# R-Tree: Spatial Representation on a Dynamic-Index Structure

**Advanced Algorithms & Data Structures**

Lecture Theme 03 – Part II

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# Overview

- Representing and handling spatial data
- The R-Tree indexing approach, style and structure
- Properties and notions
- Searching and inserting index Entry-records
- Deleting and updating
- Performance analyses
- Node splitting algorithms
- Derivatives of the R-Trees
- Applications

# Deletion

- Current index entry-records are removed from the leaves.
- Nodes that underflow are condensed, and its contents redistributed appropriately throughout the tree.
- A condense propagation may cause the tree to shorten in height.

**The main Delete routine**

- Let $E = (I, \textit{tuple-identifier})$ be a current entry to be removed.
- Let $T$ be the root of the R-Tree.
  - **[Del_1]** Find the leaf $L$ starting from $T$ that contains $E$.
  - **[Del_2]** Remove $E$ from $L$, and condense ‘underflow’ nodes.
  - **[Del_3]** Propagate MBR changes upwards.
  - **[Del_4]** Shorten tree if $T$ contains only 1 entry after condense propagation.

# Deletion – Find Leaf

→ **[Del_1]** Find the leaf *L* starting from *T* that contains *E*.

**Algorithm**: **FindLeaf** (*E*, *N*)

**Inputs**: (i) Entry *E* = (*I*, *tuple-identifier*), (ii) A valid R-Tree node *N*.

**Output**: The leaf *L* containing *E*.

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- **If** *N* is leaf **Then**
  - **If** *N* contains *E* **Then** Return *N*
  - **Else** Return NULL
- **Else**
  - Let *FS* be the set of current entries in *N*.
  - **For** each *F* = (*I*, *child-pointer*) ∈ *FS* where *F.I* overlaps *E.I* **Do**
    - Set *L* = **FindLeaf** (*E*, *F.child-pointer*)
    - **If** *L* is not NULL **Then** Return *L*
  - **Next** *F*
  - Return NULL
- **End If**

# Deletion – Remove and Adjust

→ **[Del_2]** Remove *E* from *L*, and condense 'underflow' nodes.

→ **[Del_3]** Propagate MBR changes upwards.

- *Notion (i)*: Ascend from leaf *L* to root *T* while adjusting covering rectangles MBR.
- *Notion (ii)*: If after removing the entry *E* in *L* and the number of entries in *L* becomes fewer than *m*, then the node *L* has to be eliminated and its remaining contents relocated.

# Deletion – Remove and Adjust

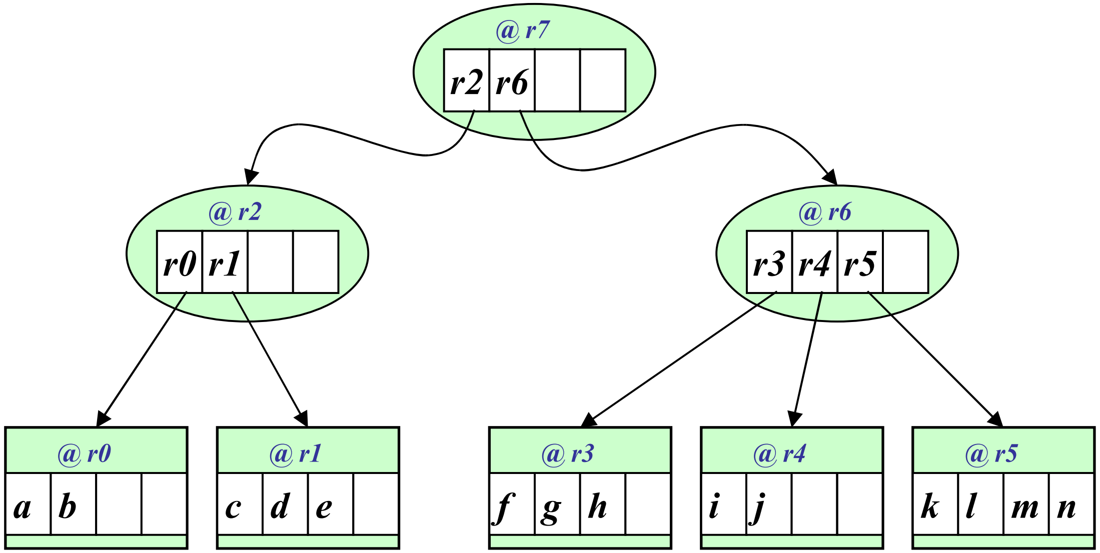
- Propagate these notions upwards by invoking **CondenseTree** (*N*, *QS*), where *N* is an R-Tree node whose entries have been modified, and *QS* is the set of eliminated nodes.
- Start the propagation by setting $N = L$, and $QS = \emptyset$.

- Re-insert the entries from the eliminated nodes in *QS* back into the tree.
- Entries from eliminated leaf nodes are re-inserted as new entries using the **Insert** routine discussed earlier.
- Entries from higher-level nodes must be placed higher in the tree so that leaves of their dependent subtrees will be on the same level as the leaves on the main tree.

# Deletion – Propagating Node Condensation

**Example**: R-Tree settings: $M = 4$, $m = 2$.

- Delete the index entry-record ***h***.
- Delete the index entry-record ***b***. Note: ***a***.*I* will form smallest MBR with ***r4***.



# Deletion – Condense Tree (I)

**Algorithm**: **CondenseTree** ($N$, $QS$)

**Inputs**: (i) A node $N$ whose entries have been modified, (ii) A set of eliminated nodes $QS$.

- **If** $N$ is NOT the root **Then**
  - Let $P_N$ be the parent node of $N$.
  - Let $E_N = (I\_N, \textit{child-pointer}\_N)$ in $P_N$.
  - **If** $N.entries < m$ **Then**
    - Delete $E_N$ from $P_N$
    - Add $N$ to $QS$
  - **Else**
    - Adjust $I\_N$ so that it tightly encloses all entry regions in $N$.
  - **End If**
  - **CondenseTree** ($P_N$, $QS$)

# Deletion – Condense Tree (II)

- **Else** If $N$ is root AND $QS$ is NOT $\varnothing$ **Then**
  - **For** each $Q \in QS$ **Do**
    - **For** each $E \in Q$ **Do**
      - **If** $Q$ is leaf **Then** **Insert** ($E$)
      - **Else** **Insert** ($E$) as a node entry at the same node level as $Q$
      - **End If**
    - **Next** $E$
  - **Next** $Q$

- **End If**

→ **[Del_4]** Shorten tree if $T$ contains only 1 entry after condense propagation.

# Deletion – Summary

**Why 're-insert' orphaned entries?**

- Alternatively, like the delete routine in B-Tree (Rosenberg & Snyder, 1981 ), an 'underflow' node can be merged with whichever adjacent sibling that will have its area increased the least, or its entries re-distributed among sibling nodes.
- Both methods can cause the nodes to split.
- Eventually all changes need to be propagated upwards, anyway.

