

Priority Queue

Queue \Rightarrow FIFO \Rightarrow First In First out

Priority Queue \Rightarrow highest Priority

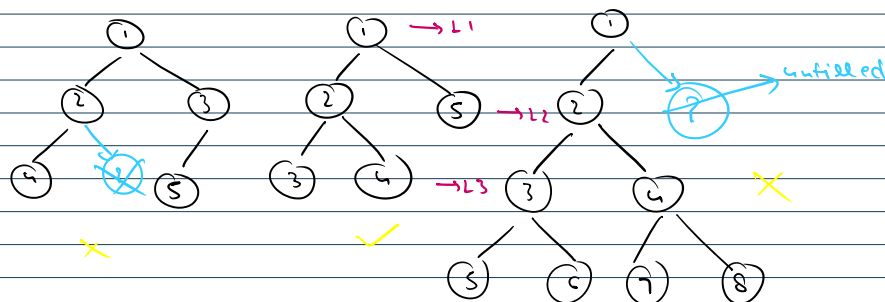
↓ Implements

Heap + complete Binary Tree

Priority Queue \rightarrow min PQ \Rightarrow min element
 \rightarrow max PQ \Rightarrow max element

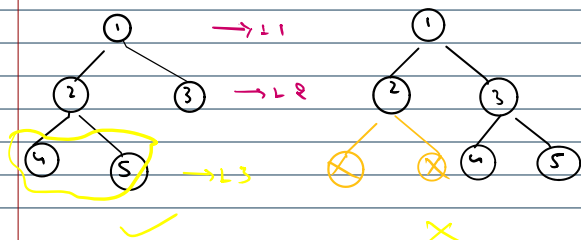
PQ \rightarrow add
 \rightarrow remove
 \rightarrow size
 \rightarrow peek
 \rightarrow empty

Complete Binary Tree



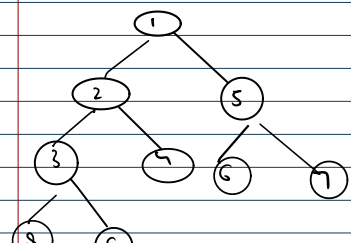
1. all levels are completely filled (except last)

2. last level filled left to right.



Complete Binary Tree

arr \Rightarrow {1, 2, 5, 3, 4, 6, 7, 8, 9}



4 \Rightarrow parent = 2

↓
4

↓
1

7 \Rightarrow parent = 5

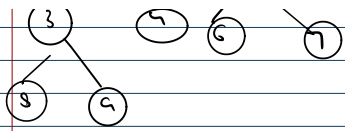
↓
6

↓
2

01 \Rightarrow parent = 3

↓
8

↓
3



$$\{i\} \Rightarrow \text{parent} = (i-1)/2$$

$$\{i\} \Rightarrow \text{child} \begin{cases} \text{lc} = 2i \rightarrow 5 \\ \text{rc} = 2i+1 \rightarrow 6 \end{cases}$$

$$\{i\} \Rightarrow \text{child} \begin{cases} \text{lc} = 8 \rightarrow 7 \\ \text{rc} = 9 \rightarrow 8 \end{cases}$$

$$\{i\} \Rightarrow \text{child} \begin{cases} \text{left} = 2i+1 \\ \text{right} = 2i+2 \end{cases}$$

