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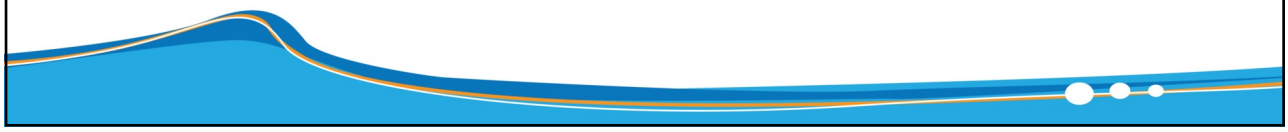
Tree Structures



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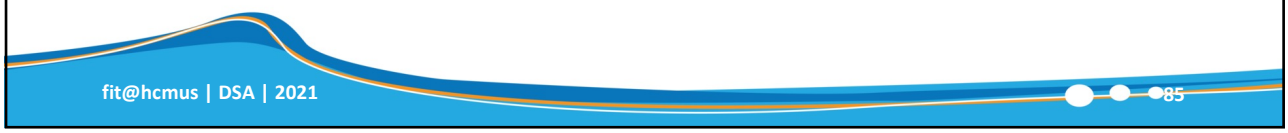
2-3 Tree and 2-3-4 Tree



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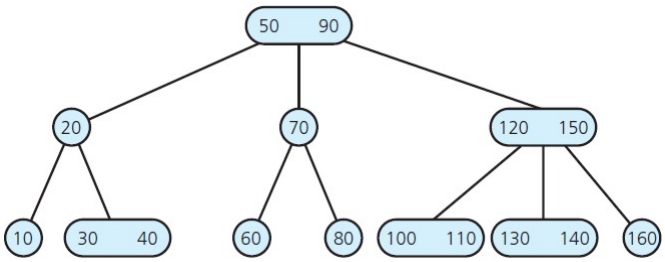
2-3 Tree



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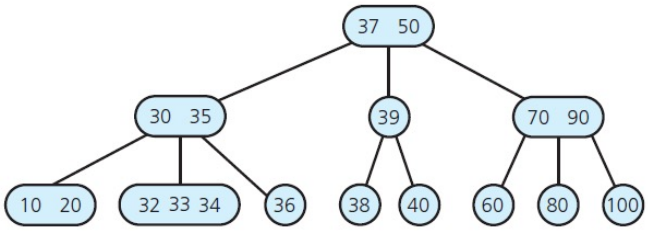
2-3-4 Tree



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2-node, 3-node, 4-node

- A `2-node` (has two children) must contain single data item greater than left child's item(s) and less than right child's item(s).
- A `3-node` (has three children) must contain two data items, `S` and `L`, such that
  - S is greater than left child's item(s) and less than middle child's item(s);
  - L is greater than middle child's item(s) and less than right child's item(s).
- A `4-node` (has our children) must contain three data items `S`, `M`, and `L` that satisfy:
  - S is greater than left child's item(s) and less than middle-left child's item(s)
  - M is greater than middle-left child's item(s) and less than middle-right child's item(s);
  - L is greater than middle-right child's item(s) and less than right child's item(s).

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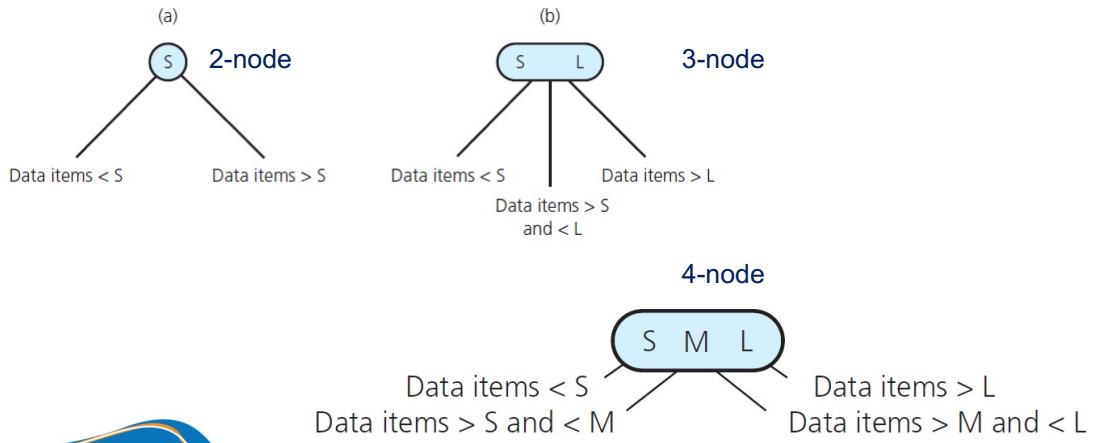
2-node, 3-node, 4-node



