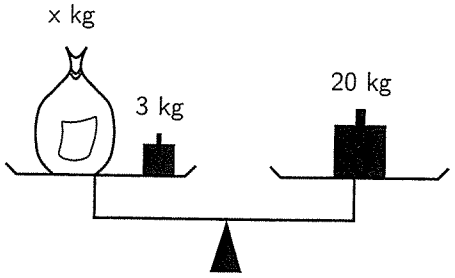
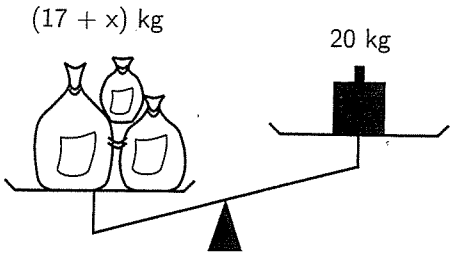
3 Gleichungen

3.1 Einfache Gleichungen und Ungleichungen



$x + 3 = 20$
$L = \{17\}$



$17 + x > 20$
$L = \{x \in \mathbb{R} \mid x > 3\} =]3, \infty[$

Zu 1 – 12: Löse in der Grundmenge $\mathbb{Z}$.

1	a) $x + 5 = 8$	b) $x + 8 = 5$	c) $x - 8 = 5$	d) $5 - x = 8$
2	a) $8 - x = 5$	b) $8 + x = x$	c) $8 - x = x$	d) $x + 2x = x$
3	a) $3x - 5 = -11$	b) $3x + 11 = -7$	c) $12 - 7x = -23$	d) $-4x = 5 + x$
4	a) $x + 13 > 0$	b) $x - 13 < 0$	c) $2x \leq 7$	d) $-3x \geq 12$
5	a) $2x \leq x$	b) $4x > -20$	c) $-4x < 20$	d) $20x < -30$
6	a) $3.6x = 15 + x$	b) $3.6 > 15 + x$	c) $15 < 3.6 + x$	d) $3.6 \leq x - 15$
7	a) $2x = -10$	b) $2x < -10$	c) $3x = -x$	d) $3x \geq -x$
8	a) $5x = x^2$	b) $2x^2 = 10x$	c) $3x^2 = 48$	d) $x^2 \leq 10x$
9	a) $x^2 \leq 20$	b) $x^2 \geq 8$	c) $x^2 \leq 100$	d) $x^2 + 100 = 0$
10	a) $3x \leq 7$	b) $4x > -3$	c) $x < -x$	d) $x^2 \leq 5x$
11	a) $5 : x = 1$	b) $x : 5 = 1$	c) $x : 5 = 0$	d) $5 : x = 0$
12	a) $x : 5 = x$	b) $5 : x = 5$	c) $5 : x = x$	d) $9 : x = x$

Zu 13–20: Löse in der Grundmenge $\mathbb{R}$.

13 a) $7x = x + 1$ b) $5x = 3x - 2$ c) $9x + 4 = 10x - 1$
d) $-4x + 1 = x - 5$

14 a) $4x + 6 = 10.8$ b) $5x - 12 = 4.5$ c) $18 - 5x = 21$
d) $10x + 1.5 = -16$

15 a) $4.5x + 10 = 2x$ b) $1.2x + 7 = 1.3x$ c) $0.75x + 1 = 19$
d) $-1.5x = 2.25$

16 a) $1.4x = -7$ b) $2.4x = 20$ c) $1.8x - 16 = 2.5x - 2$
d) $12x + 11 = 2.1x$

17 a) $x + 2.5 \leq -4$ b) $4 + x \geq -2.5$ c) $2x \leq -3.5x - 11$
d) $4.5x - 3 \geq -2$

18 a) $3x + 8 > -6$ b) $x > 2x + 1$ c) $9 - x > -0.5x + 1$ d) $-4x > 20$

19 a) $6 : x = 1.5$ b) $x : 6 = -1.5$ c) $x : 2 \geq -10$ d) $x : 5 \leq -2$

20 a) $x : 24 \geq 10$ b) $x : 1.5 \leq -12$ c) $36 : x > 10$ d) $24 : x \leq -12$

Zu 21–27: Stelle eine Gleichung auf und löse sie.

