Using quantum computers to simulate a toy problem of laser-plasma interactions

Flash GAMP

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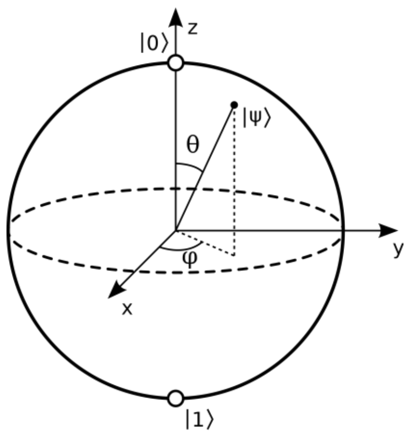
Quantum computing: a promise yet to be fulfilled

- Quantum memory can hold more information
  - Classical computing uses bits: 0 or 1, binary
    Specifying the state of n bits need n numbers, e.g. 101
  - Quantum computing uses qubits: 0 and 1 superpositions
    Specifying the state of n qubits need 2^n numbers, e.g.

$$
\begin{aligned}
|\Psi\rangle = c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle \\
+ c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle
\end{aligned}
$$



Bloch sphere represents superposition of two states

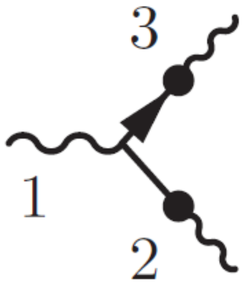
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

- Quantum algorithms may require less operations
  - Ideal quantum computers offer unitary operations
    Classical computers rely on irreversible operations
  - Notable quantum algorithms with exponential speedup:
    Quantum Fourier transform, Shor's algorithm for prime factorization,
    Grover's search, quantum random walk, quantum Hamiltonian simulations…

Idealized quantum algorithms require error correction, not yet operational

Plasma physics may benefit from quantum computing

- Problems in plasma physics are usually nonlinear
  - Lowest order: cubic couplings, common in nonlinear media
  - Examples: laser-plasma interactions, turbulence, nonlinear optics, lattice QED …
  - Classical resonant interactions described by three-wave envelope equations

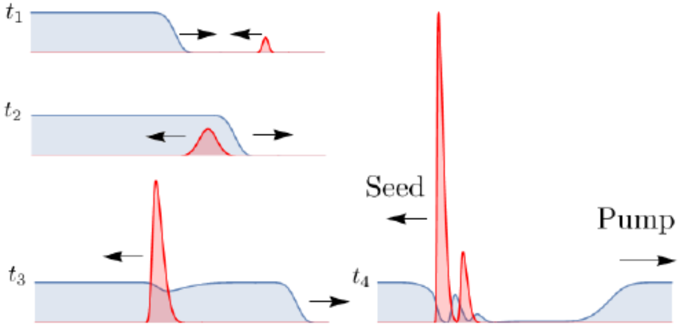


$$\omega_1 = \omega_2 + \omega_3$$
$$\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$$

$$d_t A_1 = g A_2 A_3, \quad d_t A_2 = -g^* A_1 A_3^\dagger, \quad d_t A_3 = -g^* A_1 A_2^\dagger$$

$$d_t = \partial_t + \mathbf{v}_j \cdot \nabla, \qquad \mathbf{v}_j = \partial\omega_j / \partial\mathbf{k}_j$$

Coupling coefficient



  - Quantized version $[A_j, A_l^\dagger] = \delta_{jl}$

    Interaction Hamiltonian $H_I = ig A_1^\dagger A_2 A_3 - ig^* A_1 A_2^\dagger A_3^\dagger$

- Quantum hardware usually lacks native cubic couplings: nonnative

  Can we program cubic interactions on general-purpose quantum computers? –Yes!

