæčŃçĺŃåžŔ, è£ŹäÿłçĺŃåžŔéĂŽè£Ğåŋęäźäèţřå¿Ůåě¡åŠŇäÿŊåě¡çŽĎæčŃåśĂ, äÿžäÿŃæčŃèĂĚåĄŽåĞžæŇĞåŕij[2][3]. åIJĺè£ĞåŐżçŽĎåĞăåźťéĞŇ,æIJžåŹĺåŋęäźăãĂĄäžžåůěæŹžèČ¡äźČèĞşæůśåžęå ä¡ĘåĚűåőđæŋďäÿĽèĂĚäźŃéŮťåĚűåőđæŸŕåŇĚåŘńäÿŐèćńåŇĚåŘńçŽĎåĚşçşż,åęĆåŘŇä£Ďç¡ŮæŰŕåěŮåĺČäÿĂèĹň(åŻ,



 $\mbox{\ensuremath{\mathring{a}}\mbox{\ensuremath{\mathring{Z}}}\mbox{\ensuremath{\mathring{c}}}$  5: AI, Machine Learning & Deep Learning.

## 1.2 äžžåůěæŹžèČ;

âĂIJäžžåůěæŹžèČ¡âĂİçŽĎæęĆå£ţèĞł1956åźťåIJĺė¿¿çĽźèŇĚæŰŕäijŽèőő[4]äÿŁéęŰæňąæŋčåijŔæŔŘåĞžäżěæİĕ,
éŽŔçİĂæŁĂæIJſœĽŃæőţçŽĎåŔŚåśŢäź§åIJĺäÿŊæŰŋåIJřåŔŸåŇŰ,ä¡ĘåőČçŽĎæăÿå£ČæŸſåŔſäżèæĂżçżŞäÿžâĂIJåřĘç
åŻăæŋď,äżżä¡ŢåŔſäżěåőđçŐřè£ŹäÿĂçŻœăĞçŽĎæĽŃæőţäÿŐæŰźæşŢéČ¡åŔſäżěåĘăäżěâĂIJäžžåůěæŹžèČ¡âĂİçŽĎåĔ
ä¡ĘåőđçŐřäžžåůěæŹžèČ¡çŽĎæŰźæşŢåźűäÿŊäżĚéŹŘäžŐæIJžåŹĺåŋęäźä,ä¿ŃåęĆæŮľæIJ§çŽĎåŻ¡éŹĚèśąæčŃçĺŃa
Blue. åŻ¡éŹĚèśąæčŃçŽĎæčŃåśĂåďŊæİĆåžę(35<sup>80</sup>[5])åźűéİđ缟åĹńåžđåďg,åŻäæŋďåŔſäżěéĂŽè£Ğçľůäÿ¿æşŢåřĘæſŔä

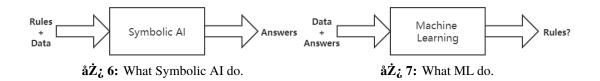
èĂŇDeeper BlueéĂŽè£Ğè£ŹäÿĂæŰźæşŢæĹŘåŁ§åIJřæĹŸèČIJäžĘåŻ¡éŹĚèśąæčŃäÿŰçŢŇåĘăåĘŻ. åIJÍDeeper BlueçŽĎäÿŃæčŃè£ĞçÍŃäÿŋ, äÿŊéIJĂèęĄåŋęäźăäżĂäźĹ, äżĔäżĚéĂŽè£ĞåĘĚç¡őçŽĎçÍŃåžŔè£ŻèąŇèőąçőŮåŊşåŔŕ. è£ŹçğŊéĂŽè£ĞçijŰåĘŹådŊæİĆèğĎåĹŹæİĕåőđçŐřäžžåůěæŹžèČ¡çŽĎæŰźæşŢèćńçğřäÿžçňęåŔůäÿżäźĽäžžåůěæŹžèČ¡

ä¡ĘåŘŐæİĕ, éŽŔçİÄäžžäżňéIJĂèęĄåďĎçŘĘçŽĎä£ąæĄŕèűŁæİĕèűŁåďŊæİĆ, åęĆåŻ¿åČŔãĂĄé§şéćŚæĹŰèĂĚæŸſ çňęåŔůäÿżäźĽäžžåůěæŹžèČ¡åůšçżŔäÿŊèČ¡åď§åďĎçŘĘè£ŹäžŻéŮőéćŸäžĘ, åŻăæŋď, äÿĂçǧŊæŰřçŽĎäžžåůěæŹžèČ¡åő åőČä¿£æŸŕâĂIJæIJžåŹĺåŋęäźăâĂİ.

## 1.3 æIJžåŹĺåŋęäźă

1950åźť, äžžåůěæŹžèČ¡åĚĹéľšAlan TuringåŔŚèąĺäžĘâĂIJComputing machinery and intelligenceâĂİäÿĂæŰĞ[6], .æŰĞäÿŋ, äżŰåŕźâĂIJèőąçőŮæIJžæŸŕåŘęèČ¡åď§åŋęäźääÿŐåĹŻæŰřâĂİè£ŹäÿĂéŮőéćŸè£ŻèąŇäžĘæĂİèĂČ, åźűäÿŤçżŹåĞžäžĘèĆŕåőŽçŽĎåŻđçŋŤ: èČ¡.

çzŔåĔÿçŽĎçňęåŔůäÿżäźĽäžžåůěæŹžèČ¡äÿŋ, æĹŚäżňè¿ŞåĚěçŽĎæŸŕæŢřæŊőäżěåŔŁçš¿å£ČçijŰåĘŹçŽĎèğĎåĹŹ è¿ŞåĞžçŽĎæŸŕäžžäżňæIJ\$æIJŻå¿ŮåĹřçŽĎçŋŤæąĹ(åŻ¿6). èĂŇåIJĺæIJžåŹĺåŋęäźăäÿŋ, è¿ŞåĚěçŽĎæŸŕæŢřæŊőåŠŇo äžžäżňåĹŹåÿŇæIJŻèőąçőŮæIJžäżŐè£ŹäžŻæŢřæŊőäÿŐçŋŤæąĹäÿŋåŋęäźăåĹřäÿĂ䞯èğĎåĹŹ(åŻ¿7).

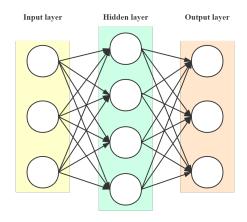


è£ŹäÿĂè£ĞçĺŃäÿŋ, èőąçőŮæIJžçŽĎåŋęäźăçşżçż\æŸŕéĂŽè£Ğåŕźè¿ŞåĚěæŢřæŊőäżěåŔŁåŕźåžŤçŽĎçŋŤæąĹçŽĎè åĹřäžĘçŐřäżč, 獎èĞşæIJĿèőÿåďŽéŮőéćŸæĹŚäżňåŔłç\šé饪æŢřæŊőèĂŇäÿŊç\šé饪çŋŤæąĹ, è£ŹæŮűæĹŚäżňäijŽål æIJ\å¿ĚäżŰèČ¡çżŹæĹŚäżňçŋŤæąĹ, è£ŹåřśæŸŕæĽÄèřŞçŽĎæŮă篌çİčåŋęäźä. äžŃåőđèąĺæŸŐ, äÿÄ䞯åźűäÿŊåÄ\éĽ Zero, 翏åĞžçŽĎçŋŤæąĹ獎èĞşèęĄæŕŤäžžçśżèĞłåůśçŽĎçŋŤæąĹåě¡å¿ĹåďŽ.

#### 1.4 æůśåžęåŋęäźă

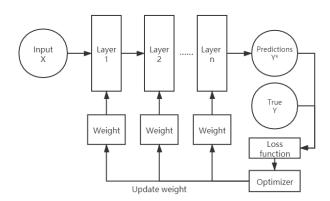
åIJÍæŹőéĂŽçŽĎæIJžåŹĺåŋęäźăéŮőéćŸäÿŋ, æĹŚäżňéIJĂèęĄèőąçőŮæIJžåŐżåŋęäźăçŽĎèğĎåĹŹ,å¿ĂåŔŕäżèèz èĂŇåIJĺè£ŹäžŻèğĎåĹŹäÿŋæIJĽäÿĂéČĺåĹĘçŽĎåďŊåŘĹåĞ¡æŢřçŽĎåďŊåŘĹæňąæŢř獎èĞşéńŸè¿¿æŢřåŊĄæňąäźČè åŕźäžŐè£ŹäžŻåďŊåŘĹåĞ¡æŢřåŊĄåĹĘåďŊæİĆçŽĎéŮőéćŸ,æĹŚäżňçğřåőČäżňäÿžæůśåžęåŋęäźăéŮőéćŸ,æůśåžęåŋęäźäçŽĎâĂIJæůśåžęâÅİ, ä¿£æŇĞçŽĎæŸŕè£ŹçğŊåďŽæňąåďŊåŘĹåĞ¡æŢřçŽĎæňąæŢř.

eĂŇåIJĺåőđeŹĚçŽĎæůsåžęåŋęäźăeŮőéćŸäÿŋ, e£ŹçğŊåďŽæňąçŽĎåďŊåŘĹåĞ¡æŢřæĂżæŸŕeĂŽe£ĞäÿĂçğŊåŘŊå åŻ; 8 ä¿£æŸŕäÿÄäÿłäÿĽåśĆçŽĎçěđçżŔç¡ŚçżIJ. âĂIJçěđçżŔç¡ŚçżIJâĂİè£ŹäÿĂæIJŕèŕŋçŽĎçĄţæĎ§æİèæžŘäžŐçěđçżŔ



åŻ; 8: A three-layer full connected neural network

èŹ¡çĎűåőČçŽĎæfŔäÿĂåśĆåŊĄåĹĘçőĂåŊŢ, ä¡ĘçżŔè£ĞèőÿåďŽåśĆçŽĎåăĘåŔăåŘŐ, å¿Ăå¿ĂåŔfäżěåfźäÿĂ䞯åŊĄåĹæŋčåČŔäžžçŽĎåďgèĎŚèŹ¡çĎűåŔłçŤśäÿĂäÿłäÿłçżĘèČđæđĎæĹŘ, ä¡ĘåŊťæIJĽåŊĄåĹĘåďŊæİĆçŽĎæŹžæĚğäÿĂæăů.
æIJĽçŻŚçİčçŽĎçěđçżŔç¡ŚçżIJæIJžåŹĺåŋęäźăçŽĎå§žæIJňåŐ§çŘĘåęĆåŻ;9, åőČéĂŽè£ĞéćĎæţŃåĂijäÿŐçIJ§åőđåĂiä;£éćĎæţŃåÄjäŔŸå¿ŮæŻťåŁäçš;åĞĘ.



åŻ; 9: The workflow of neural network ML.

