ISLAMIC UNIVERSITY

Department of CSE

Course Title: Geometry and Vector Analysis
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Assignment on Chapter 1 & 2

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1. Find the areas of triangles whose vertices are
(a) $(-3,4)$**,** $(6,2)$**,** $(4,-3)$
(b) $(a,b+c)$**,** $(b,c+a)$**,** $(c,a+b)$

2. If three points $(-1,2),(2,-1),(h,3)$ **are collinear ; show that** $h=-2$

3. If the area of quadriatel, whose angular points A,B,C,D **taken in order are** $(1,2),(-5,6),(7,-4),(k,-2)$ **be zero find the value of** k**.**

4. Show that three points $(4,2),(7,5),(9,4)$ **lie on a right line.**

5. If the area of the quadrilateral formed by the points $(1,3),(2,-5),(6,-2)$ **and** $(5,k)$ **taken in order be** 30 **, show that** $k=4$ **.**

6. Find the coordinates of the orthocentre of the triangle whose points are $(1,0),(2,-4)$ **and** $(-5,-2)$ **.**

7. If the points $(a,b),(a',b')$ **and** $(a-a',b-b')$ **are colinear, show that their joint passes through the origin and that** $ab'=ab'$ **.**

8. In what ratio is the straight line joining the pts. $(3,4)$ **and** $(8,1)$ **divides by the** X **axis. Find the abscissa of this point on** X **axis.**

9. Find the polar co-ordinates of the point whose cartesian co-ordinates are
(a) $(-\sqrt{3},1)$
(b) $(5,12)$

10. Find the cartesian co-ordinates of the points whose polar co-ordinates are
(a) $(5,-\frac{\pi}{4})$
(b) $(2,330^\circ)$

11. Find the polar distance between the points whose polar co-ordinates are
(a) $(-3,45^\circ)$ **and** $(7,105^\circ)$
(b) $(\sqrt{2},\frac{5\pi}{4})$ **and** $(2,\frac{2\pi}{3})$

12. Find the area of triangle , the polar co-ordinates of whose angular points are $(a,\theta),(2a,\theta+\frac{\pi}{3}),(3a,\theta+\frac{2\pi}{3})$

13. Change the equations to polar co-ordinates
(a) $(x^2+y^2)^2=2a^2xy$
(b) $x^4+x^2y^2-(x+y)^2=0$

14. Transform the polar co-ordinates to equations
(a) $r^2-2r(\cos\theta-\sin\theta)-7=0$
(b) $r(1-e\cos\theta)=ep$

1. (a)

Here given :

$$x_1 = -3, y_1 = 4$$
$$x_2 = 6, y_2 = 2$$
$$x_3 = 4, y_3 = -3$$
$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|$$
$$= \frac{1}{2}|(-6 - 24) + (-18 - 8) + (16 - 9)|$$
$$= 24.5$$

(b)

Here given :

$$x_1 = a, y_1 = b + c$$
$$x_2 = b, y_2 = c + a$$
$$x_3 = c, y_3 = a + b$$
$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|$$
$$= \frac{1}{2}|[a(c + a) - b(b + c)] + [b(a + b) - c(c + a)] + [c(b + c) - a(a + b)]|$$
$$= 0$$

2.

Here given :

$$x_1 = -1, y_1 = 2$$
$$x_2 = 2, y_2 = -1$$
$$x_3 = h, y_3 = 3$$

If these three points are collinear , The area made by these points must be zero. So,

$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|$$
$$= \frac{1}{2}|(1 - 4) + (6 + h) + (2h + 3)|$$
$$= 3h + 6$$

Now,

$$3h + 6 = 0$$
$$\Longrightarrow h = -2$$

(Showed)

3.

Here given :

$$\text{A} = (1, 2)$$
$$\text{B} = (-5, 6)$$
$$\text{C} = (7, -4)$$
$$\text{D} = (k, -2)$$

Now,

$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)|$$
$$= \frac{1}{2}|(6 + 10) + (20 - 42) + (-14 + 4k) + (2k + 2)|$$
$$= \frac{1}{2}(6k - 18)$$
$$= 3k - 9$$

Now,

$$3k - 9 = 0$$
$$\implies k = 3$$

4.

Here given :

$$x_1 = 4, y_1 = 2$$
$$x_2 = 7, y_2 = 5$$
$$x_3 = 9, y_3 = 7$$

In case these points lie on a straight line, the area made by these points is Zero.

Now,

$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|$$
$$= (20 - 14) + (49 - 45) + (18 - 28)$$
$$= 6 + 4 - 10$$
$$= 0$$

Therefore, these points lie on a straight line.

5.