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2-3 Tree, 2-3-4 Tree

- A 2-3 tree is not a binary tree. Neither is a 2-3-4 tree.
- A 2-3 tree, a 2-3-4 tree are never taller than a minimum-height binary tree.

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2-3 Tree

- Invented by John Hopcroft in 1970.
- 2-3 tree is a tree in which
  - Every internal node is either a 2-node or a 3-node.
  - Leaves have no children and may contain either one or two data items.

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2-3-4 Tree

- 2-3-4 tree is a tree in which
  - Every internal node is a 2-node, a 3-node or a 4-node.
  - Leaves have no children and may contain either one, two or three data items.

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Insertion Operation

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Inserting Data into a 2-3 Tree

```
// Inserts a new item into a 2-3 tree whose items are distinct and differ from the
// new item.
insertItem(23Tree: TwoThreeTree, newItem: ItemType)

   Locate the leaf, leafNode, in which newItem belongs
   Add newItem to leafNode

   if (leafNode has three items)
      split(leafNode)

// Splits node n, which contains two items. Note: If n is
// not a leaf, it has four children.
split(n: TwoThreeNode)

   if (n is the root)
      Create a new node p
   else
      Let p be the parent of n

   Replace node n with two nodes, n1 and n2, so that p is their parent
   Give n1 the item in n with the smallest value
   Give n2 the item in n with the largest value
```


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Inserting Data into a 2-3 Tree

```
split(n: TwoThreeNode)

   if (n is the root)
      Create a new node p
   else
      Let p be the parent of n

   Replace node n with two nodes, n1 and n2, so that p is their parent
   Give n1 the item in n with the smallest value
   Give n2 the item in n with the largest value

   if (n is not a leaf)
   {
      n1 becomes the parent of n's two leftmost children
      n2 becomes the parent of n's two rightmost children
   }
   Move the item in n that has the middle value up to p
   if (p now has three items)
      split(p)
```


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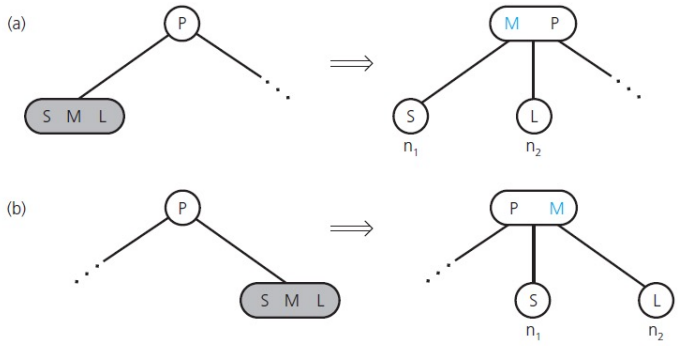
Inserting Data into a 2-3 Tree

- Splitting a leaf in a 2-3 tree when the leaf is a
  - (a) left child; (b) right child



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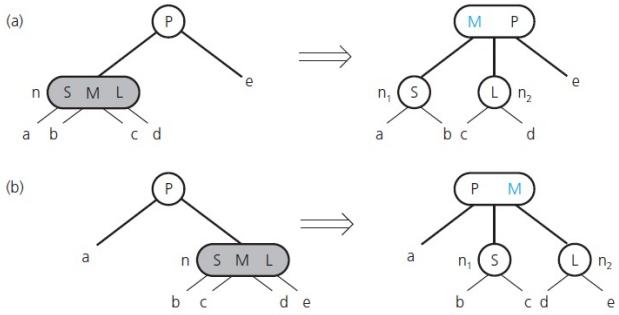
Inserting Data into a 2-3 Tree

- Splitting an internal node in a 2-3 tree when the node is a (a) left child; (b) right child



