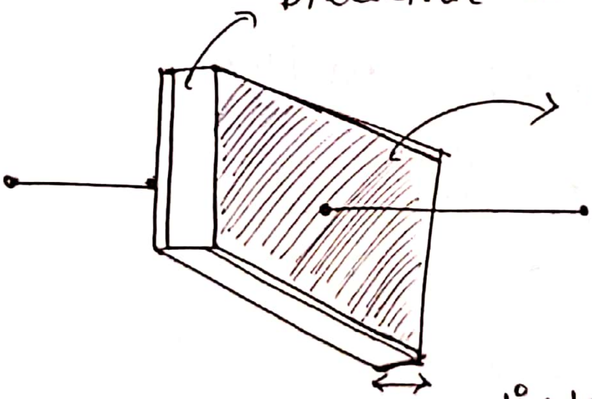
Week-8

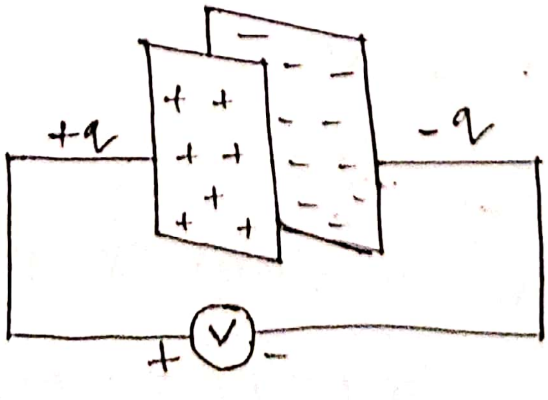
i) Capacitors
ii) Inductors
iii) Properties of both
iv) Transient response

→ Topics overview

Capacitors



Typical capacitor



Capacitor with applied voltage, V

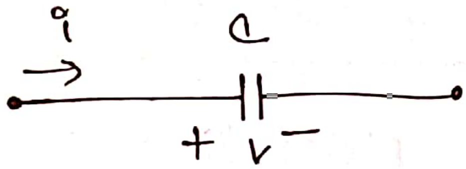
Properties ⇒

i) Capacitor is a passive element

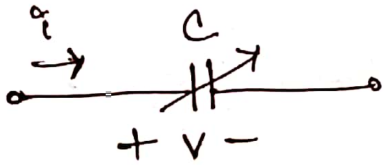
ii) It can store energy in it's electric field.

(So, we can say it has MEMORY !!!)

Symbol ⇒



Fixed capacitor.



Variable capacitor

Component equation ⇒

$$q \propto V$$

$$\Rightarrow q = CV$$

Here, C is the constant of proportionality called the CAPACITANCE.

Unit → Farad (F)

So, $C = \frac{q}{V}$

$\therefore$ 1 Farad = 1 coulomb/volt

On the contrarey,

$$C = \frac{\epsilon A}{d}$$

Here, A = surface area of each plate

d = distance betn two plates

ϵ = Permittivity of dielectric materials between plates.