## 2 æIJžåŹĺåŋęäźăçŽĎåŔŚåśŢäÿŐåžŤçŤĺ

## 2.1 åŔŚåśŢåŐĘçĺŃ

æŋčåęĆæĹŚäżňåŋęäźååŐĘåŔšç§ĕèŕĘäÿĂæăů, äżŐæIJžåŹĺåŋęäźăçŽĎåŔŚåśŢåŐĘåŔšäÿŋäžĘèğčåőČäź§æŸŕäÿĂçţ

#### 20äÿŰçžł50 - 70åźťäżč

æŋďæŮű, äžžåůěæŹžèČ¡æŋčåďĎäžŐâĂIJæŐĺçŘĘæIJ§âĂİ, åŊşäžžäżňçŤĺçijŰçĺŃèţŃäžĹæIJžåŹĺéĂżè¿ŚæŐĺçŘĘèŒäżčèąĺåůěä¡IJæIJĽ: A. NewellåŠŇH. SimonçŽĎâĂIJéĂżè¿ŚçŘĘèőžåőűâĂİ(Logic Theorist)çĺŃåžŔäżěåŔŁæŋďåŘŐçŽĬGeneral Problem Solving)çĺŃåžŔçŋĽ, èĂŇäżŰäżňäžŇäžžäź§åIJĺäźŃåŘŐèŐůå¿ŮäžĘèŐůåŻ¿çĄţåĕŰ.

20äÿŰçžł70åźťäżčäÿŋæIJ\$åijĂåğŃ, åIJĺE.A.FeigenbaumçŋĽçŽĎåĂąåŕijäÿŃ, äžžåůěæŹžèČ¡çăŤçľűè£ŻåĚĕäžĘâĂIJ åďgéĞŔçŽĎäÿŞåőűçşżçż§éŮőäÿŰ. E.A.FeigenbaumçŋĽäžžäź§èŐůå¿ŮäžĘåŻ¿çĄţåěŰ. ä¡ĘæŸŕåŘŐæİĕ, ç§ĕèrĘåůěçÍŃäź§éĄĞåĹřçŞűécĹ, äžžäzňåŔŚçŐřæĂżçżŞç§ĕèrĘåĘŊæŢŹçżŹèőąçőŮæIJžæŸŕçŻÿå¡ŞåŻř鎿çŽĎ, åŻäæŋďåžŤèrĕèő¿æşŢèőľæIJžåŹĺèĞłåůśåŋęäźăç§ĕèrĘ.èĂŇåŻ¿çĄţåIJĺ1950åźťçŽĎæŰĞçńăäÿŋ, èĆråőŽäžĘæIJžåŹĺåŋęä åŔĹéŹĘçżŋæIJĽå§žäžŐçĕđçżŔç¡ŚçżIJçŽĎâĂIJè£đæŐĕäÿżäźĽâĂİæIJžåŹĺåŋęäźããĂĄå§žäžŐéĂżè¿ŚèąĺçďžçŽĎâĂIJçı

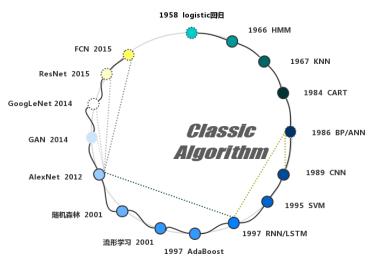
#### 20äÿŰçžł80åźťäżč

1986åźť, æIJžåŹĺåŋęäźăMachine LearningåĹŻåĹŁïijŻ1989åźť, äžžåůěæŹžèČ¡æĬČåĺĄæIJ§åĹŁArtificial IntelligenceåĞzçĽĹæIJžåŹĺåŋęäźăäÿŞè¿ŚïijŻ20äÿŰçžł80åźťäżčæIJžåŹĺåŋęäźăæĹŘäÿžäÿÄäÿłçŇňçńŃåŋęçğŚéćĘå§§, åŘŇæŮű, åŘĎçğŊæIJžåŹĺåŋęäźăæŁÄæIJŕäź§åŚĹçŐřçĬĂçŹ¿èŁśåĹİçż¡çŽĎæĂĄåŁ£.

æŋďæŮűçŽĎçăŤçľűçČŋéŮĺéČ¡èĄŽéŻĘäžŐâĂIJäżŐæăůä¿ŃäÿŋåŋęäźăâĂİ, åŇĚæŃň篌çİčåŋęäźă,
äżěåŔŁæŮă篌çİčåŋęäźă. åĚűäÿŋ, äżěçňęåŔůäÿżäźĽæIJžåŹĺåŋęäźăåŠŇå§žäžŐçěđçżŔç¡ŚçżIJçŽĎè£đæŐĕäÿżäźĽæIJž
åĽŊèĂĚåŇĚæŃňåĘşçŋŰæăŚãĂĄå§žäžŐéĂżè¿ŚçŽĎåŋęäźăçŋĽ, åŘŐèĂĚèŹ¡çĎűäžğ獧çŽĎæŸŕâĂIJ鿌çőśâĂİæĺąåđ
ä¡Ę獜äžŐèČ¡åď§æIJĽæŢĹåIJřègčåĘşèőÿåďŽçŐřåőđéŮőéćŸ, å¿ŮåĹřäžĘåź£æşŻçŽĎåžŤçŤĺ.

### 20äÿŰçžł90åźťäżč

âĂIJçż§èőąåŋęäźăâĂİæĹŘäÿžäžĘè£ŹäÿłåźťäżčçŽĎäÿżæţĄ, äżčèąĺæŁĂæIJŕæIJĽæŤŕæŇĄåŘŚæIJžSVMãĂĄâĂIJaæŤŕæŇĄåŘŚéĞŔæIJžèćńæŹőéĄŊæŐĕåŔŮåŘŐ, æăÿæŁĂåůğ(kernel trick)èćńåź£æşŻåžŤçŤĺåĹřæIJžåŹĺåŋęäźăåŘĎåďĎæäÿæŰźæşŢéĂŘæÿŘæĹŘäÿžæIJžåŹĺåŋęäźäå§žæIJňåĘĚåőź.



 $\mathring{a}\dot{Z}_{i}$  10: The evolution of the algorithm.

#### 21äÿŰçžł

âĂIJœůsåžęåŋęäźäâĂİæĹŘäÿžäžĘå¡ŞäżčçŽĎäÿżæţĄ, äżĕå¿ĂçŽĎæIJžåŹĺåŋęäźăæŁĂæIJſåſźä¡£çŤĺèĂĚèęĄæśĆæġèĂŇçŻÿæſŤäžŐäżĕå¿ĂçŽĎæIJžåŹĺåŋęäźäæŁĂæIJſ, œůsåžęåŋęäźäæŁĂæIJſåľĘåſźäžŐä¡£çŤĺėĂĚçŽĎeſŸèęĄæśĆè¡ňçäżěèĞşäžŐåŔłèęĄäÿŃåŁ§åďíèőſëőąçőŮæIJžæĬėèřČåŔĆ, æĂğèČ¡å¿Ăå¿Ääź§äÿŊäijŽå¿ĹåİŔ. åŻäæŋď,
èŹ¡çijžäźŔäÿĕæăijçŽĎçŘĘèőžå§žçąĂ, ä¡ĘåőČæŸ¿èŚŮéŹŊä¡ŐäžĘæIJžåŹĺåŋęäźäåžŤçŤĺėĂĚçŽĎeŮĺæğŻ,
äÿžåĚűêţřåŘŚåůěçĺŃåőđèùţåÿçæĬěä¿£åĹľ.

åŻ; 10 æŸŕæIJžåŹĺåŋęäźääÿŋäÿÄ䞯åĚÿåđŃçőŮæşŢçŽĎåŔŚåśŢè£ĞçĺŃ.

## 2.2 åžŤçŤĺæąĹä¿Ń

æIJżåŹĺåŋęäźăåIJĺå¡ŞäżŁåĚűåőđåůšçżŔâĂIJæŮăåďĎäÿŊåIJĺâĂİ, åőČæŹőéĄŊåžŤçŤĺäžŐäžžåůěæŹžèČ¡çŽĎåŘĎä åŇĚæŃňæŢřæŊőæŇŰæŐŸãĂĄèőąçőŮæIJžèğĘèğĽãĂĄèĞţĎűèſŋèĺĂåďĎçŘĘãĂĄèſŋé§şåŠŇæĽŃåĘŹèſĘåĹíãÃĄçŤ

åijĆåÿÿæčĂæţŃ åijĆåÿÿæŇĞçŽĎæŸŕæ§ŘäÿłæŢřæŊőåŕźèśąçŤśäžŐæţŃéĞŔèŕfåůőãĂĄæŤűéŻĘäÿŋçŽĎæŊ§åďšæĹÚ
èĂŇåřĘè£ŹäžŻåijĆåÿÿè¿ĺåĹńåĞžçŽĎè£ĞçĺŃ, éĂŽåÿÿçğřäÿžåijĆåÿÿæčĂæţŃ. åijĆåÿÿæčĂæţŃçŽĎèőŋçżČæăůæIJňéČ¡
åĄĞèő¿è£ŹäžŻæăůæIJňçŽĎ缟å¿ĄéČ¡æŸŕæIJŊäżŐéńŸæŰŕåĹĘåÿČ, åĹŹåIJĺæŋďå§žçąĂåŔŕäżěåżžçńŃäÿÄäÿłæęĆçŐ
çŤĺæİĕäijřèőąåřśæIJĽæ§ŘäÿłçĽźå¿ĄçŽĎæăůæIJňåśďäžŐéĺđåijĆåÿÿæăůæIJňçŽĎåŔŕèČ¡æĂğ. æŕŤåęĆå¡ŞçŤĺæĹůçŽĎääåŕśæIJĽåŔŕèČ¡æŸŕä£ąçŤĺåŊąèćńçŻŮåĹů, è£ŹæŸŕéŞűèąŇåřśåŔŕäżěæăźæŊőéčŐæŐğçşżçż§æİĕ韚æŋćè£ŹæăůçŽĎçŻ

çŤĺæĹů獿åČŔ çŤĺæĹů獿åČŔçZĎæŭÿå£Čåůěä¡IJåřśæŸŕ翏çŤĺæĹůæĽŞæŭĞçŋ¿, åęĆåźťé¿ĎãĂĄæĂġåĹńãĂĄåIJā åĚňåŔÿäżŐè£ŹäžŻæŭĞçŋ¿éŻĘåŘĹèČ¡æŁ¡èśąåĞžäÿĂäÿłçŤĺæĹůçŽĎä£ąæĄŕåĚĺèšŇ. äżŐèĂŇåIJĺäžġåŞĄçŽĎè£ŘèŘěåš æŭźæŊőçŤĺæĹů獿åČŔæůśåĚĕçŘĘèğčçŤĺæĹůéIJĂæśĆ, äżŐèĂŇèő¿èőąåĞžæŻťéĂĆåŘĹçŤĺæĹůçŽĎäžġåŞĄ, èő¿åőŽåďŊåŘĹçŤĺæĹůæűĹèťźèČ¡åŁŻçŽĎäżůæŭijåŇžéŮť, æŔŘåŊĞçŤĺæĹůä¡ŞéłŇçŋĽ. çŐřåIJĺçŽĎåŘĎäÿłAP-PåŕźäžŐçŤĺæĹůçŽĎäÿłæĂġåŇŰæŐĺèŊŘãĂĄæŤŕäżŸåőİçŽĎèŁİéžżä£ąçŤĺåĹĘçŋĽėČ¡åśďäžŐçŤĺæĹů獿åČŔçŽĎåĚůä