21 Berechne die gesuchte Zahl.

a) Wenn man das Vierfache einer Zahl um 9 verkleinert, erhält man gleich viel, wie wenn man ihr Dreifaches um 5 vergrössert.
b) Addiert man zu einer Zahl 15, so erhält man das Dreifache der Zahl.
c) Addiert man zu einer Zahl die um 8 grössere Zahl, so erhält man das Fünffache der Zahl.
d) Subtrahiert man von einer Zahl 20, so erhält man das Dreifache der Zahl.

22 Berechne die kleinere der beiden Zahlen.

a) Zwei Zahlen unterscheiden sich um 13; ihre Summe beträgt 100.
b) Zwei Zahlen mit der Summe -120 haben den Unterschied 12.
c) Das Doppelte der ersten Zahl ist um 5 kleiner als die zweite Zahl, die um 100 grösser ist als die erste.
d) Zwei Zahlen unterscheiden sich um 5. Ihr Produkt ist um 60 grösser als das Quadrat der kleineren Zahl.

28 a) $6x - 10 = x - 5$ b) $4x - 6 = x - 12$ c) $9x - 9 = x + 3$
d) $-x - 2 = x + 3$ e) $15x - 9 = 4x + 90$ f) $1 - 8x = 1 - 5x$

29 a) $5 - x = 25 + 3x - 4$ b) $16 + 2x = 56 - 8x - 20$ c) $3x - 9 - 5x = 9x + 24$
d) $15x - 21x = 4 + 2x$ e) $3 - 4x = 5 - 2x - 16$ f) $x - 19 = 5x + 23$

30 a) $2x - 22 - 9x = 42 + 11x - 100$ b) $2x + 7 - 16x = 8 + 6x + 39$
c) $3x - 15 - 4x = -9 + x - 13$ d) $15x - 73 - 24x = 59 - 16 + 20x$

31 a) $19x - 32 + 17x = 18x - 30 + 16x - 4$
b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$
c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

32 a) $3x - 19 - 5x + 7 - 3x - 1 = 4x - 5 + 6x + 7 - 2x - 1$
b) $19x - 24 - x = 13x - 24 + 14x - 16 - x - 17 - 8x$
c) $7x + 16 - 15x - 29 + 11x + 26 = 0$
d) $0 = 21 - 19x - 12 - 39 + 14x - 10$

33 a) $280 + 43x - 1999 = 50x - 1999$ b) $36x + 47 = 47 - x + 74$
c) $58x - 34 + 25x = 58x + 31$ d) $92 - 13x - x^2 = 52 - 3x - x^2$

34 a) $-69 + 12x - 44 = 23x - 44 + 12x$ b) $17x - 18 + 19x = 17x - 28 + 39x$
c) $x^2 + 3x - 8 = x^2 - 2x + 27$ d) $65 - 31x = 16x - 15 - 31x$

35 a) $7x - (5x + 1) = x$ b) $2x - (10 - x) = 0$
c) $0 = 8x - (4x - 5) - 29$ d) $14 - (x - 15) = 2 - (6x + 13)$

36 a) $6x + (1 - 7x) = 9 - (5x + 5)$ b) $x = (9 - x) - 7 - (x + 2)$
c) $5 - (4x + 22) - 13x = 0$ d) $12.5 - 13x = 5 - (6x + 3)$

37 a) $8(x + 1) = 11(x - 0.5)$ b) $9 - 10(7x + 2) + 13(20 - x) = 0$
c) $67 - 2(3x + 13) = 29 - 3(2x - 4)$ d) $5(4x + 9) - 6(2x - 5) = 75$

38 a) $7(100 - x) = 9(2x + 50)$ b) $11 - 11(11x - 11) + 111 = 1$
c) $4(5x - 6) - 7 = 4 - 5(6x - 7)$ d) $10 - 6(x - 14) = 20 - 3(2x - 25)$

39 a) $47(23x - 59) = 470$ b) $11(25x - 66) = 22(17x + 21)$
c) $89(5x - 48 + 3x) = 0$ d) $0 = 21(444 - 333x - 555)$

40 a) $75 + 25(13x - 5) = 125(14 - x)$ b) $38(x + 1 - 10x^2) = 380(1 - x^2)$
c) $0 = 37(37x - 47 - 9x - 37)$ d) $77(x - 22) = 7777(x - 2)$

41 a) $4x - [3x - (2x + 1) - 9] = 1$ b) $7[x - 3(5 - x)] = 3[9x - (x + 3)]$
c) $6(1 - 4(x - 2)) = 3(9 - 8x)$ d) $6x - (3 - (2x - 1) - 5) = 11$