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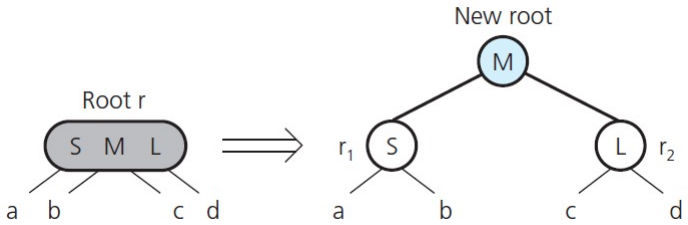
Inserting Data into a 2-3 Tree

- Splitting the root of a 2-3 tree



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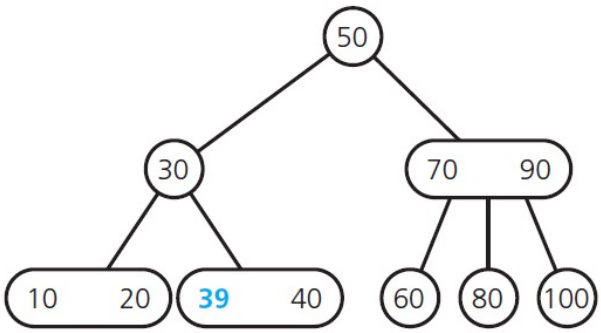
Examples

- After inserting 39 into the tree



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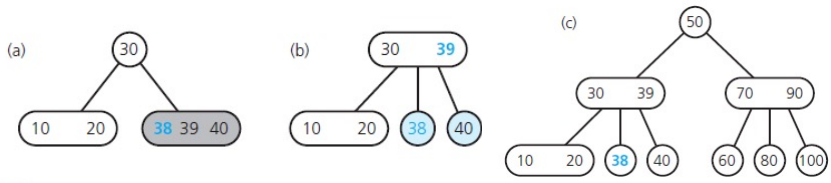
Examples

The steps for inserting 38 into the tree

(a) The located node has no room;

(b) the node splits;

(c) the resulting tree



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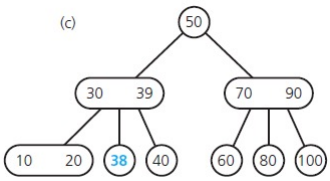
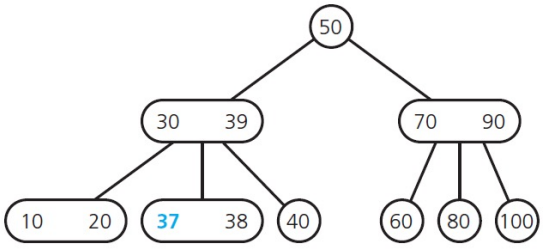
Examples

After inserting 37 into the tree



Original tree



Resulting tree

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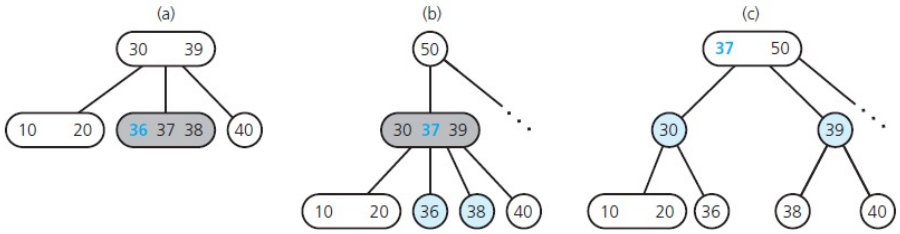
Examples

The steps for inserting 36 into the tree



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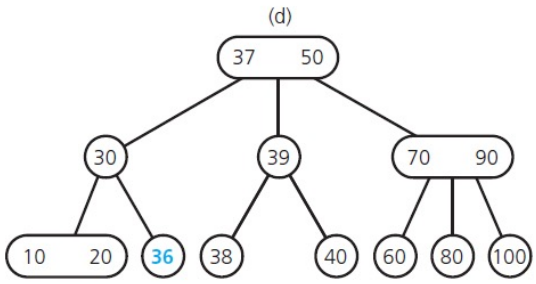
Examples

the resulting tree



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Inserting Data into a 2-3-4 Tree

- Insertion algorithm splits a node by moving one of its items up to its parent node
- Splits 4-nodes as soon as it encounters them on the way down the tree from the root to a leaf