Simulating nonnative interaction is challenging

- Standard approach to quantum Hamiltonian simulation
  - Hardware Hamiltonian: determined by device architecture

$$H_0 = \sum_{k=1}^{m} H_k$$

  - Hamiltonian of the physical system: likely different from H_0

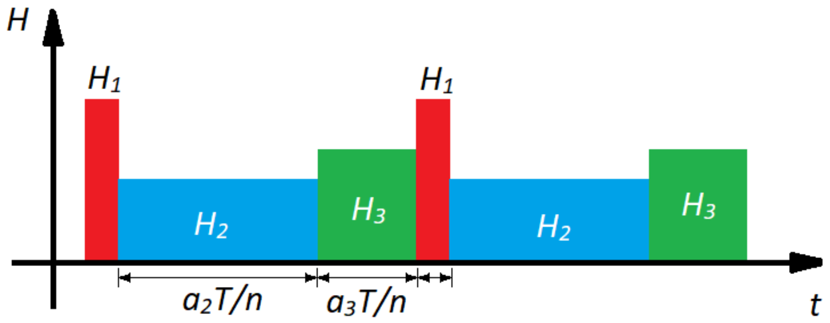
$$H = \sum_{k=1}^{m} a_k H_k$$

  - Lie-Trotter-Suzuki approximation if terms in H are natively available

$$\exp(-iHT) = \lim_{n\to\infty} [\prod_{k=1}^{m} U_k(a_k T/n)]^n$$

$$U_k(t) = \exp(-iH_k t)$$

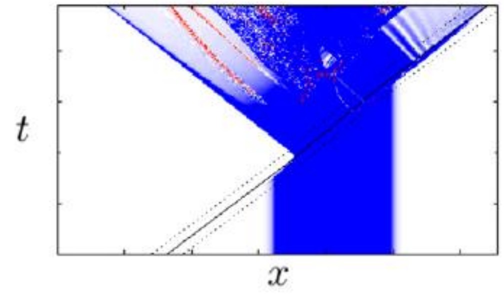
require native H_k



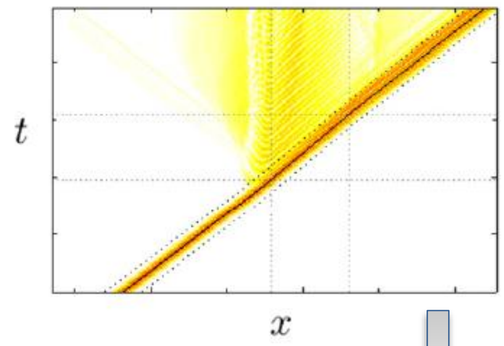
  - What if H contains terms that are nonnative?
    Implement general unitary is exponentially expensive!

It’s difficult! OK, let’s say we can do it. Does it help?

- Example application: laser-plasma interactions
  - Plasma density evolves under drive lasers
  - Laser scattering determined by plasma conditions
- Simulating real-time dynamics
  - Plasma density occupation $|\mathbf{g}(t+\Delta t/2)\rangle = \mathcal{A}(t)|\mathbf{g}(t-\Delta t/2)\rangle$
  - Laser photon occupation $\boxed{|\mathbf{a}(t+\Delta t)\rangle = \mathcal{G}(t+\Delta t/2)|\mathbf{a}(t)\rangle}$
- Sub problem: *D*-level photon occupation
  - Classical: computing next state by matrix multiplication, *O*(D^2) operations
  - Quantum: computing next state by applying cubic gates, *O(1)* operations
    - Need 1-parameter family of cubic gates
    - Initial states simple, readout only needed at final step



Plasma density

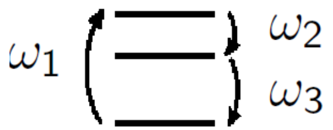


Laser intensity

Cubic gates have overhead, but once precompiled, operations cheap for each time step

