Mutual Exclusion I

Maruth Goyal

UT Austin

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- In reality, however, distributed computing is asynchronous: i.e., computation generally does not proceed in synhcronized sequences of rounds.
- This introduces a whole new swath of challenges.
- Today we will look at a primitive/fundamental challenge: mutual exclusion.
- Roughly speaking, given some resource R and processors $P_1, \ldots, P_n$ we need a protocol to ensure at most one processor is interacting with the resource at a given time.

Mutual Exclusion

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- For instance, if there is a printer on a network which prints the bytes it receives, unless mutual exclusion is enforced the streams of 2 processors may get garbled in arbitrary ways.
- We will be working in the **shared memory** model of asynchronous computing.
- i.e., all the processors will have access to some shared piece of memory. Additional constraints may be placed on who can write where, and read what.
- In practice, modern processors support many *atomic* operations such as *read-modify-write*, however, for now we will not assume their existence.

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Computing Model

- Each processor is modeled as a state machine
- The machine has possible states Σ, and a subset Σ_{start} of start states.
- The machine also has a set of "actions"
  - Each action is classified as being either "input", "output", or "internal"
  - Internal actions are further divided into those that interact with the shared memory and those that dont'.

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- Transition actions must satisy "locality":
  - An action not involving shared memory most only involve the process's local state.
  - An action accessing shared memory location x may only invole the value there and the process's local state.

Computational Model

Definition (Read-Write Variables)

In each step, a process can either read of write a single shared variable, but not both.

1. (**read**): Process i reads register x and uses the value read to modify the state of process i.
2. (**write**): Process i writes a value determined from process i's state to register x.

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Definition (Fairness Condition)

For each process i, one of the following holds:

1. The entire execution is finite, and in the final state no locally controlled action of process i is enabled.
2. The execution is infinite, and there are either infinitely many occurences of locally controlled actions of i, or else infinitely many places where no such action is enabled.

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Formalizing Mutual Exclusion

- Assume there are n users $U_1, \ldots, U_n$, each trying to access a shared resource R.
- In order to get access to R, each U_i must interact with process P_i which is part of the shared memory ME protocol.

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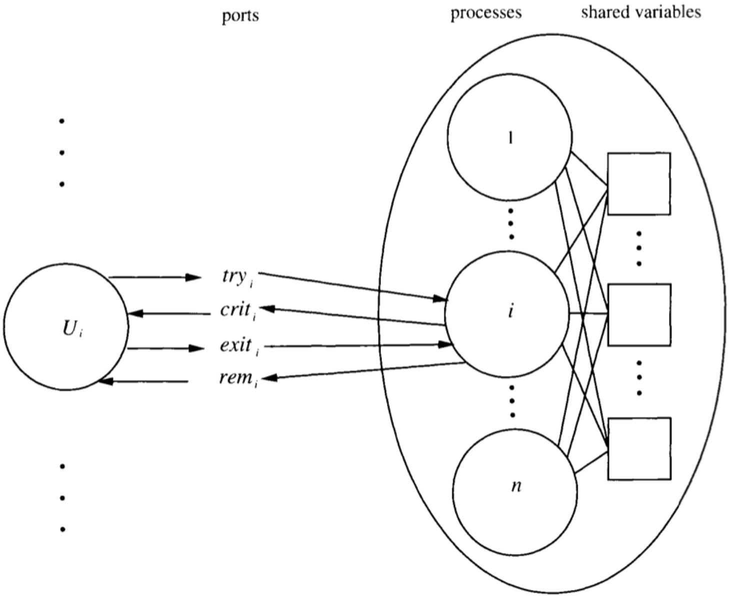
- The time between (1) and (2) is called the *Trying region* (T), (3) and (4) the *Critical region* (C), and after (4) the *remainder region* (R).

Formalizing Mutual Exclusion



Formalizing Mutual Exclusion

Definition (Solving Mutual Exclusion)

A shared memory system A solves the mutual exclusion problem for a collection of users if:

1. (**Well-formedness**): In any execution, and for any i, the subsequence describing the interaction between U_i and A is well-formed for i (i.e., follows cyclic pattern of API).