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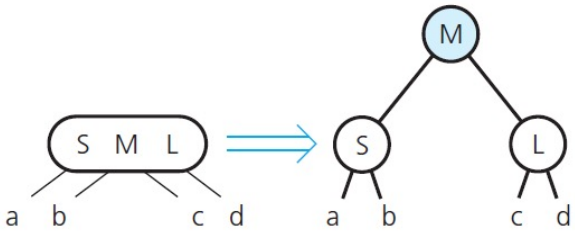
Inserting Data into a 2-3-4 Tree

- Splitting a 4-node root during insertion into a 2-3-4 tree



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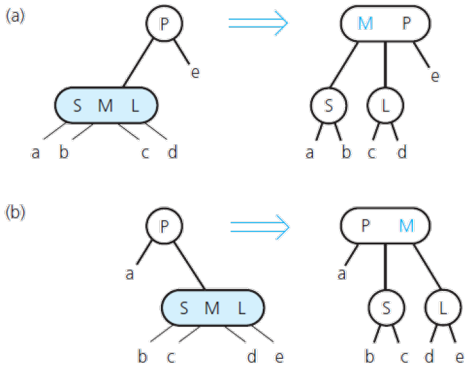
Inserting Data into a 2-3-4 Tree

- Splitting a 4-node whose parent is a 2-node during insertion into a 2-3-4 tree, when the 4-node is a (a) left child; (b) right child



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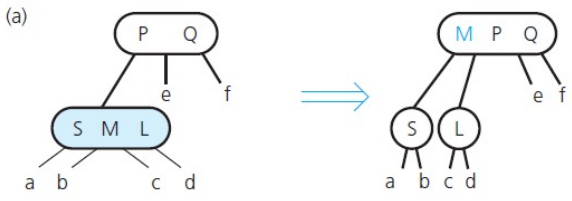
Inserting Data into a 2-3-4 Tree

- Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (a) left child



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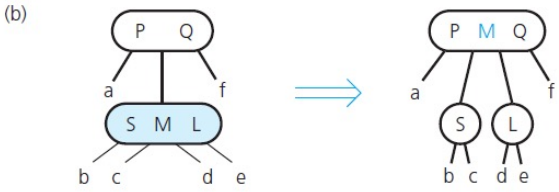
Inserting Data into a 2-3-4 Tree

- Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (b) middle child



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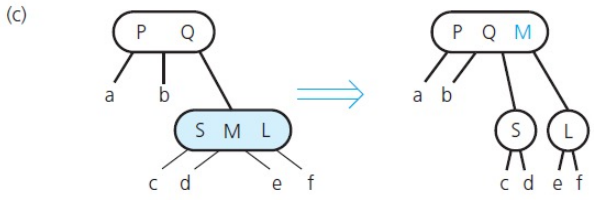
Inserting Data into a 2-3-4 Tree

- Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (c) right child



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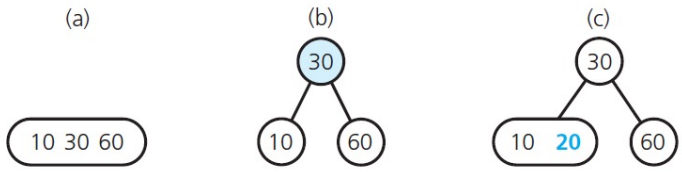
Examples

- Inserting 20 into a 2-3-4 tree

  (a) the original tree;

  (b) after splitting the node;

  (c) after inserting 20



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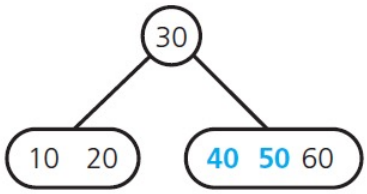
Examples

- After inserting 50 and 40 into the tree



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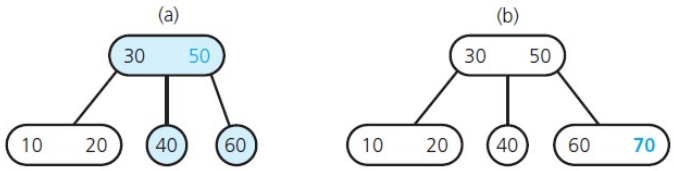
Examples

- The steps for inserting 70 into the tree

  (a) after splitting the 4-node;

