ASSIGNMENT FOR EE5101 LINEAR SYSTEMS

Control Techniques for A Continuous- Flow Stirred Tank Reactor

Arunava Manna

Matriculation Number: A0232541R

Email: arunava.manna@u.nus.edu

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National University Of Singapore

Abstract

In this project, one 3rd order LTI state space model has been considered where 2 inputs are provided, 3 outputs have to controlled and the stability of the plant has to be maintained by different control techniques. But due to the unavailability of one sensor, the component concentration has to be estimated and controlled along with the minimization of the cost. Different control techniques have been adopted in this project which are as follows:

- Pole Placement
- Optimal Control or Optimal Control
- Observer Based state estimation
- Decoupling
- Servo Control

CONTENTS

INTRODUCTION

MODELLING

$x = [C_a \;\; T \;\; T_j]^T$

C_a= The component concentration of the reactant

T = the reaction temperature in the tank

T_j = the temperature of the outflow water of the cooling jacket.

$u = [F \quad F_j]^T$ where F = outflow flow rate of the reaction

And F_j = flow rate of the water in the cooling jacket.

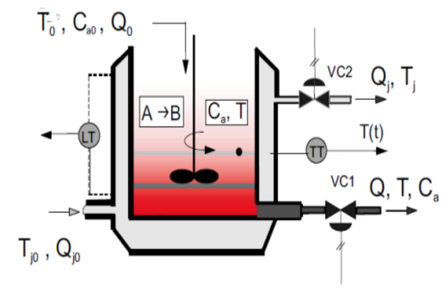


Figure 1. CSTR reactor [1]

Measurement vector, $y = [T \;\; T_j]^T$

The system is described by:

$\dot{x} = Ax + Bu + Bw$ and $y = Cx$ …. (i)

$$A = \begin{bmatrix} -1\cdot 7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -200 - ab0 & -220 - ba0 \end{bmatrix}, B = \begin{bmatrix} 3 + a & 0 \\ -30 - dc & 0 \\ 0 & -2120 - Cd0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

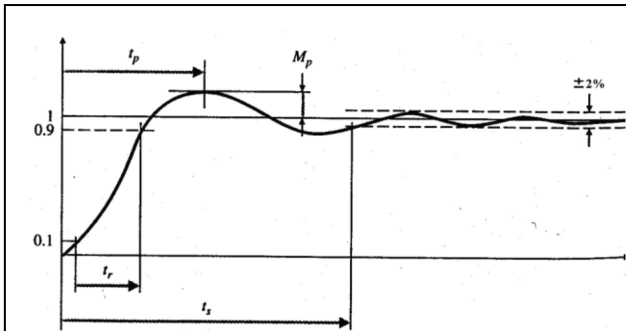
Where, a=2, b=5, c=4 and d=1

Hence after applying the above values on A, B and C, we have

$$A = \begin{bmatrix} -1\cdot 7 & -0.25 & 0 \\ 23 & -30 & 20 \\ 0 & -450 & -740 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Initial condition, $x_0 = [1 \;\; 100 \;\; 200]^T$

CONTROL SYSTEM DESIGN



Given that,

- The overshoot is less than 10%
- The 2% settling time is less than 30 seconds

Overshoot, $Mp = e^{-\frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$ $\quad => ln\, Mp < \frac{10}{100}$

=> $5.3(1 - \varepsilon^2) > \pi^2\varepsilon^2$ => $\varepsilon > 0\cdot 5912$

Let, $\boldsymbol{\varepsilon = 0\cdot 6}$

Settling time, $t_s = \frac{4\cdot 0}{\varepsilon\omega_n}$ => $\omega_n > 0.222$ let, $\boldsymbol{\omega_n = 0.3}$

So we can find the desired poles from the characteristic polynomial,

$s^2 + 2\varepsilon\omega_n s + \omega_n^2 = 0$ Hence, s=$-\xi\omega_n \pm J\omega_n\sqrt{1-\xi^2}s = -0\cdot 18 \pm J0\cdot 24$

Note: Step Reference Signals for each input channel with zero initial condition are $[1,0]$ $an\,d$ $[0,1]$

TASKS

PART1: POLE PLACEMENT USING CONTROLLABLE CANONICAL FORM

The system is described by:

$\dot{x} = Ax + Bu$ and $y = Cx$ ……… (ii)

The position of the poles, which are important property of the system (since the stability is primary requirement), can be changed by feedback.

So, state feedback, $u = -kx + Fr$

Where r is the reference input. K and F are constant matrices to be designed. For pole placement, we only need to design K.

Feedback changes the dynamic property of the system.

If the value of u is included in equation (ii) then,

$$\dot{x} = (A - Bk)x + BFr$$

First, we need to check the controllability of the plant,

So, C= [B AB A^{2}B] = $\begin{bmatrix} 5 & 0 & 2\cdot 5 & 0 & -363 & 450 \\ -44 & 0 & 1435 & -16600 & 350000 & 12780000 \\ 0 & -830 & 19800 & 614200 & -15300000 & -44704000 \end{bmatrix}$ which has 3 linearly independent columns hence, full rank and as a result (A, B) is controllable.

$$W = [B\ \ AB\ \ A^2B] = \begin{bmatrix} 5 & 0 & 2\cdot 5 & 0 & -363 & 450 \\ -44 & 0 & 1435 & -16600 & 350000 & 12780000 \\ 0 & -830 & 19800 & 614200 & -15300000 & -44704000 \end{bmatrix}$$

Next, linearly independent columns have to be taken

So, c = $[b_1\ \ Ab_1\ \ b_2] = \begin{bmatrix} 5 & 2\cdot 5 & 0 \\ -44 & 1435 & 0 \\ 0 & 19800 & -830 \end{bmatrix}$, $c^{-1} = \begin{bmatrix} 0.1970 & -0.0003 & 2\cdot 8819e-19 \\ 0.0060 & 0.0007 & 8\cdot 1658e-21 \\ .1441 & 0.0164 & -0.0012 \end{bmatrix}$

$d_1 = 2\,, d_1 + d_2 = 2 + 1 = 3$

$q_2 = [0.060\quad 0.0007\quad 8.1658e-21]$, $q_3 = [0.1441\quad 0.0164\quad -0.0012]$

$$T = \begin{bmatrix} q_2 \\ q_2 A \\ q_3 \end{bmatrix} = \begin{bmatrix} 0.0060 & 0.0070 & 8.1658e-21 \\ 0\cdot 1508 & -0\cdot 2115 & 0\cdot 1400 \\ 0\cdot 1441 & 0.0164 & -0.0012 \end{bmatrix}, T^{-1} = \begin{bmatrix} -15\cdot 6813 & 0.0645 & 7.5251 \\ 156.2983 & -0.0553 & -6.4501 \\ 253.0130 & 6\cdot 9899 & -17\cdot 8497 \end{bmatrix}$$

$\bar{A} = TAT^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -3.6063e+04 & -750.9125 & 2\cdot 2519e+03 \\ 307.4620 & 8\cdot 5075 & -20\cdot 7875 \end{bmatrix}$, one trivial(1st) and two non-trivial rows(2nd and 3rd)

$\bar{B} = \begin{bmatrix} -0\cdot 2780 & -6\cdot 776e-18 \\ 10.0600 & -116\cdot 2000 \\ -0.0011 & 0\cdot 9960 \end{bmatrix}$ $\bar{k} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$, input only effects the non-trivial rows

> Note: The dominant pole approximation is a method for approximating a (more complicated) high order system with a (simpler) system of lower order if the location of the real part of some of the system poles are sufficiently close to the origin compared to the other poles.

Since, the system is of order 3, which indicates that we must place 3 poles. We have our dominant poles from the performance specification at s $= -0 \cdot 18 \pm J0 \cdot 24$ but we also must consider one third pole.

Let the third pole is at -0.9 on the real axis which is 5 times far away from the dominant poles towards negative half of the s plane. The higher the absolute values of the real part, it goes to zero very fast which means $e^{-0.9t}$ will decay faster than $e^{-0.18t}$. Moreover, the system dynamics depends on the slow mode which the dominant poles.

Hence,

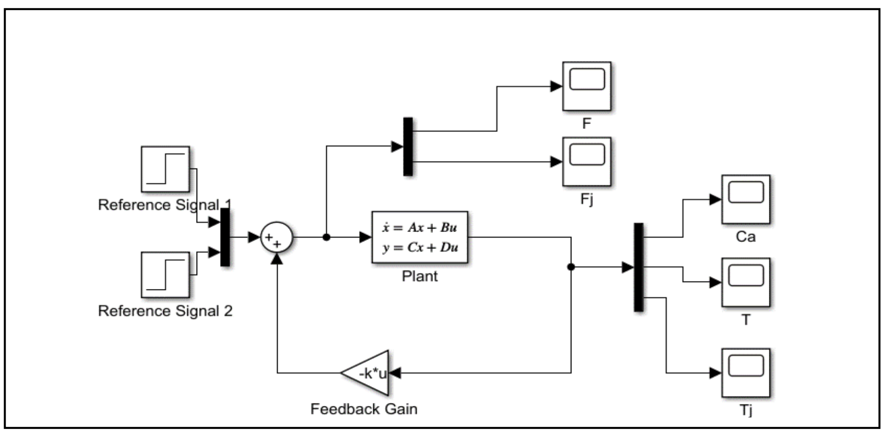
$$\text{Al} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.0810 & -0 \cdot 4140 & -1.26 \end{bmatrix}$$

If we compare the non-trivial rows of $(SI - (\bar{A} - \bar{B}\bar{k}))$ and Al, we will get,

$$\bar{k} = \begin{bmatrix} -191.7228 & 244.8082 & 19.0522 \\ 2.4159e + 04 & 0.4140 & -1.0105e + 03 \end{bmatrix}$$

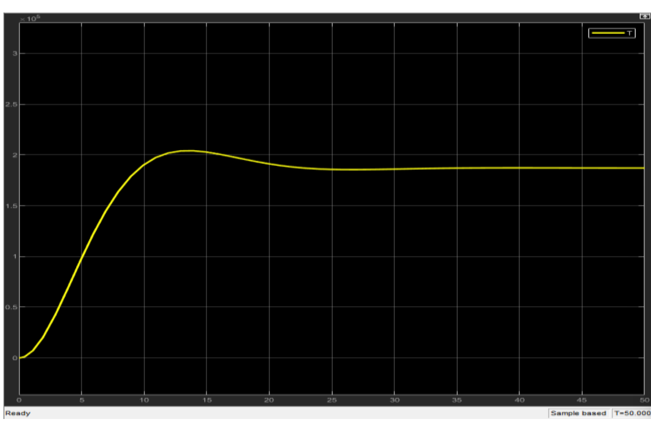
$$K = \bar{k}T = \begin{bmatrix} 2.9380 & -5 \cdot 2300 & 3 \cdot 3375 \\ 0.3160 & 0.0265 & 1 \cdot 2232 \end{bmatrix}$$

For the step response we have to consider the Simulink model,

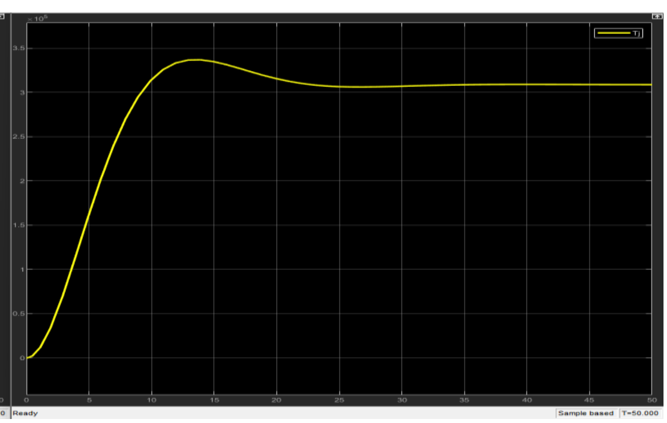


With the above model and the step inputs, the transient responses can be generated:

After passing [1,0] Step Response to the model,

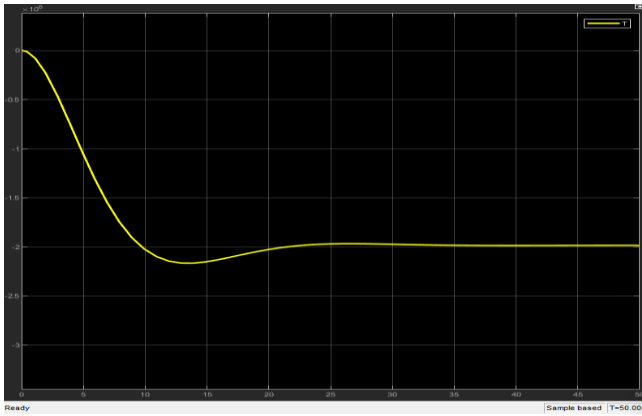


T

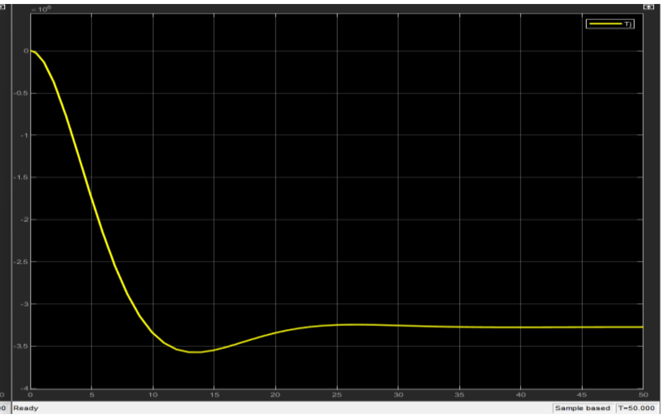


T_j

After passing [0,1] Step Input to the model,



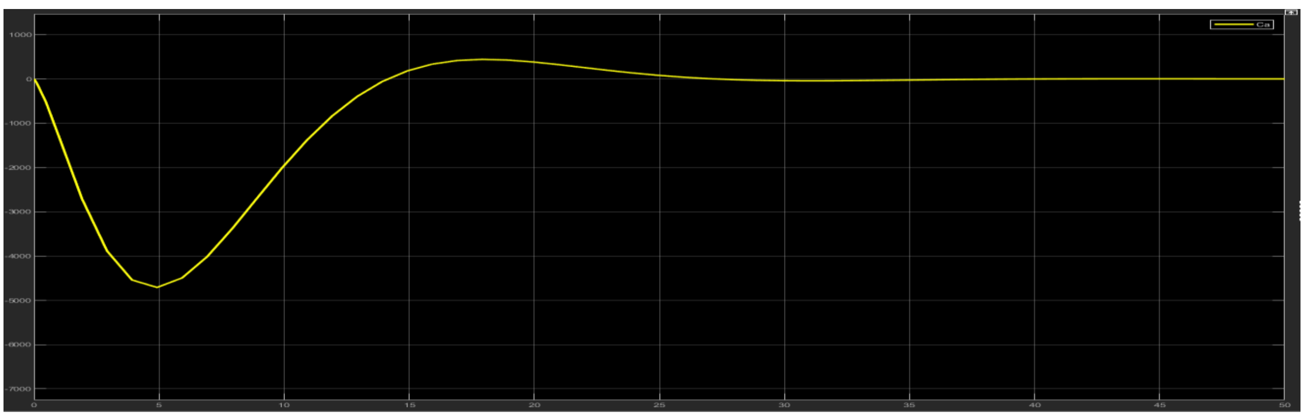
T



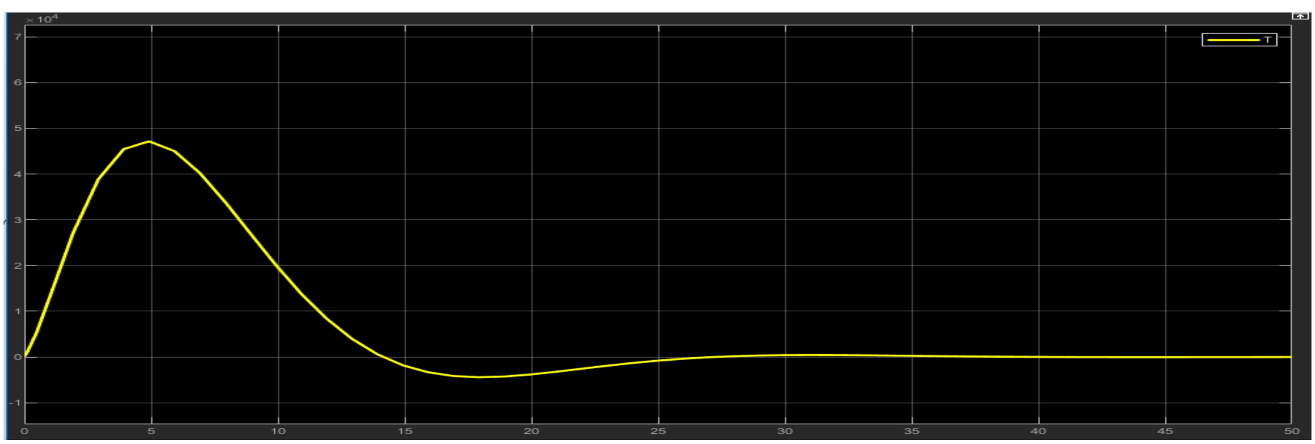
T_j

After analyzing the above responses this can be concluded the system response is according to the design specification. That is the overshoot is within 10% and the settling time is less than 30s.