<u>How capacitance depends on these</u> $\Rightarrow$

$A \uparrow \quad C \uparrow$

$d \downarrow \quad C \uparrow$

$\epsilon \uparrow \quad C \uparrow$

<u>Current - Voltage relationship</u> $\Rightarrow$

$$q = CV$$

$$\Rightarrow \frac{d(q)}{dt} = C\frac{dv}{dt}$$

$$\Rightarrow i = C\frac{dv}{dt}$$

Voltage-current relationship ⇒

$$V = \frac{1}{C} \int_{-\alpha}^{t} i \, dt$$

$$\Rightarrow V = \frac{1}{C} \int_{t_0}^{t} i \, dt + V(t_0)$$

Here, $V(t_0) = \frac{q(t_0)}{C}$

↓

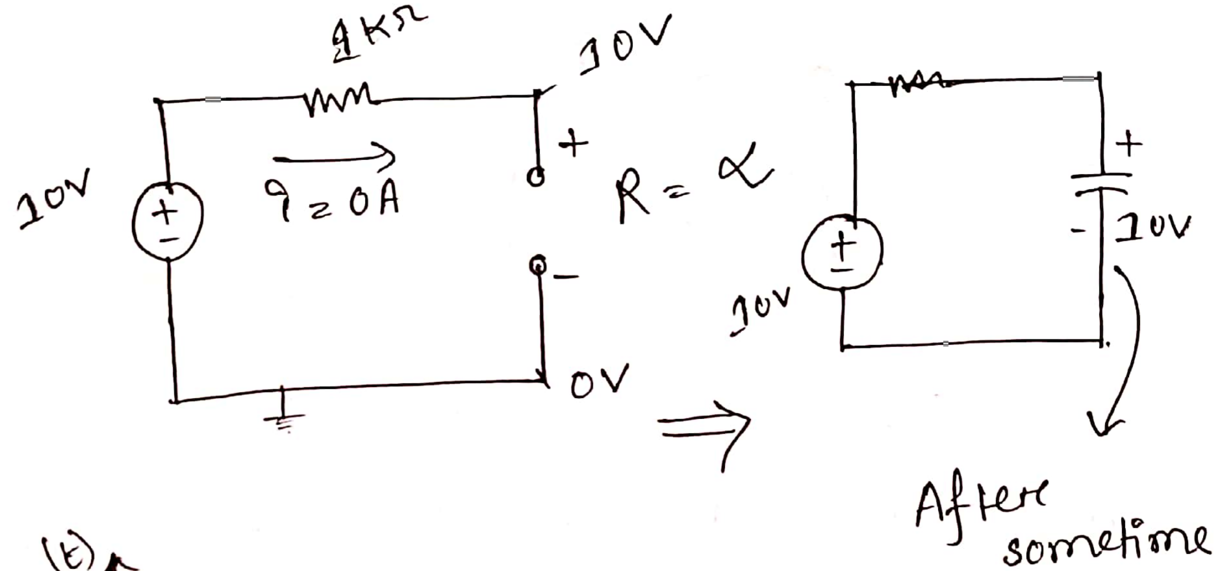
shows capacitor voltage depends on past history.