äźŧåŚŁçĆźåĞżçŐĞéćĎäijř 䞊èĄŤç¡Śåź£åŚŁæŸŕ䞊eĄŤç¡ŚåĚňåŔÿäÿżèęĄçŽĎçŻĹåĹľæĽŃæőţ, å¡ŞäżŁçŽĎçğŚæł
䞊èĄŤç¡Śåź£åŚŁäžďæŸŞçŽĎåŔŇæŰźæŸŕåź£åŚŁäÿżåŠňåłŠä¡Ş. äÿžèĞłåůśçŽĎäžğåŞĄæŁŢæŤ¿åź£åŚŁåźűäÿžåź£åŚŁ
åéĆåŘĎåďgéŮĺæĹůç¡ŚçńŹãĂĄåŘĎçğŊèőžåİŻ, åőČäżňæŔŘä¡Żåź£åŚŁçŽĎåśŢçďžåźşåŔř, åźűæŤűåŔŰåź£åŚŁèſź.
åź£åŚŁçĆźåĞżçŐĞæŸŕæŇĞåź£åŚŁçŽĎçĆźåĞżåĹřè¿¿çŐĞ, åŊşåź£åŚŁçŽĎåődéŹĚçĆźåĞżæňąæŢřéŹďäżěåź£åŚŁçŽĬ
åIJĺåődéŹĚåžŤçŤĺäÿŋ, æĹŚäżňåřĘæţůéĞŔçŽĎåŐĘåŔšçĆźåĞżæŢřæŊőä¡IJäÿžæăůæIJňæİèèőŋçżČæĺąåđŃ,
äżŐèĂŇèŕĎäijřåŘĎæŰźéİċåŻäçſťäåŕźçĆźåĞżçŐĞçŽĎå¡śåŞŊ. ä¡IJäÿžè£ŹäÿĂæĺąåđŃçŽĎåžŤçŤĺ, å¡ŞæIJĿæŰřçŽĎåź£å
åřśåŔŕäżěçŤlėőŋçżČåě¡çŽĎæĺąåđŃ, æăźæŊőåź£åŚŁäžďæŸŞåźşåŔřäijäè£ĞæİĕçŽĎçŻÿå̺缟å¿ĄéćĎäijřè£ŹæňąåśŢçd
翪åŘĹåź£åŚŁåĞžäzůèőąçőŮå¿ŮåĹřçŽĎåź£åŚŁçĆźåĞżæŤű篣, äżŐèĂŇéĂĽåĞžæŤű篣æIJĂéńŸçŽĎåź£åŠŁåŘŚå

## 2.3 UberçŽĎåĚůä;ŞåžŤçŤĺ

æĹŚäżňäżěUberäÿžä¿ŃåĄŽäÿÄäÿłåĚůä;ŞçŽĎåĹĘæđŘ, 2017åźť9æIJĹ, UberäżŃçżŊäžĘäżŰäżňçŽĎæIJžåŹĺåŋęäźä Michelangelo. åžŤçŤĺMichelangelo, UberåĄŽåĞžäžĘäÿÄ䞯æŤźåŔŸåŠŇåŔŚåśŢ.

#### **Uber Eats**

å§žäžŐMichelangelo, Uber EatsäijŽæăźæŊőçŽĎéćĎæţŃåĹřè¿¿æŮűéŮťãĂĄåŐĘåŔšæŢřæŊőäżěåŔŁéďŘéęĘçŽĎå

#### åÿĆåIJžéćĎæţŃ

UberçŽĎéń ŸåşřæIJ §åŁĺæĂĄèřČäżůæIJžåĹűėŹ¡çĎűæŸŕåÿĆåIJžåĘşåőŽçŽĎ, ä¡ĘåŕźçŤĺæĹůæİèèŕťçżİåŕźæŸŕUber UberçŽĎåÿĆåIJžåŻćéŸ §éĂŽè£ĞæIJžåŹĺåŋęäźă, åÿŇæIJŻéćĎæţŃäÿĂåďğæĽźåőćæĹůç¿ďåřĘåIJĺä¡ŢæŮűä¡ŢåIJřæIJĽæè£Źæăů, åIJĺåŁĺæĂĄåőŽäżůæIJžåĹű( surge pricing )åŘŕåŁĺäźŃåĽŊ, èőľæŻťåďŽçŽĎUberè¡ęè¿ĘèČ¡åď§æŔŘä¿ŻæIJŊåŁåIJĺæűĺäżůäźŃåĽŊ, 饪èůŕäÿŁäijŽæIJĽæŻťåďŽå¿ĚåŚ¡çŽĎåŔÿæIJžçŋĽå¿Ěè£ŹåŊşåřĘåĹřæĬěçŽĎåďgéĞŔæĽŞè¡ęéIJĂ

eĂŇåŘŇæŮű, äźŸåőćäżňäź\$äÿŊå£ĚèŃęçŋĽåĞžçğ\$è¡ę. ä¡Ęè£Źäź\$äÿŊæĎŔåŚşçİĂéĞĞçŤĺæIJžåŹĺåŋęäźăæŰźåijŔ, åŁĺæĂĄåőŽäżůæIJžåĹűäijŽæűĹåďš. åŻăäÿžåŕźåÿĆåIJžæİèèŕť, èĆŕåőŽè£ŸæŸŕäżůéńŸèĂĚå¿Ů, éĂŽè£ĞäżůæăijçŽĎæţć ä¡£ä¿ŻæśĆåŁĺæĂĄè¿¿åĹŕåźşèąą.

## écĎèőaåĹřè¿¿æŮűéŮť(ETAs)

årźåĚňåŔÿæİčert, æIJĂéĞŊèęĄçŽĎæŇĞæăĞäźŃäÿĂårśæŸråŘĎçğŊéćĎäijřæŮűéŮť. çš¿çąőçŽĎéćĎäijřæŮűéŮťæ è£ŹäžŻæŇĞæăĞèćńè¿ŞåĚĕæŮăæŢřåĚűäżŰçŽĎåĘĚéČĺçşżçż§äÿŋ, æİčåŊŔåŁľåĹďãőŽäżůæăijåŠŇèůr瞣.

UberçŽĎåIJřåŻ¿æIJŊåŁąåŻć韧åijĂåŔŚäžĘäÿĂäÿłåďŊæİĆçŽĎåĹĘæőţèůŕ瞣çşżçż§,çŤĺæİĕéćĎäijřå§žæIJňçŽĎ è£ŹäžŻå§žæIJňçŽĎéćĎäijřæŮűéŮťåĚůæIJĽçŻÿåŘŇçśżåđŃçŽĎ鍏èŕŕ. åIJřåŻ¿æIJŊåŁąåŻć韧åŔŚçŐřäżŰäżňåŔŕäżó åźűçŤĺéćĎæţŃçŽĎ鍏èŕŕæİĕè£ŻėąŇä£őæŋč. 獜äžŐè£ŹäÿłæĺąåđŃæŋčéĂŘäÿłåžŤçŤĺåIJĺåŘĎäÿłå§ŐåÿĆ, Uber åŻć韧åŔŚçŐřéćĎäijřåĹřè¿¿æŮűéŮťçŽĎåĞĘçąőæĂğåďgåźĚæŔŘåŊĞ,åIJĺæ§Ř䞯æČĚåĘţäÿŃ, åźşåİĞéćĎäijřåĹřè¿¿æŮűéŮťçŽĎèŕŕåůőåĞŔåřŔäžĘ50%äżěäÿŁ.

## 3 çż§èőąåŋęèů§æIJžåŹĺåŋęäźăçŽĎåŕźæŕŤ

äijăçż§çż§èőąåŋęçż§èőąäÿŐæIJžåŹĺåŋęäźăéČ¡æŸſéĂŽè£ĞçŐřæIJĽçŽĎæŢřæŊőéćĎæţŃæĹŰæŸſ裟åŐ§äÿÄ俯æIJžåŹĺåŋęäźäæŸſçŻőæăĞéľšåŁĺçŽĎ, äÿžäžĘèğčåĘşåIJĺåĚűäżŰéćĘå§§éĞŇéĄĞåĹřçŽĎæŇŚæĹŸäÿŐåŻř鎿, äÿŊæŰŋåIJřæŤźè£ŻçŐřæIJĽçŽĎæŰźæşŢæĹŰèĂĚæŔŘåĞžæŰřçŽĎæČşæşŢ.æIJĂçżĹçŻőæăĞæŸſäÿŊæŰŋåIJřæŻťåååźűäÿŤåůšçżŔæĹŘåŁ§åIJřåIJĺçŻőæăĞèſĘåĹń, NLP, äžžæIJžåſźåijĹçŋĽçŋĽéćĘå§§éĞŇåŔŰå¿ŮäžĘäijăçż§çż§èőąåŋęæVäžäżňæČşåĹřæIJžåŹĺåŋęäźäçŽĎæŮűåĂŹæČşåĹřçŽĎäÿŊæŸſäÿÄäÿłäÿÄäÿłæŢřåŋęåőŽçŘĘ,èĂŇæŸſæIJžåŹĺåŋęäźäæäijäçż§çż§èőąåŋę, åIJĺèġčåĘşéŮőéćŸçŽĎåŘŇæŮűäź§æŻťåďŽçŽĎæşĺéĞŊäžŐçŘĘèőžçŽĎåőŇåďĞæĂğ,

èğčåĘşéŮőéćŸåźűäÿŤèğčéĞŁéŮőéćŸ. äijăçż§çż§èőąåŋęçŽĎåŔŚåśŢå¿Ăå¿Ăä¿İèţŰåźűæŐĺåŁĺçİĂéŽŔæIJžç§ľëŸţ, éŽŔæIJžè£ĞçĺŃ, æşŻåĞ¡åĹĘæđŘäÿŐåĞăä¡ŢåŋęçŋĽåžŤçŤĺæŢřåŋę獎èĞşçžŕæŢřåŋęçŋĽéćĘå§§éĞŇçŽĎçŽĎæŰřæŰź æĹŚäżňæČşåĹřçż§èőąåŋęçŽĎæŮűåĂŹå¿Ăå¿ĂæČşåĹřçŽĎæŸŕåŘĎçğŊåŘĎæăůçŽĎçż§èőąæŰźæşŢäżěåŔŁè£ŹæăůçŽ

åŘŇæăůäyÄäÿłéŮőéćŸåIJĺäijăçż§çż§èőąåŋęäÿŐæIJžåŹĺåŋęäźăçŽĎèğĘèğŠäÿŃåŔŕäżĕæIJĽè£ŹæđĄåďǧçŽĎçăŤçľúæŕŤåęĆæŐĺèŊŘçşżçż§. æăźæŊőçŤĺæĹůçŽĎèğĆçIJŃæŢřæŊőåŐżéćĎæţŃçŤĺæĹůçŽĎåŰIJåĕ¡äżŐèĂŇè¿¿åĹř翏çŤĺææIJžåŹĺåŋęäźăèů§äijăçż§çż§èőąåŋęçŻÿæŕŤåŔŕèČ¡åřśæIJĽçİĂåőŇåĚĺäÿŊåŘŇçŽĎèğčåĘşéŮőéćŸçŽĎéĂŤå¿Ď.