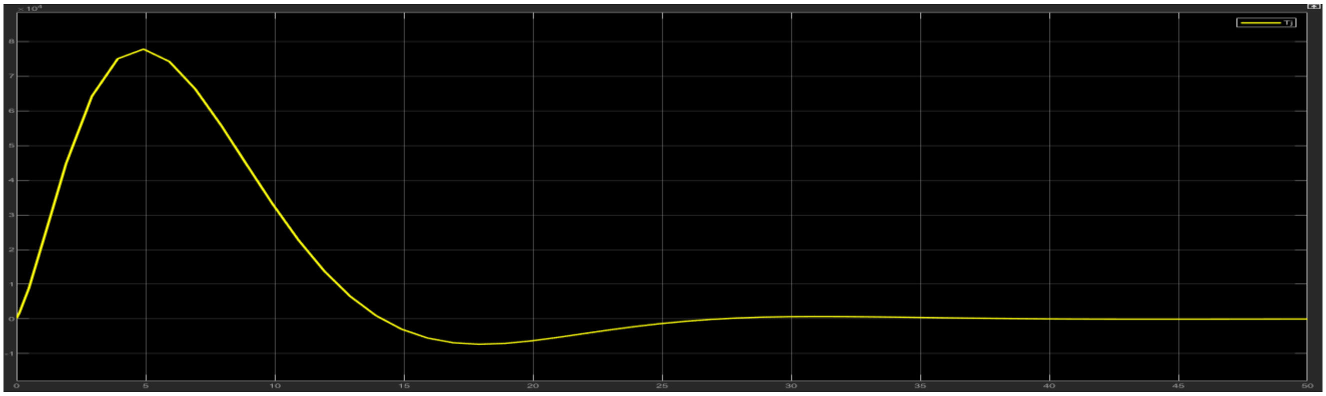
After simulation for 50secs with the above values and SIMULINK model (also considering that the **initial state is not zero with zero external** inputs), **the three responses can be shown below**:



Ca, component concentration of the reactant



T, reaction temperature in the tank



Tj, temperature of the outflow water of the cooling jacket

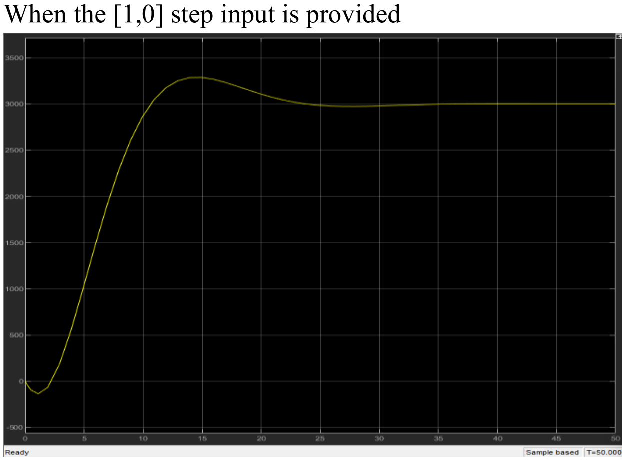
- Explanation:

  From the above observations, it can be concluded that during non-zero initial state and zero initial state the system is always staying within the limits of the system specification.

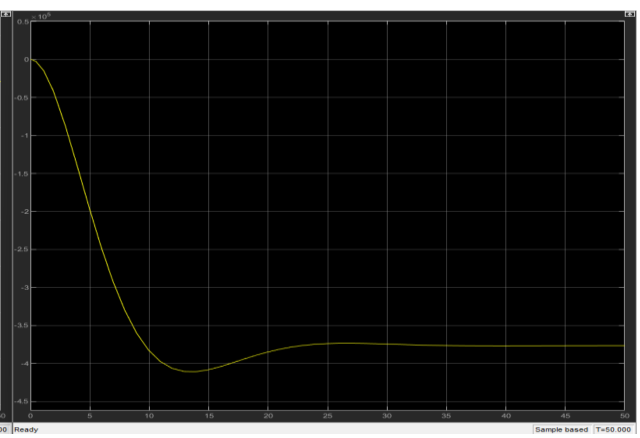
- Discuss effects of the positions of the poles on system performance and monitor control signal size.

  With the present poles the control signals ($U = \left[F \;\; F_j\right]^T$, where F is the outlet flow rate of the reaction; Fj is the flow rate of the water in the cooling jacket) have the below forms,

  When the [1,0] step input is provided

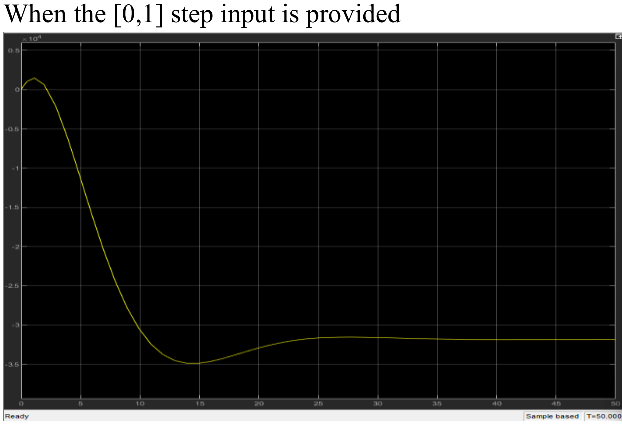


F

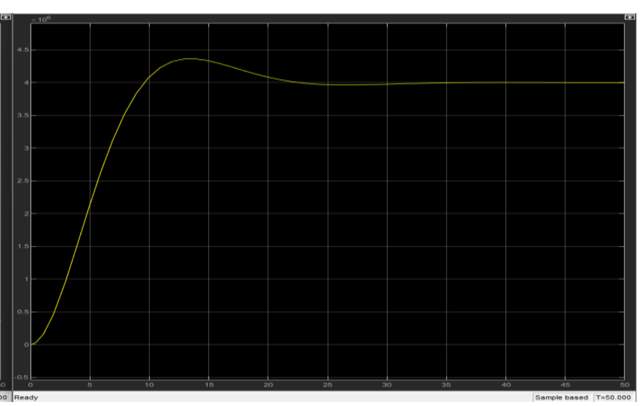


F_j

When the [0,1] step input is provided



F



F_j

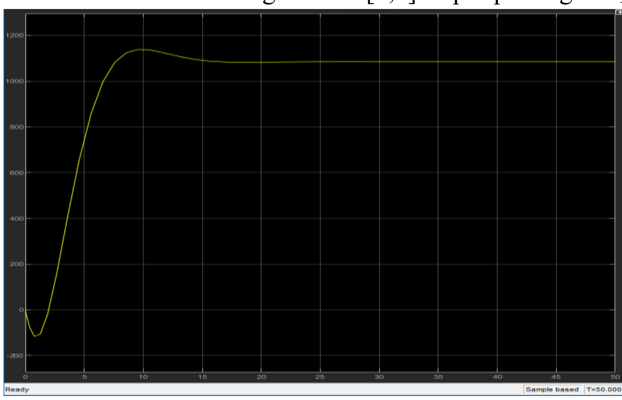
The system response is already shown above, which has almost 9.1 - 9.2% overshoot and around 20secs of settling time.
Let's consider

$$\varepsilon = 0 \cdot 7$$

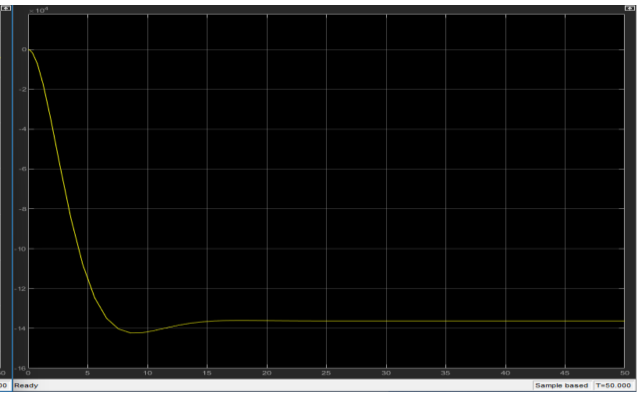
$$\omega_n = 0.5$$

S= -0.3500 ± 0.3571i and the third pole is at -1.75

Which has the control signal wen [1,0] step input is given,

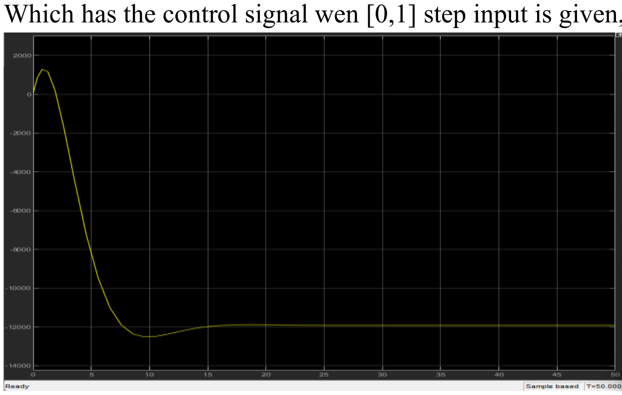


F

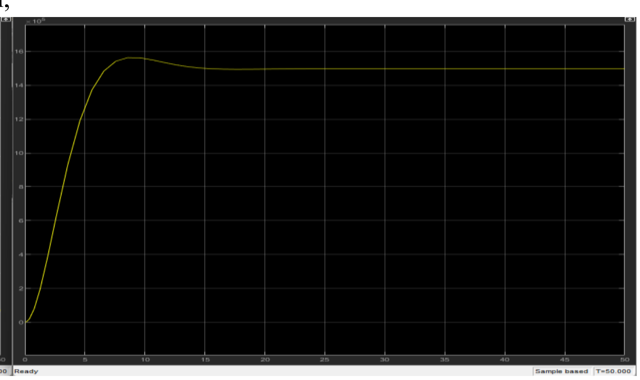


F_j

Which has the control signal wen [0,1] step input is given,



F



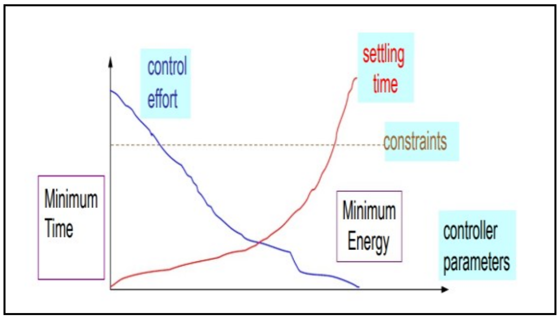
F_j

From the above signals it can be sad that the rise time is decreasing along with the settling time with the increase in two variables and with the move in the pole towards the negative side of the s plan.
Moreover, the system has even lesser overshoot and settling time compared to the previous case, which are nearly 4.98-5% and 12secs correspondingly.

PART 2: LQR METHOD

For the plant, we want to regulate the estate to x=0, with simple state feedback control: u=-kx. However large gain means higher amount of control efforts. Hence, we would like to keep u as small as possible. To keep a balance between speed and cost, we want to minimize the following equation:

$J = \frac{1}{2}\int_0^{\alpha}(qx^2 + ru^2)\,dt$



The weighting factors q and r express the relative importance of keeping x and u near zero. The larger the q is, the larger will be the gain K, and the faster will x(t) approach zero. However, the peak magnitude of u will be larger with higher energy cost, r. The parameter q is selected to achieve a compromise between those effects.

First step: checking if the matrix (A, B) is controllable, Where,

C= [B AB A^2B] which is full rank, so the matrices are controllable.

Second Step: Assumption of Q and R. For our convenience, we are considering Q and R to be identity matrices of dimensions (3,3) and (2,2) respectively.

i.e., $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where Q and R both are symmetric and positive definite matrices.

