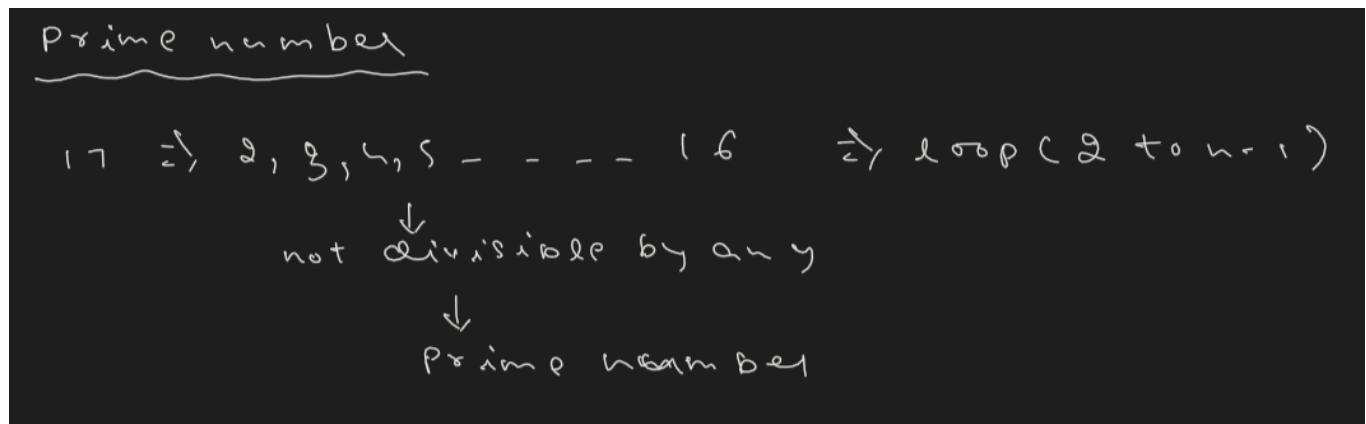


🎯 Count Primes

<https://leetcode.com/problems/count-primes/>



```
class Solution {  
    public int countPrimes(int n) {  
        int count = 0;  
        for (int i = 2; i < n; i++) {  
            if (isPrime(i)) {  
                count++;  
            }  
        }  
        return count;  
    }  
  
    public boolean isPrime(int x) {  
        for (int d = 2; d <= x-1; d++) {  
            if (x % d == 0) {  
                return false;  
            }  
        }  
        return true;  
    }  
}
```

The brute-force solution we implemented above results in a **Time Limit Exceeded (TLE)** error. This happens because, in the worst-case scenario, its time complexity is $O(n^2)$.

Now, consider the problem constraints:

The maximum value of n can be up to 10^6 .

So, in the worst case, the number of operations would be on the order of 10^{12} (i.e., $10^6 \times 10^6$).

We know that a typical computer can execute about **10⁸ operations per second**. Since 10^{12} is far beyond that, this brute-force solution is simply too slow and will inevitably lead to a TLE error.

To check whether a number n is **prime**, I don't need to try dividing it by all numbers up to n . Instead, I can reduce the number of checks to \sqrt{n} (**square root of n**).

Why?

If n is **not** a prime number, then it can be expressed as the product of two integers:

$$n=a \times b$$

There are three possible cases:

1. Case 1: $a = b$

In this case:

$$n=a \times a = a^2 \Rightarrow a = \sqrt{n}$$

2. Case 2: $a > b$

As a increases, b must decrease to maintain the equality $a \times b = n$.

So b becomes smaller than \sqrt{n} .

3. Case 3: $a < b$

Similarly, since a is smaller, it must be **less than** \sqrt{n} to keep the product equal to n .

Final Conclusion:

In all three cases, at least one factor (either a or b) will be $\leq \sqrt{n}$.