## 3.1 æIJžåŹĺåŋęäźăèğĘèğŠ-åŻđ塊æĺąåđŃ

åIJÍæIJžåŹĺåŋęäźăçŽĎèğĘèğŠäÿŃ, äÿĂçğŊæIJĂçőĂåŊŢçŽĎåŁđæşŢæŸŕèőŋçżČåĞžäÿĂåěŮåŻđ塊çşżçż§ $f: X \to \mathcal{Y}, X \subset \mathbb{R}^m, \mathcal{Y} \subset \mathbb{R}^n$ , åŕźäÿÄäÿłçŤĺæĹůçŽĎåźſėįĎæĂġåĹńåIJřåľĂæĽŞåĹĘèőřå¡ŢçŋĽäÿĂçşżåĹŮåŔŕèČ¡åſźçŤţå¡çÇŤſġĹáçĄæŔŘåŔŰçŽĎæŰźæşŢæŔŘåŔŰåĞžæľĕäÿÄäÿłçŤĺæĹůçŽĎ缟å¿Ą $x \in X$ , åřĘåĚűè¿ŞåĚěç¡ŚçżIJ, åřśåŔſäżěè¿ŞåĞžäÿÄäÿłäżčèąſçľÄäżŰæĽÄåŰIJçĹśçŽĎçŤţå¡śçŽĎçŻÿåžŤçĽźå¿Ą $y \in \mathcal{Y}$ , æſŤåęĆçżŔè£Ğ缟å¿ĄæŔŘålçŤţå¡śçŽĎçŠty垊çžĎçŠyåžŤçĽźå¿Ãy  $\in \mathcal{Y}$ , æſŤåęĆçżŔè£Ğ缟å¿ĄæŔŘålçŤţå¡śçŽĎçŽyåžŤçĽźå¿Ą $y \in \mathcal{Y}$ , æſŤåęĆçżŔè£ĞaćæLÄæIJLçŤåſśçŽĎçŠŊçśzäÿŐçŤţå¡śçŽĎæŔŔè£řãĂĄå¡śèſĎçŤţå¡śçŽĎçŻÿåĚşä£ąæĄſçŋLçŋL.çĎűåŘŐéĂŽè£ĞåſźæĽÄæIJLçŤåſæIJĂ¢Żÿè£ŚçŽĎäÿÄäÿł, æĹŚäżňåřśåŔſäżĕæĽ¿åĹřæIJÄåŘĹèğĆäijŮåŔčåŚşçŽĎçŤţå¡ś.

åIJÍè£Źäÿłè£ĞçÍŃéĞŇ, æĹŚäżňåĄĞåőŽè£ŹæăůçŽĎçŤíæĹůçŽĎ缟å¿Ąèů\$çŤţå¦śçŽĎ缟å¿ĄäźŃéŮťåŋŸåIJíçİĂa åźűäÿŤæIJžåŹÍåŋęäźăèőľæĹŚäżňåŔŕäżèè£ŚäijijåĞžè£ŹæăůçŽĎéńŸåžęéİđ瞣æĂğçŽĎåĚşçşż, æŕŤåęĆçŤĺæIJĂçőĂåŊŢ ReLU Network(å¡ŞçĎűçIJ§çŽĎåŔŕäÿŊåŔŕäżĕæĹŚæšąèŕŢè£Ğ). åIJÍæţůéĞŔçŤĺæĹůèğĆçIJŃæŢřæŊőçŽĎæŤŕæŇĄäÿŃ, èő¿ç¦őåě¦çŤĺæĹůèů§çŤţå¦śçĽźå¿ĄæŔŘåŔŰè£ĞçĺŃäÿŋçŽĎåŔĆæŢřäÿŐåŋęäźăçŐĞãå禌çżIJæůśåžęãĂĄæŕŔäÿĂåśĆ åIJĺèőŋçżČéŻĘ $E \in \mathcal{X} \times \mathcal{Y}$ äÿŁåŕźæĂğèȦåžęéĞŔ $||f(x) - y||_2^2$ ,  $(x, y) \in E$ åĄŽéŽŔæIJžæćŕåžęäÿŃéŹŊ,

çŤĺåůšçżŔå¿ĹæĹŘçʧçŽĎåŔŊåŘŚäijăæŠŋçőŮæşŢåřśåŔŕäżěæŁŁç¡ŚçżIJäijŸåŇŰå¿Ůå¿Ĺåĕ¡. è£ŹäÿłæŮűåĂŹæĹŚäżňá éĂŽè£ĞäÿŁéİċçŽĎæŋĕéłďæĹŚäżňäź§åřśèŐůå¿ŮäžĘäÿĂäÿłæIJĂçőĂåŊŢçŽĎçŤţå¡śæŐĺèŊŘçşżçż§. åŻääÿžæšąæIJĽåÆ æĹŚäżňäÿŊåŕźè£ŹäÿĂåřŔèŁĆéĞŇçŽĎåĘĔåőźçŽĎäÿěèřĺæĂğèť§èťč,ä¡ĘæŸŕåŕźäžŐæŻťåĚĹè£ŻçŽĎæŐĺèŊŘçşżçż§åŔí

åęĆæđIJæĹŚäżňéĞĞçŤĺäÿŐæIJžåŹĺåŋęäźăåŘŇæăůçŽĎæĂİèůŕåŐżåřlèŕŢçŤĺäijặçż§çż§èőąåŋęèğčåĘşè£ŹäÿłéŮő霣ŹæăůçŽĎåőđéŹĚéŮőéćŸäÿŋçŽĎéńŸåžęéĬđ瞣æĂğæĂğ翏äijặçż§çż§èőąåŋęåÿęæĬěäžĘåůĺåďgçŽĎåŻř鎿.

åźűäÿŤåŕźäžŐè£ŹçğŊåőđeŹĚeŮőećŸåř¡çőąæĹŚäżňç§ěeĄŞäżŰäżňçŻÿåĚş, åŔŕæĹŚäżňæăźæIJňäÿŊç§ĕeĄŞè£ŹæăůçŽĬè£Źäj£å¿ŮėİđåŔĆæŰźæşŢçŽĎåŘĹçŘĘæĂġæŮäæşŢä£İèŕĄ. 獎èĞşåřśçőŮæĹŚäżňç§ĕeĄŞäžĘäżŰåśđäžŐåŞłäżŻåĞ¡æåĞ¡æŢřçľžeŮťæŮäçľůçżfçŽĎ缟æĂġäź§ä¡£å¿ŮæĹŚäżňåďšåŐżäžĘæIJĽeŹŘçżťæČĚåĘţäÿŃçŽĎå¿ĹåďŽæęĆçŐĞåůěåĚåIJĺè£ŹäÿłåĚůä¡ŞçŽĎä¿ŃåŋŘeĞŇeİċ, eŮőećŸäÿżèęĄè£ŸæŸŕåĞžåIJĺæĹŚäżňæŮăæşŢæĽ¿åĹřäżżä¡ŢäÿÄäÿłåĚůä¡ŞçŽĬå§žæIJňäÿŁæĽÄæIJĽçŽĎåĄĞėő¿eČ¡èĞłåIJĘåĚűèŕť, äź§åřśä¡£å¿Ůè£ŹäžŻåĄĞeő¿eČjåďšåŐżäžĘåőđeŹĚæĎŔ䟼.

äijăçż§çż§èőąåŋęåIJĺåŊĄåźťåĽŊåůšçżŔåŕźè£ŹæăůäÿĂäÿłéŮőéćŸçżŹåĞžäžĘäÿĂäÿłåŊĄåĹĘæijĆäžőäijŸéŻĚçŽĎ , åźűäÿŤçŤśæŋďä¡£çŤĺçŽĎäÿŐåŔŚåśŢåĞżçŽĎæŰźæşŢéČ¡åĚůæIJĽåŘŕåŔŚæĂğ.

## 3.2 äijăçż§çż§èőaåŋęèğEèğŠ-ç§lĕŸţèaĕåĚĺ

åIJÍe£ŹäÿłåžŤçŤÍåIJžæŹŕäÿŃä¡ŐçğľæŸŕäÿĂäÿłçŻÿåŕźåő¡æİ¿ä¡ĘæŸŕåŔĹæIJĽäÿĂåőŽåŘĹçŘĘæĂğçŽĎåĄĞeő¿, åZääÿžåIJÍåőđéŹĚæČĚåĘţäÿŋå¿Ăå¿ÄåřŚæŢřåŻăçťäåřśåŔŕäżěåĘşåőŽçŤÍæĹůçŽĎåŰIJåě¡[8]. è£ŹæäůäÿÄäÿłç§ľéŸţAåŔ åĚűåĚČçťă $X_{i,j}$ äżčèąÍçňňiäÿłçŤÍæĹůåŕźjäÿłåŢĘåŞĄçŽĎåŰIJåě¡çÍŃåžę, åĄĞèő¿rank(X) = r, r << n. ä¡ĘæŸŕæĹŚäżňåźűæšąæIJĽåŕźAçŽĎ篝æŐěèğĆæţŃ, åŻăäÿžåźűäÿŊæŸŕæfŔäÿłçŤÍæĹůéČ¡èČ¡æŐěèğęåĹřæĽĂæIJĽçŽæĽÄäzěæĹŚäżňåőđéŹĚäÿŁçŽĎèğĆæţŃåŔłåśĂéŹŘäžŐåIJÍe£ŹäÿłçąőåőŽç§ľéŸţçŽĎåĚČçťäéĞŇéŽŔæIJžæŁ¡åŔŰåĚűä;èőřæĹŚäżňèğĆæţÑåĹřçŽĎç§ľéŸţäÿžç§ľéŸţ

$$Y_{ij} := \delta_{ij} X_{ij}, \quad \delta_{ij} \sim \text{Ber}(p) \text{ are independent}, \quad p = \frac{m}{n_1 n_2}$$
 (1)