Here given :

$$\text{A} = (1, 3)$$
$$\text{B} = (2, -5)$$
$$\text{C} = (6, -2)$$
$$\text{D} = (5, k)$$

Now,

$$\text{Area} = \frac{1}{2}|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)|$$

$$= \frac{1}{2}|(-5-6)+(-4+30)+(6k+10)+(15-k)|$$
$$= \frac{1}{2}(5k+40)$$

Now,

$$= \frac{1}{2}(5k+40) = 30$$
$$\implies k = 4$$

$$(showed)$$

6.

Let,

$$\text{A} = (1,0)$$
$$\text{B} = (2,-4)$$
$$\text{C} = (-5,-2)$$

The orthocenter is the intersection point of the altitudes drawn from the vertices of the triangle to the opposite sides.

Now, equation to **BC** is

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$
$$\implies \frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$
$$\implies \frac{y+4}{-4+2} = \frac{x-2}{2+5}$$
$$\implies 2x+7y+24=0$$
$$....(i)$$

Equation of Perpendicular to **BC** from **A** is

$$7x-2y+k=0$$
$$\text{This line goes through the point } (1,0)$$
$$\text{So,}7(1)-2(0)+k=0$$
$$\implies k=-7$$
$$\text{And the equation becomes, } 7x-2y-7=0$$
$$....(ii)$$

Similarly, Equation of **AC** is

$$4x=y-4=0$$
$$....(iii)$$

And Equation of Perpendicular to **AC** from **B** is

$$x-4y-3=0$$
$$....(iv)$$

From equation (ii) and (iv) we get ,

$$\text{The intersection point is } = (\frac{11}{13}, \frac{-7}{13})$$

This point is the orthocentre of the triangle.

7.

Here three points are

$$(a, b)$$

$$(a^{'}, b^{'})$$

$$\text{and } (a - a^{'}, b - b^{'})$$

If three points are collinear, they lie on the same line.

The join of (a, b) and $(a^{'}, b^{'})$ is

$$\frac{x - a}{a - a^{'}} = \frac{y - b}{b - b^{'}}$$

Since it passes through $(a - a^{'}, b - b^{'})$

$$\implies \frac{a - a^{'} - a}{a - a^{'}} = \frac{b - b^{'} - b}{b - b^{'}}$$

$$\implies ab^{'} = a'b$$

so, $x(b - b^{'}) = y(a - a^{'})$, which passes through the origin.

8.

Let, the line joining the points $(3, 4)$ and $(8, 1)$ divided by the x-axis at $(x_1, 0)$ in the ratio $(k : 1)$.

$$\text{so, } \frac{k.y_2 + 1.y_1}{k + 1} = 0$$

$$\implies \frac{k.1 + 4}{k + 1} = 0$$

$$\implies k + 4 = 0$$

$$\implies k = -4$$

That means the required ratio is $-4 : 1$.

Now,

$$x = \frac{k.x_2 + 1.x_1}{k + 1}$$

$$\implies x = \frac{-4.8 + 3}{-4 + 1}$$

$$\implies x = \frac{29}{3}$$

This is the abscissa of this point on X axis.

9. **(a)**

Here the given cartesian co-ordinate is $(-\sqrt{3}, 1)$
Now,

$$r = \sqrt{x^2 + y^2}$$
$$\implies r = \sqrt{(\sqrt{3})^2 + 1^2}$$
$$\implies r = 2$$

And,

$$\tan\theta = \frac{y}{x}$$
$$\implies \tan\theta = \frac{1}{-\sqrt{3}}$$
$$\implies \theta = \frac{5\pi}{6}$$

So, the polar co-ordinate is $(2, \frac{5\pi}{6})$.

(b)

Here the given cartesian co-ordinate is $(5, 12)$
Now,

$$r = \sqrt{x^2 + y^2}$$
$$\implies r = \sqrt{5^2 + 12^2}$$
$$\implies r = 13$$

And,

$$\tan\theta = \frac{y}{x}$$
$$\implies \tan\theta = \frac{12}{5}$$
$$\implies \theta = \tan^{-}(\frac{12}{5})$$

So, the polar co-ordinate is $(13, \tan^{-}(\frac{12}{5}))$.

10. **(a)**

Here the given polar co-ordinate is $(5, -\frac{\pi}{4})$
Now,

$$x = 5\cos(-\frac{\pi}{4})$$
$$\implies x = \frac{5}{\sqrt{2}}$$

And,

$$y = 5\sin(-\frac{\pi}{4})$$
$$\implies y = -\frac{5}{\sqrt{2}}$$

The cartesin co-ordinates are $(\frac{5}{\sqrt{2}}, -\frac{5}{\sqrt{2}})$.