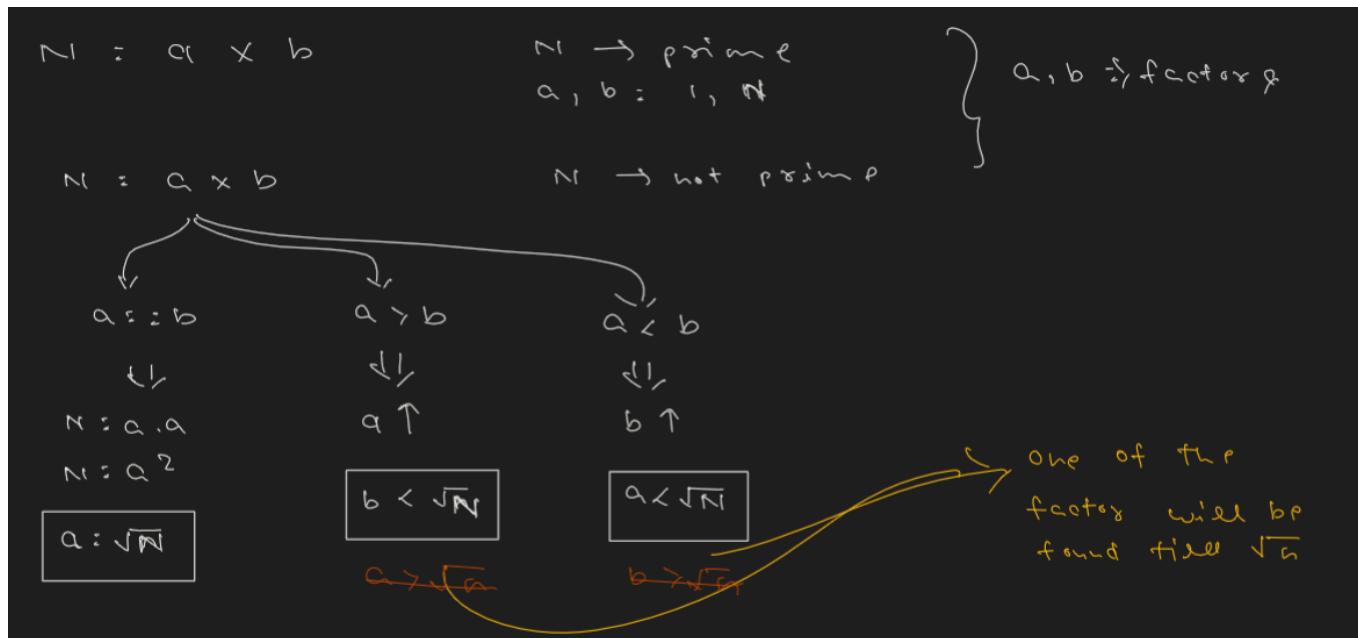
So, if a number n has any factor other than 1 and itself, you will find at least one such factor $\leq \sqrt{n}$.

Ex: $36 \Rightarrow 2, 4$

Therefore:

To check if n is prime, it's enough to try dividing it only by numbers from 2 to \sqrt{n} .

If none of them divide n , then n is **prime**.



$81 \Rightarrow \sqrt{81} = 9$ $27 \times 3 = 81$

$\rightarrow 1, 3, 9, \cancel{3}, \cancel{9}$

$32 \Rightarrow 1, 2, 4, \cancel{8}, \cancel{16}, \cancel{32}$

$\sqrt{32} = 5 + \epsilon$

$$i < \sqrt{n}$$

11

12 < 5

三六一

sem. inequalities

```
public boolean isPrime(int x) {  
    if (x <= 1) return false;  
  
    for (int d = 2; d * d <= x; d++) {  
        if (x % d == 0) {  
            return false;  
        }  
    }  
    return true;  
}
```

The smart brute-force solution we implemented above results in a **Time Limit Exceeded (TLE)** error. This happens because, in the worst-case scenario, its time complexity is $O(n^* \sqrt{n}) \Rightarrow O(n^{3/2})$.

Now, consider the problem constraints:

The maximum value of n can be up to 10^6 .

So, in the worst case, the number of operations would be on the order of 10^9 (i.e., $10^{6 \times 3/2}$).

We know that a typical computer can execute about **10^8 operations per second**. Since 10^8 is far beyond that, this smart brute-force solution is simply too slow and will inevitably lead to a TLE error.

What is the Sieve of Eratosthenes?