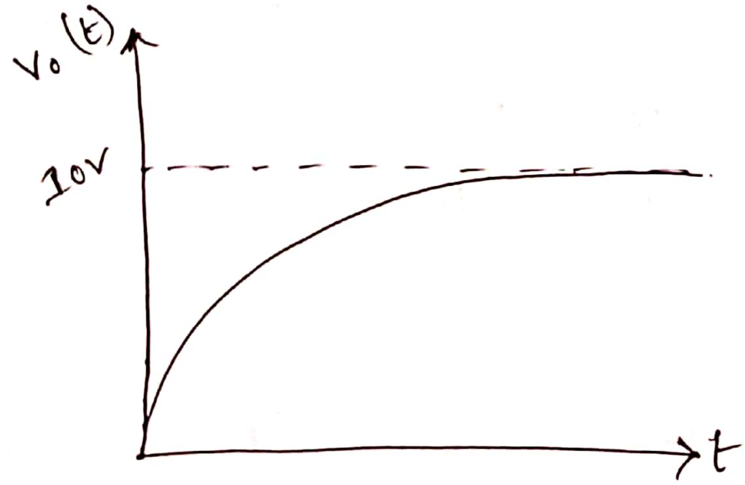
Power, $P = Vi = CV \frac{dv}{dt}$

$$W = Pt = \frac{1}{2} CV^2 = \frac{q^2}{2C}$$

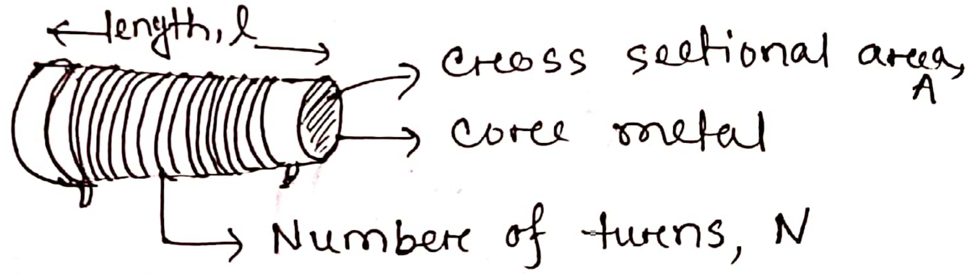
$$\boxed{\int_{-\alpha}^{\alpha} P \, dt}$$

Open circuit to DC =>





Inductors



Typical form of Inductor

Properties =>

(I) Store energy

(II) Passive element

Inductor follows Henry's law.

$$\phi \propto i$$

$$\Rightarrow \frac{d\phi}{dt} = L\frac{di}{dt}$$

$$\therefore v = L\frac{di}{dt}$$

Here, L is the constant of proportionality called inductance.

$$L = \frac{N^2 \mu A}{l}$$

<u>current-voltage relationship</u> $\Rightarrow$

$$di = \frac{1}{L} v\, dt$$

$$\Rightarrow i = \frac{1}{L}\int_{-\alpha}^{t} v(t)\, dt$$

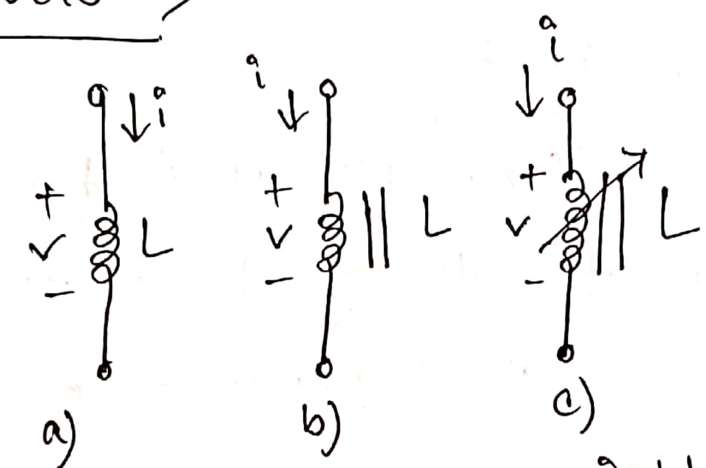
$$\therefore i = \frac{1}{L}\int_{t_0}^{t} v(t)\, dt + i(t_0)$$

Inductor stores energy in it's magnetic field.