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2. (**Mutual exclusion**): There is no reachable system state (that is, a combination of an automaton state for A and states for all the U_i) in which more than one user is in the critical region C.
3. (**Progress**): At any point in *fair execution*:
    1. If at least one user is in T and no user is in C, then at some later point *some* user enters C
    2. If at least one user is in E, then at some later point some user enters R.

Formalizing Mutual Exclusion

- The well-formedness and mutual exclusion properties are **safety** properties, whilst the progress property is a **liveness** property. (eventually something good happens).

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Formalizing Mutual Exclusion

- The well-formedness and mutual exclusion properties are **safety** properties, whilst the progress property is a **liveness** property. (eventually something good happens).
- Note progress assumes a fair execution: i.e., every user/process keeps executing
- This definition does have a "problem":
  1. **Starvation**: progress only requires that *some* user enters C. i.e., it's fine if there's a processor forever stuck in the trying phase so long as there exists a process enterring C

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Dijkstra's ME Algorithm

- Essentially a distributed Spin-Lock without atomics (*nasty*).
- Not the most elegant, or efficient algorithm, neither does it satisfy the strongest conditions.
- First example of such algorithm though, good to study.

Dijkstra's ME Algorithm

Process i:

** Remainder region **

try_i
L: $flag(i) := 1$
while $turn \neq i$ do
    if $flag(turn) = 0$ then $turn := i$
$flag(i) := 2$
for $j \neq i$ do
    if $flag(j) = 2$ then goto L
$crit_i$

** Critical region **

$exit_i$
$flag(i) := 0$
rem_i

Dijkstra's ME Algorithm

- In the first loop all processors try to see if the processor which was previously in the critical section has left.
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- If so, they "plant their flag" by setting the `turn` to their `id`.
- Next, they set their flag to 2 as a way of indicating that they are about to try and enter the critical section next.
- Now each process checks that no one else is simultaneously planning the same thing. If so, they go back to the start.
  1. **Note:** The next time, whichever processor set `turn` last will skip the first loop, and directly go the second loop. It will also succeed in this loop and enter the critical section.
  2. Note every processor which set `turn` before that processor will be stuck in the first loop till it exits, since that processor's `flag` was already set to 2.

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- Essentially *last-come-first-serve*-ish. Everyone tries first, but then if multiple went at the same time, whoever went last goes first.

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Dijkstra's ME Analysis

- Proof of corectness essentially by case-work.

Theorem

Dijkstra's ME algorithm guarantees well-formedness

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Proof.

1. By contradiction. Suppose U_i, U_j simultaneously exist in the critical region.
2. Both i, j must set their `flag` to 2 before enterring the critical region.
3. `flag` will be 2 for both until they exit the region.

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2. Both i, j must set their `flag` to 2 before enterring the critical region.
3. `flag` will be 2 for both until they exit the region.
4. Suppose WLOG U_i enters first. But then, `flag`(i) must be 2, and thus j's test in the while loop must fail.

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Proof.

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3. Call processes stuck in T *contenders*.
4. For each contender i, `flag(i)` ≥ 1.

□

Dijkstra's ME Analysis

Lemma

$turn$ *eventually acquires a contender's index.*

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Proof.

1. By contradiction. Suppose it acquires j, which is not a contender, and thus $\texttt{flag}(j) = 0$.
2. Then, for any contender i, in the first while loop it will see that $\texttt{flag}(\text{turn}) = 0$, and set the flag to i.

□

Dijkstra's ME Analysis

Proof of Main thm cont'd.

1. The value of `turn` eventually stabilizes to the value i, of some contender.

$\square$

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Proof of Main thm cont'd.

1. The value of `turn` eventually stabilizes to the value i, of some contender.
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Proof of Main thm cont'd.

1. The value of `turn` eventually stabilizes to the value i, of some contender.
2. For any contender $j \neq i$ it gets stuck in the first loop. Thus, their `flag` is set to 1.
3. Contender i proceeds and sets its flag to 2, and is the only one to satisfy this and thus will enter the critical region.

□

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Summary

- Introduced Shared Memory asynchronous model of computation.
- Motivated and Introduced Mutual Exclusion.
- Presented Dijkstra's ME algorithm.
- Analyzed safety and liveness properties of Dijkstra's algorithm.

References I

Questions?

Thank You!