If R is semi positive then gain will become infinity

To solve the LQR problem, the key method is ARE (Algebric Riccatti Equation), which is,

$$A^T p + PA + Q = PBR^{-1}B^T P$$

To solve the ARE equation, the systematic way would be the ***eigenvalue-eigenvector*** based algorithm,

Let, consider a 6 × 6 matrix ***Hamiltonian matrix***,

$$\Gamma = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$$

After including those values, we will have,

$$\Gamma = \begin{bmatrix} -1\cdot 7 & -0.25 & 0 & -25 & 220 & 0 \\ 23 & -30 & 20 & 220 & -1936 & 0 \\ 0 & -450 & -740 & 0 & 0 & -688900 \\ -1 & 0 & 0 & 1.7 & -23 & 0 \\ 0 & -1 & 0 & 0.25 & 30 & 450 \\ 0 & 0 & -1 & 0 & -20 & 740 \end{bmatrix}$$

After applying eig() function of MATLAB, the eigenvalue and eigenvectors can be visualized of Γ

Hence, the eigen values are, $\begin{bmatrix} -1.1034e+03 \\ 1\cdot 1034e+03 \\ -61\cdot 8599 \\ -3\cdot 5984 \\ 3.5984 \\ 61.8599 \end{bmatrix}$,

Since, Γ is Hamiltonian, if it does not have any eigenvalues on the imaginary axis, then exactly half of its eigenvalues have a negative real part. If we denote the 6×3 matrix whose columns form a basis of the corresponding subspace, in matrix block-matrix notation, as

$$\begin{bmatrix} v \\ u \end{bmatrix}$$

Then $P = uv^{-1}$

In our calculation, $v = \begin{bmatrix} 4.1728e-05 & 0.0351 & 0.6456 \\ -0.0190 & -0.9190 & 0.6373 \\ 0.9998 & 0.3925 & -0.3704 \end{bmatrix}$ and

$$u = \begin{bmatrix} -4.7730e-06 & -0.0034 & 0\cdot 1989 \\ -2.3115e-04 & -0.0110 & 0.0178 \\ 5.3897e-04 & 2.1399e-04 & -2.0383e-05 \end{bmatrix}$$

Hence, $P = \begin{bmatrix} 0.2934 & 0.0151 & 2.7204e-04 \\ 0.0151 & 0.012^6 & 8.1352e-06 \\ 2.7024e-04 & 8.1352e-06 & 5.4011e-04 \end{bmatrix}$

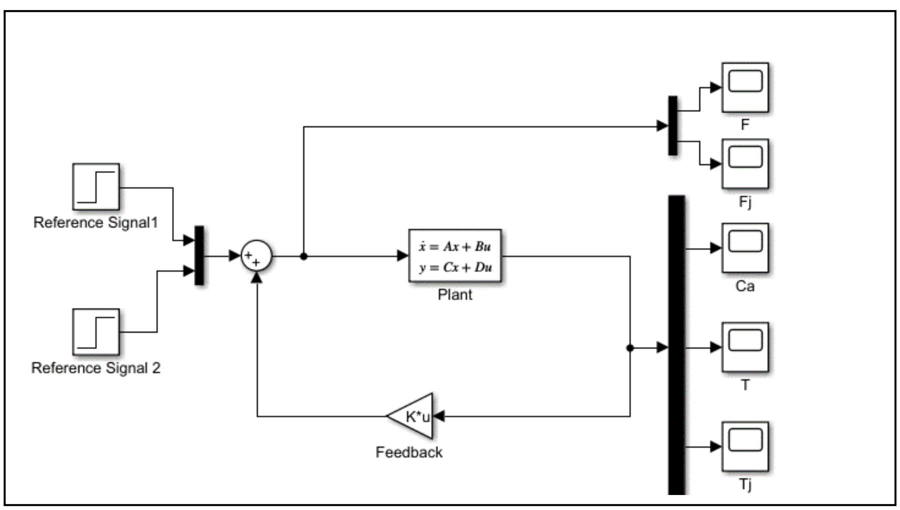
Now the optimal control is

$u(t) = -R^{-1}B^T Px(t)$ where $k = R^{-1}B^T P$

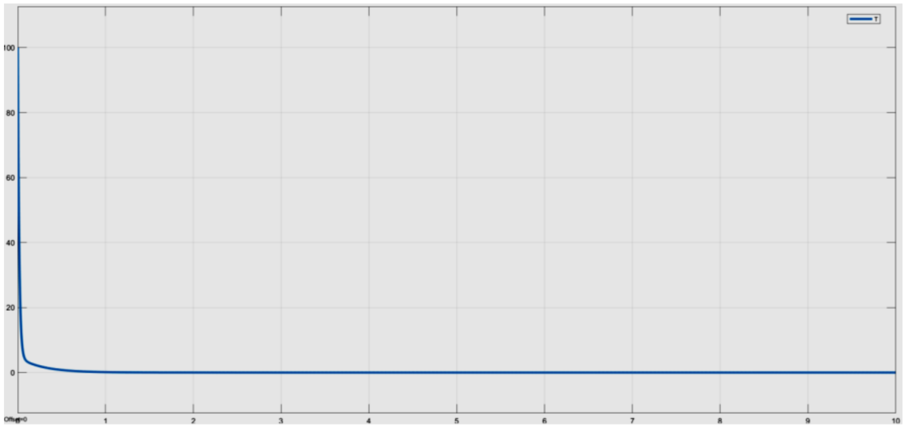
From our calculation,

$$k = \begin{bmatrix} 0.8036 & -0.4788 & 9.9326e-04 \\ -0.2243 & -0.068 & -0.4483 \end{bmatrix}$$

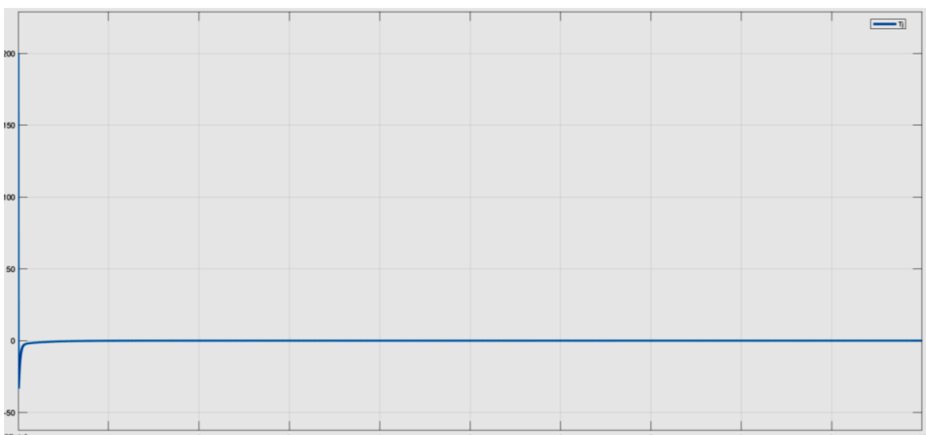
Simulink Model:



Considering Q and R as identity matrix, the output response with non-zero initial state with zero external inputs is following:



T, Reaction temperature in the tank

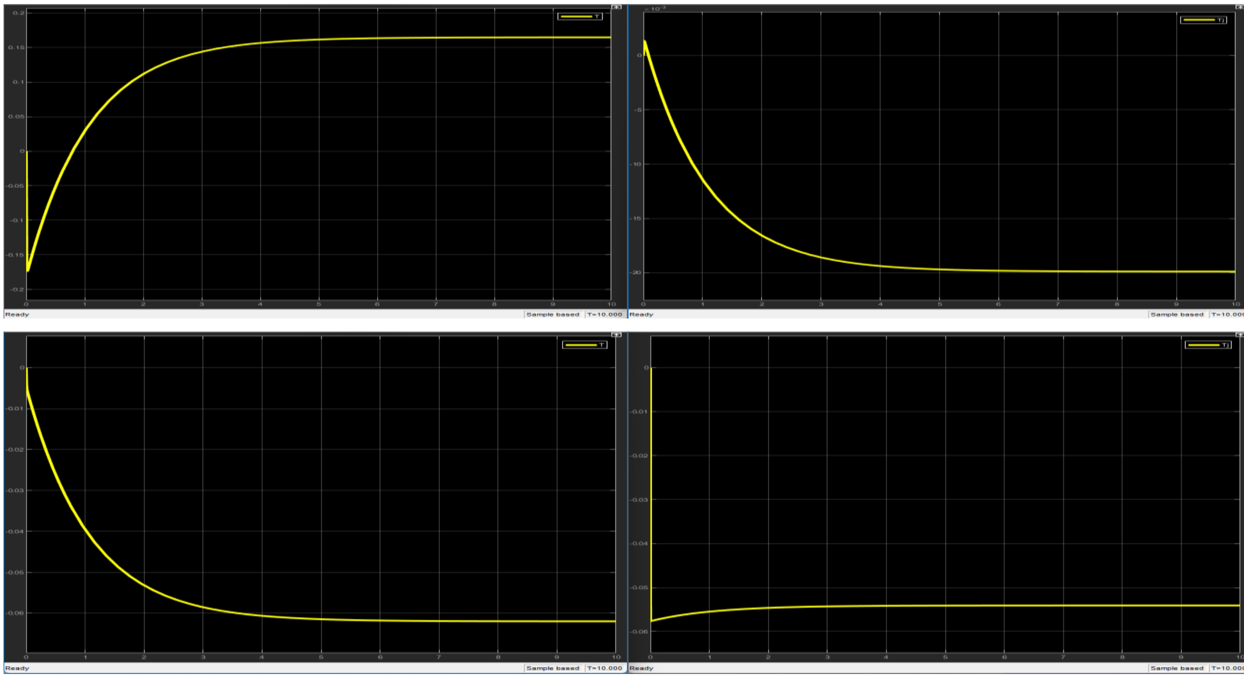


T_j , Temperature of the outflow water of the cooling jacket

❖ Discuss effects of weightings Q and R on system performance and monitor control signal size

With the above conditions the system response during transient condition doesn't follow the system specification, which was studied for the design of the controller So, we need to tune Q and R to get the response according to the design specification.Here, we have tried **trial and error method** to find the probable values of Q and R and the system response, control signal is generated accordingly. Let's, keep the previous R value and set the Q values as `[0 0 0;0 30 0;0 0 300]`, the system responses have nearly 0% overshoot for [1,0] step input and around 6% for the [0,1]. However, the settling very less which more or less 5secs. The step responses are following:

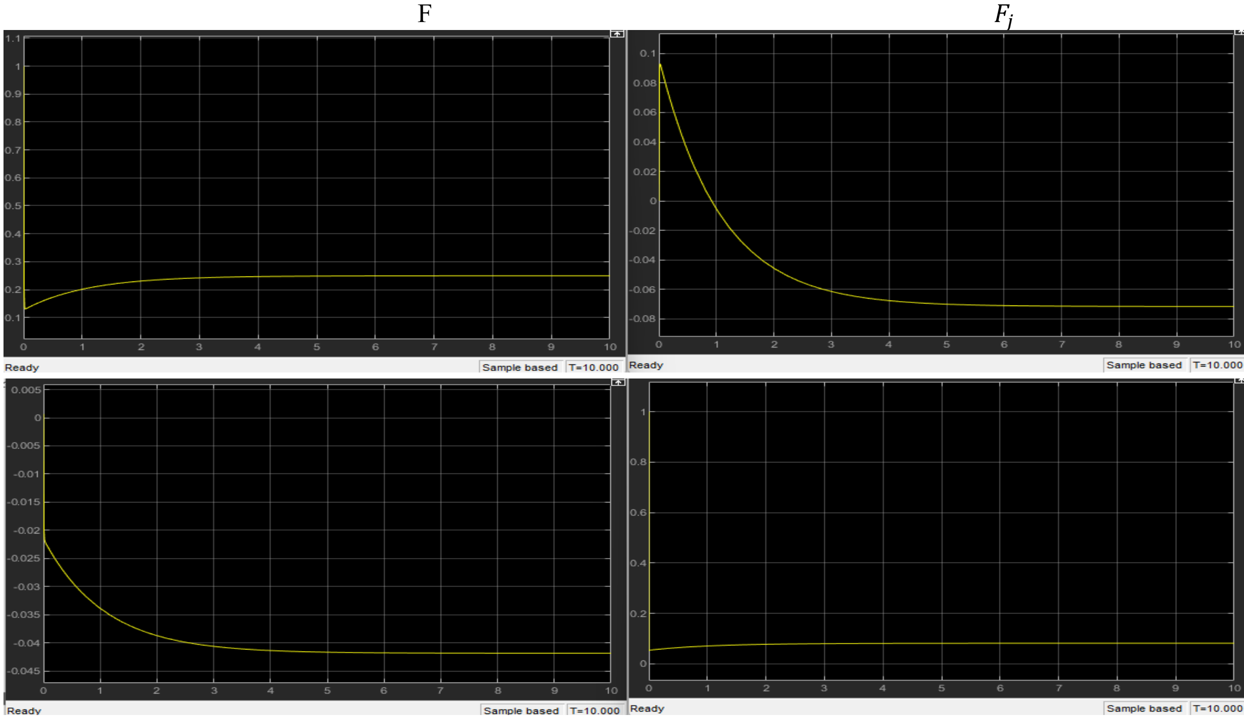
T T_j



[1,0]

[0,1]

F F_j

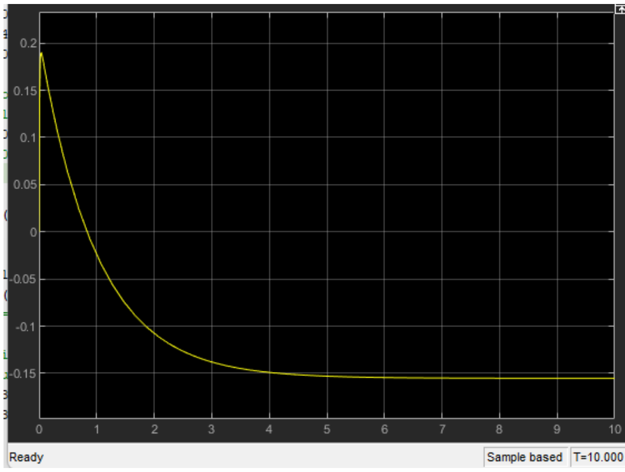


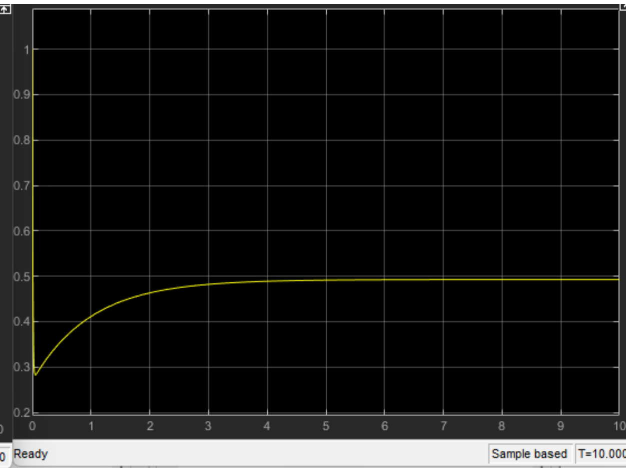
[1,0]

[0,1]

Again, if we try with Q values like [0 0 0;0 60 0;0 0 900] and same R value, then it can be seen that the overshoot is increasing more, and the control signals are same.