42 a) $52 + 7x - [8(x + 3) - 3(12 - x)] - 11(5 - 3x) = 9(1 - x) + 10x$
b) $100 - [2(x - 3) + 3(20 - x)] = 28[4(x - 5) - 3(x - 4)]$
c) $40 - (3(2x - 4) - 2(2x - 3)) = 60 - 2(x + 5) - 4$
d) $500 - (2x + 500 - (3x + 100 - x)) = 10(2x - 1)$

43 a) $(x + 1)(x + 7) = (x + 2)(x + 3)$ b) $(x - 5)(x - 2) = (x - 4)(x + 3)$
c) $(x - 5)(x - 2) = (x - 4)(x - 3)$ d) $(x + 1)(x + 6) = (x + 4)^2$
e) $(x + 3)(x - 5) = (x - 3)^2$ f) $(x + 8)(x - 1) = (x - 8)(x + 1)$

44 a) $(15x - 3)^2 = x(225x - 15)$ b) $(17x - 30)^2 = 17x(17x - 10)$
c) $x^2 - 3x + 14 = x(x + 7)$ d) $(2x - 3)^2 = (2x + 3)^2 + 12$

45 a) $2(x + 2)(x + 5) = (2x + 7)(x + 3)$ b) $3(x + 1)(x + 4) = (3x + 6)(x + 3)$
c) $3(x + 1)(x + 4) = (3x + 6)(x - 3)$ d) $(x + 7)(x + 8) - (x - 7)(x - 8) = 45$

46 a) $2x^2 - (x + 3)(x - 3) = (x + 1)^2 - 2x + 8$
b) $(x + 2)(x - 3) - 3(2x - 3) = (x - 6)^2 + 2$
c) $(x - 3)(2x - 5) + 4(2 - x) + 12 = 2(1 - x)^2$
d) $2x(x - 5) - (x - 5)^2 = (x - 10)^2 + 20x - 125$

47 a) $(2x - 3)^2 - (x - 5)^2 - 3x(x - 7) + 17 = 0$
b) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$
c) $(5x - 1)^2 - x[10x - 3(x - 4)] = 18x^2 - 21$
d) $(2x - 3)(3 + 2x) - [4 - 5(x - 1)] \cdot x = 9x^2$

48 a) $(x - 1)(x - 2)(x - 3) - (x - 1)(x - 2)(x - 4) = x^2$
b) $(x - 5)(x - 4)(x - 2) - (x - 5)(x - 6)(x - 2) = 2x(x - 1)$
c) $(x - 3)^2(x + 4) - (x - 3)(x + 4)(x - 7) - 4x(x - 1) = 0$
d) $(2x - 3)^3 - (2x - 3)^2(2x - 7) = (4x - 8)^2$

Zu 49–55: $a \cdot b = 0 \Leftrightarrow a = 0$ oder $b = 0$[1].

49 a) $7777(2x - 37) = 0$ b) $1234(444 - 10x) = 0$
c) $x(12x + 96) = 0$ d) $35x(7x + 91) = 0$

50 a) $(x - 3)(x - 4) = 0$ b) $(x + 5)(x - 2) = 0$
c) $(3x + 12)(x - 11) = 0$ d) $(2x + 7)(5x - 8) = 0$

51 a) $(x - 6)(2x + 9) = 0$ b) $(5x - 2)(4x + 3) = 0$
c) $(120 - 8x)(12 + 8x) = 0$ d) $(x + 2.5)(5x - 2) = 0$

52 a) $x(x - 9)(2x + 13)(3x - 15) = 0$ b) $(5x + 7)(6x - 90)(9x + 60) = 0$
c) $(x - 7.5)(7.5 - x)(4x + 10) = 0$ d) $3x(4 + x)(16 - 5x)(16x + 24) = 0$

53 a) $(4x + 3 + 7x)(15 - 7x - 1) = 0$ b) $x(3x + 17 - 20x)(25 + 7x + 3) = 0$
c) $111x(6x + 60 - x)(4x - 10) = 0$ d) $(11x + 12 - 3x)(11 + 12x - 3) = 0$