$$p = vi = L\left(\frac{di}{dt}\right) i$$

$$w = \frac{1}{2} L i^2$$

Symbols ⇒

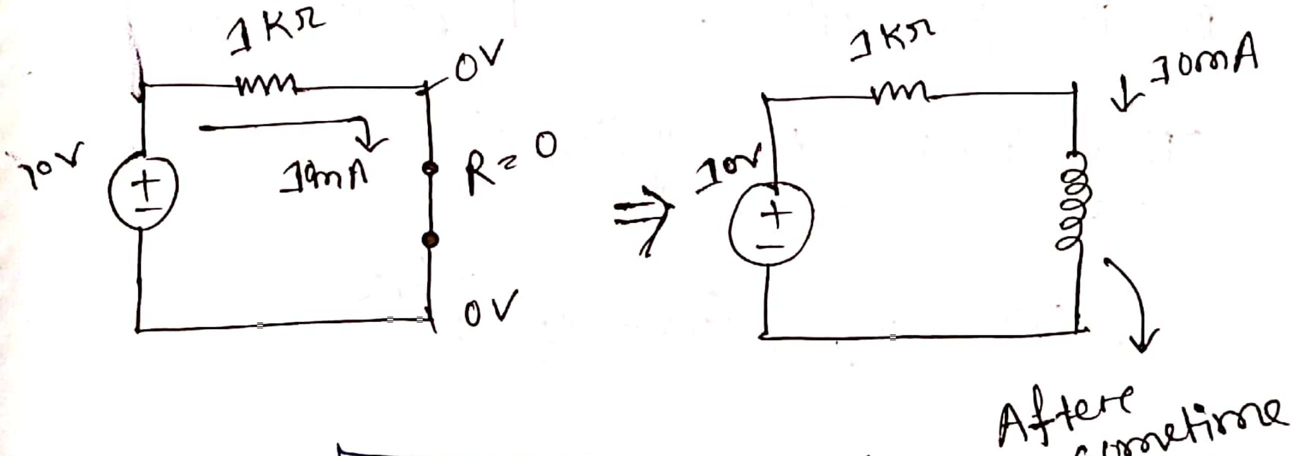


a) air-core

b) iron-core

c) variable iron-core

Short Circuit to DC ⇒



Transient Analysis

It determines a circuit's response over a period of time defined by the user. Since capacitor / inductor v / i cannot change abruptly, so.....

Time constant ⇒

Time required for the response to decay to a factor of 1/e or 36.8 percent of it's initial value.

For capacitor, $\tau = RC$ [unit-sec]

For Inductor, $\tau = L/R$

Basic formula for Exponential Transient Analysis ⇒

$$x(t) = x_f + (x_i - x_f)\, e^{-t/\tau}$$

Here, x_i = initial value

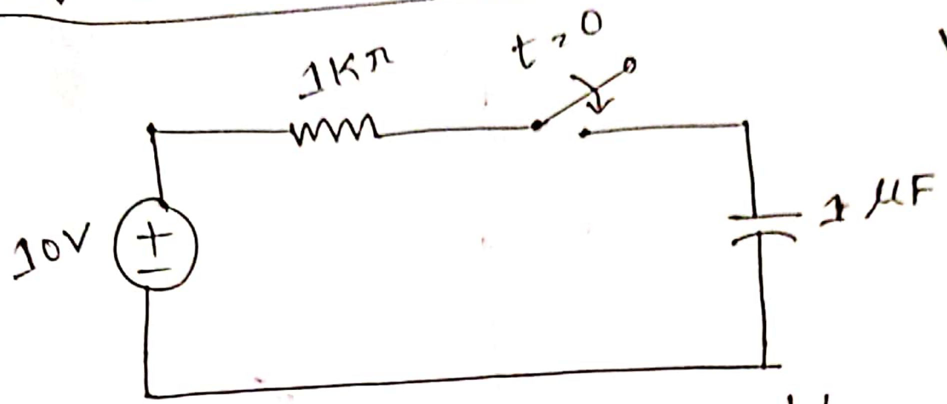
x_f = final "

$x(t)$ = v / i at time t

t = given time

τ = time constant

Capacitor Example :)