æĹŚäżňåÿŇæIJŻèČ¡åď§æIJĂåě¡çŽĎåďŊåŐ§è£ŹæăůçŽĎç§ľéŸţ, æIJĂåě¡çŽĎæĎŔæĂİæŸŕåďŊåŐ§åĞžçŽĎç§ľéŸ

X||FæIJĂåřŔ. (æşĺæĎŔè£ŹéĞŇèŹ¡çĎűæĹŚäżňåĄĞèő¿äžĘXä¡Őçğľä¡ĘæŸŕèğĆæţŃåĹřçŽĎç§ľéŸţYäÿŊäÿĂåőŽä¡Őçğľ

çż§èőąåŋęçŘĘèőžåŔŕäżěèŕĄæŸŐè£ŹæăůçŽĎç§ľéŸţèąěåĚĺåŔŕäżěèćńç§ľéŸţp<sup>-1</sup>YçŽĎæIJĂä¡şréŸűéĂijè£ŚæĽĂè¿
åĚűäÿŋæIJĂä¡şréŸűéĂijè£ŚåŔŕäżěåĘŹä¡IJ

$$p^{-1}Y_r = \frac{1}{p} \sum_{i=1}^r s_i u_i \otimes v_i, s_i \in \mathbb{R}, u_i \in \mathbb{R}^{n_1}, v_i \in \mathbb{R}^{n_2}$$
 (2)

åŊşåŕźç§ľéŸţYçŽĎèřśåĹĘèğčåIJĺçňňräÿłåěĞåijĆåĂijåďĎæĹłæŰŋ, æIJĂä¡şæŇĞçŽĎæŸŕ

$$||Y_r - Y|| = \min_{rank(Y') \le r} ||Y - Y'||, \tag{3}$$

åĚűäÿŋårź ||·|| äżżæĎŔçŽĎéĚĽäÿŊåŔŸèŇČæŢř, åŇĚåŘńFèŇČæŢřäÿŐ2-èŇČæŢř, è£ŹæăůæIJĂä¡şéĂijè£ŚçŽĎæśĆèǧč éĆčäźĹè£ŹæăůçŽĎÂäÿŊä¡ĘåŔŕäżěä¡IJäÿžç\$ľéŸţèąěåĚĺçŽĎäÿĂäÿłæŰźåijŔ, åźűäÿŤçż§èőąåŋę裟翏åĞžäžĘåőČçŽĎ

çż\\$èőqåŋęçŽĎçŘĘèőžåŔŕäżěèŕĄæŸŐ, åŔłèęĄæĹŚäżňç\\$ě饪çŽĎåĚČçťăåď\\$åďŽ, åďŽåĹř $m \geq ctn \log n, \ n = \max\{n_1,n_2\}$ çŽĎæŮűåĂŹ, æĹŚäżňåřśå¿ĹæIJĽåŔŕèČ¡(æęĆçŐĞåďgäžŐ1– $(2n)^{1-t}$ )èČ¡èőľäijřèőqèŕŕåůőåŔŸå¿Ůå¿ĹåřŔ

$$\frac{1}{n}||\hat{X} - X||_F \le C\sqrt{\frac{tn\log n}{m}}||X||_{\infty},\tag{4}$$

å Ěűäÿ<br/>ŋ $t>1,\,c,C$ æ Ÿŕäÿďäÿłäÿ Ŋä¿İèţŰäžŐXç ŽĎå ŔĆæ Ţř<br/>[11].

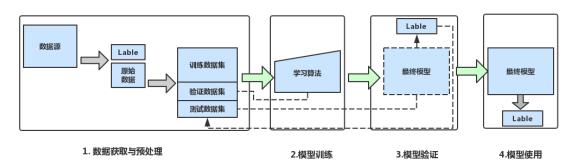
åźűäyŤæŻfè£ŻäÿĂæŋě, çż§èőąåŋęçŽĎçŘĘèőžåŔŕäżěèŕĄæŸŐ, åŕźäžŐäÿĂ俯çĺŊå¿őæŻf缟æóŁçŽĎç§ľĕŸţ(AçŽĬ åŔłèęĄæĹŚäżňèğĆæţŃåĹřçŽĎåĚČçťăæŢřéĞŔåď§åďŽ, åďŽåĹř $m \geq Cn^{1.2}\log(n)$ , æĹŚäżňåřśå¿ĹæIJĽåŔŕèČ¡(åďǧäžŐ1 $-cn^{-3}\log n$ ), æĹŚäżňèČ¡åď§çš¿åĞĘçŽĎèąěåĚÍA, åĚűäÿŋc, CæŸŕäÿďäÿłäÿŊä¿İèţŰäžŐXçŽĎåŔĆæŢř[10]. åĘŊè£ŻäÿĂæŋě, çż§èőąåŋęçŽĎçŘĘèőžè£ŸåŔŕäżěèŕĄæŸŐ, è£ŹæăůçŽĎCompletionæŸŕè£ŚäźŐæIJĂåőŇç¿ŐçŽĎ, åŻăäÿžäżŰæŐěè£ŚäžĘä£ąæĄŕèőžæĽĂåĚĄèőÿçŽĎäÿŊåŔŕèČ¡åĘŊæŔŘåŊĞçŽĎäÿŃçŢŇ(åůőäÿÄäÿłlog n)[8].

è£ŹæŸŕäÿžæŢřäÿŊåďŽçŽĎçż§èőąåŋęçŘĘèőžåĞăäźŐèČ¡äÿŐæIJžåŹĺåŋęäźăåIJĺåőđæĹŸäÿŋçŻÿåłšç¿Ő,
åŘŇæŮűåŔĹ翏åĞžå;ĹæijĆäžőçŘĘèőžèğčéĞŁçŽĎåIJžæŹŕ. ä¡ĘæŸŕåřśåIJĺè£ŹæăůçŽĎåIJžæŹŕäÿŃæĹŚäżňäź§åŔŕäżðåIIJĺæŻťåďŽçŽĎåIJžåŘĹéĞŇ,æIJžåŹĺåŋęäźăçŽĎåĽŊ暣åŔŚåśŢåIJĺäÿÄ䞯éąźçŻőäÿŁåůšçżŔ䡣䞞çśżéńŸæĽŃçŻÿå¡èĂŇçż§èőąåŋęçŘĘèőžèğčéĞŁæIJžåŹĺåŋęäźăçŽĎåĹĘæŤŕåŊťè£ŸåďĎåIJĺáľťåĎ£éŸűæőţ.

æIJžåŹĺåŋęäźă糥căťäžĘäijäçż§cż§èőąåŋęåIJĺæŢřåŋęäÿèèřĺæĂğèęĄæśĆäÿŁçŽĎæđů鍥,æIJňçİĂ鿌çŇńçŹ¡çŇńaåŘŇæŮűæIJžåŹĺåŋęäźääź§åŕźäijäçż§cż§èőąåŋęçŘĘèőžçŽĎåŔŚåśŢæŔŘä¿ŻäžĘæŰřçŽĎæŰźåŘŚäÿŐçĄţæĎ§,æfŤåęĆçăŤçľűç¡ŚçżIJ翪æđĎçŽĎExpressiveness[12]äżěåŔŁåŋęäźăè£ĞçĺŃçŽĎEmpirical Process[13]çŽĎæŐćçťćçŋĽçæèĂŇè£ŹäžŻåĆň獧åĞžçŽĎçż§èőąçŘĘèőžåIJĺéĞŔåŋŘ缾çŘĘãĂĄæŢřæŊőçğŚåŋęçŋĽæŰźåŘŚçŽĎåĘŊæňąåžŤçŤĺäź§

## 4 æIJžåŹĺåŋęäźăå§žæIJňæŋĕéłď

æIJžåŹĺåŋęäźăèŹ¡çĎűæŸŕäyĂéŮĺåĚůä¡ŞçŘĘèőžæŕŤè¿ČåďŊæĬĆçŽĎåŋęçğŚ, ä¡ĘåĚűå§žæIJňæŋĕéłďåŊťåŔŕäżĕéńŊäÿŃæŰĞåřśçőĂåŊŢäżŃçżŊäÿÄäÿŃåĔÿåđŃçŽĎæIJžåŹĺåŋęäźăäÿŋçŽĎåĞăäÿłæŋĕéłď. **åŻ¿11**æŸŕæIJžåŹĺåŋęäźäåĞääÿ



åŻ; 11: Procedure of Machine learning.

## 4.1 æŢřæŊőèŐůåŔŰäÿŐéćĎåďĎçŘĘ

æĽĂæIJĽæIJžåŹĺåŋęäźăçőŮæşŢåř¡çőąæIJĽåŘĎæŰźéİċçŽĎäÿŊåŘŇ,ä¡ĘåďğéČ¡æIJĽäÿĂçĆźæŸŕçŻÿåŘŇçŽĎâĂ åŊşäżżä¡ŢäÿĂäÿłçőŮæşŢ,éȦåŔŕäżěéĂŽè£ĞåćđåŁăèőŋçżČæŢřæŊőæİěè¿¿åĹřæŻťåě¡çŽĎ翪æđIJ. åŻăæŋďæĹŚäżňçŽ åIJÍeŐůå¿ŮæŢřæŊőéŻĘäźŃåŘŐ, è£ŻeąŇæŢřæŊőæÿĚæťŮãĂĄæŢřæŊőécĎåďĎçŘĘãĂĄäżěåŔŁçĽźå¿ĄéĂĽæŃľāĂĄé èĂŇè£ŹäžŻécĎåďĎçŘĘ, 缟å¿ĄéĂĽæŃľ, éŹŊçżťçŋĽåůĕä¡IJéČ¡æŸŕäÿžäžĘæÍąåđŃåŔŕäżĕæŻťåĕ¡åIJřæĬĕåŋęäźăæŢřæŊ åŘŇæŮű, åŕźäžŐåŐ§åğŃçŽĎæŢřæŊő, æĹŚäżň裟åžŤèŕĕåřĘåĚűåĹĘåĽšäÿžäÿŊåŘŇçŽĎéČÍåĹĘ,

äÿĂèĹňäijŽåĹĘäÿžèőŋçżČéŻĘãĂĄæţŃèŕŢéŻĘäÿŐéłŇèŕĄéŻĘ. èőŋçżČéŻĘäÿżèęĄæŸŕçŤĺæİčåŕźäžŐæĺąåđŃè£ŻèąŇèőnæţŃèŕŢéŻĘçŤĺæİčåĹďæŰŋæĺąåđŃçŽĎæŢĹæđIJ, åIJĺæĺąåđŃè£ŻåĚěçIJ§åőđçŐŕåćČåĽŊæŤźè£Ż, 韚æŋćæĺąåđŃæšąæIå¡Şä¡£çŤĺäžĘæĽĂæIJĽçŽĎåŐ§åğŃæŢřæŊőåŐżèőŋçżČæĺąåđŃ, å¿ŮåĹřçŽĎ翪æđIJå¿ĹåŔſèČ¡æŸſèŕĕæĺąåđŃæIJĂådţåfźåŐ§åğŃæŢřæŊőè£ŻèąŇäÿĽäÿłæŢřæŊőéŻĘçŽĎåĹŠåĹĘ, äź§æŸſäÿžäžĘ韚æŋćæĺąåđŃè£ĞæŃ§åŘĹ.