Heap

A heap is a CBT that satisfies the heap property

max heap \Rightarrow root = max element

min heap \Rightarrow root = min element

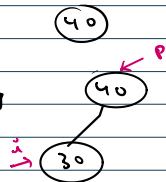
max Heap

arr $\rightarrow \{40, 30, 50, 10, 20, 60, 70, 5, 15, 25\}$

$h \Rightarrow \{ \}$

$40 \Rightarrow h \Rightarrow \{40\}$

$30 \Rightarrow h \Rightarrow \{40, 30\}$

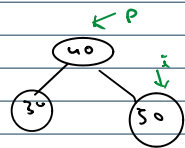


$$i=1 \Rightarrow p = (i-1)/2 = 0$$

$h[i] > h[p] \Rightarrow \text{swap (float up)}$

$$30 > 40 \rightarrow \times \text{ (No swap)}$$

$50 \Rightarrow h \Rightarrow \{40, 30, 50\}$

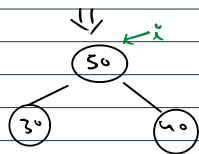


$$i=2 \Rightarrow p = (i-1)/2 = 0.5 = 0$$

$h[i] > h[p] \Rightarrow \text{swap (float up)}$

$$50 > 40 \rightarrow \checkmark \text{ (swap)}$$

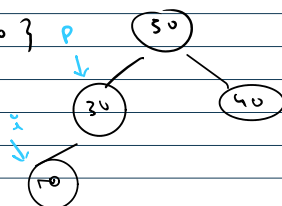
$\{50, 30, 40\}$



\Rightarrow call float up again

\checkmark

$10 \Rightarrow h \Rightarrow \{50, 30, 40, 10\}$

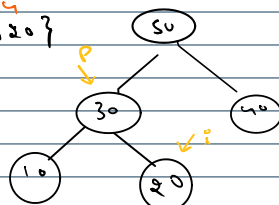


$$i=3 \Rightarrow p = (i-1)/2 = 1$$

$h[i] > h[p] \Rightarrow ?$

$$10 > 30 \Rightarrow \times \Rightarrow \text{no swap}$$

$20 \Rightarrow h \Rightarrow \{50, 30, 40, 10, 20\}$

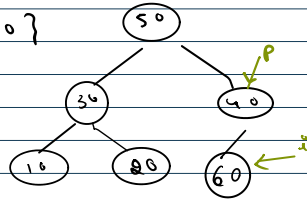


$$i=4 \Rightarrow p = (i-1)/2 = 1.5 = 1$$

$h[i] > h[p] \Rightarrow ?$

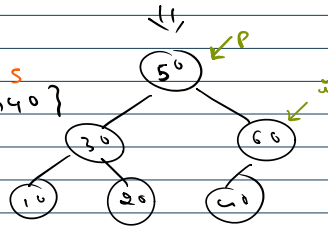
$$20 > 30 \Rightarrow \times \Rightarrow \text{no swap}$$

60 \Rightarrow h \Rightarrow { 50, 30, 40, 10, 20, 60 }



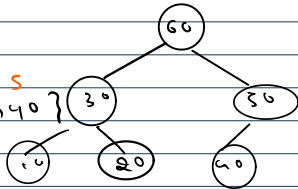
$i=5 \Rightarrow p=(i-1)/2=2$
 $h[i] > h[p] \Rightarrow ?$
 $60 > 40 \Rightarrow \checkmark$
 \hookrightarrow swap

h \Rightarrow { 50, 30, 60, 10, 20, 40 }



$i=2 \Rightarrow p=(i-1)/2=0$
 $h[i] > h[p] \Rightarrow ?$
 $60 > 50 \Rightarrow \checkmark$
 \hookrightarrow swap

h \Rightarrow { 60, 30, 50, 10, 20, 40 }

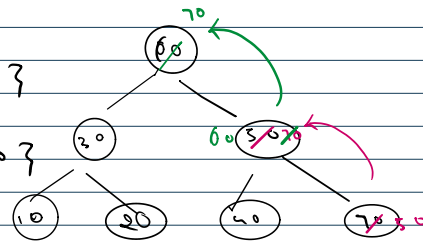


\Rightarrow { 70, 5, 15, 25 }

70 \Rightarrow h \Rightarrow { 60, 30, 50, 10, 20, 40, 70 }

{ 60, 30, 10, 10, 20, 40, 50 }

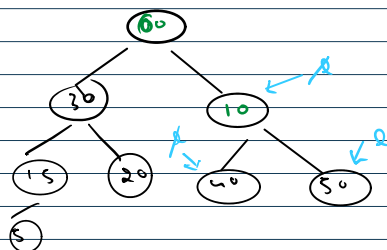
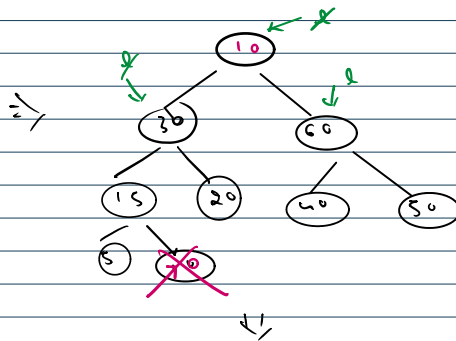
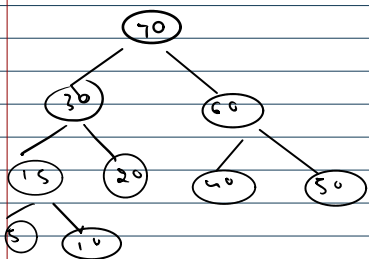
{ 70, 30, 60, 10, 20, 40, 50 }

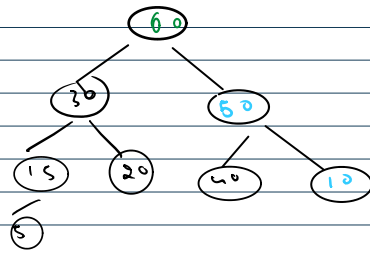


Removal from Heap

h \Rightarrow { 70, 30, 60, 15, 20, 40, 50, 5, 10 }

h \Rightarrow { 10, 30, 60, 15, 20, 40, 50, 5, ~~10~~ }





Remove again