  (b) after inserting 70



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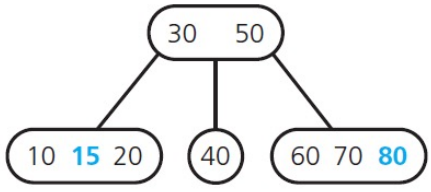
Examples

- After inserting 80 and 15 into the tree



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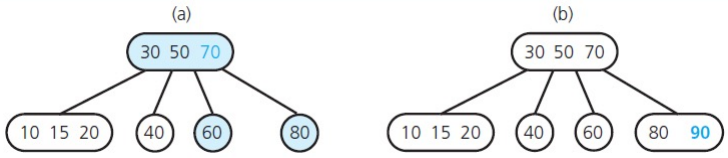
Examples

- The steps for inserting 90 into the tree



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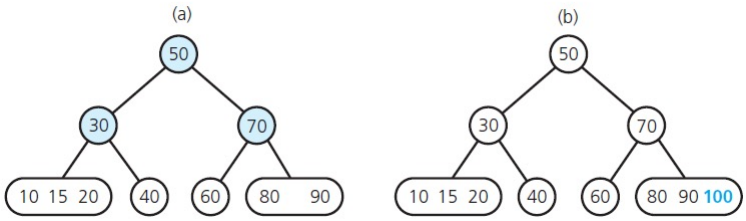
Examples

- The steps for inserting 100 into the tree



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Removal Operation

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Removing Data from a 2-3 Tree

```
// Removes the given data item from a 2-3 tree. Returns true if successful
// or false if no such item exists.
removeItem(23Tree: TwoThreeTree, dataItem: ItemType): boolean

   Attempt to locate dataItem
   if (dataItem is found)
   {
      if (dataItem is not in a leaf)
         Swap dataItem with its inorder successor, which will be in a leaf leafNode

      // The removal always begins at a leaf
      Remove dataItem from leaf leafNode
      if (leafNode now has no items)
         fixTree(leafNode)
      return true
   }
   else
      return false
```


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Removing Data from a 2-3 Tree

```
    else
        return false

// Completes the removal when node n is empty by either deleting the root,
// redistributing values, or merging nodes. Note: If n is internal, it has one child.
fixTree(n: TwoThreeNode)

    if (n is the root)
        Delete the root
    else
    {
        Let p be the parent of n
        if (some sibling of n has two items)
        {
            Distribute items appropriately among n, the sibling, and p
```


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Removing Data from a 2-3 Tree

```
            if (n is internal)
                Move the appropriate child from sibling to n
        }
        else // Merge the node
        {
            Choose an adjacent sibling s of n
            Bring the appropriate item down from p into s
            if (n is internal)
                Move n's child to s
            Remove node n
            if (p is now empty)
                fixTree(p)
        }
    }
```


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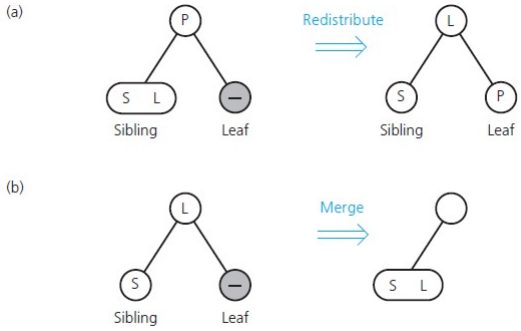
Removing Data from a 2-3 Tree

(a) Redistributing values;

(b) merging a leaf;



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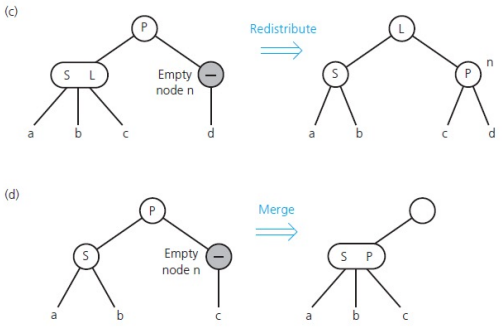
Removing Data from a 2-3 Tree

(c) redistributing values and children

(d) merging internal nodes



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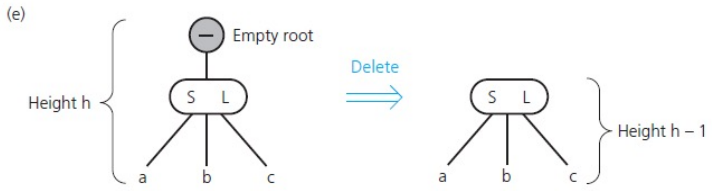
Removing Data from a 2-3 Tree