# Deletion – Summary

**→ Re-insertion accomplishes the same thing, and:**

- It is simpler to implement (and at comparable efficiency).
- It incrementally refines the spatial structure of the tree.
- It prevents gradual deterioration if each entry was located permanently under the same parent node.

# Updating Changes in Spatial Objects


- When a spatial object in a leaf entry *E* (pointed to by *E.tuple-identifier*) changes in size, its MBR contained in *E.I* must reflect this change.

- BUT, instead of simply updating the value for *E.I*, we should:
  1. Delete *E* from the tree.
  2. Create a new entry *E'* for the affected change in the spatial object.
  3. Insert *E'* into the tree using the **Insert()** routine.

# Updating Changes in Spatial Objects

**Why? Because…**

- Rather than percolating a stagnant change from the leaf level upwards, we let *E*’ find its way down the tree to the most appropriate location.
- See notion on ‘optimal placement emphasis’.

**Is it more expensive?**

- Case 1: Percolate new change from Leaf.
- Case 2: Delete and re-insert new change from Root.

# Performance with respect to Parameter *m*

- A high value of $m$, nearer to $M$, is useful when the underlying database is mostly used for search inquiries with very few updates.
- The height of the tree will be kept to a minimum.
- High search performance is maintained.
- However, the risk of overflow and underflow is also high.

- A small value of $m$ is good when frequent updates and modifications of the underlying database is required.
- The nodes are less dense.
- Maintenance is less costly.

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# Node Splitting

- Happens when the node-overflow condition is triggered.
- We need to divide the $M$+1 entries between $N$ and $N$' (equally).

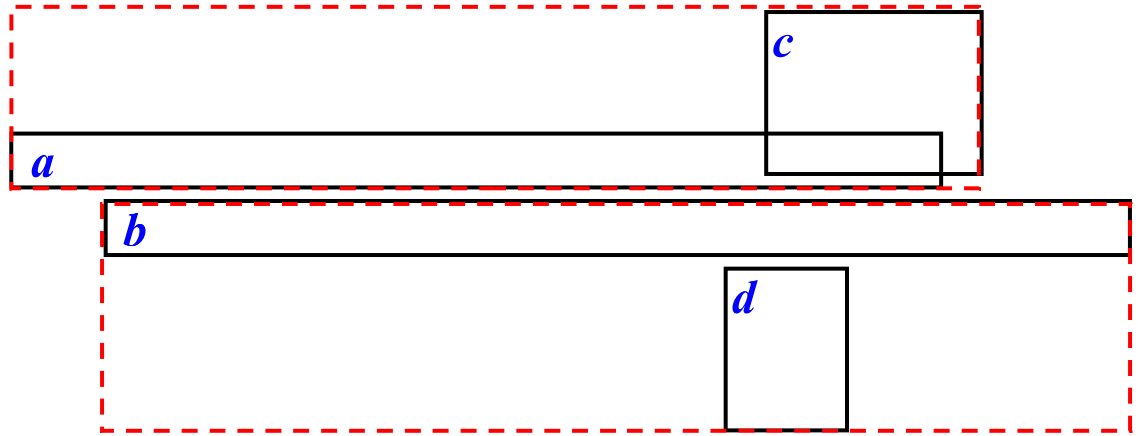
- *Notion*: The consolidated entries in $N$ and $N$' must make it as unlikely as possible that both nodes will need to be examined on subsequent searches.
- Already implemented and used in **ChooseLeaf()** in the **Insert()** routine.

- **Objective**: To minimise the resulting MBRs of $N$ and $N$' after consolidation.
- *Supplement*: The smaller the covering regions, the smaller the possibility that they overlap with other MBRs.