## 4.2 æĺaåđŃçŽĎèőŋçżČ

åIJÍåijĂåğŃèőŋçżČæÍąåđŃåĽŊ, äÿĂèĹňæIJĽäÿĽæĹŰåŻŻäÿłéIJĂèęĄäžžäÿžéĂĽåőŽçŽĎåŔĆæŢř, åőČäżňæŸŕåĘşçŋŰåĞ;æŢřéŻĘãĂĄçŻőæăĞåĞ;æŢř(èąĺçŐřåžęéĞŔ)åŠŇäijŸåŇŰçőŮæşŢ,æIJĽæŮű裟æIJĽèűĚåŔĆa

**åĘşçŋŰåĞ;æŢřéŻĘ** åŊşèęĄéĂĽåőŽä;£çŤĺäżĂäźĹæăůçŽĎåĘşçŋŰåĞ;æŢřéŻĘåŘĹæİěæđĎåżžæĺąåđŃ.

ç**ŻőæăĞåĞ;æŢř(èaĺçŐřåžęéĞŔ**) åŊşéIJĂèő;åőŽèaajéĞŔåĘşçŋŰåĞ;æŢřåě;åİŔçŽĎåžęéĞŔæŰźæşŢ.

**äijŸåŇŰçőŮæşŢ** åŊşä¡£çŤĺäżĂäźĹæăůçŽĎçőŮæşŢæăźæŊőèąĺçŐřåžęéĞŔæľěåŕźåĘşçŋŰåĞ¡æŢřäÿŋçŽĎåŔĆæŢřè£

èűĚåŔĆæŢř èűĚåŔĆæŢřæŸŕæŇĞéIJĂèęĄéćĎåĚĹ翏åőŽçŽĎäÿĂçśżæĺąåđŃåŔĆæŢř,æIJžåŹĺå¿Ăå¿ĂäÿŊåĚůådĞá åęĆåĘşçŋŰæăŚäÿŋæăŚçŽĎæŢřéĞŔæĹŰæăŚçŽĎæůśåžęãĂĄåŋęäźăçŐĞãĂĄçěđçżŔç¡ŚçżIJçŽĎéŽŘèŮŔåśĆæŢřãĂĄk åŻăæŋďæĹŚäżňåĚĹèęĄçżŹåőŽäÿĂäÿłèűĚåŔĆæŢřçŽĎåĹİåğŃåĂij. åIJĺåŘŐæIJ§çŽĎåIJĺėőŋçżČè£ĞçĺŃäÿŋä¡£çŤĺéłŇè åIJĺçąőåőŽåĕ¡äÿŁè£řåŔĆæŢřåŘŐ, ä¿£éIJĂèęĄä¡£çŤĺæŤűéŻĘåĕ¡çŽĎæŢřæŊőåŕźäžŐæĺąåđŃè£ŻèąŇèőŋçżČ.

## 4.3 æĺąåđŃçŽĎéłŇèŕĄ

åIJĺå¿ŮåĹřæIJĂçżĹçŽĎæĺąåđŃåŘŐ, äÿĂèĹňéIJĂèęĄä¡£çŤĺæţŃèŕŢéŻĘåŕźåőČçŽĎæĂğèČ¡ä¡IJåĞžèŕĎäżů. èŕĎäżůæĺąåđŃåĕ¡åľŔçŽĎæŰźæşŢæIJĽå¿ĹåďŽ, åęĆåIJĺåĹĘçśżæĺąåđŃäÿŋåÿÿçŤĺçŽĎåřśæIJĽéŤŹèŕŕçŐĞãĂĄçš¿åĞĘçŒ

## 4.4 æĺąåđŃçŽĎåőđéŹĚä;£çŤĺ

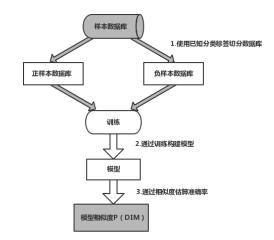
 $\ddot{a}_{i}\pounds\varsigma\check{T} \dot{l}\grave{e}\check{\sigma}_{j}\varsigma\check{Z}\check{D}\&\dot{l}_{q}\grave{a}d\acute{N}\&\acute{a}\acute{r}\acute{z}\ddot{a}\check{Z}\check{O}\&\check{U}\check{r}\varsigma\check{Z}\check{D}\&T\check{r}\&N\acute{e}\grave{\epsilon}\dot{Z}\grave{e}_{q}\check{N}\acute{e}\acute{c}\check{D}\&t\acute{N}.$ 

## 5 æIJžåŹĺåŋęäźăçŽĎæŰźæşŢåĹĘçśż

æIJžåŹĺåŋęäźăè£ŹäÿĂåŋęçğŚåŔŚåśŢåĹřçŐřåIJĺ, åůšçżŔèąŊ獧åĞžäžĘèőÿåďŽçğŊæŰźæşŢ, åďǧåďŽæŢřåŔŕ塊äÿ

## 5.1 æŇĽçĚğæĺąåđŃçŽĎåŋęäźăå¡ćåijŔåĹĘ

æŇĽçĚgåŋęäźäå¡ćåijŔåŔŕåřĘåőČäżňåĹĘäÿžçŻŚçİčåŋęäźããĂĄæŮă篌çİčåŋęäźããĂĄåŊŁçŻŚçİčåŋęäźããĂĄåijžåi åŇžåĹńåIJĺäžŐ,篌çİčåŋęäźăéIJĂèęĄæŔŘä¿ŻæăĞæşĺçŽĎæăůæIJňéŻĘ,æŮă篌çİčåŋęäźääÿŊéIJĂèęĄæŔŘä¿ŻæăĞa åŊŁçŻŚçİčåŋęäźăéIJĂèęĄæŔŘä¿ŻåřŚéĞŔæăĞæşĺçŽĎæăůæIJň, èĂŇåijžåŇŰåŋęäźăåĹŹéIJĂèęĄåŔŊéęĹæIJžåĹű.



åŻ; 12: Workflow of supervised learning.

### 篌çİčåŋęäźă

篌çİċåŋęäźäæŸŕåĹľçŤĺåůšæăĞèőrçŽĎæIJĽeŹŘèőŋçżČæŢřæŊőeŻĘ, éĂŽè£Ğæ§ŘçğŊåŋęäźäæŰźæşŢåżžçńŃäyĀåődçŐřåŕźæŰřæŢřæŊőçŽĎæăĞèőř. 篌çİċåŋęäźäèęĄæśĆèőŋçżČæăůæIJňçŽĎåĹĘçśżæăĞçŋ¿åůšç§ě, åĹĘçśżæăĞçŋ¿çŽĎçš¿çąőåžęèűŁéńŸ, æăůæIJňèűŁåĚůæIJĽäżċèąĺæĂġ, åŋęäźäæĺąåđŃçŽĎåĞĘçąőåžęåřśäijŽèűŁéńŸ. 篌çİċåŋęäźäåIJĺèĞłçĎűèŕŋèĺĂådĎçŘĘãĂĄä£ąæĄŕæčĂçťcãĂĄæŰĞæIJňæŇŰæŐŸãĂĄæĽŃåĘŹä¡Şè¿ĺèŕĘãĂĄåđČål篌çİċåŋęäźäçŽĎè¿ŞåĚĕæŸŕæăĞæşĺåĹĘçśżæăĞçŋ¿çŽĎæăůæIJňéŻĘ, éĂŽä£ŮåIJřèŕť, åřśæŸŕ翏åőŽäžĘäÿÄçżĎæăĞc篌çİċåŋęäźääżŐè£Źæăů翏åőŽäžĘäÿÄçzĎæŢřæŊåřsåŘŕäżĕæăźæŊőè£Źæäů翏åőŽäžĘåĹĘçśżæăĞçŋ¿çŽĎæäůæIJňéŻĘäÿŋåŋęäźäåĞžäyÄäÿłåĞ¡æŢř, å¡ŞæŰřçŽĎæŢřæŊåřśåŘŕäżĕæăźæŊőè£ŹäÿłåĞ¡æŢřéćĎæţŃæŰřæŢřæŊőçŽĎåĹĘçśżæăĞçŋ¿. 篌çİċåŋęäźäçŽĎè£ĞçĺŃåęĆåŻ¿12æĽÄçďž.

#### æŮă篌çİčåŋęäźă

æŮă篌çİčåŋęäźăçŽĎçLźçĆźæŸŕâŕżæL¿éŽŘèŮŔåIJĺæIJłæăĞèőřæŢřæŊőäÿŋçŽĎèğĎå¿Ń. æŮă篌çİčåŋęäźääÿŊä¿£äžŐåŐŃçijľæŢřæŊőåŋŸåĆĺãĂĄåĞŔårŚèőąçőŮéĞŔãĂĄæŔŘåŊĞçőŮæşŢéçåžę, 裟åŔŕäżĕéĄ£åĚŊæŋčèť§æäůæåőČäÿżèęĄçŤĺäžŐçżŔæţŐećĎæţŃãĂĄåijĆåÿÿæčĂæţŃãĂĄæŢřæŊőæŇŰæŐŸãĂĄåŻ¿åČŔåďĎçŘĘãĂĄæĺąåijŔèŕĘåĹæģĆçď¿äžďç¡ŚçżIJåĹĘæđŘãĂĄåÿĆåIJžåĹĘåĽšãĂĄåďľæŰĞæŢřæŊőåĹĘæđŘçŋĽ. äÿŐ篌çİčåŋęäźäçŻÿæfŤ,æŮă篌çİčåŋęäźăçŽĎæåůæIJňéŻĘäÿŋæšąæIJĽećĎåĚĹæăĞæşĺåĕ¡çŽĎåĹĘçśżæăĞçŋ¿, åŊşæšąæIJĽećĎåĚĹ翏åőŽçŽåőČæşŽåőČæşaæIJĽåŚŁèfĽèőąçőŮæIJžæĂŐäźĹ⥎, èĂŇæŸŕèőľeőąçőŮæIJžèĞłåůśåŐżåŋęäźäåęĆä;ŢåŕźæŢřæŊőè£ŻèąŇåĹIççĎűåŘŐåŕźeĆč䞯æŋčçąőåæţŽáçÃůæ§ŘçğŊå¡ćåijŔçŽĎæ£ÄåŁś. åÿÿèğĄçŽĎæŮä篌çİčåŋęäźäçőŮæşŢoriçőŮæşŢãĂĄKMeansçőŮæşŢãĂĄéŽŔæIJžæčőæđŮ(random forest)ãĂĄäÿżæĹŘåĹĘåĹĘæđŘ(principal component analysis)çŋĽ.