(b)

Here the given polar co-ordinate is $(2, 330°)$

Now,

$$x = 2\cos(330°)$$
$$\implies x = 2\cos(360° - 30°)$$

$$\implies x = 2\cos(30°)$$
$$\implies x = \sqrt{3}$$

And,

$$y = 2\sin(330°)$$
$$\implies y = 2\sin(360° - 30°)$$

$$\implies y = -2\sin(30°)$$
$$\implies y = -1$$

The cartesin co-ordinates are $(\sqrt{3}, -1)$.

11. **(a)**

Given polar co-ordinates are

$$(-3, 45°) \text{ and } (7, 105°)$$

Now,

$$\begin{aligned}\text{Distance } &= \sqrt{(r_1)^2 + (r_2)^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ &= \sqrt{(-3)^2 + (7)^2 - 2.(-3).7\cos(105° - 45°)} \\ &= \sqrt{79}\end{aligned}$$

(b)

Given polar co-ordinates are

$$(\sqrt{2}, \frac{5\pi}{4}) \text{ and } (2, \frac{2\pi}{3})$$

Now,

$$\begin{aligned}\text{Distance } &= \sqrt{(r_1)^2 + (r_2)^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ &= \sqrt{(\sqrt{2})^2 + (2)^2 - 2.(\sqrt{2}).2\cos(\frac{5\pi}{4} - \frac{2\pi}{3})} \\ &= \sqrt{6 - 4\sqrt{2}\cos(\frac{7\pi}{12})} \\ &= \sqrt{6 - 4\sqrt{2}(-\frac{\sqrt{3}+1}{2\sqrt{2}})} \\ &= \sqrt{(\sqrt{3}+1)^2} \\ &= 1 + \sqrt{3}\end{aligned}$$

12.

Let,

$$\text{A} = (a, \theta)$$
$$\text{B} = (2a, \theta + \frac{\pi}{3})$$
$$\text{C} = (3a, \theta + \frac{2\pi}{3})$$

Now,

$$\begin{aligned}\triangle \text{ ABC } &= \frac{1}{2}|r_1 r_2 sin(\theta_2 - \theta_1) + r_2 r_3 sin(\theta_3 - \theta_2) + r_3 r_1 sin(\theta_1 - \theta_3)| \\ &= \frac{1}{2}|2a^2 \sin(\frac{\pi}{3}) + 6a^2 \sin(\frac{\pi}{3}) + 3a^2 \sin(-\frac{2\pi}{3})| \\ &= \frac{1}{2}(2a^2\frac{\sqrt{3}}{2} + 6a^2\frac{\sqrt{3}}{2} - 3a^2\frac{\sqrt{3}}{2}) \\ &= \frac{5\sqrt{3}}{4}a^2\end{aligned}$$

13 (a)

Given equation

$$(x^2 + y^2)^2 = 2a^2 xy$$

Putting $x = r\cos\theta$ and $y = r\sin\theta$ we get,

$$\begin{aligned}((r\cos^2\theta) + (r\sin^2\theta))^2 &= 2a^2.r\cos\theta.r\sin\theta \\ \Longrightarrow (r^2(\cos^2\theta + \sin^2\theta))^2 &= 2a^2r^2\sin\theta\cos\theta \\ \Longrightarrow r^4 &= 2a^2r^2\sin\theta\cos\theta \\ \Longrightarrow r^2 &= a^2\sin 2\theta\end{aligned}$$

(b)

Given equation

$$x^4 + x^2y^2 - (x+y)^2 = 0$$

Putting $x = r\cos\theta$ and $y = r\sin\theta$ we get,

$$\begin{aligned}r^2\cos^2\theta r^2 - (r^2 + 2r^2\cos\theta\sin\theta) &= 0 \\ \Longrightarrow r^2\cos^2\theta &= 1 + \sin 2\theta \\ \Longrightarrow r^2 &= \frac{(\cos\theta + \sin\theta)^2}{\cos^2\theta} \\ \Longrightarrow r &= \pm\frac{\cos\theta + \sin\theta}{\cos\theta} \\ \Longrightarrow r &= \pm\tan\theta\end{aligned}$$

14. (a)

Here given,

$$r^2 - 2r(\cos\theta - \sin\theta) - 7 = 0$$
$$\implies r^2 - 2(r\cos\theta - r\sin\theta) - 7 = 0$$

Putting $r\cos\theta = x$ and $r\sin\theta = y$ we get,

$$x^2 + y^2 - 2(x - y) - 7 = 0$$
$$\implies x^2 + y^2 - 2x + 2y - 7 = 0$$

(b)

Here given,

$$r(1 - e\cos\theta) = ep$$
$$\implies r - e\,r\cos\theta = ep$$

Putting $r\cos\theta = x$ and $r\sin\theta = y$ we get,

$$r - ex = ep$$
$$\implies r = e(x + p)$$
$$\implies r^2 = e^2(x + p)^2$$
$$\implies x^2 + y^2 = e^2(x + p)^2$$

END