54 a) $x^2 - 5x + 6 = 0$ b) $x^2 - 9x + 20 = 0$
c) $x^2 - x - 20 = 0$ d) $x^2 - 5x - 24 = 0$
e) $x^2 - 2x - 63 = 0$ f) $x^2 - 5x - 14 = 0$

55 a) $x^2 + 13x - 68 = 0$ b) $x^2 - 21x + 68 = 0$
c) $x^2 - 5x - 50 = 0$ d) $x^2 - 15x + 50 = 0$
e) $x^2 - 7x - 120 = 0$ f) $x^2 - 29x + 120 = 0$

Bruchgleichungen ohne Variable im Nenner

56 a) $\frac{x}{4} + \frac{1}{5} = \frac{x}{2} + \frac{x}{6}$ b) $\frac{2x}{3} - \frac{4x}{9} = 31 - \frac{3x}{2}$
c) $\frac{2x}{9} - \frac{5x}{6} - \frac{11}{12} = 0$ d) $\frac{15x}{16} - \frac{11x}{12} - 25 = 0$

57 a) $\frac{x+3}{5} = \frac{2x-8}{3}$ b) $\frac{x+1}{4} = \frac{x-1}{3}$
c) $\frac{x+3}{4} + \frac{1-3x}{7} = 0$ d) $\frac{4x-3}{6} - \frac{3x-8}{5} = 0$

58 (a) $\frac{x-2}{3} + \frac{x-2}{2} = \frac{5}{6}$ b) $\frac{2x+1}{3} + \frac{2x-1}{4} = \frac{5}{4}$
c) $\frac{x-15}{5} + \frac{3x-4}{4} = 15$ (d) $\frac{2x+7}{5} + \frac{4x+3}{3} = 5$

[1] Siehe auch Kap. 4.3 Nummern 109–116 unter "Verlustumformungen".

59 a) $\frac{x-5}{4} - \frac{x-5}{8} = \frac{1}{8}$ b) $\frac{2x-4}{3} - \frac{2x-4}{5} = \frac{2}{15}$

c) $\frac{3x-11}{15} - \frac{2x+3}{18} = 0.4$ d) $\frac{4x+1}{26} - \frac{2x-1}{39} = 0$

60 a) $\frac{x+5}{4} - \frac{1-x}{6} = 4$ b) $\frac{19x}{25} - \frac{13x+1}{15} = 2$

c) $\frac{x-15}{18} - \frac{6x+5}{14} - \frac{x}{7} = 0$ d) $\frac{4x-5}{3} - \frac{2x-1}{6} = \frac{x}{2} - 1$

61 a) $\frac{2x-3}{6} - \frac{3x+1}{4} - \frac{x+3}{5} = 1$ b) $\frac{3x-19}{15} - \frac{x}{18} = \frac{x-12}{10}$

c) $\frac{x-21}{14} - \frac{x-28}{21} - \frac{x-14}{28} = \frac{1}{7}$ d) $2x - 3 - \frac{x-5}{17} = \frac{x}{51}$

62 a) $7.75 - \frac{3x-7}{8} = 13 - \frac{2x+3}{4}$ b) $6 - \frac{x}{6} = \frac{x-7}{3} + 2x$

c) $4 - \frac{5x-1}{6} + \frac{2}{3} = \frac{34x-1}{3} - 7$ d) $x + \frac{2}{5} - \frac{3x-0.5}{10} = \frac{3x}{4} - 2.5$

63 a) $\frac{8x-3}{8} - \frac{8+3x}{3} = 0$ b) $\frac{6x-1.5}{12} + \frac{1-4x}{8} = 0$