Whereas, if we increase R, the control signal magnitude is increasing, it can be visualized with different values of R as follows:

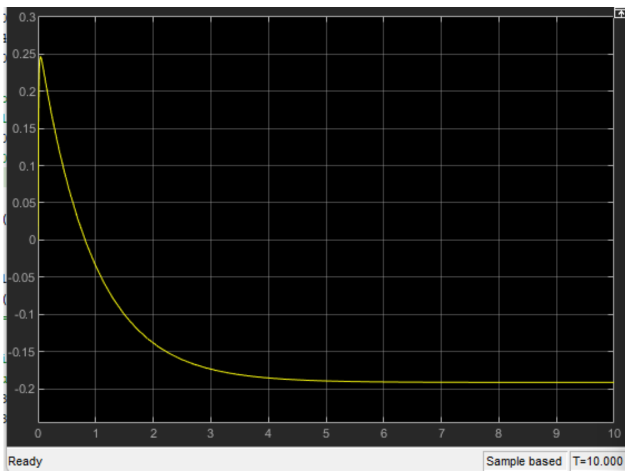


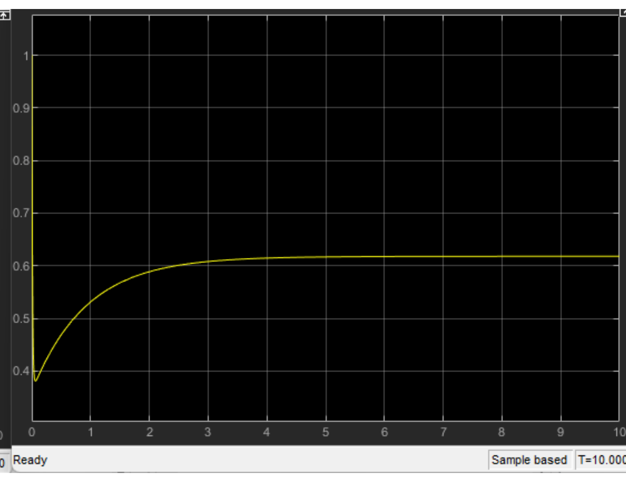


STEP INPUT:
[1,0]

R=

[10 0;0 10]

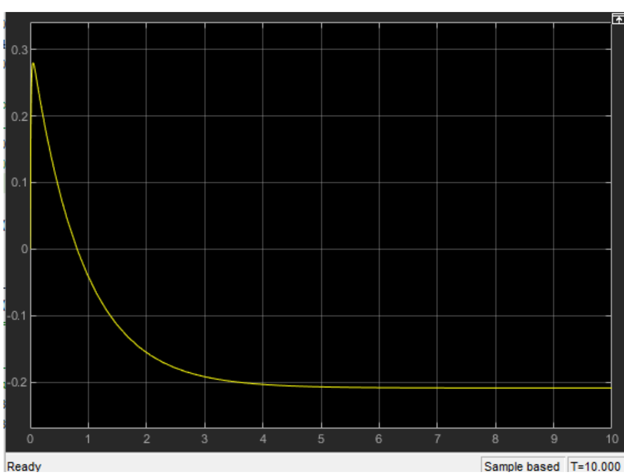


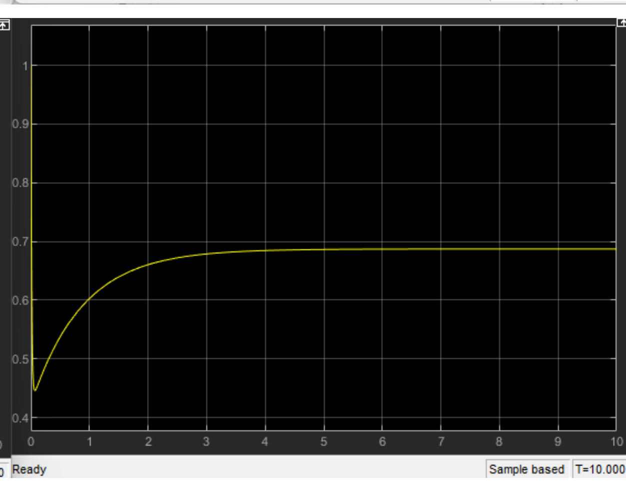


STEP INPUT:
[1,0]

R=

[20 0;0 20]





STEP INPUT:
[1,0]

R=

[30 0;0 30]

So, we can conclude that, if we tune Q more precisely, the system will perform in a well condition whereas, proper R value will send the desired control signal to the plant. The larger the elements of Q are, the larger are the elements of the gain matrix K, and the faster the state variables approach zero. On the other hand, the larger the elements of R, the smaller the elements of K which leads to slower response but smaller energy cost

PART 3: LQR BASED OBSERVER

According to our plant that is, $y = Cx$, y has a dimension of (2x1), X is a matrix of (3x1) and C is of (2x3).

C has a rank of 2.

We want to take advantage of the 2 state variables, that are available through y and construct an observer of order (3-2)=1, lower than 3.

Let $\xi = Tx$, where T is of dimension (3-2) x3 i.e., 1x3.

Combining the two equations results in

$\begin{bmatrix} y \\ \xi \end{bmatrix} = \begin{bmatrix} C \\ T \end{bmatrix} x$, $\begin{bmatrix} C \\ T \end{bmatrix}$ is of 3x3 and non-singular

The state can be obtained by inverting $\begin{bmatrix} C \\ T \end{bmatrix}$.

Hence, $x = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} y \\ \varepsilon \end{bmatrix}$ …... (1)

Which implies that x is a combination of plant's output and the observer output.

For our system, $x_1(t)$ needs to be constructed since other states can be observed from y.

Consider $T = [t_1 \;\; t_2 \;\; t_3]$ since we already know that T is of dimension 1x3

we will have, $\begin{bmatrix} y(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t_1 & t_2 & t_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$ where the main constrain is that $t_1 \neq 0$ to maintain the whole rank of the matrix which is 3.

The form of the observer is, $\dot{\xi} = d\xi + eu + gy$

Where ξ, d are scalars.

Let d=-10. With e=TB, the observer will have the below form,

$$\dot{\xi} = -10\xi + TBu + gy \qquad \text{……. (2)}$$

$$= -10\xi + [t, \;\; t_2 \;\; t_3] \begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix} u + gy \quad \text{….. (3)}$$

g and T values can be found from $dT - TA + gC = 0$, or

$$TA \qquad - \qquad dT \qquad = \qquad gC$$

$$[t_1 \;\; t_2 \;\; t_3] \begin{bmatrix} 1 \cdot 7 & -0.25 & 0 \\ 20 & -30 & 20 \\ 0 & -450 & -740 \end{bmatrix} + 10[t_1 \;\; t_2 \;\; t_3] = [g_1 \;\; g_2] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{……. (4)}$$

let, $g_1 = 1 \;\; and \;\; g_2 = 1$

after comparing the left and right side of the equation, we will have from the MATLAB calculation, $T = [0.1694 \; -0.0110 \; -0.0017]$

$$\begin{bmatrix} y(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.1694 & -0.0110 & -0.0017 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Which is a full rank matrix.

After which, the final observer designed can be deduced to,

$$\begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.1694 & -0.0110 & -0.0017 \end{bmatrix}^{-1} \begin{bmatrix} y(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} 0.0649 & 0.01 & 5.9032 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y(t) \\ \xi(t) \end{bmatrix}$$

Or $\hat{x}_1(t) = 0.0649 y_2(t) + 0.01 y_3(t) + 5.9032 \xi(t)$

$\hat{x}_2(t) = y_2(t) \;\; and \;\; \hat{x}_3(t) = y_3(t)$

Since, we have all the states, the state feedback can be designed as,

$$u = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} k_{11}\hat{x}_1 + k_{12}\hat{x}_2 + k_{13}\hat{x}_3 \\ k_{21}\hat{x}_1 + k_{22}\hat{x}_2 + k_{23}\hat{x}_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$u_1 = k_{11}(0.0640 y_2(t) + 0.01 y_3(t) + 5.9032 \xi(t)) + k_{12} y_2(t) + k_{13} y_3(t)$ ……………….. (5)

$u_2 = k_{21}(0.0640 y_2(t) + 0.01 y_3(t) + 5.9032 \xi(t)) + k_{22} y_2(t) + k_{23} y_3(t)$ ………………… (6)

$\dot{\xi} = -10\xi + [t, \; t_2 \;\; t_3] \begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix} u + gy = -10\xi + [2 \cdot 3914 \;\;\; 1.7170] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + [1 \;\;\; 1] \begin{bmatrix} y_2 \\ y_3 \end{bmatrix}$… (7)

From equation 5,6 and 7 we will have,

$$\dot{\xi} = 0.4128 y_3(t) - 0.6663 y_2(t) - 15.8790 \xi(t)$$

The closed loop system will have the below form,

$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} -1 \cdot 7 & -0.25 & 0 & 0 \\ 23 & -30 & 20 & 0 \\ 0 & -450 & -740 & 0 \\ 0 & -0.6663 & 0.4128 & -15.8790 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + Bu$ ………………….. (8)

From ***Part2 LQR controller***, we can find out the values of k with Q = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and R=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and after including the values in equation 5 and 6, we will get,

$$u = \begin{bmatrix} -0.1447 & 0.0479 & 27.6145 \\ 0.1 & -0.4614 & -7.7079 \end{bmatrix} \begin{bmatrix} y_2(t) \\ y_3(t) \\ \xi(t) \end{bmatrix}$$

$$\text{Bu}=\begin{bmatrix} -0.7236 & 0.2397 & 138.0723 \\ 6.3675 & -2.1093 & -1.2150e+03 \\ 83.0150 & 382.9556 & 6.3976e+03 \end{bmatrix}\begin{bmatrix} y_2(t) \\ y_3(t) \\ \xi(t) \end{bmatrix}$$

$$\text{Bu}=\begin{bmatrix} -0.7236 & 0.2397 & 138.0723 \\ 6.3675 & -2.1093 & -1.2150e+03 \\ 83.0150 & 382.9556 & 6.3976e+03 \end{bmatrix}\begin{bmatrix} y_2(t) \\ y_3(t) \\ \xi(t) \end{bmatrix}$$

$$=\begin{bmatrix} 0 & -0.7236 & 0.2397 & 138.0723 \\ 0 & 6.3675 & -2.1093 & -1.2150e+03 \\ 0 & 83.0150 & 382.9556 & 6.3976e+03 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \xi \end{bmatrix}$$

Hence from equation 8 we will get,

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} -1.7 & -0.9736 & 0.2397 & 138.0723 \\ 23 & -11.1935 & 19.6027 & -1.2150e+03 \\ 0 & -366.9850 & -357.0434 & 6.3976e+03 \\ 0 & -0.6663 & 0.4128 & 15.8790 \end{bmatrix}\begin{bmatrix} x \\ \xi \end{bmatrix}$$

$$y = [0\ 1\ 1\ 0]\begin{bmatrix} x \\ \varepsilon \end{bmatrix}$$

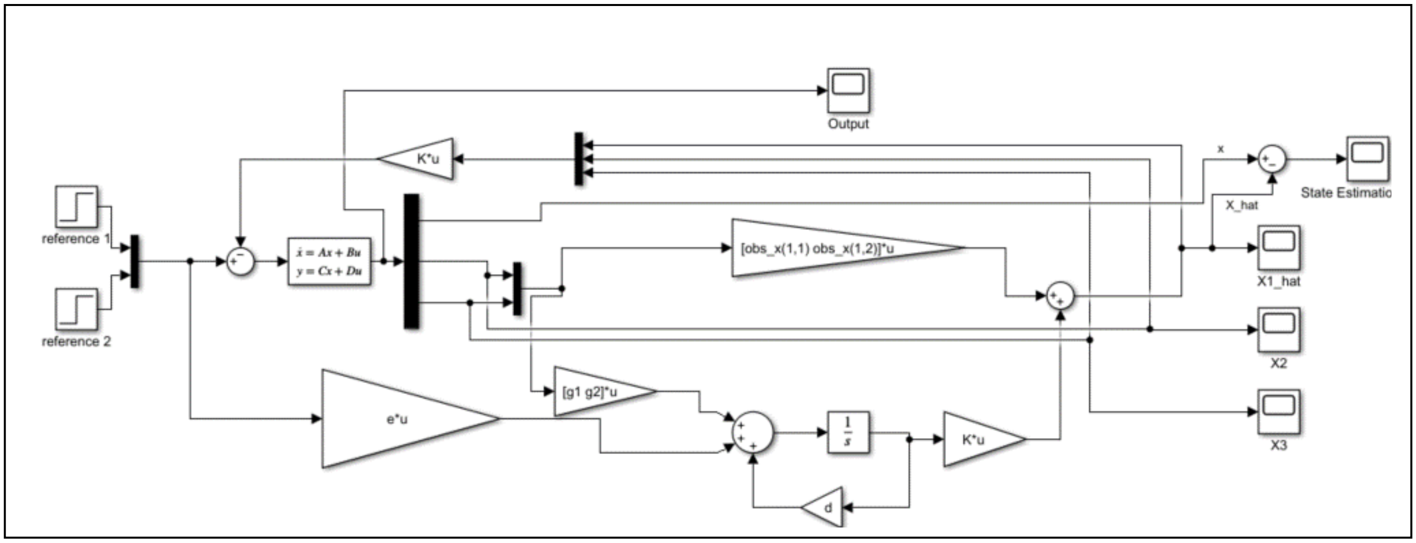
Simulink Model:

Using the below SIMULINK model (without the feedback, step time = 50secs), we are observing only the first response which is c_a or the component concentration of the reactant since the only two outputs in y can be measured.

With the change in different variables like G, the state estimation error is different.

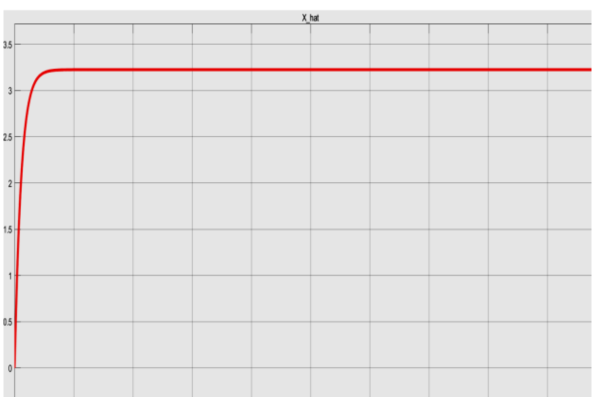
State estimation error= $\tilde{x} = x - \hat{x}$, where x = real state variable and $\hat{x}$ = observed state variable

For the above d, G values the observed response (without feedback) of the c_a is as follows:



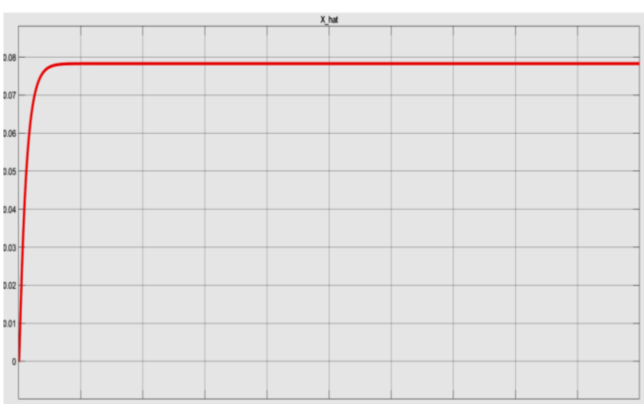
State response of the observer

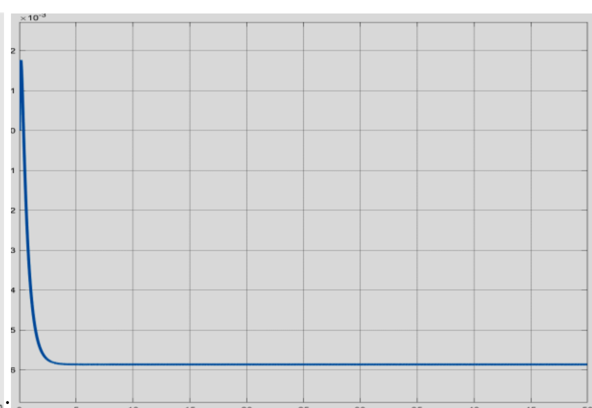
$\hat{x}_1$



[1,0] state

State Estimation Error [1,0] state





[0,1] state

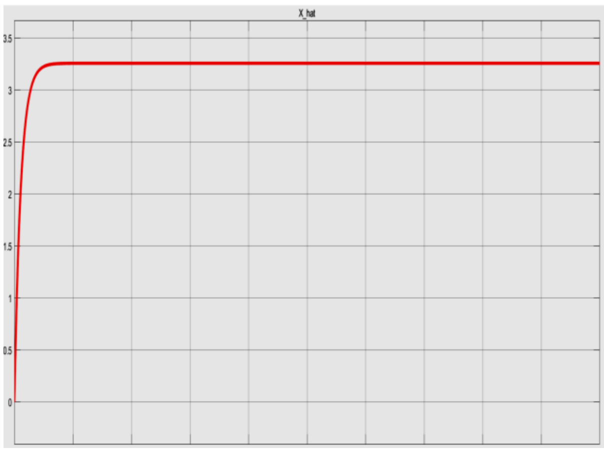
State Estimation Error [0,1] state

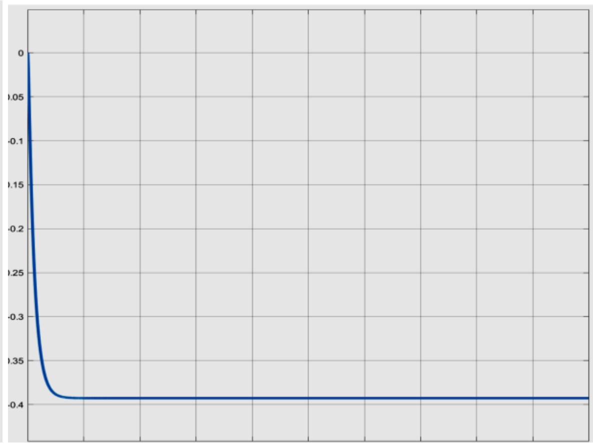
Whereas if we change the values of d and G, the observed state will have different response.