The **Sieve of Eratosthenes** is an ancient and efficient algorithm used to **find all prime numbers up to a given number n** .

Instead of checking each number individually (like in brute-force or `isPrime()`), it uses a clever **marking technique** to eliminate non-primes in bulk — significantly reducing the time complexity.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Sieve of Eratosthenes

D

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

Prime numbers

2 3 5 7

13 17 23

29 31 37

41 43 47

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7}$$

$$\sqrt{120} = 11$$

$$n \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots \right]$$

$$\downarrow \\ n (\log_2(\log_2 N))$$

{
O(n)}

$$n \cdot \log_2(\log_2 N)$$

$$n = 1024 = 2^{10}$$

$$\log_2^b = b \cdot \log_2 a$$

$$\log_a a = 1$$

$$2^{10} \cdot \log_2(\log_2 2^{10})$$

$$2^{10} \cdot \log_2(10 \cdot \log_2 2^7)$$

🧠 How It Works:

1. Create a boolean array

- Make an array `isPrime[]` of size `n` and set all entries to `true`.
- Mark `0` and `1` as `false` since they are not prime.

2. Start from 2

- Begin with the first prime number `p = 2`.

3. Mark multiples as non-prime

- For each `p`, mark all multiples of `p` starting from `p*p` as `false` (i.e., not prime).
- For example, for `p = 2`, mark `4, 6, 8, ...`
For `p = 3`, mark `9, 12, 15, ...`, etc.

4. Repeat until \sqrt{n}

- Only go up to `p * p < n`, because any composite number less than `n` will have a factor $\leq \sqrt{n}$.

5. Count the `true` values

- The remaining `true` values in `isPrime[]` are all primes.



Time & Space Complexity:

- Time:** $O(n \log \log n)$
- Space:** $O(n)$

Very efficient for values of n up to 10^6 or more.

```
public class Main {
    public static void main(String[] args) {
        int n = 100;
        System.out.println(primeSieve(n));
    }

    // 1 -> not prime | 0 -> prime
    public static int primeSieve(int n) {
        int[] prime = new int[n];
        prime[0] = prime[1] = 1; // 0 & 1 is not a prime number

        for (int i = 2; i*i < prime.length; i++) {
            if (prime[i] == 0) {
                for (int j = i*i; j < prime.length; j+=i) {
                    prime[j] = 1;
                }
            }
        }

        int c = 0;
        for (int i = 2; i < prime.length; i++) {
            if (prime[i] == 0) {
                c++;
            }
        }
        return c;
    }
}
```

Note: Dry Run for $n = 30$.

```
class Solution {
    public int countPrimes(int n) {
        if(n <= 1){
            return 0;
        }

        int[] prime = new int[n];
        prime[0] = prime[1] = 1; // 0 & 1 is not a prime number

        for (int i = 2; i*i < prime.length; i++) {
            if (prime[i] == 0) {
                for (int j = i*i; j < prime.length; j+=i) {
                    prime[j] = 1;
                }
            }
        }

        int c = 0;
        for (int i = 2; i < prime.length; i++) {
```

```

        if (prime[i] == 0) {
            c++;
        }
    }
    return c;
}

}