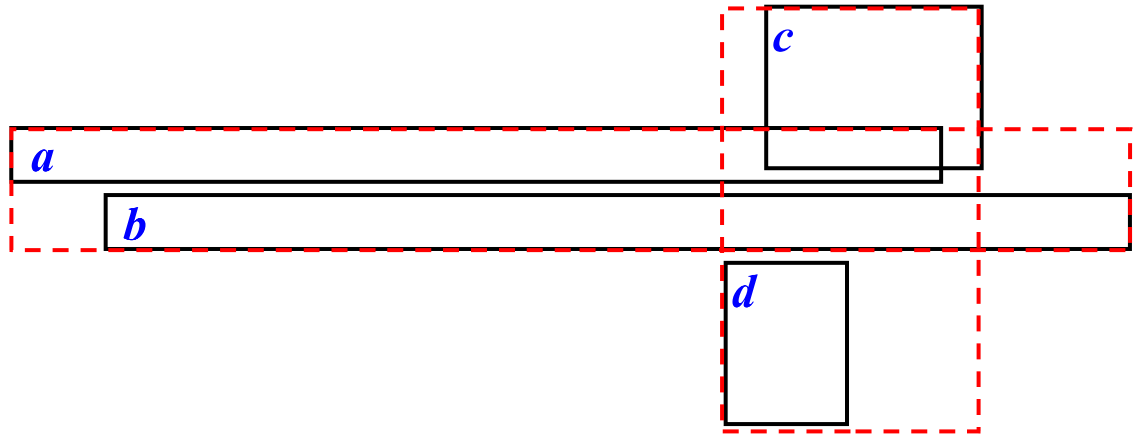
# Node Splitting



Bad split




Good split



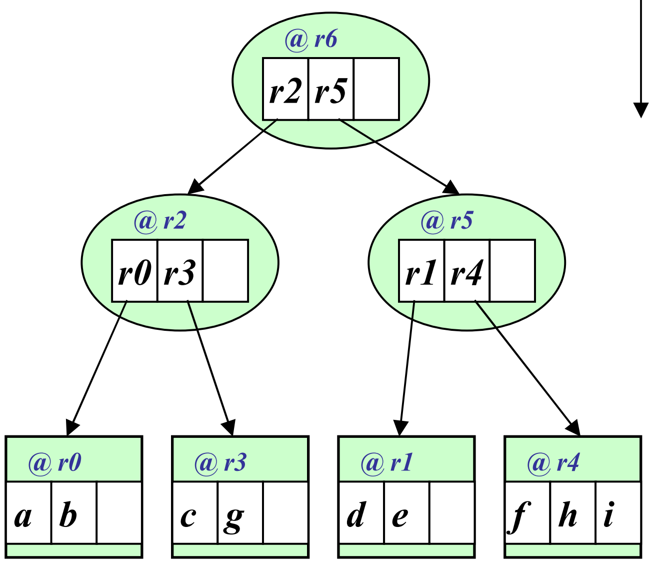
# Node Splitting – Naïve

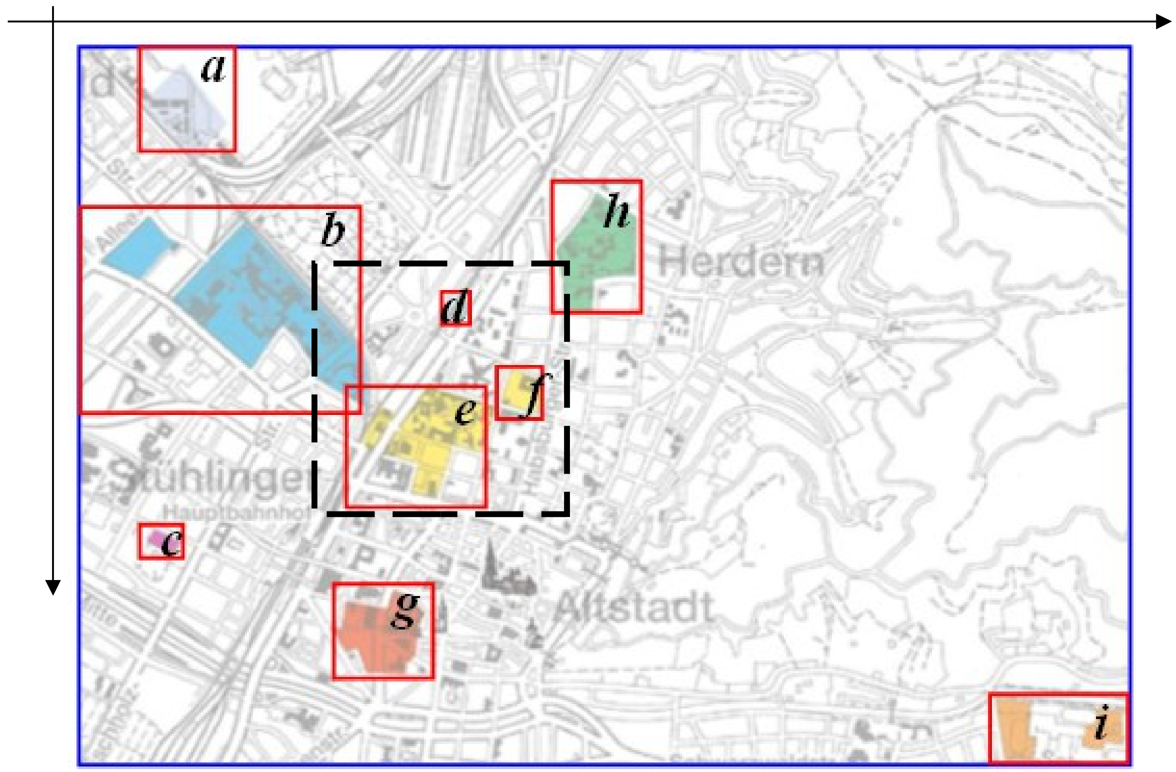
- The greedy and straight forward algorithm to find the minimum area node split is by brute force.
- Generate all possible groupings and take the best one.
- Total possibilities (for *M* entries, 2 nodes):

- Advantage:

- Disadvantage:

# Optimised Example: Map of Freiburg




$I(a) = \{[30, 105], [0, 80]\}$

$I(b) = \{[0, 220], [125, 280]\}$

$I(c) = \{[60, 80], [370, 390]\}$

$I(d) = \{[270, 300], [190, 215]\}$

$I(e) = \{[210, 315], [260, 350]\}$

$I(f) = \{[320, 360], [245, 285]\}$

$I(g) = \{[195, 270], [415, 480]\}$

$I(h) = \{[360, 430], [115, 205]\}$

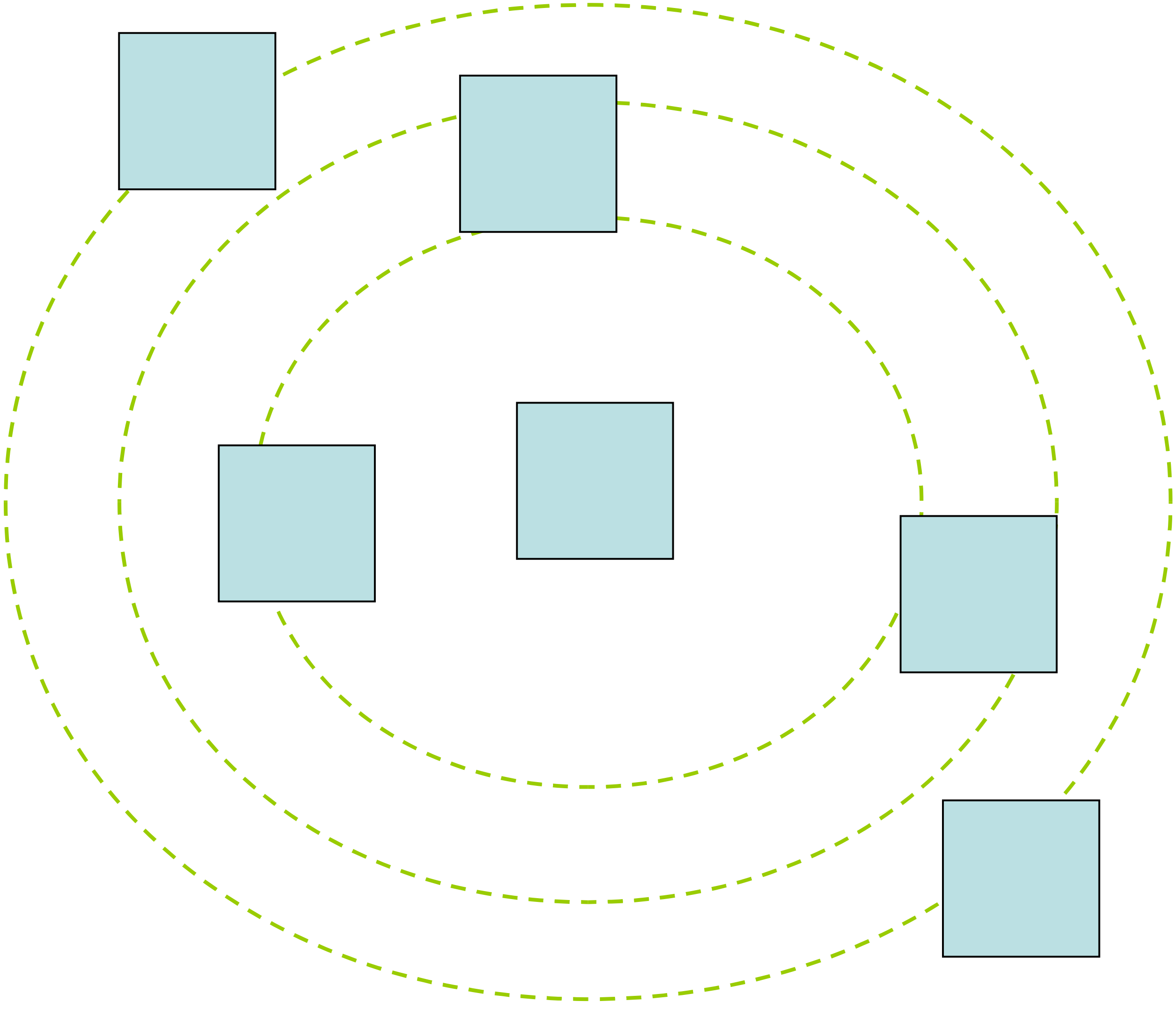
$I(i) = \{[700, 815], [500, 560]\}$

# Node Splitting – Quadratic Cost

- The Quadratic-Cost algorithm attempts to find a small-area split, but cannot guarantee the smallest possible area.