Let's consider,

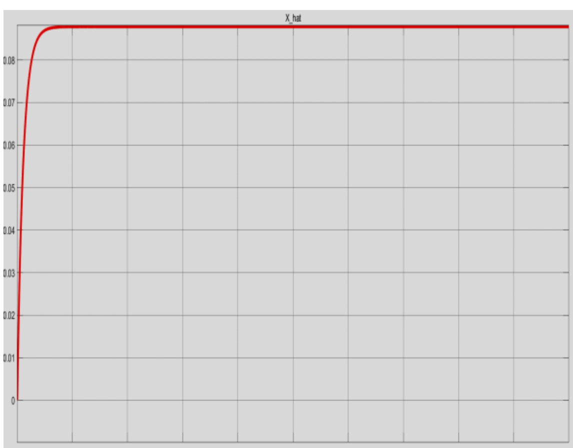
$d = -20$, $g1=g2=10$

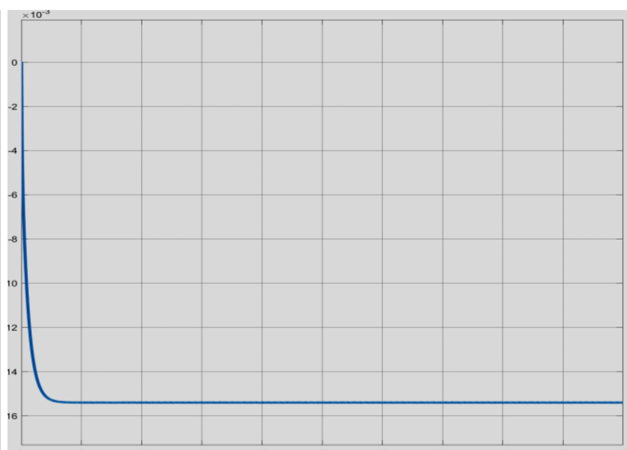
$\hat{x}_1$ State Estimation Error



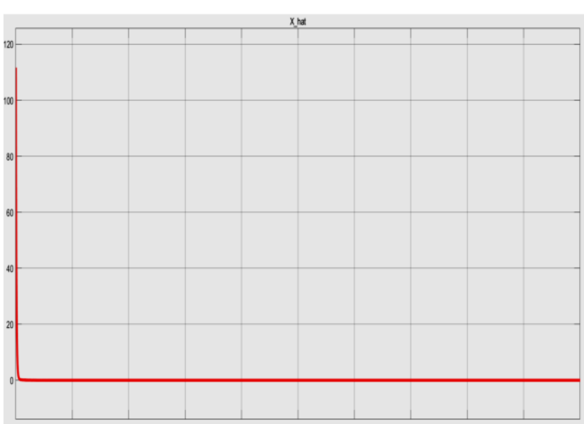


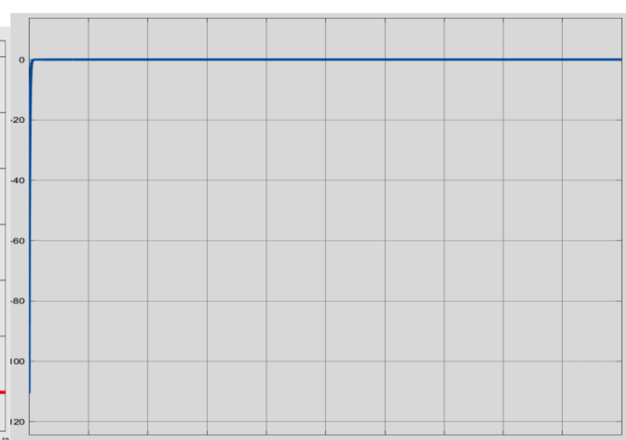
[1,0] state





[0,1] state



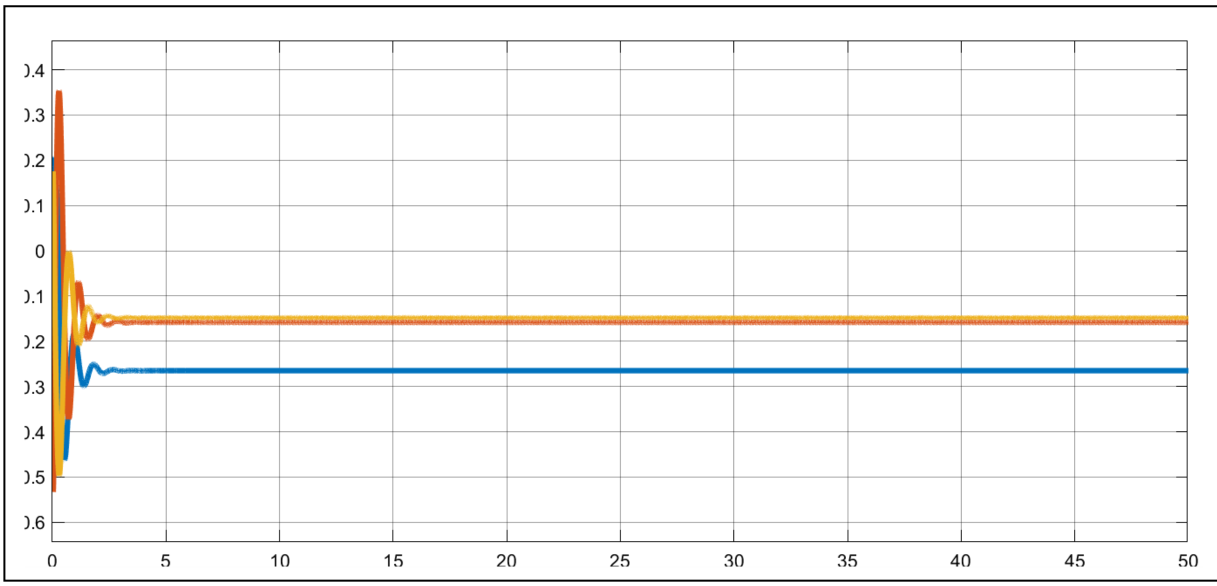


With Initial Condition

Hence, it can be said that with proper values of d and G the observer will estimate the state preciously. From the above observation during the non-zero initial state the state estimation error is going to zero that means the estimation error is zero. So the state is perfectly observed although during transient state there are some errors which can be reduced by tuning the parameters.

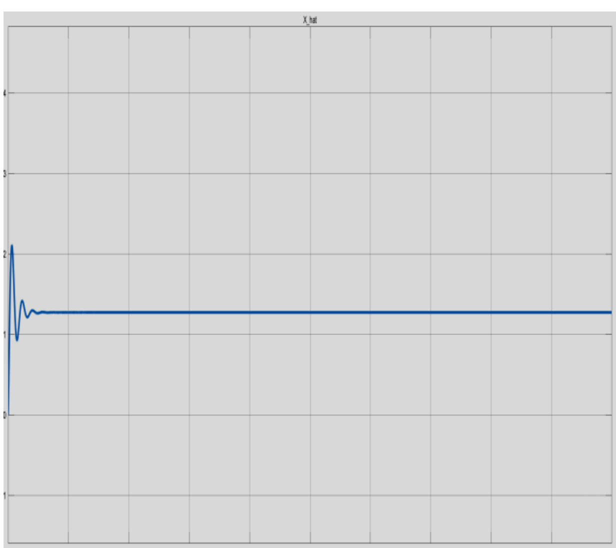
Closed Loop Performance:

With the feedback the system has a different response. Since we will have too many variables to tune. We can use the previous d, G values and will tune Q and R value of LQR to see how the system is responding, After including the previous identity matrices of Q and

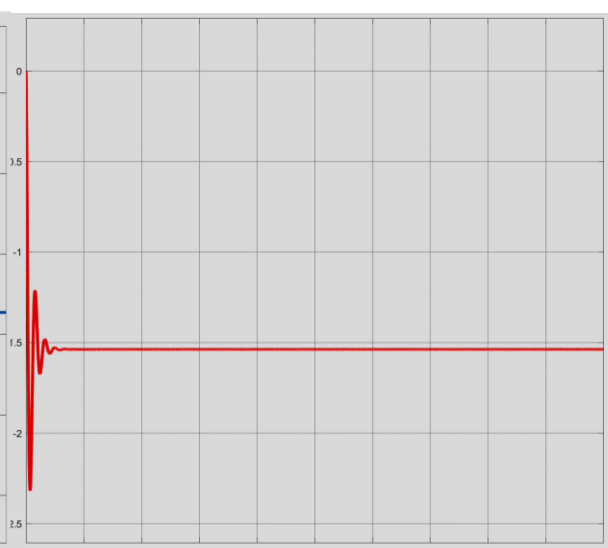


Ca, T, Tj

R, the responses of the output and the state are:

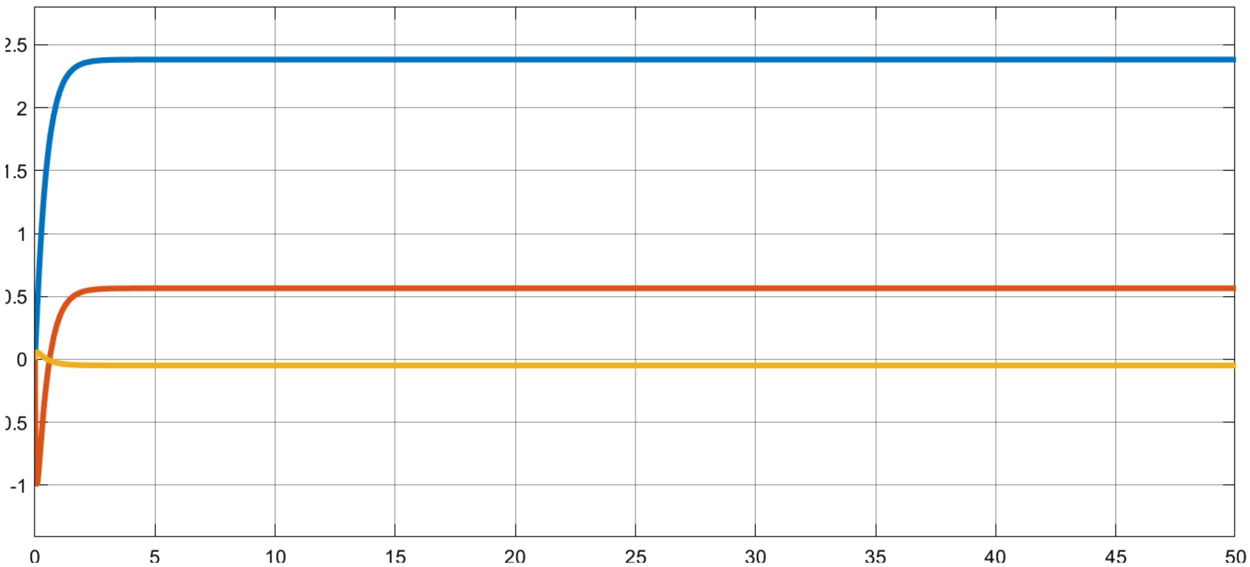


$\hat{x}_1$



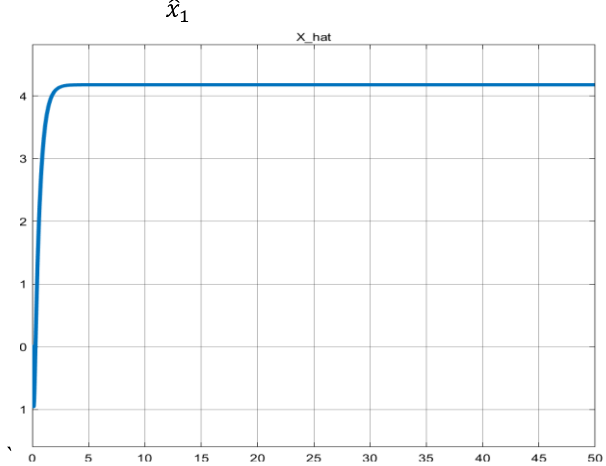
State Estimation Error

Let’s consider a different Q value which is [0 0 0; 0 0 0; 0 0 10] and same R value

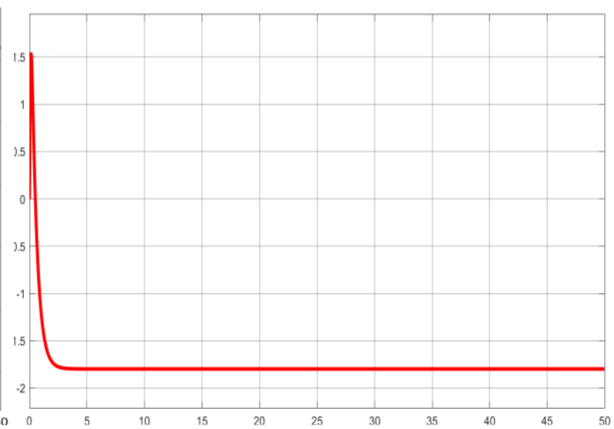


Ca, T, Tj

$\hat{x}_1$

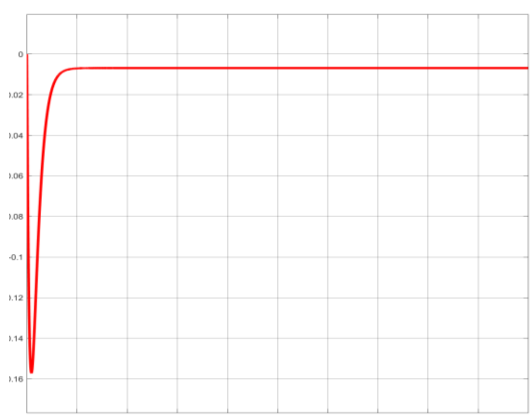


State Estimation Error



State Estimation Error

[1,0] state



With feedback and the values of different variables like below the state-estimation error is minimum:

Q=[0 0 0;0 10 0;0 0 1], R=[50 0;0 50], G=[10 10], D=-3. The state-estimation graph is,

With proper tunning of the weights for the LQR method, the state estimation error can be minimized.

It can be seen from the observations is that the performance can be improved by changing the 4 variables and system can work at steady state.

Using the Q= [0 0 0;0 0 0;0 0 5], R=[1 0;0 1], d=-20, g1=g2=10, the closed loop system has poles at s= - - 0.9129 $\pm$ 3.7239i and - 19.7152 - 2.2114e-37i. which means that the system us stable.

PART 4: DECOUPLED BY STATE FEEDBACK

The open loop transfer function matrix: $G(s) = C(sI - A)^{-1}B$

Let the state feedback controller, $u = -kx + Fr$

The closed loop system is $\dot{x} = (A - BK)x + BFr$

$y = Cx$

And the transfer function matrix of the feedback system is,

$$H(s) = C(sI - A + Bk)^{-1}BF$$

The objective is to make H(s) to a diagonal matrix which is so called decoupled matrix.