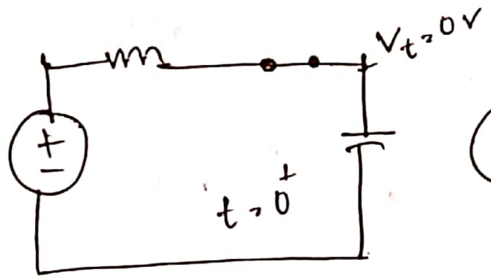
$v(t) = ?$

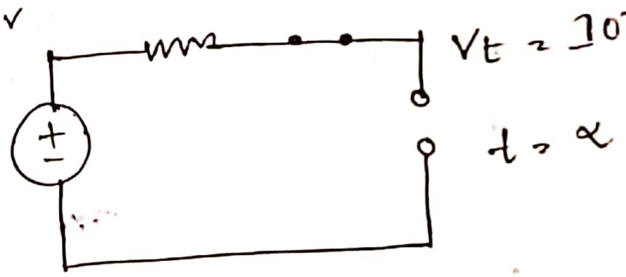
$I(t) = ?$

$$v(t) = V_f + (V_i - V_f)\, e^{-t/\tau}$$

Here, $\tau = RC = 1\,K\Omega \times 1\,\mu F$

$= 10^{-3}\ s$





$$I_i = \frac{10 - 0}{1K}\ A = 10\ mA$$

$$I_f = 0\ A$$

$$\therefore v(t) = V_f + (V_i - V_f)\, e^{-t/10^{-3}}$$

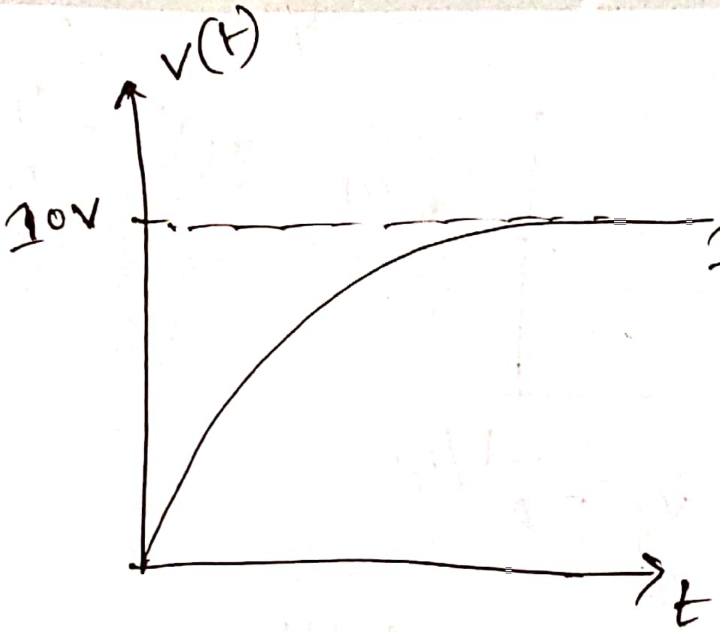
$$= 10 + (0 - 10)\, e^{-1000t}\ V$$