Algorithm for cubic problem: mapping in action space

- Naive mapping in **energy space**
  - Direct mapping from resonant levels in energy space to hardware space restricted by
    (1) Tunability of level spacings and coupling
    (2) Unwanted terms in native Hamiltonian
    (3) Inefficient representation: 0 or 1 per qubit



$$|n_j\rangle = \frac{(A_j^\dagger)^{n_j}}{\sqrt{n_j!}}|0\rangle$$

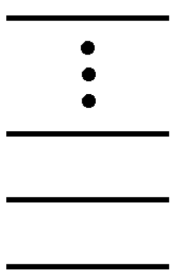
- More versatile mapping in **action space**
  - Action operators commute with Hamiltonian

$$S_2 = n_1 + n_3, \quad S_3 = n_1 + n_2$$

$$[H, S_2] = [H, S_3] = 0$$

  - Simultaneous eigen states of H, S_2 and S_3

$$|\psi\rangle = \sum_{j=0}^{\min(s_2,s_3)} c_j |s_2 - j, s_3 - s_2 + j, j\rangle$$



$$|\psi_j\rangle = |s_2 - j, s_3 - s_2 + j, j\rangle$$

$$|n_1, n_2, n_3\rangle = |n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle$$

Temporal three-wave problem = Hamiltonian simulation

- Occupation amplitudes satisfy Schrödinger equations

$$i\partial_t c_j = igh_{j+\frac{1}{2}}c_{j+1} - ig^* h_{j-\frac{1}{2}}c_{j-1}$$

$$h_{j-\frac{1}{2}} = \sqrt{j(s_2+1-j)(s_3-s_2+j)}$$

$$h_{-\frac{1}{2}} = h_{D+\frac{1}{2}} = 0$$

- Observables can be post processed from occupation probabilities

$$\langle n_1\rangle = \sum_{j=0}^{s_2}(s_2 - j)|c_j|^2$$

$$\langle n_2\rangle = \sum_{j=0}^{s_2}(s_3 - s_2 + j)|c_j|^2$$

$$\langle n_3\rangle = \sum_{j=0}^{s_2} j|c_j|^2$$

- Quantum number operators satisfy Heisenberg equations

$$\begin{aligned}\partial_t^2 n_1 &= -\partial_t^2 n_2 = -\partial_t^2 n_3 \\ &= 2|g|^2\left[s_2 s_3 - (2s_2 + 2s_3 + 1)n_1 + 3n_1^2\right]\end{aligned}$$

- Classical expectation values satisfy slightly different equations

$$\begin{aligned}\partial_t^2\langle n_1\rangle &= -\partial_t^2\langle n_2\rangle = -\partial_t^2\langle n_3\rangle \\ &= 2|g|^2\left[s_2 s_3 - (2s_2 + 2s_3)\langle n_1\rangle + 3\langle n_1\rangle^2\right]\end{aligned}$$

- Quantum system behaves like classical when wave packet is localized, and spontaneous emission is subdominant

Simplest nontrivial case requires *D* = 3 = (1+1/2) qubits

- Readily realizable on hardware for s_2=2 and s_3=s. Hamiltonian matrix tridiagonal

$$h(\theta, s) = \begin{pmatrix} 0 & e^{i\theta}\sqrt{2(s-1)} & 0 \\ e^{-i\theta}\sqrt{2(s-1)} & 0 & e^{i\theta}\sqrt{2s} \\ 0 & e^{-i\theta}\sqrt{2s} & 0 \end{pmatrix}$$

$$h = H/|g|$$
$$\exp(i\theta) = ig/|g|$$

$$|2, s-2, 0\rangle = (1,0,0)^{\mathrm{T}}$$
$$|1, s-1, 1\rangle = (0,1,0)^{\mathrm{T}}$$
$$|0, s, 2\rangle = (0,0,1)^{\mathrm{T}}$$