- *Notion (i)*: Pick 2 *seed*-entries that are most distant from each other, and put them in 2 separate nodes *N* and *N'*.
- *Notion (ii)*: The remaining entries are picked and then assigned to either groups one at a time.
  - *Notion (iia)*: Manner of picking next entry to insert: Outer boundary to the inner circle.
  - *Notion (iib)*: Manner of assigning: Add to node whose MBR will have to be enlarged the least (resolve ties by picking node whose MBR is smaller).

# Node Splitting – Quadratic Cost



# Node Splitting – Quadratic Cost: Pick Seeds

**Algorithm**: **PickSeeds** (*ES*)

**Input**: *ES* : A set of $M$+1 entries.

**Outputs**: 2 entries *Ei* and *Ej* that are furthest apart.

---

- Let $ES = \{E_0, E_1, \ldots, E_M\}$
- **For** each *Ei* from $E_0$ to $E_{M-1}$ **Do**
  - **For** each *Ej* from *Ei* to $E_M$ **Do**
    - Compose a region *R* that includes *Ei.I* and *Ej.I*
    - Calculate $\partial = area(R) - area(Ei.I) - area(Ej.I)$
  - **Next** *Ej*
- **Next** *Ei*

- <u>Return</u> *Ei* and *Ej* with the largest $\partial$

# Node Splitting – Quadratic Cost: Pick Next

**Algorithm**: **PickNext** $(ES, N, N')$

**Inputs**: $ES$ : A set of entries not yet in $N$ or $N'$.

**Output**: The next entry $E$ to be considered for assignment.

---

- Let $ES = \{E_0, E_1, \ldots, E_k\}$
- **For** each $E$ from $E_0$ to $E_k$ **Do**
  - Calculate $\partial N$ = area increase req. in the MBR of $N$ to include $E.I$
  - Calculate $\partial N'$ = area increase req. in the MBR of $N'$ to include $E.I$
- **Next** $E$

- <u>Return</u> $E$ with the maximum difference between $\partial N$ and $\partial N'$

# Node Splitting – Quadratic Split Algorithm

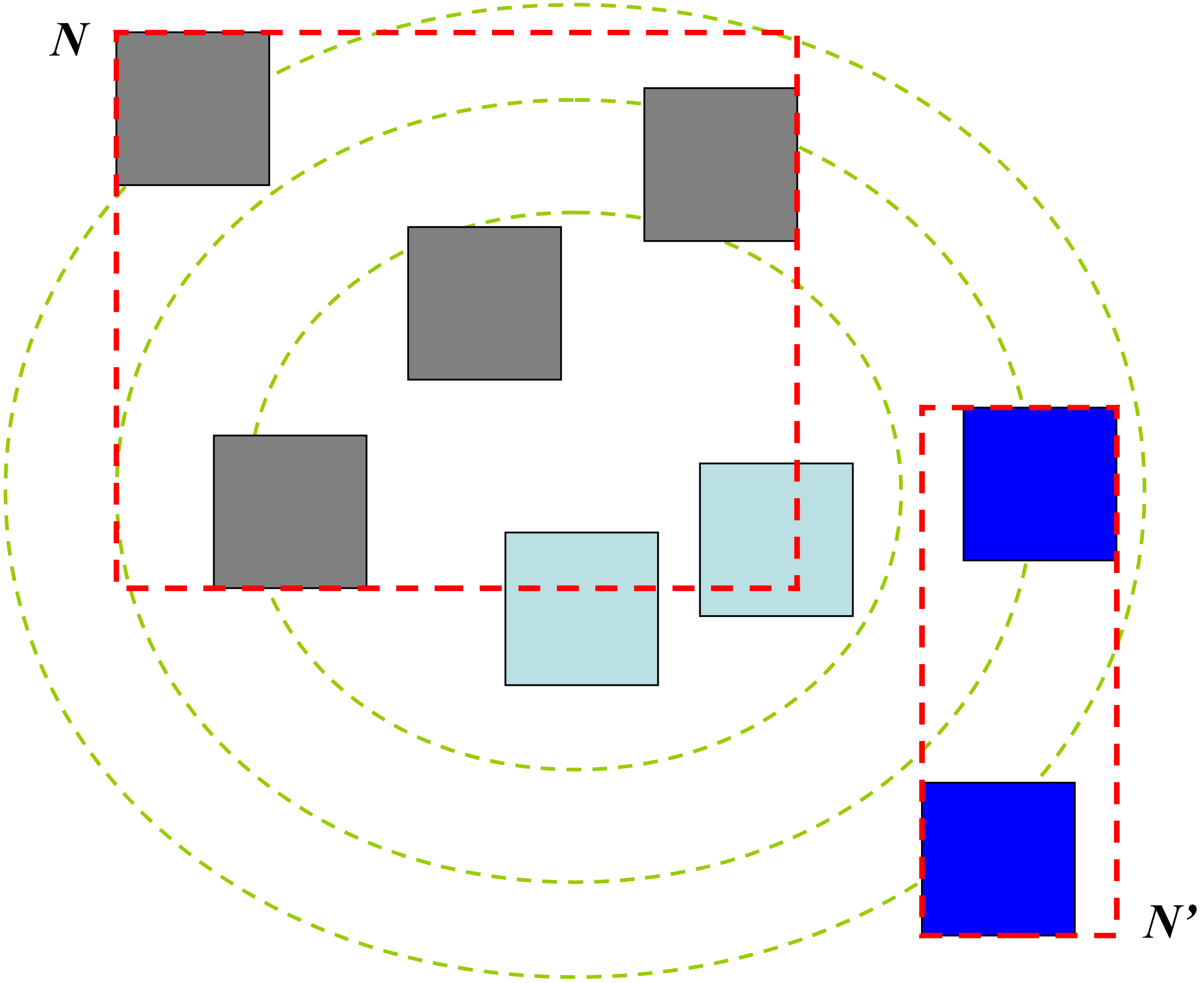
**Algorithm**: **QuadraticSplit** ($ES$, $N$, $N'$)

**Inputs**: $ES$ : A set of entries ES to be split into $N$ and $N'$.

---

- Let $\{E_A, E_B\}$ = **PickSeeds** ($ES$), be the first entries in $N$ and $N'$ respectively
- Let $ES' = ES - \{E_A, E_B\}$.
- **While** $ES'$ still has entries to be assigned **Do**
  - **If** either $N$ or $N'$ has $m$ entries **Then**
    - Add all of $ES'$ into either $N$ or $N'$ that has fewer than $m$ entries
  - **Else**
    - Let $E$ = **PickNext** ($ES'$, $N$, $N'$)
    - Add $E$ into either $N$ or $N'$ following ***Notion (iib)***
  - **End If**
  - Update ES'
- **Loop**

# Node Splitting – Quadratic Cost



# Node Splitting – Linear Split

- The Linear-Split algorithm is the same as the Quadratic-Split algorithm, but
  - Uses **LinearPickSeeds()** instead of **PickSeeds()**.
  - Randomly chooses next entry for assignment instead of using **PickNext()**.