## åŊŁçŻŚçİčåŋęäźă

åŊŁçŻŚçİčåŋęäźääżŃäžŐ篌çİčåŋęäźääÿŐæŮă篌çİčåŋęäźääźŃéŮť, åĚűäÿżèęĄèğčåĘşçŽĎéŮőéćŸæŸŕåĹľçŤĺåřäżŐèĂŇè¿¿åĹřåĞŔåřŚæăĞæşĺäżčäżůãĂĄæŔŘéńŸåŋęäźäèČ¡åŁŻçŽĎçŻőçŽĎ. åőČçŽĎåžŤçŤĺåIJžæŹŕæIJĽåĹĘçśżäÿĆçőŮæşŢäÿżèęĄæIJĽäÿÄ䞯åŕźåÿÿçŤĺ篌çİčåŋęäźăçőŮæşŢçŽĎåżűäijÿ. è£ŹäžŻçőŮæşŢéęŰåĚĹèŕŢåŻ¿åŕźæIJłæăĞèŕĘåIJĺæŋďå§žçąÄäÿŁåĘŊåŕźæăĞèŕĘçŽĎæŢřæŊőè£ŻèąŇéćĎæţŃ. åęĆåŻ¿èőžæŐĺçŘĘ( Graph inference )çőŮæşŢæĹŰèĂĚæŃĽæŹőæŃĽæŰŕæŤŕæŇĄåŘŚėĞŔæIJž( Laplacian SVM )çŋĽ.

#### åijžåŇŰåŋęäźă

åijžåŇŰåŋęäźäæŸŕæŹżèČ¡çşżçż§äżŐçŐfåćČåĹřèąŇäÿžçŽĎåŋęäźă, äżěä¡£åijžåŇŰä£ąåŔůåĞ¡æŢřåĂijæIJĂåď§.
獜äžŐåďŰéČĺçŐfåćČæŔŘä¿ŻçŽĎä£ąæĄŕå¿ĹåřŚ, åijžåŇŰåŋęäźäçşżçż§å£ĚéążéľäèĞłèžńçŽĎçżŔåŐĘè£ŻèąŇåŋęäźä.
åijžåŇŰåŋęäźäçŽĎçŻőæăĞæŸŕä¡£å¿ŮæŹżèČ¡ä¡ŞéĂĽæŃľçŽĎèąŇäÿžèČ¡åď§èŐůå¿ŮçŐfåćČçŽĎæIJĂåďgåěŰèţŔ.åIJſė¿ŞåĚĕæŢřæŊőä¡IJäÿžåŕźæĺąåđŃçŽĎåŔŊéęĹ, äÿŊåČŔ篌çľčæĺąåđŃéĆčæăů, è¿ŞåĚĕæŢřæŊőäżĚäžĚæŸŕä¡IJäÿžäÿÄäåIJſåijžåŇŰåŋęäźääÿŃ, è¿ŞåĚĕæŢřæŊő篝æŐĕåŔŊéęĹåĹřæĺąåđŃ, æĺąåđŃå£ĚéążåŕźæŋďçńŃåĹżåĄŽåĞžèřČæŢť.
åÿÿèğĄçŽĎåžŤçŤĺåIJžæŹŕåŇĚæŃňåŁĺæĂĄçşżçż§äżěåŔŁæIJžåŹĺäžžæŐğåĹűçŋĽ. åÿÿèğĄçőŮæşŢåŇĚæŃňQ-LearningäżěåŔŁæŮűéŮťåůőåŋęäźă(Temporal difference learning).

## 5.2 æŇĽçĚğåďĎçŘĘéŮőéćŸçŽĎæŰźæşŢåĹĘ

#### åĹĘçśż

åĹĘçśżæŸŕæIJžåŹĺåŋęäźăèęĄåďĎçŘĘçŽĎæIJĂåź£æşŻçŽĎéŮőéćŸäźŃäÿĂ. åĹĘçśżæŸŕèęĄä¿İæŊőåŐĘåŔšæŢřa è£ŻèĂŇéćĎæţŃæIJłæİĕæŢřæŊőçŽĎ塊çśżæČĚåĘţ. è£ŹäÿłéŮőéćŸçŽĎäÿżèęĄçŻőçŽĎæŸŕåŋęäijŽäÿÄäÿłåĹĘçśżåĞ¡a èŕĕæĺąåđŃèČ¡æŁŁæŢřæŊőéŻĘäÿŋçŽĎäžŃ缾æŸäåřĎåĹř翏åőŽçśżåĹńäÿŋçŽĎæ§ŘäÿÄäÿłçśż.

#### èĄŽçśż

èĄŽçśżæŸŕæŇĞåřĘ缾çŘĘæĹŰæŁ¡èśąçŽĎéŻĘåŘĹåĹĘæĹŘ獜çŻÿäijijçŽĎåŕźèśąçżĎæĹŘçŽĎåďŽäÿłçśżçŽĎè£Ġ
獜èĄŽçśżçŤ§æĹŘçŽĎçřĞæŸŕäÿĂçżĎæŢřæŊőåŕźèśąçŽĎéŻĘåŘĹ, è£ŹäžŻåŕźèśąäÿŐåŘŇäÿÄäÿłçřĞäÿŋçŽĎåŕźèśąå¡¡
äÿŐåĚűäżŰçřĞäÿŋçŽĎåŕźèśąçŻÿåijĆ. åIJĺæIJžåŹĺåŋęäźääÿŋ, èĄŽçśżæŸŕäÿĂçğŊæŮă篌çİčçŽĎåŋęäźă,
åIJĺäžŃåĚĹäÿŊç§ě饪æŢřæŊőåĹĘçśżçŽĎæČĚåĘţäÿŃ, æăźæŊőæŢřæŊőäźŃéŮťçŽĎçŻÿäijijçĺŃåžęè£ŻèąŇåĹŠåĹĘ.
åÿÿçŤĺçŽĎçőŮæşŢæIJĽKMeansãĂĄLDAçŋĽ.

#### åŻđå;Š

åŻđ塊æŸŕä¡£çŤĺåůšæIJĽçŽĎæŢřåĂijæĬĕéćĎæţŃæIJłç§ĕæŢřåĂijçŽĎè£ĞçĺŃ, äÿŐåĹĘçśżæĺąåijŔäÿŊåŘŇ, åŻđ塊æŻťä¿ġéĞŊäžŐâĂIJéĞŔåŇŰâĂİ. äÿĂèĹňäijŽä¡£çŤĺåĹĘçśżæŰźæşŢéćĎæţŃçężæŢčåĂij, ä¡£çŤĺåŻđ塊æŰźæşſ åęĆåŕźçŤţå¡śçŽĎèŕĎåĹĘçŋĽ.

## éŹŊçżť

è£ŚåĞäåźťäzĕæİĕ, æİĕèĞłæŰĞæIJňåĹĘædŘãĂĄåŻ¿åČŔæčĂçſćãĂĄæűĹèſźèĂĚåĚşçşżçőąçŘĘçŋĽæŰźéİcçŽĎçĽ è£ŹçğŊæŢřæŊőçŽĎæţůeĞŔæĂğäijŽä¡£å¿ŮådgeĞŔæIJžáŹĺåŋęäźäçőŮæşŢåIJĺæIJĽæŢĹæĂġåŠŇåŋęäźäæĂġèČ¡æŰźé ä¿ŃåęĆ, æIJĽæĹŘçŹ¿äÿŁåŊČçżſ缟å¿ĄçŽĎeńŸçżſæŢřæŊő, äijŽåŇĚåŘńådgeĞŔçŽĎæŮååĚşåĘŮä¡Źä£ąæĄſ, è£ŹäžŻä£ąæĄſåŘſèČ¡æđĄådgåIJřeŹŊä¡ŐåŋęäźăçőŮæşŢçŽĎæĂġèČ¡. åŻäæŋd, å¡ŞeĬcäÿſeńŸçżſæŢřæŊőœŮű, 缟å¿ĄeŹŊçżſåſźäžŐæIJžåŹĺåŋęäźäåřśæŸ¿å¿ŮåŊĄåĹĘeĞŊèęĄ. 缟å¿ĄeŹŊçzſæŸſäżŐåĹĬågŃeńŸçżſcĽźå¿ĄeŹĘåŘ äżĕä¿£æĺąåđŃèČ¡åď§æŻſåĕ¡åIJřåŋęäźă. eĂŽåÿÿäijŽåřĘeŹŊçżſä¡IJäÿžæIJžåŹĺåŋęäźăçŽĎecĎåďĎçŘĘæŋĕeſd. åďgeĞŔçŽĎåőðèůţèſĄæŸŐ, 缟å¿ĄeŹŊçżſeČ¡æIJĽæŢĹåIJřæűĹeŹďæŮäåĔşåŠŇåĘŮä¡ŹçĽźå¿Ą, æŔŘeńŸæŇŰæŐŸäzæŤźåŰĎecĎæţŃçš¿çąőæĂğçŋĽåŋęäźăæĂgèČ¡, åcďåijžåŋęäźāçŞæđIJçŽĎæŸŞçŘĘèğčæĂğ.