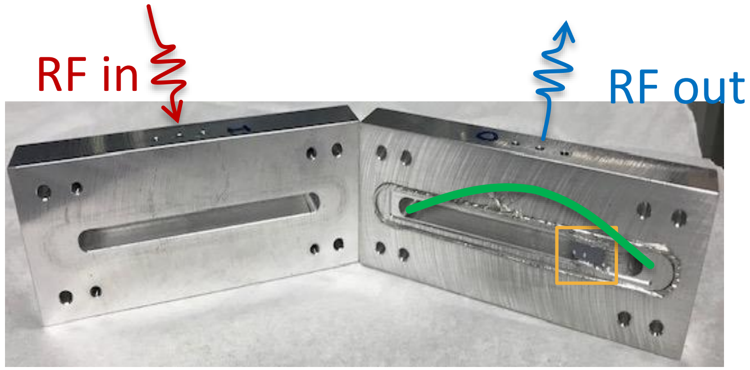
- Solution to 3-level problem known analytically. Unitary to be implemented on hardware

$$U = \begin{pmatrix} \frac{(s-1)\cos\lambda\tau+s}{2s-1} & -ie^{i\theta}\sqrt{\frac{s-1}{2s-1}}\sin\lambda\tau & e^{2i\theta}\frac{\sqrt{s(s-1)}}{2s-1}(\cos\lambda\tau-1) \\ -ie^{-i\theta}\sqrt{\frac{s-1}{2s-1}}\sin\lambda\tau & \cos\lambda\tau & -ie^{i\theta}\sqrt{\frac{s}{2s-1}}\sin\lambda\tau \\ e^{-2i\theta}\frac{\sqrt{s(s-1)}}{2s-1}(\cos\lambda\tau-1) & -ie^{-i\theta}\sqrt{\frac{s}{2s-1}}\sin\lambda\tau & \frac{s\cos\lambda\tau+s-1}{2s-1} \end{pmatrix}$$

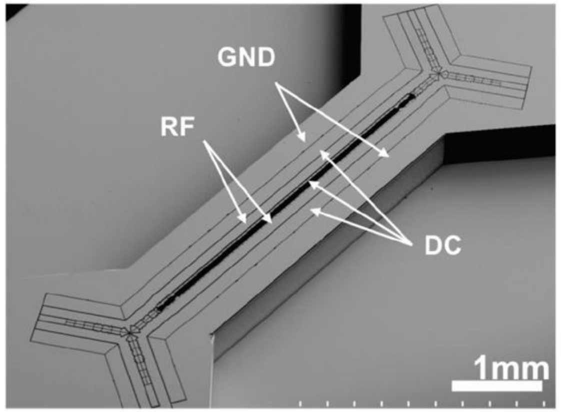
$$U = \exp(-ih\tau)$$
$$\lambda = \sqrt{2(2s-1)}$$
$$U_2 = \begin{pmatrix} U & \mathbf{0}_3^T \\ \mathbf{0}_3 & 1 \end{pmatrix}$$

Many competing device architectures under development

- Leading performers
  - **Superconducting qubits**: nonlinear oscillators controlled by microwave pulses. Fabrication techniques already in use in semiconductor industry
  - **Trapped-ion qubits**: ions trapped on surface and controlled by electrodes, lasers, and microwave. Connection topology potentially more versatile
  - **Photonic qubits**: photon states manipulated by a network of interferometers. Miniaturized and programable phase shifters, beam splitters …
  - Other technologies less mature: neutral atom, quantum dots, nuclear spin …
- Noisy intermediate-scale quantum (NISQ) era: many qubits available, but not fault tolerant …



Superconducting qudit at LLNL



Ion trap at Sandia National Laboratories
“QUANTUM COMPUTING Progress and Prospects”, National Academies of Sciences, Engineering, and Medicine (2019)