(e) deleting the root



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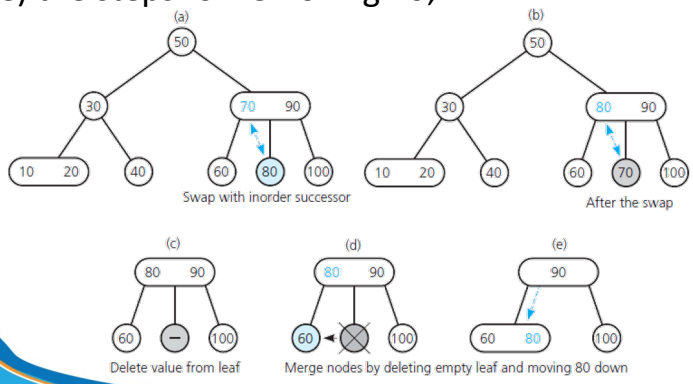
Examples

(a) A 2-3 tree;

(b), (c), (d), (e) the steps for removing 70;



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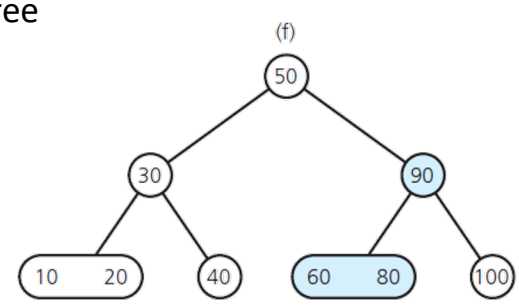
Examples

- the resulting tree



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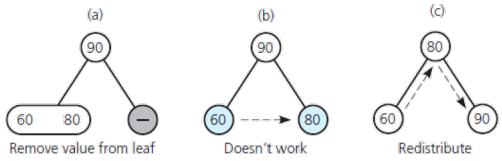
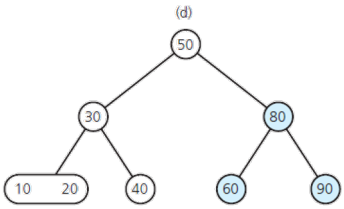
Examples

(a), (b), (c) The steps for removing 100 from the tree;

(d) the resulting tree





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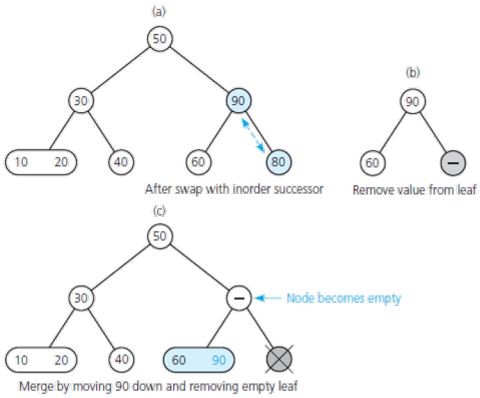
Examples

The steps for removing 80 from the tree



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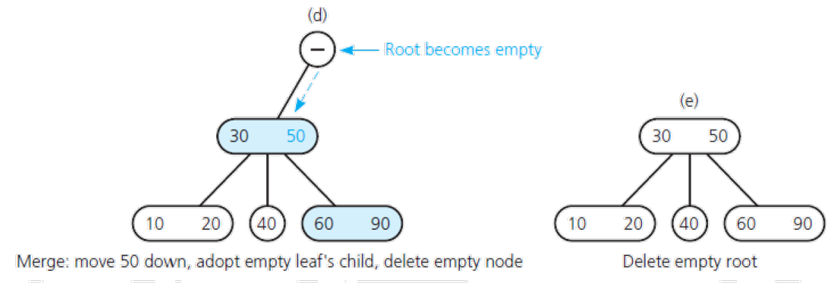
Examples

The steps for removing 80 from the tree



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Removing Data from a 2-3-4 Tree

- Removal algorithm has same beginning as removal algorithm for a 2-3 tree
- Locate the node `n` that contains the item `I` you want to remove.
- Find `I` 's in-order successor and swap it with `I` so that the removal will always be at a leaf.
- If leaf is either a 3-node or a 4-node, remove `I` .