The closed loop transfer function is related to open loop transfer function matrix in the below way,

$$H(s) = G(S)[I + k(sI - A)^{-1}B]^{-1}F$$

Where G(s) has to be nonsingular along with H(s).

For our model, we have,

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

One proceeds as follows:

$i = 1\,,\ C_1^T = [0\ 1\ 0]$

$$C_1^T B = [0\ 1\ 0]\begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix} = [-44\ \ 0] \neq 0 \rightarrow \sigma_1 = 1$$

$i = 2, c_2^T = [0\ \ 0\ 1]$

$$C_2^T B = [0\ 0\ 1]\begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix} = [0\ \ -830] \neq 0 \rightarrow \sigma_2 = 1$$

After expanding each row of the G(s), the G(s) can be written as,

G(s)=diag($s^{-\sigma_1}, s^{-\sigma_2}$) $[B^* + C^*(sI - A)^{-1}B]$

Where H(s)= diag($s^{-\sigma_1}, s^{-\sigma_2}$) is called an integrator decoupled system.

$C^* = \begin{bmatrix} C_1^T A^{\sigma_1} \\ C_2^T A^{\sigma_2} \end{bmatrix}, B^* = \begin{bmatrix} C_1^T A^{\sigma_1 - 1} B \\ C_2^T A^{\sigma_2 - 1} B \end{bmatrix}$ where $\sigma_1 = 1, \sigma_2 = 1$

$c^* = \begin{bmatrix} 23 & -30 & 20 \\ 0 & -450 & -740 \end{bmatrix}, \; B^* = \begin{bmatrix} -44 & 0 \\ 0 & -830 \end{bmatrix}$

$F = (B^*)^{-1}, k = (B^*)^{-1} C^*$

Hence, $F = \begin{bmatrix} -0.0227 & 0 \\ 0 & -0.0012 \end{bmatrix}, k = \begin{bmatrix} -0 \cdot 5227 & 0 \cdot 6818 & -0 \cdot 4545 \\ 0 & 0.5422 & 0.8916 \end{bmatrix}$

The feedback system becomes

$\dot{x} = (A - Bk)x + BFr$

$$= \begin{bmatrix} 0.9136 & -3 \cdot 6591 & 2.2727 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} -0 \cdot 1136 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} r$$

$y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ x ,

$H = C[sI - A + B * k]^{-1} BF$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.9136 & 3.6591 & -2.2727 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix}^{-1} \begin{bmatrix} 0.1136 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/s & 0 \\ 0 & 1/s \end{bmatrix}$$

From the above expression we could make the closed loop transfer function to our desired one.

And decoupled the system. But the above system is not stable. Hence, to make the system stable we need the poles at the negative half of the imaginary axis.

For that, we need to decouple the system with pole placement. Where,

$H(s) = \begin{bmatrix} \frac{1}{\phi_{f_1}} & 0 \\ 0 & \frac{1}{\phi_{f_2}} \end{bmatrix}$ after considering $\phi_{f_1} = s + 4, \phi_{f_2} = s + 6$

the $H(s) = \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+6} \end{bmatrix}$

from here we just need to change the way to compute C^*

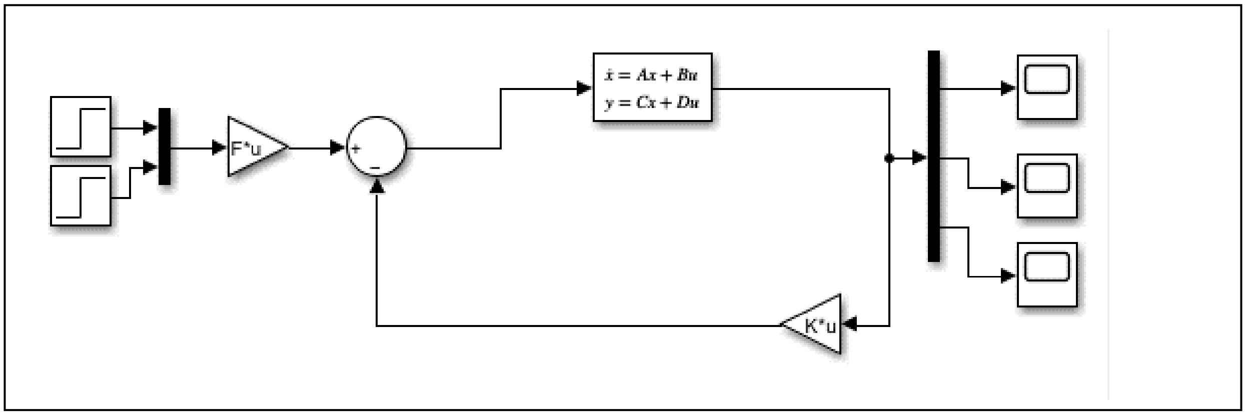
$$C^{**} = \begin{bmatrix} C_1^T \phi_{f_1}(A) \\ C_2^T \phi_{f_2}(A) \end{bmatrix} \begin{bmatrix} C_1^T (A + 4I) \\ C_2^T (A + 6I) \end{bmatrix} = \begin{bmatrix} 23 & -26 & 0 \\ 0 & -450 & -734 \end{bmatrix}$$

$$k = (B^*)^{-1}C^{**} = \begin{bmatrix} -0\cdot5227 & 0.5909 & -0.4545 \\ 0 & 0.5422 & 0.8843 \end{bmatrix}$$

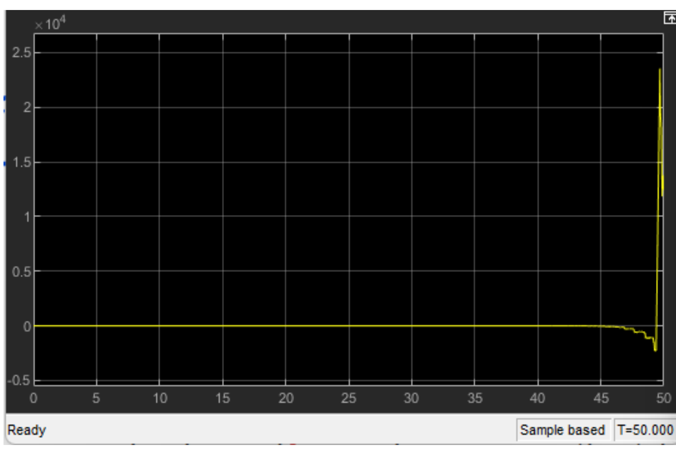
Verification,

$$H(s) = C[sI - A + B * k]^{-1}BF = \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+6} \end{bmatrix}$$

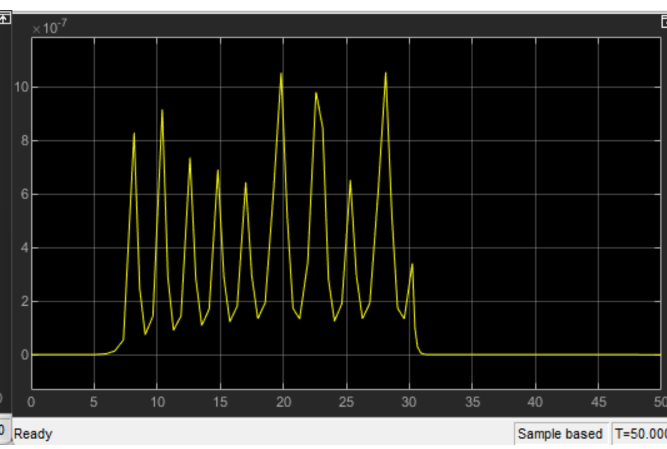
Simulink model:



After simulating the above model for 50s the transient response is generated as follows:

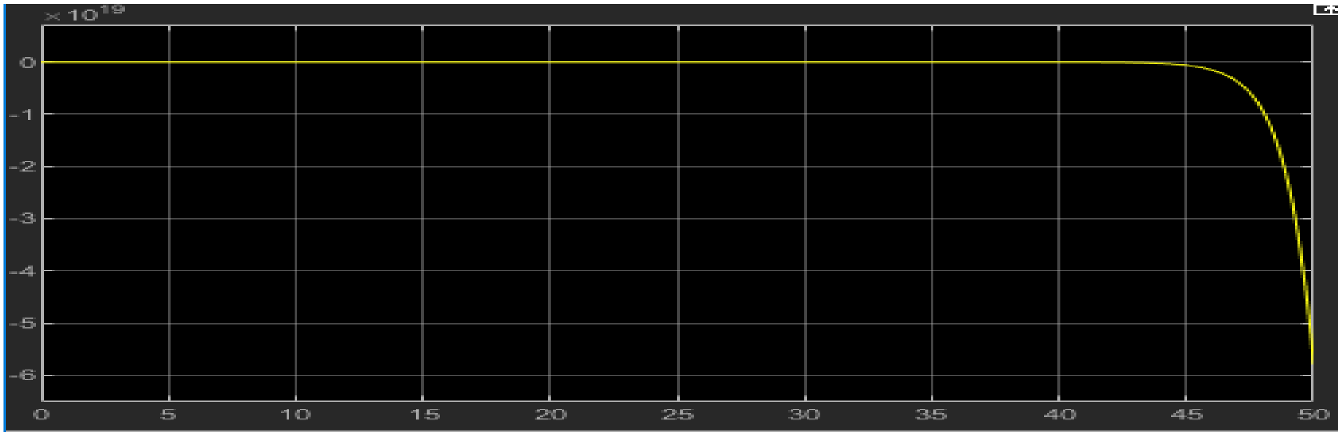


T



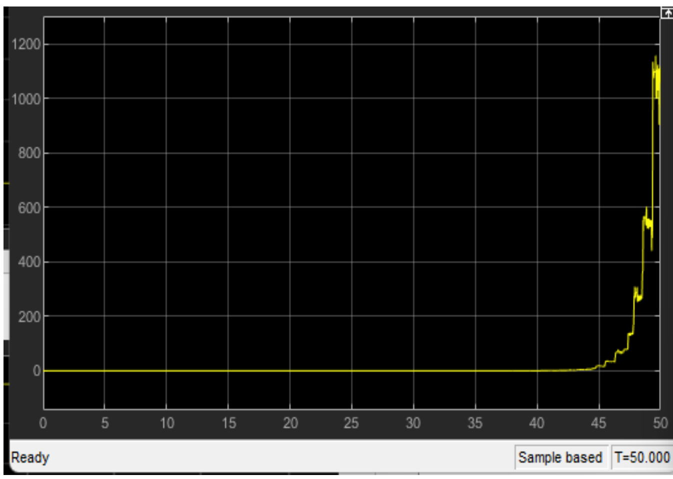
[1,0]

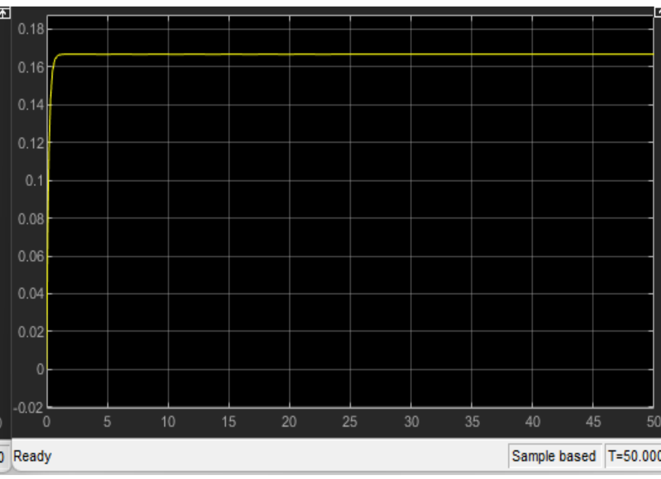
T_j



[1,0]

c_a

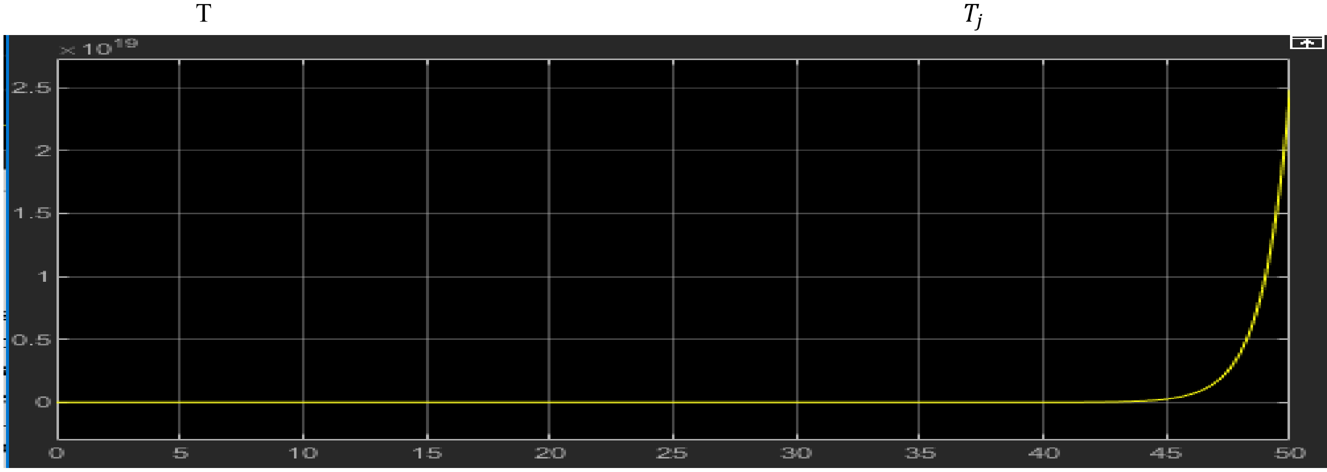




[0,1]

T

T_j



[0,1]

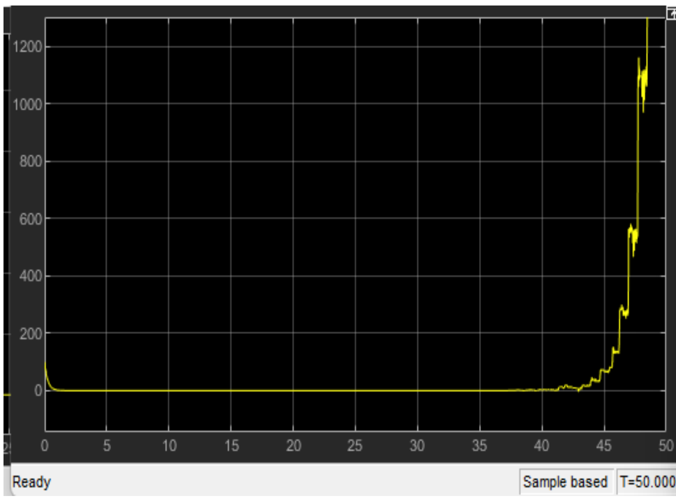
c_a

Moreover, the poles of the closed loop decoupled system are at 0.9136, -4 and -6

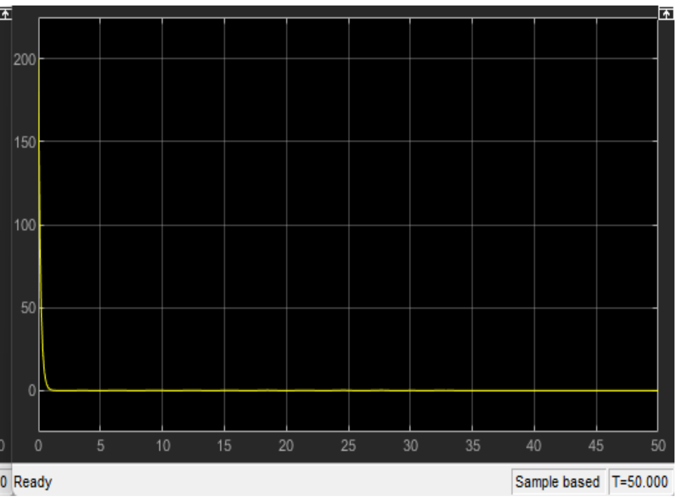
After evaluating the transient responses and the pole locations, according to Lyapunov equation for the LTI system, all the poles of a system should locate at the left half of the s plane. For the reason being such location of one pole, which can't be changed without the change in the plant, it can be concluded that the decoupled system is internally unstable.