```

🎯 Palindromic Substrings

<https://leetcode.com/problems/palindromic-substrings/>

```

class Solution {
    public int countSubstrings(String s) {
        int c = 0;
        for(int i=0; i<s.length(); i++){
            String op = "";
            for(int j=i; j<s.length(); j++){
                op = op + s.charAt(j);
                // System.out.println(op);
                if(isPalindrome(op)){
                    c = c+1;
                }
            }
        }
        return c;
    }

    public static boolean isPalindrome(String x){
        int i = 0;
        int j = x.length()-1;

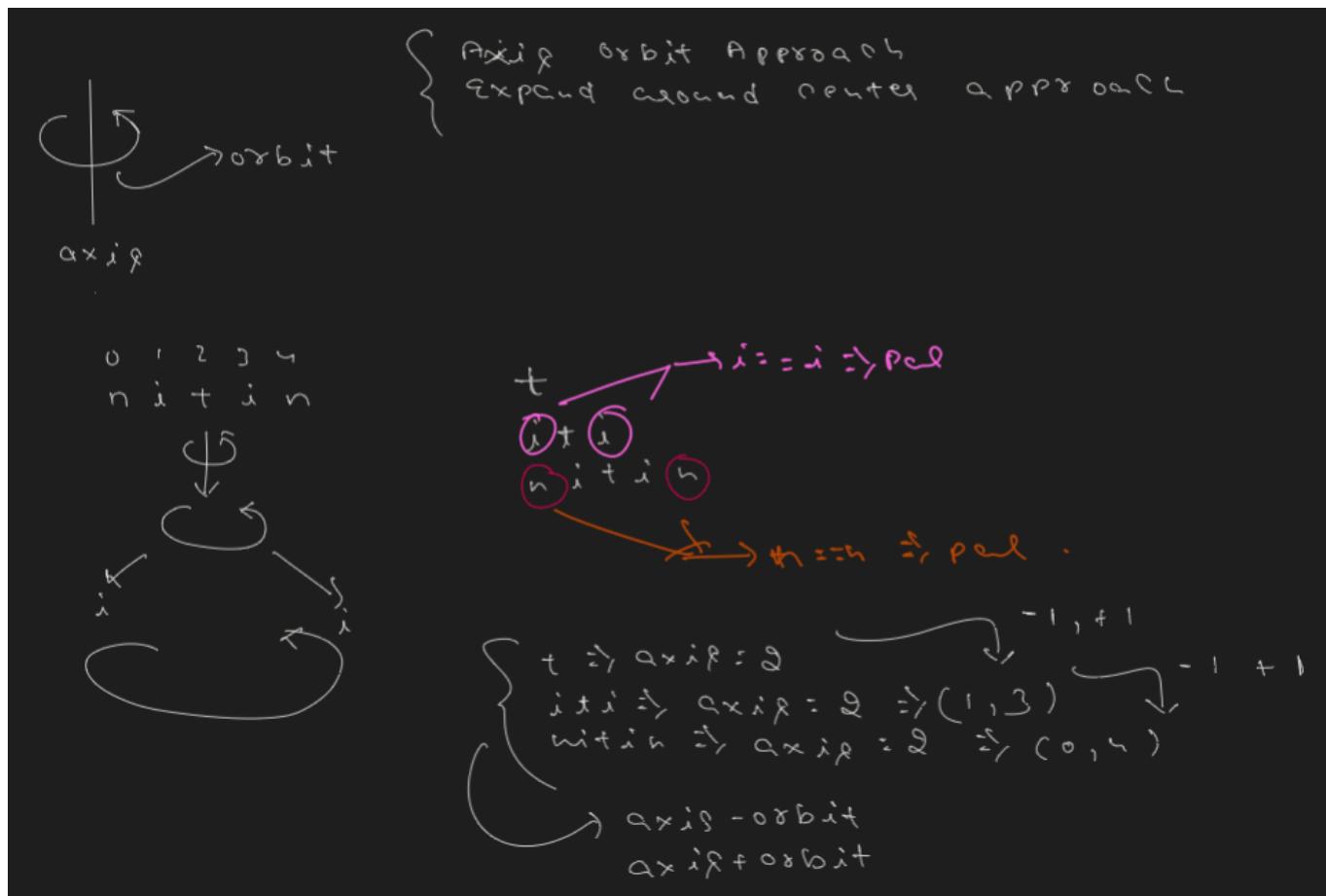
        while(i<j){
            if(x.charAt(i) != x.charAt(j)){
                return false;
            }
            i++;
            j--;
        }
        return true;
    }
}

```

Variations of the Palindromic Substring Problem:

1. Count all palindromic substrings
2. Find the longest palindromic substring
3. Count all odd-length palindromic substrings
4. Count all even-length palindromic substrings
5. Find the longest odd-length palindromic substring
6. Find the longest even-length palindromic substring

Expand Around Center/ Axis Orbit Approach



🧠 Intuition

Suppose we take the character '**t**' as the **axis** (i.e., the center) of a potential palindrome.

✓ Step 1: Start with Orbit = 0

- At **orbit = 0**, we are only looking at the center itself, which is '**t**'.
- A single character is always a palindrome.
- So, the substring "**t**" is a valid palindrome.