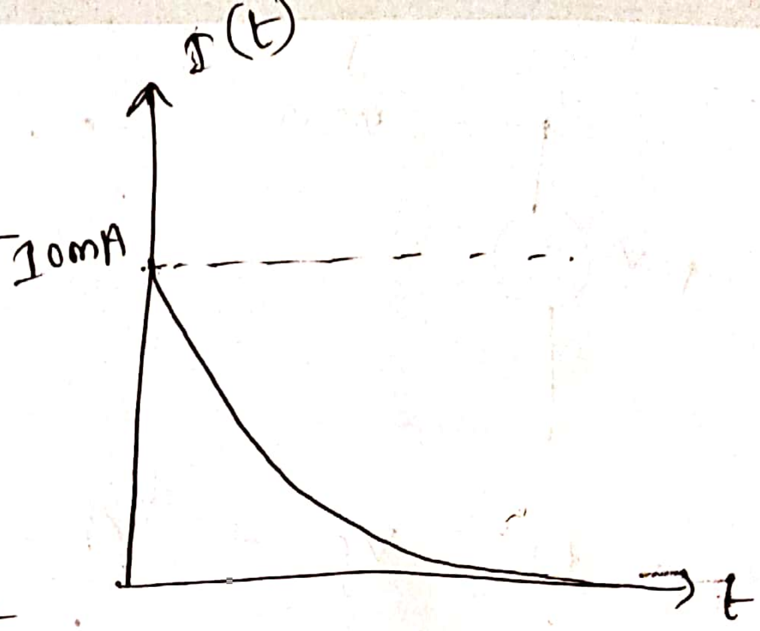
$$= 10\,(1 - e^{-1000t})\ V$$

$$I(t) = I_f + (I_i - I_f)\, e^{-t/10^{-3}}$$

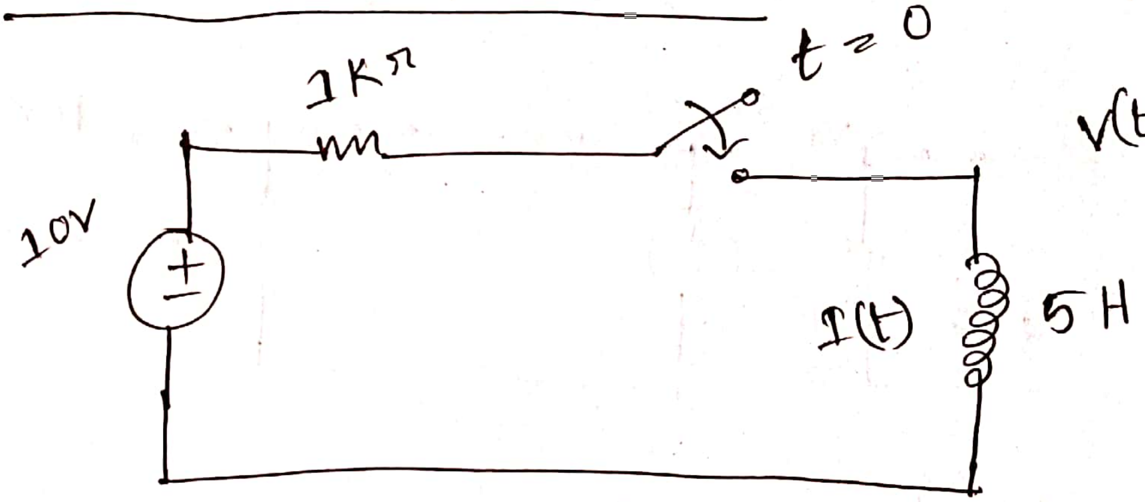
$$= 0 + (10 - 0)\, e^{-t/10^{-3}}$$

$$= 10\, e^{-1000t}\ mA$$



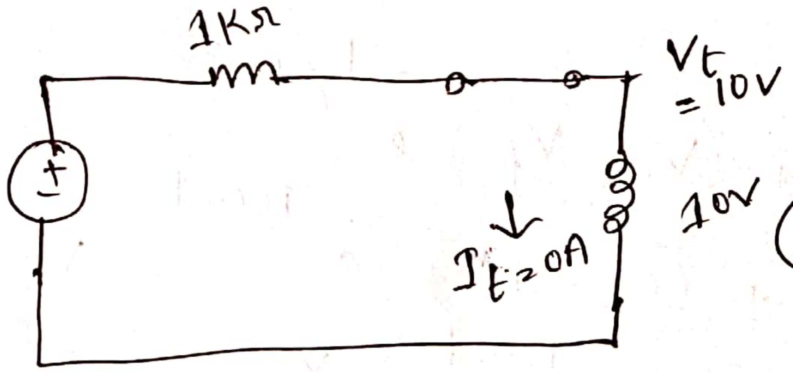


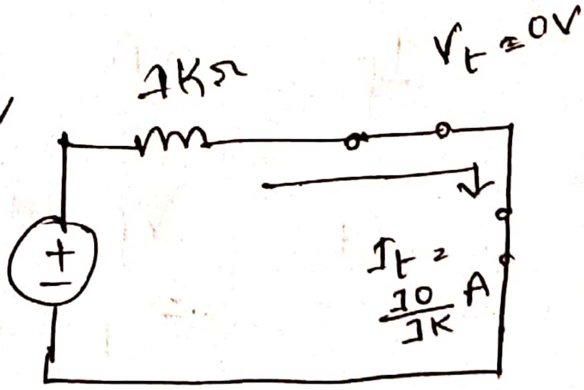
Inductor Example =)



v(t) = ?

$$\tau = \frac{L}{R} = \frac{5}{1K} = 0.005\ S$$





$I_i = 0A$

$I_f = 10mA$