Given $\dot{x} = Ax$. The system is said to be stable i.s.L. if and only if all eigenvalues of A have zero or negative real parts and those with zero real parts has no Jordan block of order 2 or higher. The system is Asymptotically Stable i.s.L. if and only if all the eigenvalues of A have negative real parts.

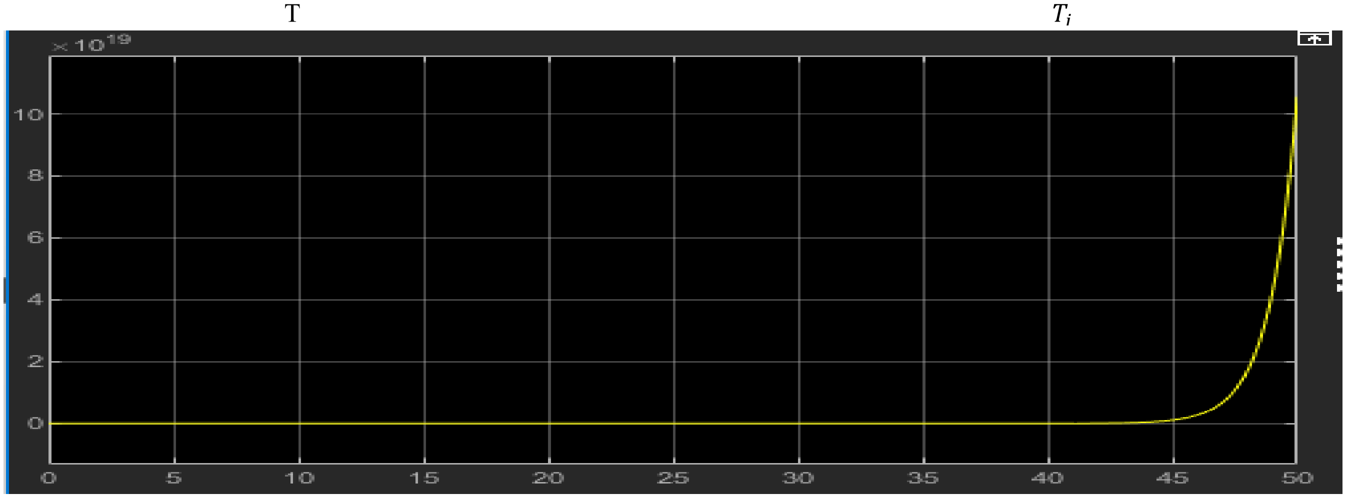
The responses with the initial condition are as follows:



T



T_i



c_a

PART5: SERVO CONTROL

Servo Control problem is the problem where we require both asymptotic tracking and regulation.

Asymptotic Tracking: $\lim_{t\to\alpha}(steady\ state\ error) = \lim_{t\to a} e(t) = \lim_{t\to\alpha}\big(r(t) - y(t)\big) = 0$ for

$r(t) \neq 0, w(t) = 0$

Asymptotic Regulation: $\lim_{t\to\alpha} (steady\ state\ error) = \lim_{t\to a} e(t) = \lim_{t\to\alpha}\left(r(t) - y(t)\right) = \lim_{t\to\alpha}\left(-y(t)\right) = 0$

For $r(t) = 0, w(t) \neq 0$

Design = Servo Mechanism + Stabilization

The calculation for the multivariate Integral Control:

Consider our plant which is,

$$\begin{aligned} \dot{x} &= Ax + Bu + B_w w \\ y &= Cx \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

The error is defined as, $e = r - y$

W and r both are of step type.

One integrator to each channel is required to be introduced to achieve zero steady state error, like SISO integral control.

$$v(t) = \int_0^t e(t)\,dt\ \dot{v}(t) = e(t) = r - y(t) = r - Cx(t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

From equations 1 and 2, we will get

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}\begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} B_\omega \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ I \end{pmatrix} r$$

$$y = [c \quad 0]\begin{bmatrix} x \\ v \end{bmatrix} = \bar{c}\bar{x}$$

The $Q_C = \begin{bmatrix} B & AB & A^2B \ldots \ldots . . \\ 0 & -CB & -CAB \ldots \ldots . \end{bmatrix}$ Is controllable if and only if (i) the plant is controllable and

(ii) $rank\begin{pmatrix} A & B \\ -C & 0 \end{pmatrix} = n + m$

For our plant,

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \left(\begin{bmatrix} -1\cdot 7 & -0.25 & 0 & 0 & 0 \\ 23 & -30 & 20 & 0 & 0 \\ 0 & -450 & -740 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}\right)\begin{pmatrix} x \\ v \end{pmatrix} + \begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} w + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix}$$

(i) From the previous parts, it's known that the plant is controllable

(ii) $rank\begin{pmatrix} A & B \\ -C & 0 \end{pmatrix} = 5 = \mathrm{n} + \mathrm{m}$

Our plant is satisfying both of the above conditions, so the augmented system is controllable.

Since, the augmented system is non-singular and thus controllable, it can be stabilized by the state feedback control law:

$u = -k\bar{x} = -[k_1 \quad k_2]\begin{bmatrix} x \\ v \end{bmatrix}$

Or $u(t) = -k_1 x - k_2 \int_0^t e\, d\tau$

So, the resultant feedback system is,

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{pmatrix} A - Bk_1 & -Bk_2 \\ -C & 0 \end{pmatrix}\begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} B_\omega \\ 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ I \end{pmatrix} r$$

$y = [C \quad 0]\begin{bmatrix} x \\ v \end{bmatrix}$ -> $e(t) = \dot{v}(t) = 0$

The optimal control is minimizing $J = \int_0^\alpha (\bar{x}^T Q \bar{x} + u^T R u)\, dt$

Yields the feedback gain for the state feedback u= -Kx

$$k = R^{-1}\bar{B}^T P$$

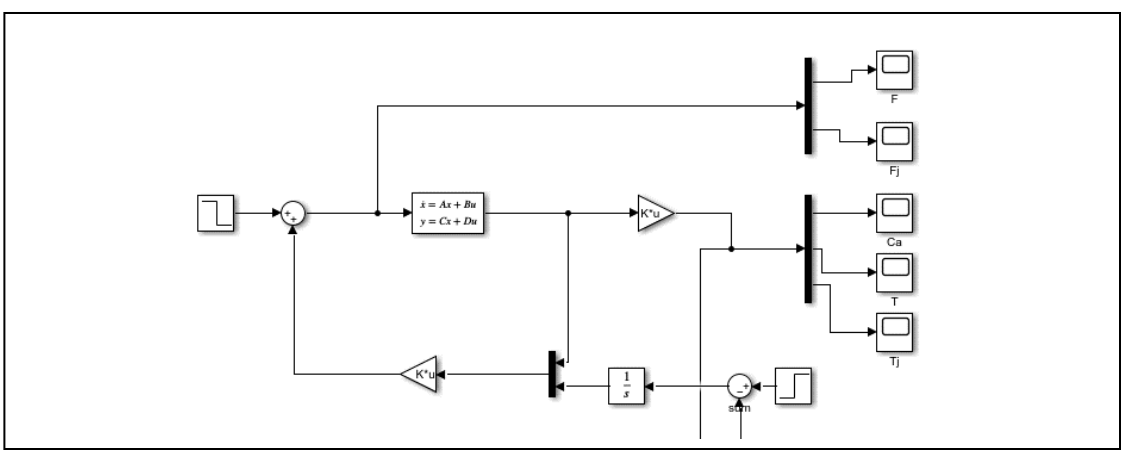
Using the ***LQR method of Part2*** and considering $Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

we will get

$$k = \begin{bmatrix} 0.0272 & -0.0015 & 0 & -0.0605 & 0.0366 \\ -0.0008 & -0.0001 & -0.0029 & 0.0366 & 0.0605 \end{bmatrix}$$

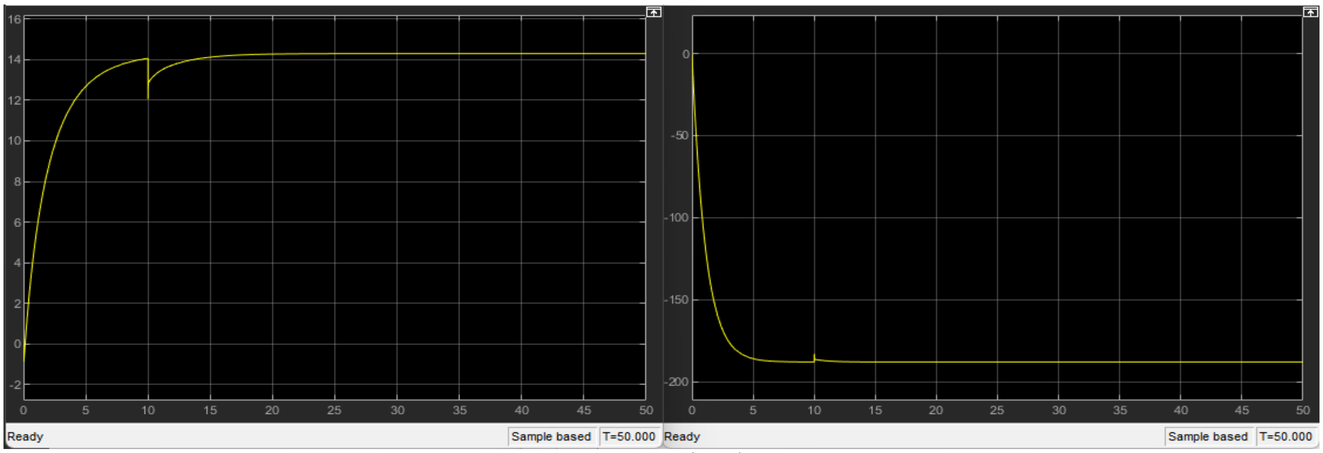
Given the operating set point of two outputs is $y_{sp} = [100{,}150]^T$ and step disturbance, $n = [-2{,}5]^T$

Simulink Model:

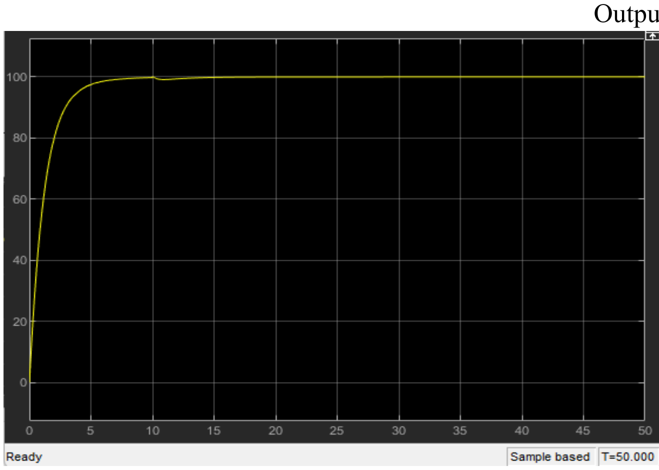


After including the set point $y_{s_P} = [100,150]^T$ and the step disturbance $w = [2,5]^T$ which takes effect after $t_d = 10s$ afterwards, the control signal and output signal from the SIMULINK model are:

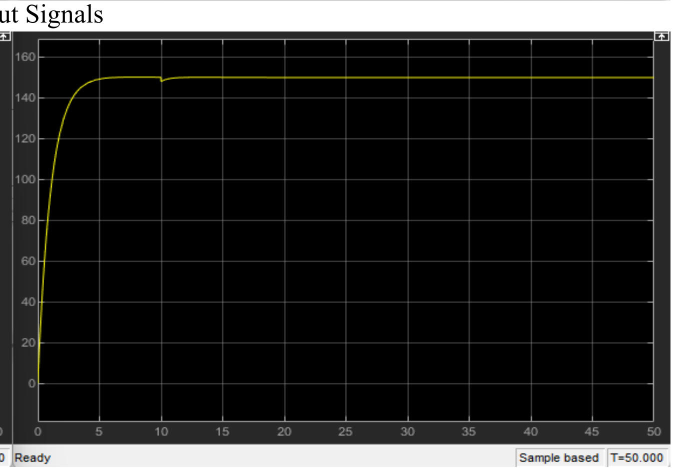
Control Signals



Output Signals



T



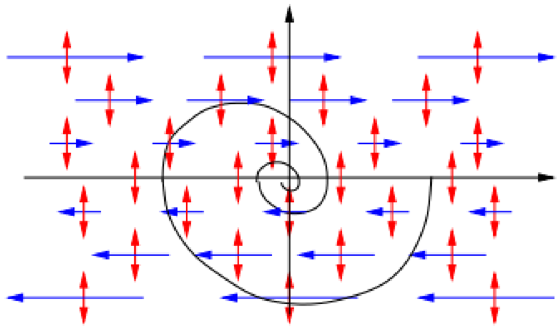
T_i

PART6

According to me, we will not be able to maintain the state variables at the set point $x_{sP} = [5,250,300]^T$ from the initial state x_0,

Reason:

If we maintain the state variables at any set point in the state space, then it can only pass through the point in a transient fashion. But if we steer the system to an equilibrium point, it is possible to remain their infinitely (since $\dot{x}_1 = 0, \dot{x}_2 = 0 \; and \; \dot{x}_3 = 0$).



with proper control strategy we can bring the system to the equilibrium line.

(b)

However, we may only want to keep the state variables at steady state close enough to the set point x_{sp}.