Realize cubic gates using standard gates

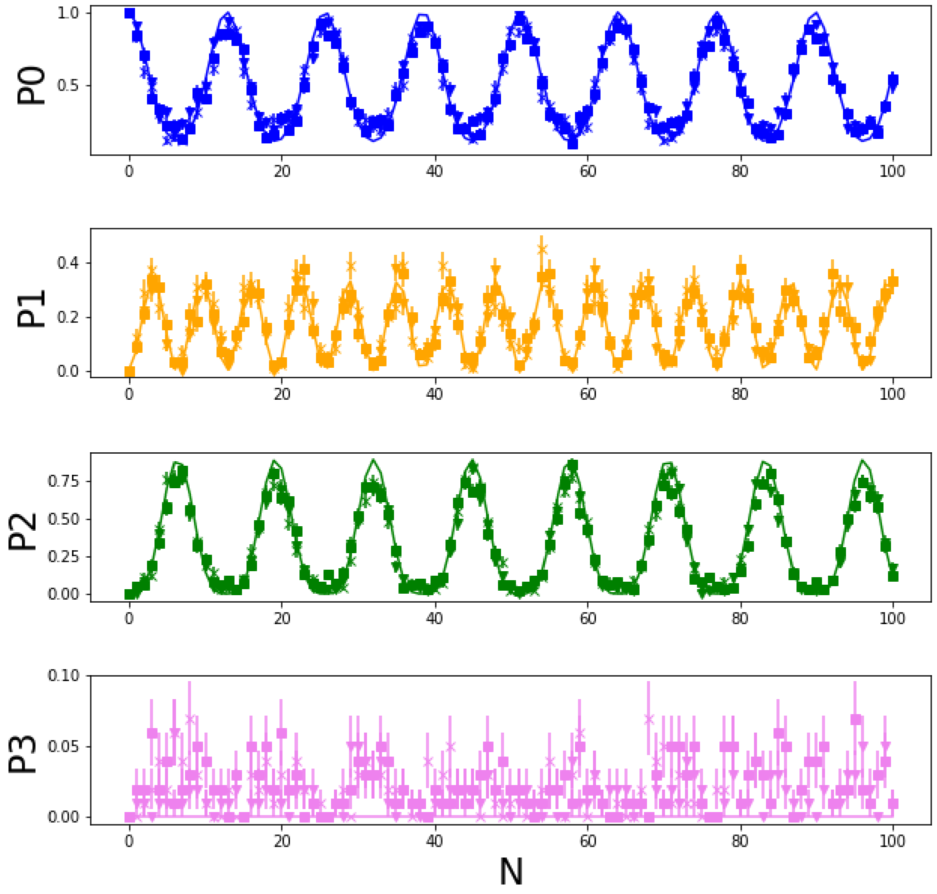
Quantum Cloud Services rigetti

Hardware: superconducting transmon qubits on 2D lattice, utilized Aspen-4-2Q-A

- Imbed 3 levels in 2 qubits:
  Utilize 00, 01, and 10 states
- Control by standard gates, approximate single-step unitary matrix by native gates[1]

```
RZ(pi) 0
RX(1.570796326794897) 0
RZ(-0.9553166181245063) 0
RX(1.5707963267948948) 1
CZ 0 1
RX(pi/2) 0
RZ(0.48989794855663593) 0
RX(-pi/2) 0
RZ(1.42746659829489) 1
RX(pi/2) 1
RZ(0.39864643091397856) 1
RX(-pi/2) 1
CZ 0 1
RZ(-2.186276035465287) 0
RX(pi/2) 0
RZ(-1.7141260552949023) 1
RX(-1.5707963267948928) 1
```

1. arXiv:1608.03355, (2017)