- *Notion*: Examine each Interval [*ka*, *kb*] spanning each dimension for every Entry, and select 2 Entries that are most distant from each other.

# Node Splitting – Linear Pick Seeds

**Algorithm**: **LinearPickSeeds** $(ES)$

**Input**: $ES$ : A set of $M$+1 entries.

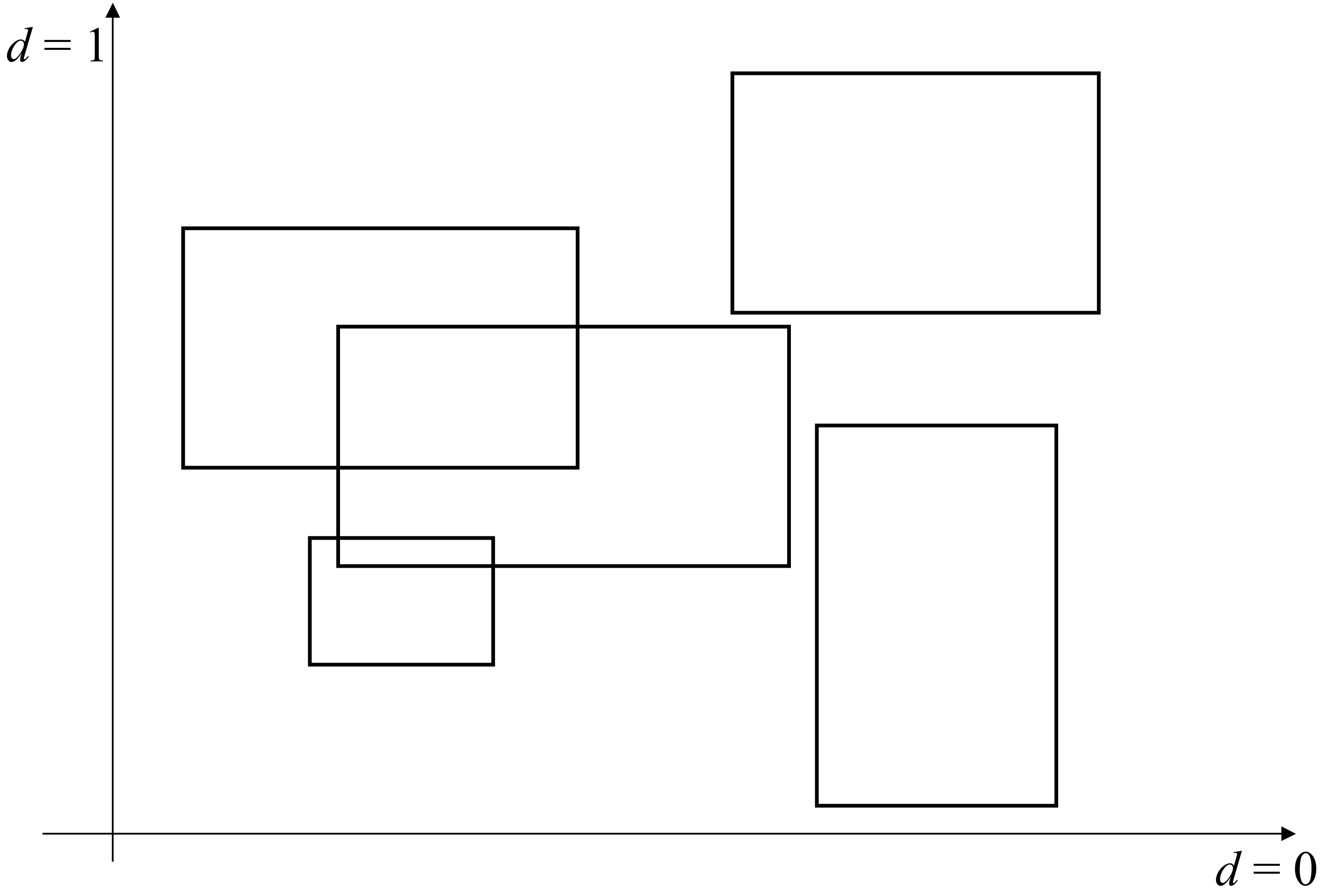
**Outputs**: 2 entries $Ei$ and $Ej$ that are deemed furthest apart.

---

- Let $ES = \{E_0, E_1, \ldots, E_M\}$
- **For** each dimension $d$ from 1 to $n$ **Do**
  - Let $Ei$ and $Ej$ be the 2 entries farthest away from each other in dimension $d$, such that $Ei.I = [da_i, db_i]$ has the highest lower-bound $da_i$ and $Ej.I = [da_j, db_j]$ has the lowest upper-bound $da_j$
  - Calculate $wid\_d = da_j - da_i$
  - Normalise $wid\_d$ to the width of the entire set on this $d$
- **Next** $d$

- Return $Ei$ and $Ej$ with the largest $wid\_d$

# Node Splitting – Linear Pick Seeds



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  - Quadratic-Split
  - Linear-Split
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# Derivatives of the R-Tree

- Concepts and index-structure of the original R-Tree remains the same.
- Derivatives of the R-Trees emphasise on improvements on optimisation methods to pack rectangles based on other imposed constraints.

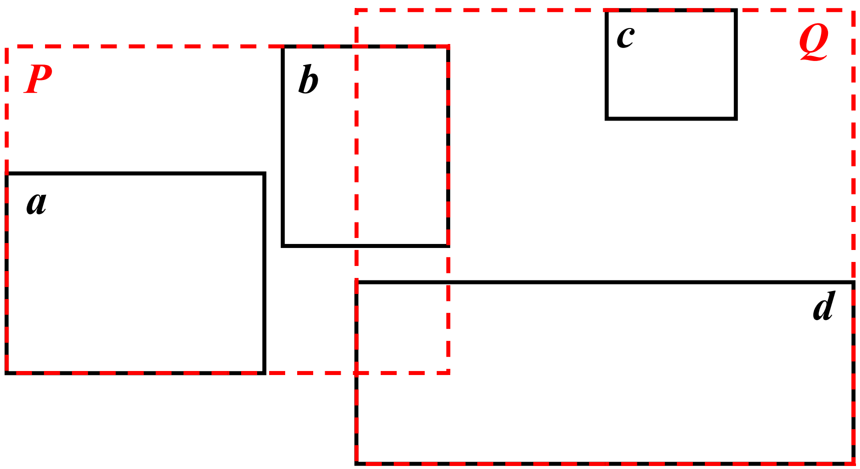
- 2 main constraint-issues addressed in the literature:
  - *Coverage*: Total area of all the MBRs of all leaf nodes.
  - *Overlap*: Total area contained within 2 or more leaf MBRs.
- For efficient R-Tree searching: Both *coverage* and *overlap* must be minimised.