⟳ Step 2: Expand Outwards – Orbit = 1

- Now, we look in **both directions** from the center:
 - **axis - 1** and **axis + 1**
- Suppose both characters at these positions are '**i**' and '**i**'.
- Since both are the same, the substring "**iti**" is a palindrome too.

⟳ Step 3: Expand Further – Orbit = 2

- Next, we check again:
 - **axis - 2** and **axis + 2**
- Suppose we find '**n**' and '**n**' at these positions.
- Again, both are equal, so the full substring "**nitin**" is also a palindrome.

✳️ Summary of the Idea:

- We **start from a center** (the axis).
- We **expand symmetrically outward** (using orbit).
- At each step, we check if the characters on both sides are equal.

- If they are, we found another palindromic substring.
- This process continues until the characters don't match or we go out of bounds.

💡 Key Insight:

If a smaller substring is a palindrome and we **wrap matching characters around it**, the new substring is also a palindrome.

odd length palindromic substrings

$0 \ 1 \ 2 \ 3 \ 4$ $n i t i n$ \downarrow $a x i s = 0$ $0 - 0 = 0$ $0 + 0 = 0$	$0 \ 1 \ 2 \ 3 \ 4$ $n i t i n$ \downarrow $a x i s = 1$ $1 - 0 = 1$ $1 + 0 = 1$	$0 \ 1 \ 2 \ 3 \ 4$ $n i t i n$ \downarrow $a x i s = 2$ $2 - 0 = 2$ $2 + 0 = 2$
---------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------

$0 - 1 = -1$ $0 + 1 = 1$	$1 - 1 = 0$ $1 + 1 = 2$	$2 - 1 = 1$ $2 + 1 = 3$
-----------------------------	----------------------------	----------------------------

$\{ \quad \} \quad \{ \quad \} \quad \{ \quad \}$

$m \quad i \quad t \quad i = i \quad i t i$

$0 \ 1 \ 2 \ 3 \ 4$ $n i t i n$ \downarrow $a x i s = 3$ $3 - 0 = 3$ $3 + 0 = 3$	$0 \ 1 \ 2 \ 3 \ 4$ $n i t i n$ \downarrow $a x i s = 4$ $4 - 0 = 4$ $4 + 0 = 4$	$\left\{ \begin{array}{l} a x i s - o b i t \\ a x i s + o b i t \end{array} \right.$
---------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------

$3 - 1 = 2$ $3 + 1 = 4$	$4 - 1 = 3$ $4 + 1 = 5$
----------------------------	----------------------------

$\{ \quad \} \quad \{ \quad \}$

$t \neq h \quad h \quad X \quad X$

orbit out of bound or char mismatch

Even length palindromic substring

0 1 2 3
n a a n
↓
axis = 0.5

0 1 2 3
n a a n
↓
axis = 1.5

● ● ● ●

$$\begin{aligned} 0.5 - 0.5 &= 0 \quad \left\{ \begin{array}{l} n \neq a \\ 1.5 - 0.5 = 1 \end{array} \right. \quad \left\{ \begin{array}{l} a = a \\ 1.5 + 0.5 = 2 \end{array} \right. \quad \left\{ \begin{array}{l} 1.5 - 2 \cdot 5 = -1.5 \\ 1.5 + 2 \cdot 5 = 4 \end{array} \right. \\ 0.5 + 0.5 &= 1 \quad \left(\times \right) \quad \left(\textcircled{aa} \right) \end{aligned}$$