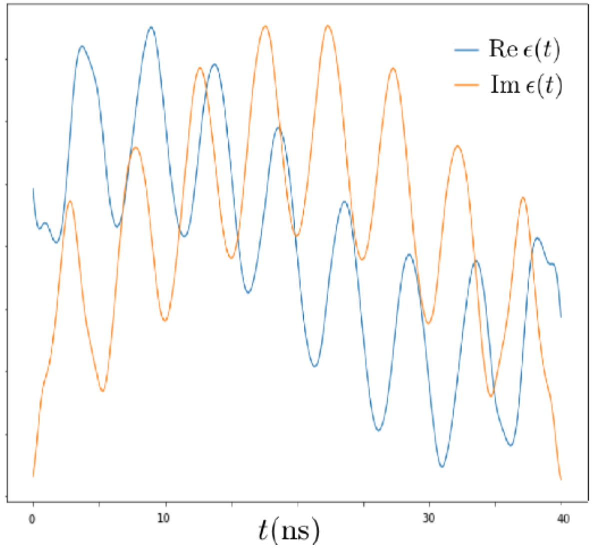


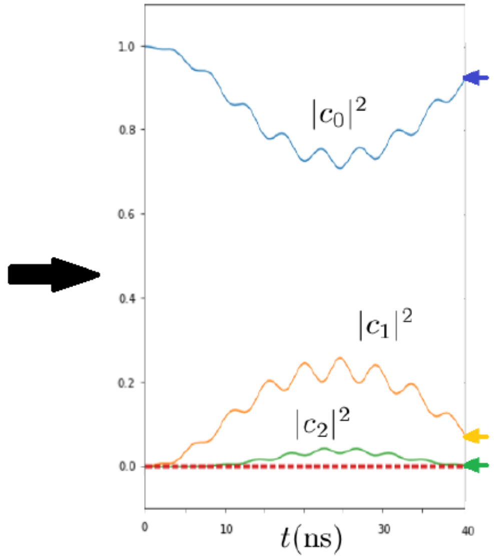
Tracks exact dynamics if allow compiler simplification $[U(\Delta\tau)]^N = U(N\Delta\tau)$

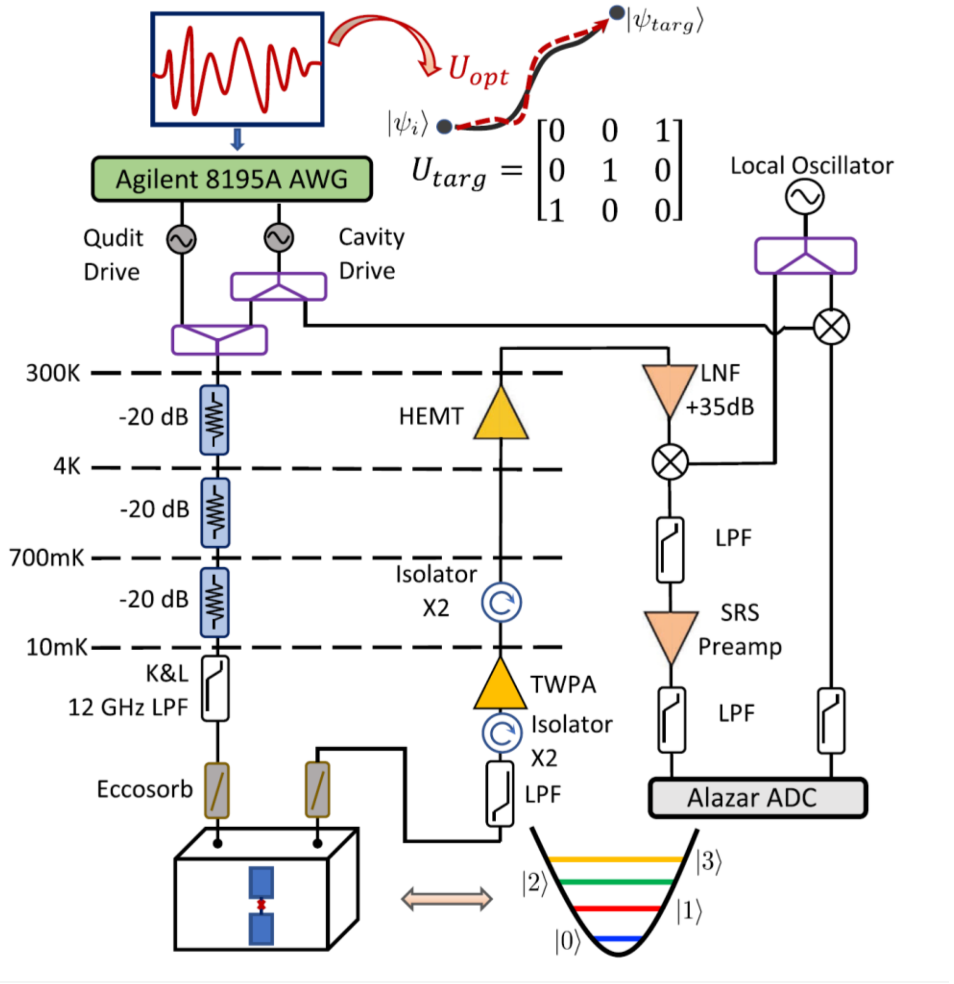
Realize cubic gates using optimal control