Einige Ungleichungen

64 a) $3x + 5 > 2x - 6$ b) $3 - x < 4x + 5$

c) $4 - 2x \leq 3x + 4$ d) $-3x + 5 \leq -5 + x$

65 a) $0.5x - 1.2 \geq 4.5x + 3.4$ b) $0.7x - 1.5 \leq 1.5 + 0.7x$

c) $-\frac{3}{4}x < \frac{1}{2}x$ d) $\frac{1}{3}x + \frac{1}{5}x - 0.8 > 0$

66 a) $\frac{5-2x}{3} < 0$ b) $\frac{3+2x}{4} \geq -2$

c) $\frac{x-1}{3} > \frac{1-x}{3}$ d) $\frac{3x-5}{3} < \frac{2x-1}{2}$

67 a) $(x+1)(x-3) > (x-4)^2$

b) $(x-3)(x+3) - (x-2) - (x-2)^2 \leq x - 1$

c) $(5x-1)^2 + 21 > 18x^2 + x[10x - 3(x-4)]$

d) $x - (2x-1)(3x+4) \leq (x+1)^2 - (7x-1)(x+1)$

83 a) $ax + bx = a + b$ b) $mx - nx = 2m - 2n$

84 a) $qx - x = q^2 - 1$ b) $cy + 2y = c^2 + 4c + 4$

85 a) $(a - b)x = a^2 - 2ab + b^2$ b) $4(c - 1)z = 4c^2 - 4$

86 a) $cx - x = 1 - c$ b) $by + cy = mb + mc$

87 a) $ay + by = a - cy$ b) $qx = rx + x + 1$

88 a) $4(ax - b) = 2(ax + 2a - bx)$ b) $2(bx - cx) = x + bx - c$

89 a) $4x - 2(ax + b) = b(x - 1)$ b) $5ax - c = a(x + 1) - 2(c + x)$

90 a) $(x - b)^2 = (x - a)^2$ b) $(m - x)^2 = (m + x)^2$

91 a) $(x - 4a)^2 - x(x + b) = b(7x + 16b)$ b) $(x + 3p)^2 = 2x(x + 3p) - x(x - 1)$

92 a) $2mx(mx - n) - 2(mx - n)^2 = mn(m + n) - 2n^2$
b) $(p + q)(p - q)x + (p + q)^2x - (p - q)^2x = p^2x + 4p - q$

93 a) $2a^2 - 2b^2 - (a - b)^2(x - a) = (a^2 - 2ab + b^2)(x + a)$
b) $-2x + (x - p)(x + p) + (p + 1)^2 = (x - p)^2 + 2$

94 a) $p(q - x) + q(r - x) - q(p - x) = rx$
b) $3a(y + 2b) - 2b(y - 3a) = 18a^2$

95 a) $b(a + z) - 2z(a + b) = a(b - z) + 2b(a - z)$
b) $2b(z + c) - 2c(3z - b) = 6c(b + z) - 4c(2z + c)$

96 a) $(x + a + 1)^2 - (x + a - 1)^2 + 2x - a = 0$
b) $(x - a - b)^2 - (x + a + b)^2 + 8a^2 + 8ab = 0$

97 a) $(x + c + d)^2 - (x - c - d)^2 = nc + nd$
b) $(x - n - p)^2 - (x + n)^2 + 2np + p^2 = 0$

98 a) $a(x - 2b) = 0$ b) $(ax + c)(a + c) = 0$ c) $s(x - a) + c(x - a) = 0$

99 a) $4(x + 3a)(x - 4a) = 0$ b) $(2c + x)(3c - 2x) = 0$ c) $px(2x + p)(4x - 5p) = 0$

100 $px(px + 4)(4x + p)(2x - 3q)(x + q)(4x - 22) = 0$

Zu 101–104: Löse die Gleichung für die angegebenen Parameterwerte.

101 $4ax - x(a-1) = x - 6a^2$ $\quad a = -15,\ -0.5,\ 20,\ 1000$

102 $(c+3)x - 2c(x+1) + 4c(x-1) = 6$ $\quad c = -5,\ -99,\ 520,\ 1200$

103 $(t+1)^2x - t^2x = t + 1 - x$ $\quad t = 13,\ 99,\ 9999$

104 $(p+2)(p-3)x + 9 = p(p-1)x - 6p$ $\quad p = 2.5,\ -1.5,\ 98.5$

Zu 105–109: Löse die Gleichung nach jeder Variablen auf, ohne Diskussion von Sonderfällen.

105 a) $s = vt$ $\quad$ b) $A = \frac{1}{2}gh$ $\quad$ c) $A = \frac{a+c}{2} \cdot h$

106 a) $Z = K \cdot \frac{p}{100}$ $\quad$ b) $K_1 = K_0 + K_0 \cdot \frac{p}{100}$

107 a) $L = 2R + R\alpha$ $\quad$ b) $b = \frac{2\pi R}{360^\circ} \cdot \alpha$

108 a) $A = \frac{a+b+c}{2} \cdot \varrho$ $\quad$ b) $A = \frac{abc}{4R}$

109 a) $m = \frac{a+2b+3c+4d}{10}$ $\quad$ b) $S = 2(ab+ac+bc)$