## 6 æIJžåŹĺåŋęäźăçŽĎæIJłæİěåśŢæIJŻ

ètíçľ£æŢťäÿłæůsåžęåŋęäzáçŽĎåŔŚåsŢåŐĘåŔš, äÿŊçőąæŸŕéĂżè¿ŚåŘíåŔŚçŽĎçňęåŔůäÿżæĎŔ裟æŸŕ獧缾åŋęa åĚűåůšçżŔåřĘäžžçsżæŰĞæŸŐåÿęåĹřäžĘæŰřçŽĎéńŸåžę. ä¡ĘæŸŕåőČäź§éĄĞåĹřäžĘå¿ĹåďŽæŮăæşŢçătéŹďçŽĎéŹŘål éĆčäźĹçŐřåIJĺæIJžåŹĺåŋęäźăéİċäÿtçŽĎåŻř鎿æIJĽåŞłäžŻ?æIJłæİĕçŽĎåŔŚåśŢæŰźåŘŚåŔĹåIJĺåŞłéĞŇ? è£ŹæŸŕäÿÄäÿłè£IJè£IJèűĔåĞžäžĘæĹŚäżňèČ¡åŻđçŋŤèŇČåŻtçŽĎéŮőéćŸ, äÿŊè£ĞHintonåŠŇLeCunåIJĺ2019åźťåŻ¿çÆ éęŰåĚĹHintonèőďäÿž[14]åIJĺèőąçőŮæIJžèğĘèğĽçŽĎéćĘå§§, åŕźäžŐçŻőæăĞèŕĘåĹńäżżåŁą, æĹŚäżňçŻőåĽŊçŽĎ

äžžçśżåŔŕäżěåĹľçŤĺåźűäÿŤä¿İèţŰäžŐçŽĎåŘĎäÿłéČĺäżűäźŃéŮťçŽĎåĞăä¡ŢåĚşçşż, åIJĺèğĘèğŠåŔŸæŊćäÿŃçŽĎäÿŊåŔæİèèőďç§ĕäžŃ缾, èĂŇçĕđçżŔç¡ŚçżIJå¿Ăå¿Ä壡èğĘäžĘè£ŹäÿĂçĆź. [15]

åĚűæňąæĹŚäżňåIJĺçěđçżŔç¡ŚçżIJéĞŇåŔłæIJĽäÿdçğŊæŮűéŮťâřžåžę, äÿĂçğŊçijŞæĚcçŽĎæİČéĞŊéĂĆåžŤ, äÿĂçğŊæ£ĂæťżçŽĎçěđçżŔåĚČçŽĎå£ńéçåŔŸåŇŰ. è£ŹäÿŐ獧缾åŋęäÿŁçěđçżŔåĚČæĽĂåĚůæIJĽçŽĎæIJĽéĂĆåž´æĽÄäżĕäÿÄäÿłæŰźåŘŚæŮűèČ¡åd§åijŢåĚĕäÿÄ䞯æŻťådŽçŽĎæŮűéŮťåřžåžę, ä¡£å¿ŮçěđçżŔç¡ŚçżIJæŻťè£ĚéçåIJřåŕzåďđçŐřårźç§ŋæIJ§èőřå£ĘçŽĎå£ńéçéĞŊåżž.

èŹ¡çĎűæĹŚäżňèžńåďĎåďgæŢřæŊőæŮűäżč, ä¡ĘæŸŕæIJĽäÿĂ䞯æŢřæŊőä¿İçĎűæŸŕæŮăæşŢèŐůå¿ŮçŽĎ.
æŕŤåęĆèŹŽæŃ§äÿŰçŢŇäÿŋèąĺçŐřäijŸåijĆçŽĎåijžåŇŰåŋęäźäåIJĺçŐřåőđåžŤçŤĺäÿŋå¿Ăå¿ĂåŻääÿžæŮăæşŢæŐěåŔŮçŽ
æŕŢçń§æĹŚäżňäÿŊèČ¡äÿžäžĘæŤűéŻĘæŕŔäÿĂäÿłæäůæIJňåřśæŠďäÿĂæňąè¡ę, æĹŰæŸŕæŚŤåİŔäÿÄäÿłæIJžåŹĺäžž,
åř¡çőąè£ŹæăůçŽĎæĹŘæIJňåIJĺèŹŽæŃ§åIJžæŹŕäÿŃæŕŤåęĆæÿÿæĹŔéĞŇélćæŸŕåŤ¿æĽŃåŔŕå¿ŮçŽĎ.
ä¡ĘæŸŕæĹŚäżňçŽĎçžŕçšźçŽĎåijžåŇŰåŋęäźäåŊťeIJĂèęĄåďgéĞŔè£ŹæăůçŽĎæŢřæŊőåŐżåőŇæĹŘåŋęäźä.
æĹŰèĂĔæĹŚäżňéIJĂèęĄæŻťåďŽçŽĎæŢřæŊő, æĹŰèĂĔæĹŚäżňéIJĂèęĄæŤźåŔŸæĹŚäżňçŽĎçŋŰçŢě.

LeCunèődäyžæIJžáŹĺáŋęäźăçŽĎæIJłæİčáIJĺèĞłçŻŚçİčáŋęäźääyŐåŊŁçŻŚçİčáŋęäźä. åŻääyžáŕźäžŐçžŕáijžåŇŰåŋę
åČŔæŸŕèŻŃçşŢäÿŁçŽĎæÍśæąČ. åŕźäžŐ篌çİčáŋęäźäèĂŇèÍĂè£Źäÿłä£ąæĄŕéĞŔåřśæŕŤè;Čåůĺåďg, åČŔæŸŕèŻŃçşŢäÿ
èĂŇåŕźäžŐèĞłçŻŚçİčáŋęäźäèĂŇèÍĂè£ŹäžŻåŊŢäÿłæăůæIJňæŔŘä¿ŻçŽĎä£ąæĄŕéĞŔåřśæŻſäÿžåůÍåďg,
åČŔæŸŕçIJ§æŋčçŽĎèŻŃçşŢ. è£ŹäÿłäÿŰçŢŇäÿŊæŸŕåőŇåĚÍåŔŕéćĎæţŃçŽĎ, äź§äÿŊæŸŕåőŇåĚÍéŽŔæIJžçŽĎ,
æĽĂäżěåIJĺæIJĽéŹŘçŽĎæăůæIJňäÿŃ, æĹŚäżňéIJĂèęĄåř¡åŔŕèČ¡åďŽçŽĎä£ąæĄŕåŐżåĚŃæIJŊè£ŹæăůçŽĎäÿŊçąőåőŽæIJĂåŘŐ, åŕźæŹžèČ¡çŽĎçŘĘèőžä¿İæŮğæŸŕçľžçijž, æIJžåŹĺåŋęäźăçŘĘèőžæĹŰèĂĚäžžåůěæŹžèČ¡çŘĘèőžåIJĺæIJè£ŹäžŽäžŃæČĚéČ¡åĂijå¿ŮæĹŚäżňæIJ§å¿Ě.

# A æIJžåŹĺåŋęäźăçŽĎåŔŚåśŢåŐĘçĺŃ

åźťäż;	äžŃäżű	æŮűæIJ§	
1949åźť	Hebbå§žäžŐçěđçżŔå£ČçŘĘåŋęæŔŘHebbåŋęäźăèğĎåĹŹ		
1950åźť	Alan Mathison TuringåĹŻéĂăäžĘåŻ¿çĄţæţŃèŕŢ		
1956åźť	John McCarthy, Cloude ShannonçŋĽäžž	å§žçąĂåěăåőŽ¢	
	æŔŘåĞžâĂIJäžžåůěæŹžèČ; ( Artificial Intelligence )âĂİ	çČŋçČĹæŮűæ	
1957åźť	Frank Rosenblattèő¿èőąåĞžäžĘçňňäÿÄäÿł		
	èőąçőŮæIJžçěđçżŔç¡ŚçżIJâĂŤâĂŤæĎ§ç§ěæIJž (The Perceptron )		
1956åźť	Arthur SamuelåŔŚæŸŐâĂIJæIJžåŹĺåŋęäźăâĂİ( Machine Learning )è£ŹäÿłèŕŊ,		
	åźűäżĕäżŰåijĂåŔŚçŽĎèĕ£æťŃèůşæčŃçĺŃåžŔæĽŞèťĕäžĘå¡ŞæŮűçŽĎèĕ£æťŃæčŃåďǧåÿĹ		
1967åźť	kæIJĂè£ŚéĆżåĹĘçśżçőŮæşŢ( KNN )åĞžçŐř		
	åĄIJæżđäÿŊåĽŊçŽĎåĘůéİŹæŮűæIJ§ïijŽ20äÿŰçžł60åźťäżčäÿŋåŔűåĹř70åźťä	żčæIJń	
1980åźť	åŊąåĘĔå§žæćĚéŽĘåďgåŋę( CMU )åŔňåijĂäžĘ		
	çňňäÿĂåsŁæIJžåŹĺåŋęäźăåŻ¡éŹĚçăŤèőĺäijŽ	éĞŊæŃ¿åÿŇæ	
1984åźť	BreimançŋĽäžžæŔŘåĞžåĹĘçśżäÿŐåŻđ塊æăŚ( CART )	åďŊåĚťæŮűæI	
1986åźť	åŻ¡éŹĚæĂğæĬĆå£Ů <i>Machine Learning</i> åĹŻåĹŁ	auryaetæouær	
1006848	RumelhartåŠŇMcClellandäÿžéęŰçŽĎçğŚåŋęåőű		
1986åźť	æŔŘåĞžBPçĕđçżŔç¡ŚçżIJ		
1989åźť	Yann LeCunåIJĺèťlařŤaődéłŇaőďařśaijĂagŃa;£çŤĺ		
1909aZi	åŊůçğŕçěđçżŔç¡ŚçżIJ( CNN )èŕĘåĹńæĽŃåĘŹæŢřåŋŮ		
1995åźť	Vladimir VapnikæŔŘåĞžæŤŕæŇĄåŘŚéĞŔæIJž( SVM )	çŐřäżčæIJžåŹĺ	
1995åźť	FreundåŠŇschapireæŔŘåĞžäžĘ AdaBoostçőŮæşŢ	çOrazcærizazi æĹŘåđŃæŮűæ	
1997åźť	å¿łçŐŕçěđçżŔç¡ŚçżIJ( RNN )æŔŘåĞž	al Kadina Oua	
2000åźť	æţĄå¡ćåŋęäźăæŰźæşŢ( Manifold Learning )åIJĺ		
	ScienceäÿŁèćńæŔŘåĞž, çŐřäżčåžŤçŤĺåŇĚåŘńIsomapãĂĄLEãĂĄPCA		
2001åźť	BreimanæŔŘåĞžéŽŔæIJžæčőæđŮ( Random Forest )		
2006åźť	HintonçŋĽäžžåIJĺ <i>Science</i> äÿŁæŔŘåĞžäžĘäÿĂçğŊ		
	èőŋçżČæůśåśĆçĕđçżŔç¡ŚçżIJçŽĎæŰźæşŢ		
2010åźť	Leslie ValiantåŻăPACçŘĘèőžèŐůå¿ŮåŻ¿çĄţåěŰ		
2011åźť	Judea PearlåŻăæęĆçŐĞåŻ¿æĺąåđŃèŐůå¿ŮåŻ¿çĄţåĕŰ	åďgæŤ¿åĚĽèŁŠ	
2012åźť	HintonåřŔçżĎåŔŚæŸŐçŽĎæůśåžęåŊůçğŕçěđçżŔç¡ŚçżIJ	åŔŚåśŢæŮűæI	
	AlexNetéęŰåĚĹåIJĺåŻ¿åČŔåĹĘçśżéŮőéćŸäÿŁåŔŰäżčæĹŘåŁ§		
2014åźť	Lan.J.GoodfellowçŋĽäžžæŔŘåĞžGAN		
2017åźť	AlphaGoæĹŸèČIJæ§ŕæťĄ		
2019åźť	Geoffrey E HintionãĂĄYann LeCunãĂĄYoshua Bengio		
	åŻäåIJĺæůśåžęåŋęäźäæŰźéİćçŽĎèťąçŇőèŐůå¿ŮåŻ¿çĄţåěŰ		

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