LLNL QuDIT[1]

- Hardware: transmon inside a 3D microwave cavity, utilize three levels of a single qudit
- Control by customized waveform, single-step pulse optimized using GRAPE algorithm[2]

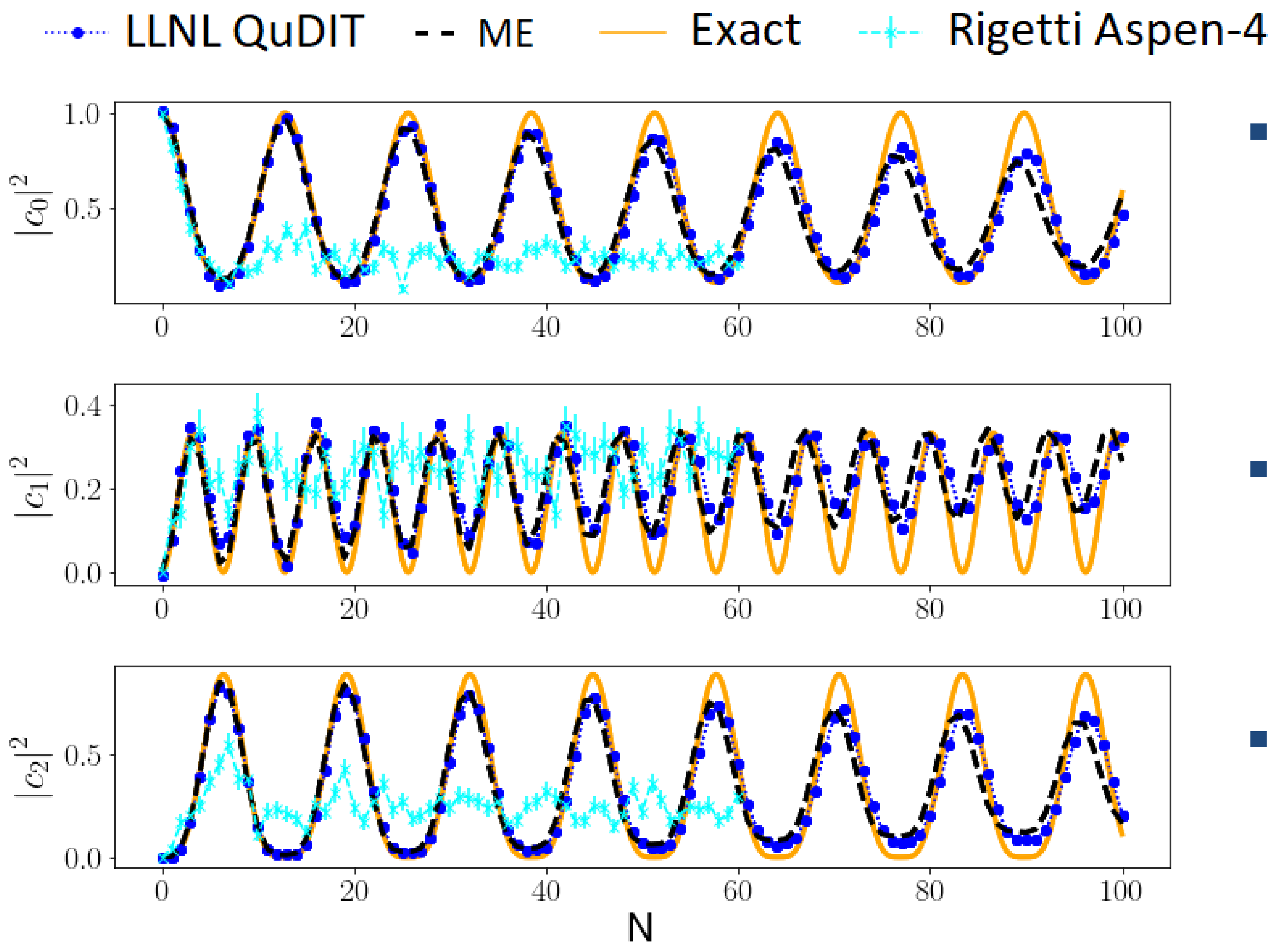






1. Phys. Rev. Lett. 125, 170502 (2020)
2. Comput. Phys. Commun. 184, 1234 (2013)

Repeat precompiled gates *N* times to simulate dynamics



- Long-time evolution captured by customized gate. Results match Master Equation (ME) simulation and are close to Exact solution.
- Short-time dynamics requires shallow gates, sequences of standard gates perform OK
- Decay and dephasing limit the fidelity after ~100 gate repetitions, for both standard/customized gates

Interpolated control pulses can achieve high fidelity

- Numerical optimizations expensive, shortcut by interpolation. Works well for 3-level parametric gates

Target Hamiltonian

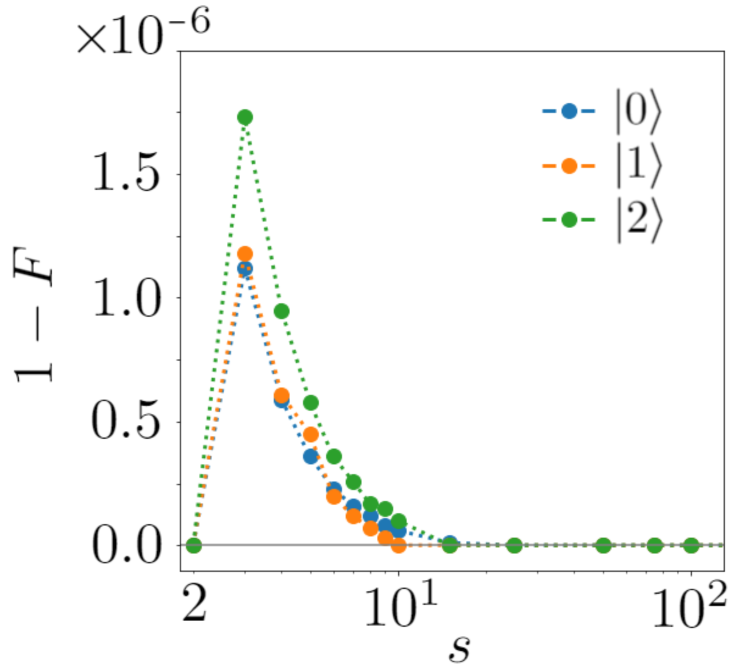
$$h(s) = \frac{\sqrt{2s}}{(1-2^{-1/2})}\left[(1-\xi)K(2) + (\xi - 1/\sqrt{2})K(\infty)\right]$$

$$\xi(s) = \sqrt{1-1/s}$$

$$K(2) = \begin{pmatrix} 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad K(\infty) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Control pulse

$$\boxed{\epsilon_{\mathrm{I}}(s) = [(1-\xi)\epsilon_{\mathrm{O}}(2) + (\xi - 1/\sqrt{2})\epsilon_{\mathrm{O}}(\infty)]/(1-1/\sqrt{2})}$$



Fidelity:

$$F(\rho,\sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

density matrices after applying
σ : optimized pulse
ρ : interpolated pulse

Cubic gates as building blocks for future applications

- Cubic gates can be programmed, no need for native cubic couplings
  Action space mapping reduces nonlinear problem to Hamiltonian simulation
- ➢ **Quantum computers useful for simulating nonlinear interactions**

- Simulations realized on Rigetti Aspen-4 using standard gates, on LLNL QuDIT using customized gates, and both limited by decoherence up to ~100 gates
- ➢ **Compilation using customized gates offer higher fidelity on NISQ hardware**

- Future directions:
  - Generalize mapping to other nonlinear interactions
  - Implement gates on hardware with more qubits/higher fidelity
  - Utilize N-wave gates as building blocks for realistic applications

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