- Overlap condition is more critical than coverage to achieve maximum efficiency in searching.
- However, it is difficult to control overlaps during dynamic splits.

# Derivative I: R+ Tree

→ **Notion**: Avoid overlaps among MBRs at the expense of space.

- Rectangles are decomposed into smaller sub-rectangles.
  - A leaf-MBR *L.I* that overlaps other MBRs is broken into several non-overlapping rectangles (whose union makes up *L.I*).
  - All pointers of the sub-rectangles point to the same object *L.tuple-identifier*.
- Sub-rectangles chosen so that no MBR at any level needs to be enlarged.

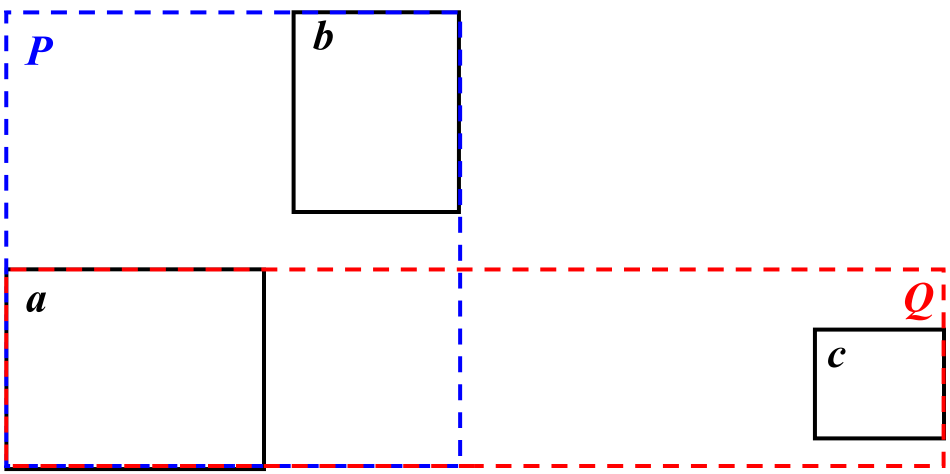


# Derivative II: R*-Tree

→ **Notion**: Avoid overlaps among MBRs, while optimising space.

- Similar constraints as in the R+ Tree, but differ in choosing the path of least resistance when inserting new index-records.
- Favours smallest variation in margin length, in addition to choosing the smallest encompassing area.

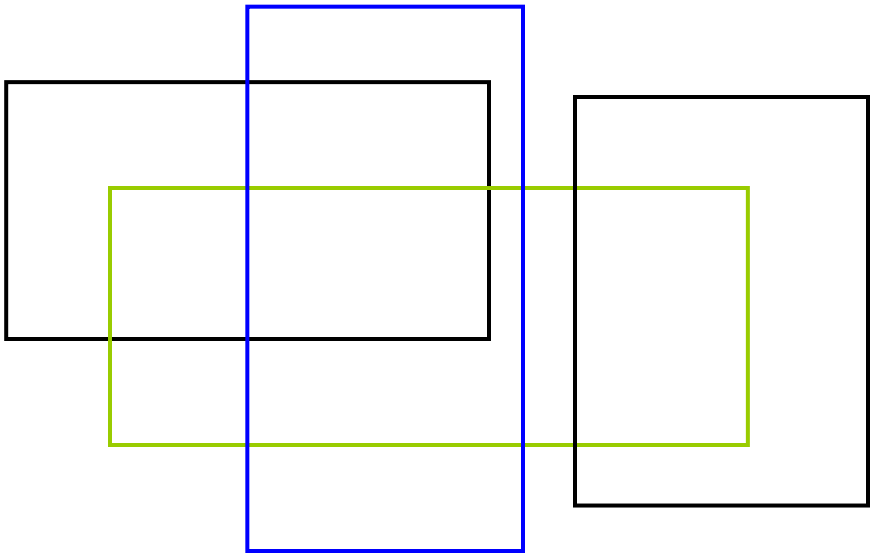
**Example**: Suppose Area($P$) ≈ Area($Q$)



# Derivative III: X-Tree

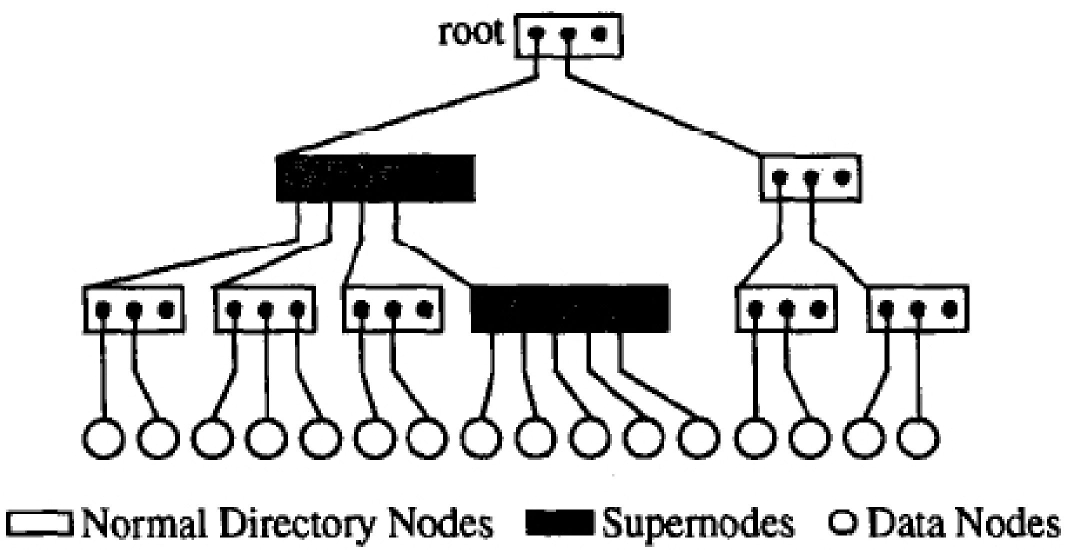
→ **Notion**: (A variant specifically designed for dealing with high-dimensional space) Dynamically organise the tree such that portions of the data which would produce *high overlap* are organised linearly in extended *supernodes*.

- Employs the concept of weighted overlap, measured by the number of rectangles within an overlapping region.



# Derivative III: X-Tree

- The X-Tree uses extended variable-size supernodes with $\alpha M$ entries, $\alpha > 1$.
- Supernodes are used to avoid splits with high overlap values.
- Studies show that overlaps increase with dimension.
- Supernodes are created during insertion only if there is no possibility to avoid high overlaps.