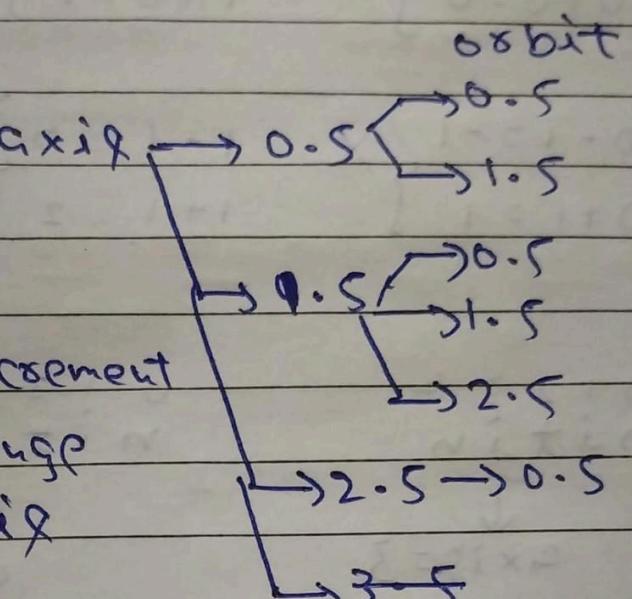
$$\begin{aligned} 1.5 - 1.5 &= 0 \quad \left\{ \begin{array}{l} n = n \\ \text{(naan)} \end{array} \right. \\ 1.5 + 1.5 &= 3 \end{aligned}$$

0 1 2 3
n a a n
↓
axis = 2.5

n a a n
↓
axis = 3.5

$$\begin{aligned} 2.5 - 0.5 &= 2 \quad \left\{ \begin{array}{l} a \neq n \\ 1.5 - 0.5 = 1 \end{array} \right. \\ 2.5 + 0.5 &= 3 \quad \left(\times \right) \end{aligned}$$

axis - orbit
axis + orbit



orbit out of bound or change axis mismatch

```

public class Main {
    public static void main(String[] args) {
        String s = "naan";
        System.out.println(palindromicSubstringCount(s));
    }

    public static int palindromicSubstringCount(String s) {
    }
}
  
```

```

        // odd length substring → O(N2)
        int odd = 0;
        for (int axis = 0; axis < s.length(); axis++) {
            for (int orbit = 0; axis - orbit >= 0 && axis + orbit < s.length(); orbit++) {
                if (s.charAt(axis - orbit) != s.charAt(axis + orbit)) {
                    break;
                }
                odd++;
            }
        }

        // even length substring → O(N2)
        int even = 0;
        for (double axis = 0.5; axis < s.length(); axis++) {
            for (double orbit = 0.5; axis - orbit >= 0 && axis + orbit < s.length(); orbit++) {
                if (s.charAt((int) (axis - orbit)) != s.charAt((int) (axis + orbit))) {
                    break;
                }
                even++;
            }
        }
        return odd + even;
    }

}

```

```

class Solution {
    public int countSubstrings(String s) {
        return palindromicSubstringCount(s);
    }

    public static int palindromicSubstringCount(String s) {
        // odd length substring
        int odd = 0;
        for (int axis = 0; axis < s.length(); axis++) {
            for (int orbit = 0; axis - orbit >= 0 && axis + orbit < s.length(); orbit++) {
                if (s.charAt(axis - orbit) != s.charAt(axis + orbit)) {
                    break;
                }
                odd++;
            }
        }

        // even length substring
        int even = 0;
        for (double axis = 0.5; axis < s.length(); axis++) {
            for (double orbit = 0.5; axis - orbit >= 0 && axis + orbit < s.length();
orbit++) {
                if (s.charAt((int) (axis - orbit)) != s.charAt((int) (axis + orbit))) {
                    break;
                }
                even++;
            }
        }
    }
}