Now, we have the state-space equation as,

$\dot{x}_s = Ax_s + Bu$, x_s is the state vector at steady state

Since at equilibrium, $\dot{x} = 0$, hence

$$Ax_s + Bu = 0$$

$$x_s = A^{-1}Bu$$

The objective function is,

$J(x_s) = \frac{1}{2}(x_s - x_{sp})^T w(x_s - x_{sp})$ where $w = diag(a + b + 1, c + 4, d + 5)$

a=2, b=5, c=4, d=1

$$w = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}, A^{-1}B = \begin{bmatrix} -0.5446 & 0.0032 & 0.0001 \\ -0.2971 & -0.0220 & -0.0006 \\ 0.1806 & 0.0134 & -0.0010 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -44 & 0 \\ 0 & -830 \end{bmatrix} = \begin{bmatrix} 2 \cdot 8648 & -0.0724 \\ -0.5192 & 0.4925 \\ 0.3157 & 0.8221 \end{bmatrix}$$

$x_{sp} = [5,250,300]^T$

Now, after including all the variables, the objective function will be,

$$J(x_s) = 0.5 \text{ x } (\begin{bmatrix} 2 \cdot 8648 & -0.0724 \\ -0.5192 & 0.4925 \\ 0.3157 & 0.8221 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} 5 \\ 250 \\ 300 \end{bmatrix})^T \text{ x } \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{ x } (\begin{bmatrix} 2 \cdot 8648 & -0.0724 \\ -0.5192 & 0.4925 \\ 0.3157 & 0.8221 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} 5 \\ 250 \\ 300 \end{bmatrix})$$

$$J(x_s) = 0.5 \text{ x } \begin{bmatrix} 2.8648u_1 - 0.0724u_2 - 5 \\ -0 \cdot 5192u_1 + 0 \cdot 4925u_2 - 250 \\ 0.315ru_1 + 0.8221u_2 - 300 \end{bmatrix}^T \text{ x } \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix} \text{ x } \begin{bmatrix} 2.8648u_1 - 0.0724u_2 - 5 \\ -0 \cdot 5192u_1 + 0 \cdot 4925u_2 - 250 \\ 0.315ru_1 + 0.8221u_2 - 300 \end{bmatrix}$$

To minimize above objective function, we need to differentiate the function with respect to $u_1 and\ u_2$

As a result, we have to calculate,

$\frac{\partial J(s)}{\partial u_1} = 0 \quad , \frac{\partial J(s)}{\partial u_2} = 0$

After using MATLAB for the above calculation, the values of both the variables are,

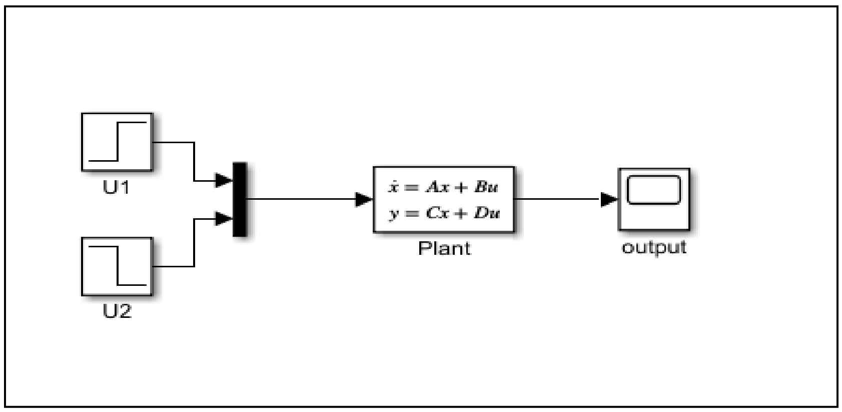
$$u_1 = 15 \cdot 5813, u_2 = -410.7783$$

After including the values in the equation of $x_s = A^{-1}Bu$, the equilibrium points can be extracted.

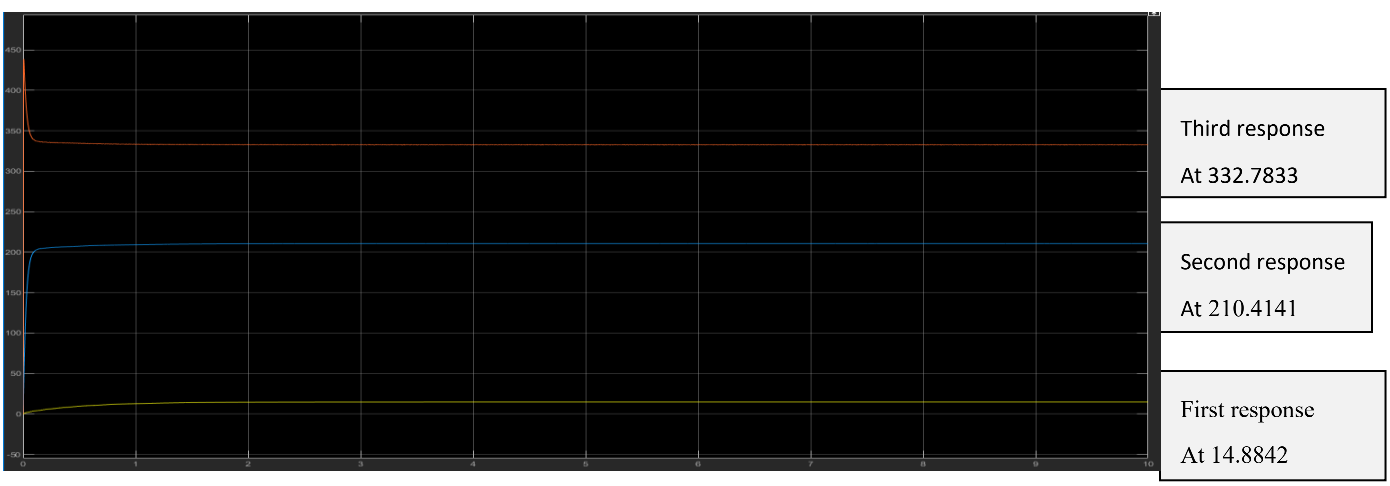
Hence, $\begin{bmatrix} x_{s_1} \\ x_{s_2} \\ x_{s_3} \end{bmatrix} = \begin{bmatrix} 2 \cdot 8648 & -0.0724 \\ -0.5192 & 0.4925 \\ 0.3157 & 0.8221 \end{bmatrix} \begin{bmatrix} 15 \cdot 5813 \\ -410.7783 \end{bmatrix} = \begin{bmatrix} 14.8842 \\ 210.4141 \\ 332.7833 \end{bmatrix}$

The distance between the equilibrium points and the set point is d = $x_s - x_{sp} = \begin{bmatrix} 9 \cdot 8842 \\ -39.5859 \\ 32 \cdot 7833 \end{bmatrix}$

Simulink Model:



Response: after simulating the above model for 10secs, the response can be visualized as,



From the above calculation, the value of the objective function at steady state is 9883.2

Whereas if we consider the initial state and calculate the objective function then the value would be

$(x_0 - x_{sp})^T \times w \times (x_0 - x_{sp})$ = 120064

For LQR the objective is to bring the state from non-zero initial value to zero (equilibrium point).

For zero value, the J will become, $\left(-x_{sp}\right)^T \times w \times \left(-x_{sp}\right)$ that is 520100.

As a result, we can conclude that the J is minimum with the values derived from the above calculations of Part6

REFERENCE

- All the Class Notes and Tutorials
- Algebraic Riccati equation - Wikipedia
- Help Center for MATLAB, Simulink and other MathWorks products
- Decoupling Control Design for the Module Suspension Control System in Maglev Train (hindawi.com)
- http://www.cds.caltech.edu/~murray/books/AM05/pdf/am06-statefbk_16Sep06.pdf

APPENDIX

POLE PLACEMENT

```
clc
clear
format short;
A=[-1.7 -0.25 0; 23 -30 20; 0 -450 -
740];
b=[5 0; -44 0; 0 -830];
C=[0 1 0;0 0 1];
d=0;
x0=[1;100;200];
w=[b A*b A^2*b];
c=[w(:,1) w(:,3) w(:,2)];
c_inv=inv(c);
T=[c_inv(2,:);c_inv(2,:)*A;c_inv(3,:)];
T_inv=inv(T);
A_bar=T*A*T_inv;
B_bar=T*b;

%%%%%%% pole calculation %%%%%%%%%%%
sig=0.6;
w=0.25;
lambda=-(sig*w)+(w*(1-sig^2)^0.5)*i
syms s;
equ1=(s-
(real(lambda)))^2+(imag(lambda))^2;
equ2=(s+5*real(-lambda));
pol_equ=expand(equ1*equ2);
pol_equ1=sym2poly(pol_equ);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Al=[0 1 0;0 0 1; -pol_equ1(1,4) -
pol_equ1(1,3) -pol_equ1(1,2)];
kbar=mldivide(B_bar,(A_bar-Al));
% x0=[0;0;0];
k=kbar*T
sys_new=ss((A-b*k),b,C,d);
% initial(sys_new,x0);
[p,z]=pzmap(sys_new);
t=0:0.01:2;
u=[ones(size(t));zeros(size(t))];
[p,z]=pzmap(sys_new);
```

LQR METHOD

```
clc
clear
A=[-1.7 -0.25 0; 23 -30 20; 0 -450 -
740];
b=[5 0; -44 0; 0 -830];
C=[0 1 0;0 0 1];
d=0;
Q=[0 0 0;0 60 0;0 0 900];
R=[10 0;0 10];
a1=A;
b1=-b*inv(R)*b';
c1=-Q;
d1=-A';
tau=[a1 b1;c1 d1];
[u,v]=eig(tau);
%%%%u has negative eigen values in 1,3
and 4th column, so we will divide
%%the divide those eigen vectors into
two parts which will be v and u of
%%the Pv=u.
v1=zeros(3,3);
u1=zeros(3,3);
count=1;
for i=1:6
    if(v(i,i)<0)
        v1(:,count)=real(u(1:3,i));
        u1(:,count)=real(u(4:6,i));
        count=count+1;
    end
end
P=u1*inv(v1);
K=inv(R)*b'*P
sys=ss((A-b*K),b,C,d);
step(sys);
l=stepinfo(sys);
l(1,1)
```

OBSERVER BASED MODEL

```
""" statespace matrices  """;

A=[-1.7 -0.25 0; 23 -30 20; 0 -450 -
740];
b=[5 0; -44 0; 0 -830];
c=[0 1 0;0 0 1];
D=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%


"""   LQR   """ ;

Q=[0 0 0;0 0 0;0 0 10];
R=[1 0;0 1];
a1=A;
b1=-b*inv(R)*b';
c1=-Q;
d1=-A';
tau=[a1 b1;c1 d1];
[u,v]=eig(tau);
%%%%u has negative eigen values in 1,3
and 4th column, so we will divide
%%the divide those eigen vectors into
two parts which will be v and u of
%%the Pv=u.
v1=zeros(3,3);
u1=zeros(3,3);
count=1;
for i=1:6
    if(v(i,i)<0)
        v1(:,count)=real(u(1:3,i));
        u1(:,count)=real(u(4:6,i));
        count=count+1;
    end
end
P=u1*inv(v1);
K=inv(R)*b'*P

""""""""""" observer design
""""""""""";

syms t1 t2 t3 g1 g2;
g1=10;
g2=10;
d=-20;
equ1=-1.7*t1+20*t2-d*t1;
equ2=-0.25*t1-30*t2-450*t3-d*t2-g1;
equ3=20*t2-740*t3-d*t3-g2;
[t1,t2,t3]=solve(equ1,equ2,equ3,t1,t2,t
3);
t=[double(t1) double(t2) double(t3)];
e=t*b;
new_c1=[c;t];
obs_x=inv(new_c1);
```

```
""" Observer state and closed loop
calculation """ ;

k11=double(K(1,1));k12=double(K(1,2)
);k13=double(K(1,3));
k21=double(K(2,1));k22=double(K(2,2)
);k23=double(K(2,3));
coef1=obs_x(1,1)*k11+k12;
coef2=obs_x(1,2)*k11+k13;
coef3=obs_x(1,3)*k11;
coef4=obs_x(1,1)*k21+k22;
coef5=obs_x(1,2)*k21+k23;
coef6=obs_x(1,3)*k21;
sympref('FloatingPointOutput',true);
syms y1 y2 sig;
u2=coef4*y1+coef5*y2+coef6*sig;
u1=coef1*y1+coef2*y2+coef3*sig;
sig_dot=d*sig+[1.3310
1.4110]*[u1;u2]+[1 1]*[y1;y2]
sig_dot_mat=coeffs(sig_dot)
u=[coef1 coef2 coef3;coef4 coef5
coef6];
u_new=b*u;
new_A=[-1.7 -0.25 0 0; 23 -30 20 0;
0 -450 -740 0;0 sig_dot_mat(1,2)
sig_dot_mat(1,1) sig_dot_mat(1,3)];
Bu=[zeros(3,1) u_new;zeros(1,4)];
A_close=new_A+Bu;
new_c=[0 1 1 0];
eig(A_close)
```

DECOUPLING PROBLEM

```
clc
clear
A=[-1.7 -0.25 0; 23 -30 20; 0 -450 -
740];
B=[5 0; -44 0; 0 -830];
C=[0 1 0;0 0 1];
D=[0 0; 0 0; 0 0];
x0=[1;100;200];

%intergrator - decoupled by state
feedback
syms s;

sympref('FloatingPointOutput',true)
GS = C*inv(s*eye(3)-A)*B;

%Computing Bstar
J1 = C(1,:)*B;
J2 = C(2,:)*B ;
Bstar=[ J1(1,:) ; J2(1,:)];
CK1=det(Bstar); %check if Bstar is
singular

%computing C star
I1= C(1,:)*A;
I2= C(2,:)*A;
Cstar = [I1(1,:) ; I2(1,:)];

%computing C double star with Pole
I11= C(1,:)*A + 4*C(1,:)*eye(3);
I21= C(2,:)*A + 6*C(2,:)*eye(3);
Cdstar1 = ([I11(1,:) ; I21(1,:)]);

F = inv(Bstar);
K = F*Cstar;
K1 = (F*Cdstar1)

HS = C*inv(s*eye(3)-A + B*K)*B*F;

HS1 = C*inv(s*eye(3)-A + B*K1)*B*F;
simplify(HS1);
diag(HS1);
Acl = (A-B*K1);
E=eig(Acl)
```

SERVO CONTROL

```
clc
clear
%""" The augmented system """%
A=[-1.7 -0.25 0 0 0;23 -30 20 0 0;0 -
450 -740 0 0;0 -1 0 0 0;0 0 -1 0 0];
b=[5 0;-44 0;0 -830;0 0;0 0];
c=[0 1 0 0 0;0 0 1 0 0];
d=0;
bw=[-2;5;0;0;0];
br=[0;0;0;0;1];


%""" LQR """%
Q=[1 0 0 0 0;0 5 0 0 0;0 0 5 0 0;0 0 0
5 0;0 0 0 0 5];
R=[1 0;0 1];
a1=A;
b1=-b*inv(R)*b';
c1=-Q;
d1=-A';
tau=[a1 b1;c1 d1];
[u,v]=eig(tau);
v1=zeros(5,5);
u1=zeros(5,5);
count=1;
for i=1:10
    if(v(i,i)<0)
        v1(:,count)=real(u(1:5,i));
        u1(:,count)=real(u(6:10,i));
        count=count+1;
    end
end
P=u1*inv(v1);
K=inv(R)*b'*P
```

PART 6

```
clear;
clc;
sympref('FloatingPointOutput',true);
syms u1 u2 x
a=2;
b=5;
c=4;
d=1;
A=[-1.7 -0.25 0; 23 -30 20; 0 -450 -740];
B=[5 0; -44 0; 0 -830];
Cd=[1 0 0;0 1 0;0 0 1];
D=0;
x0=[1; 100; 200];
xsp=[5;250;300];
W=[a+b+1 0 0;0 c+4 0;0 0 d+5];

"""  At Steady State """ ;
U=[u1;u2];
xs=-inv(A)*B*U; %At steady state xdot=0
dis=xs-xsp;
J=0.5*dis'*W*dis;
J1=diff(J,u1)
J2=diff(J,u2)
[U1,U2]=vpasolve([J1 == 0, J2 == 0],[u1,u2])
U=[U1;U2]

""" J calculation with equilibrium points """ ;
xs=-inv(A)*B*U
dis=xs-xsp
J=0.5*dis'*W*dis

""" J calculation with initial condition """;
dis1=x0-xsp;
J1=0.5*dis1'*W*dis1

dis2=-xsp;
J2=0.5*dis2'*W*dis2
```