Berchtold, Keim and Kriegel. (1996)

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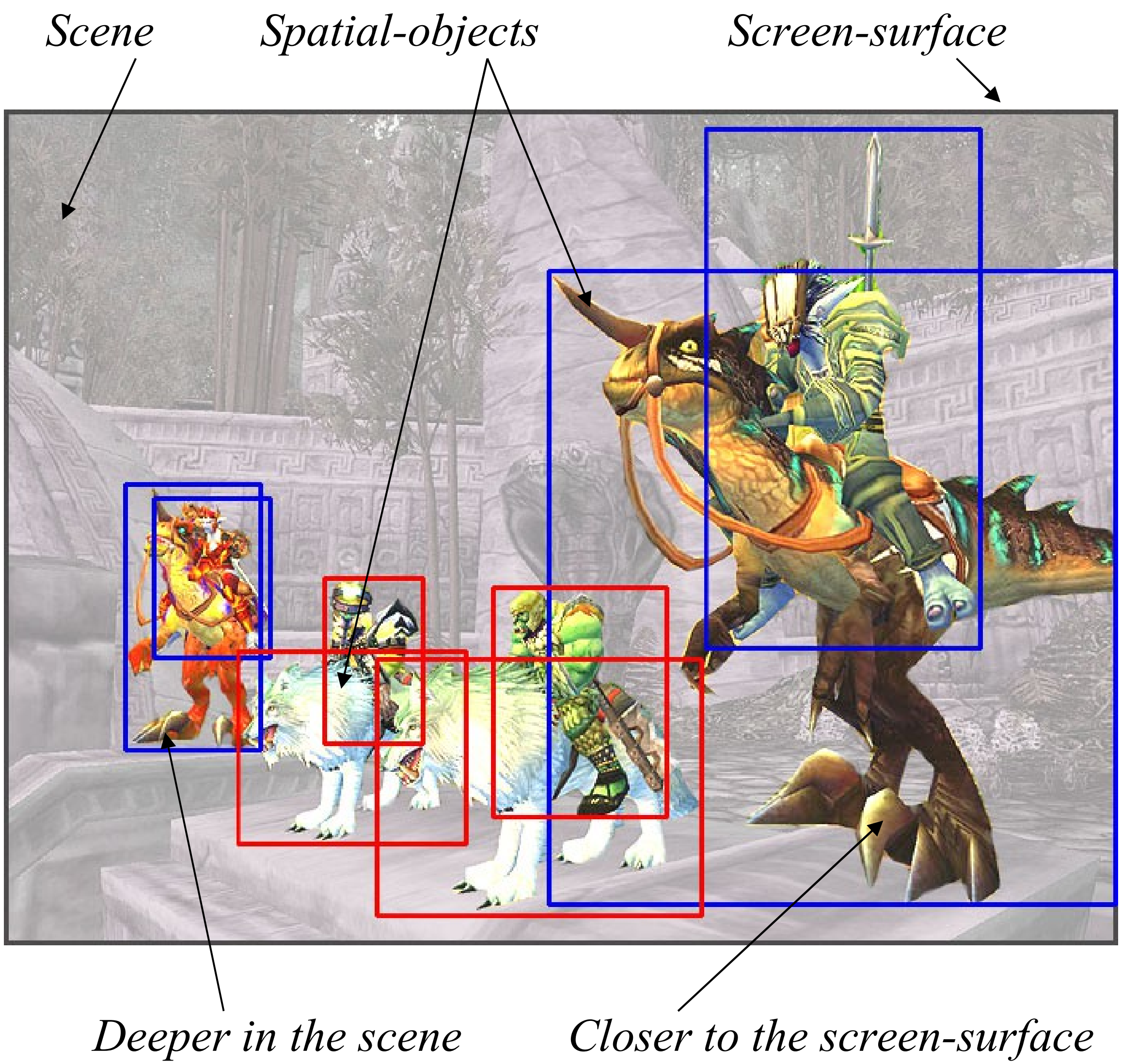
  - R+ Tree
  - R*-Tree
  - X-Tree
- Applications

# Interactive Graphic-Objects (IGO) on 2D Screen




**Consider**

- Placing (spatial) 3D-graphic-objects in a scene.
- Screen display is 2D.
- Objects have depth-perception.

**Interactions**

- Clicking (2D Point) returns ONE correctly selected object.

**R-Tree Structure**

- Assumptions?
- Support dimension?
- Augment information?

# IGO: Assumptions

i. That the *scene* is static; i.e. camera view on *screen-surface* don’t change.

ii. That we are dealing only with one snapshot at time $t_s$.

iii. That the (*spatial*) *objects* are static at $t_s$; following (ii ).

iv. That 3D *objects* and *scene* are flattened for 2D display on *screen-surface*.

v. That depth-perception is realised on size of *objects*; they appear relatively smaller when deeper in the scene, and relatively larger otherwise.

vi. That *objects* closer to the *screen-surface* are always drawn on top of *objects* that are deeper in the *scene*.

vii. That within the MBR of the *objects*, we can tell whether or not the *object* itself has been clicked on (e.g. via Quad-trees).

# IGO: Problem Formulation

**Objective:** Clicking must return the correct object viewed on 2D screen.

- Case 1: Non-overlapping objects. Straight forward
- Case 2: Overlapping objects.
  - Notion (i): Perform a stabbing query – use point $p$ as query point and return all leaf entries stabbed by $p$.
  - Notion (ii): Return the ONE object closest to the screen-surface.

**Additional Requirements**

- Augment 'layer' values in the R-Tree nodes; such that objects deeper in the scene would have lower 'layer' values than objects closer to the screen surface.
- Modify entry $E_N = (I, *pointer, \boldsymbol{v_l}, \boldsymbol{v_h})$, where $\boldsymbol{v_l}$ is the lowest 'layer' value at the subtree of node $N$, and $\boldsymbol{v_h}$ is the highest 'layer' value.