```

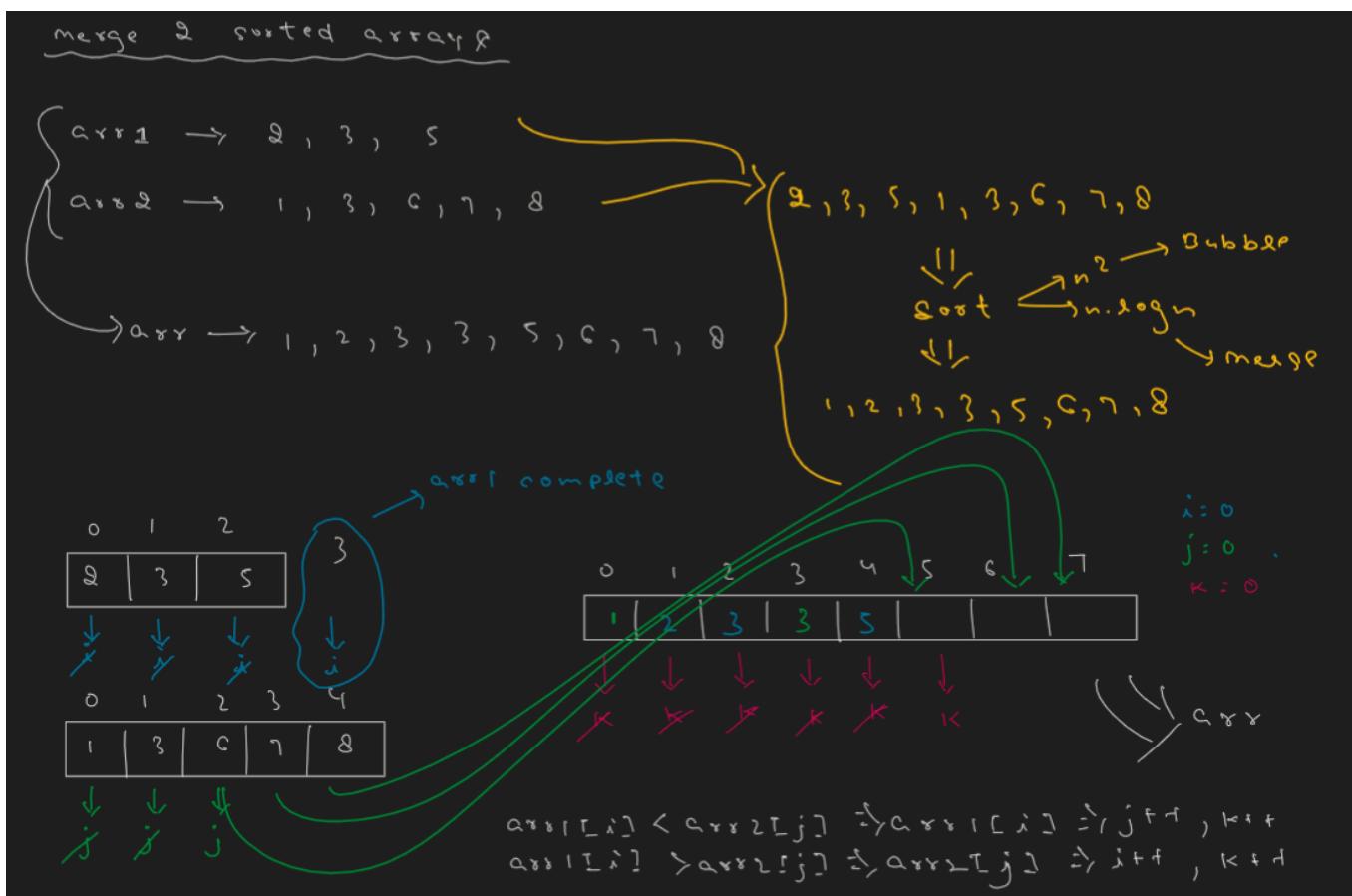
```

        }
    }
    return odd + even;
}
}

```

🎯 Merge Sorted Array

<https://leetcode.com/problems/merge-sorted-array/>



```

public class Main {
    public static void main(String[] args) {
        int[] arr1 = { 2, 3, 5 };
        int[] arr2 = { 1, 3, 6, 7, 8 };
        int[] arr = merge(arr1, arr2);
        for (int i = 0; i < arr.length; i++) {
            System.out.print(arr[i] + " ");
        }
    }

    public static int[] merge(int[] arr1, int[] arr2) {
        int n = arr1.length;
        int m = arr2.length;

        int[] arr = new int[n+m];

```

```
int k = 0;

while(i<n && j<m){
    if(arr1[i] < arr2[j]){
        arr[k] = arr1[i];
        i++;
    }
    else{
        arr[k] = arr2[j];
        j++;
    }
    k++;
}

while(i<n){
    arr[k] = arr1[i];
    k++;
    i++;
}

while(j<m){
    arr[k] = arr2[j];
    k++;
    j++;
}

return arr;
}

}
```