# IGO: Structure – Spatial View

**Level Order:** Deepest in *scene* (0), closest to *screen-surface* (7).

$I(\boldsymbol{a}) = \{[76, 234], [160, 402]\}$

$I(\boldsymbol{b}) = \{[93, 168], [243, 344]\}$

$I(\boldsymbol{c}) = \{[147, 290], [340, 460]\}$

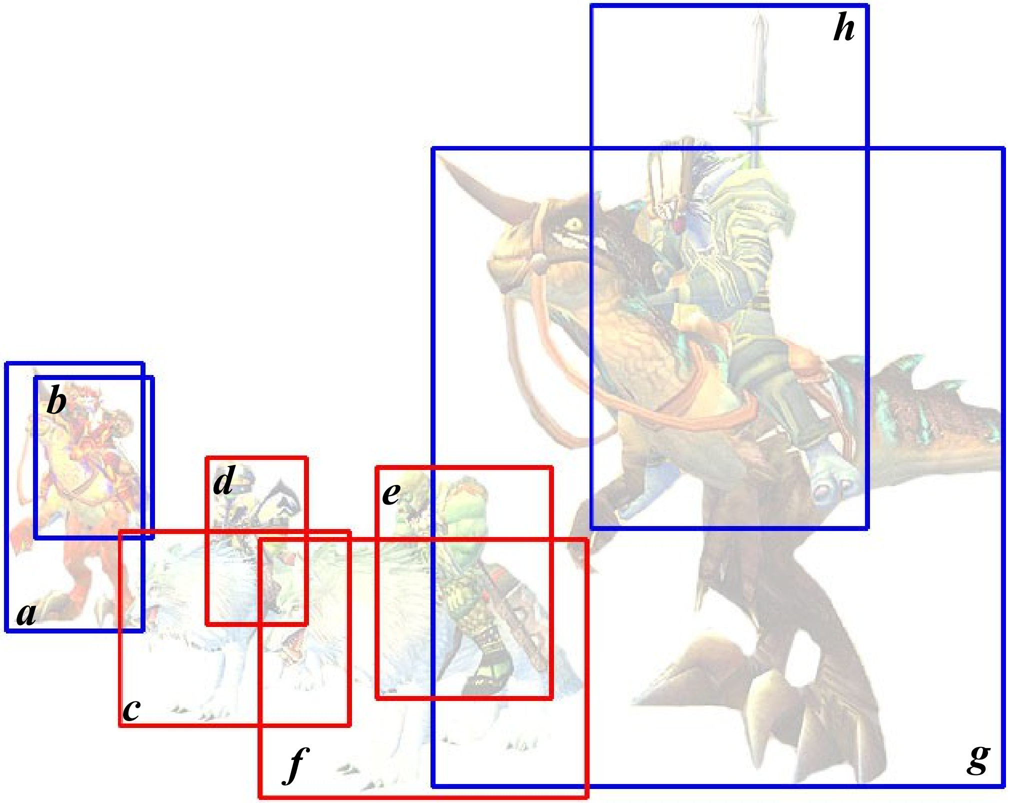
$I(\boldsymbol{d}) = \{[202, 264], [294, 398]\}$

$I(\boldsymbol{e}) = \{[307, 416], [299, 444]\}$

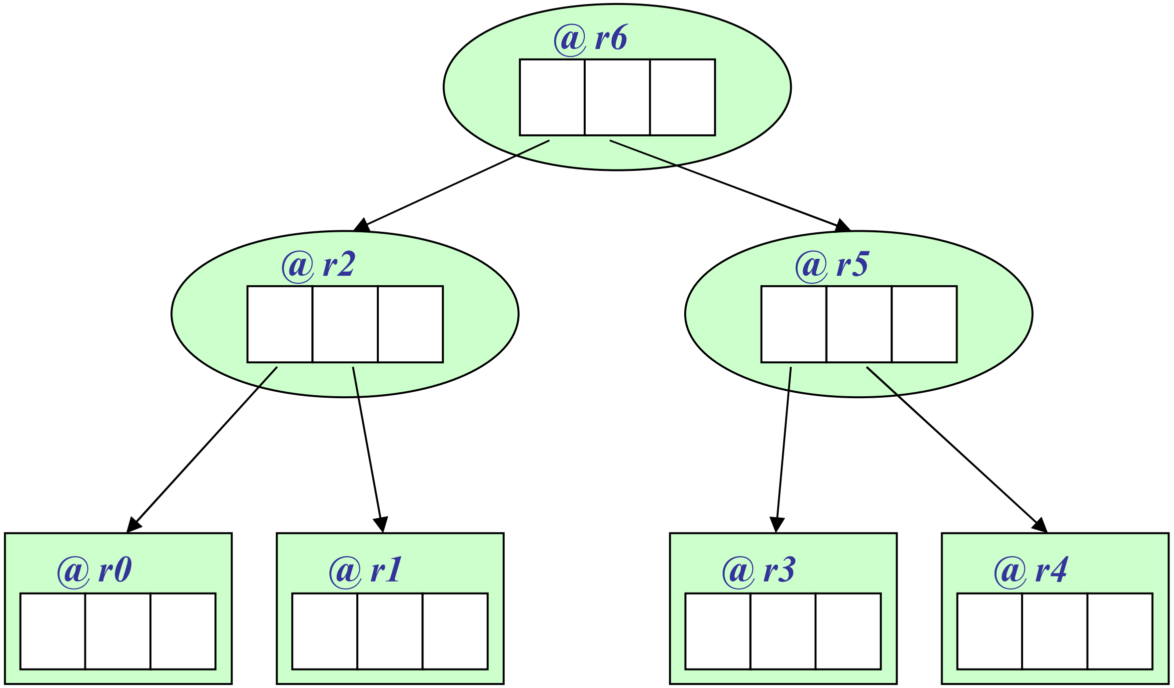
$I(\boldsymbol{f}) = \{[234, 439], [245, 507]\}$

$I(\boldsymbol{g}) = \{[342, 700], [100, 500]\}$

$I(\boldsymbol{h}) = \{[440, 613], [12, 338]\}$



# IGO: Structure – Tree View



# IGO: Search Strategy

- **Algorithm**: **IGOSearch** ($N$, $p$)
- **Inputs**: (i) A node $N$ in the R-Tree, (ii) The clicked point $p$.
- **Output**: The leaf entry $E_L$ stabbed by $p$ which is on the top-most layer.

---

- **If** $N$ is Leaf **Then**
  - Let $ES$ be the set of entries in $N$ whose spatial objects are stabbed by $p$.
  - **If** $ES = \varnothing$ **Then** Return NULL
  - **Else** Return $E_L = (I, \textit{child-pointer}, vl, vh) \in ES$ whose $vh$ is the highest

# IGO: Search Strategy

- **Else**
  - Let $ES$ be the set of child-node entries in $N$ whose $I$'s contain $p$.
  - Arrange $ES$ in decreasing order of $vh$.
  - **For** each $E_N = (I_N, \textit{child-pointer}\_{N}, vl_N, vh_N) \in ES$ **Do**
    - Store $E_L$ = **IGOSearch** ($\textit{child-pointer}\_{N}$, $p$)
  - **Next** $E_N$
  - Return $E_L = (I, \textit{child-pointer}, vl, vh) \in ES$ whose $vh$ is the highest
- **End If**

# Summary & Conclusions

- The R-Tree has similar features and properties to the B-Tree.
- They remain height balanced while maintaining adjustable rectangles that ignore ‘dead spaces’.
- The structure and composition in R-Trees are dynamically driven by the spatial objects they represent.
- The spatial objects stored at the leaf-level are considered ‘atomic’, as far as the search is concerned; i.e. they are not further decomposed into pictorial primitives.
- The performance of the R-Tree is based on the optimality of packing lower level rectangles in higher level nodes.
- Splitting nodes contribute to local improvement of rectangular arrangements.
- Condensing nodes contribute to global refinement of rectangular arrangements.

# References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