16-720 Homework 2: Feature Descriptor & Homographies & RANSAC

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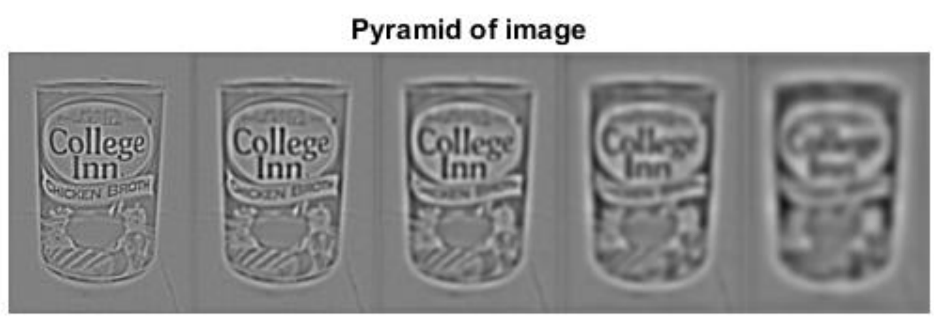
Q1.1

In this section, I use difference of Gaussian (DoG) to approximately represent Laplacian of Gaussian. The function is given and my result is shown as below:

Q1.2

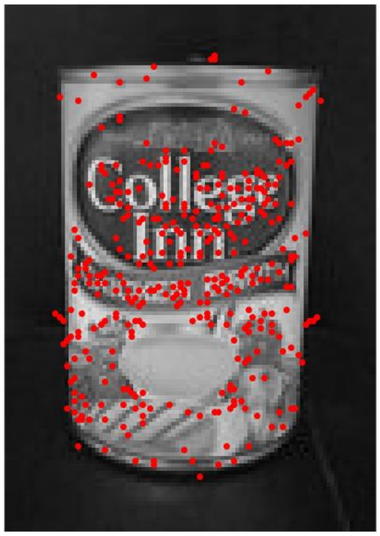
Then I use this N layer of Gaussian pyramid to create DoG by subtracting the n+1th layer by nth layer so I can create N-1 layer of DoG images. The results are shown as below:

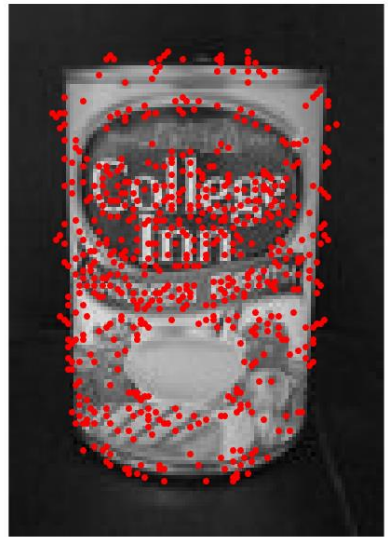


Actually I believe there is a typo in the write-up which should be G(l)-G(l-1) instead of vice versa.

Q1.3 & Q1.4

Then I calculate the principal curvature which represent how likely a pixel is to be at the edge instead of a corner. By using this principal curvature and DoG magnitude, I remove those extrema which does not satisfy our restriction. The pictures below shows a contrast of interest points detection with and without edge detection.

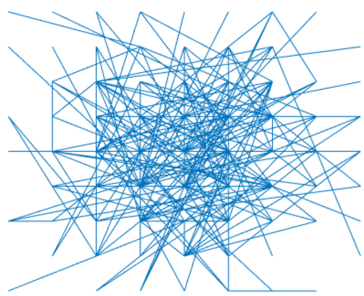




The effect of edge suppression shows significant effect on filtering key points.

Q2.1

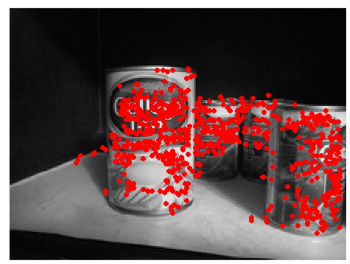
Here I use the second approach in the paper, which is that both x and y are selected based on independent Gaussian distribution. The pattern is plotted as below:



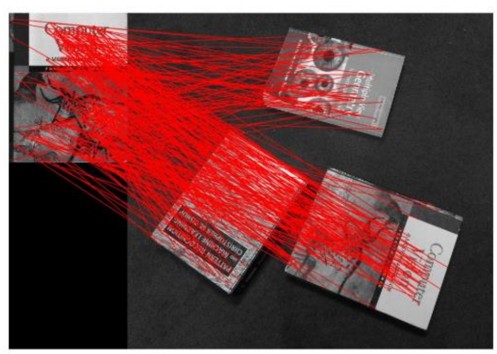
Q2.4

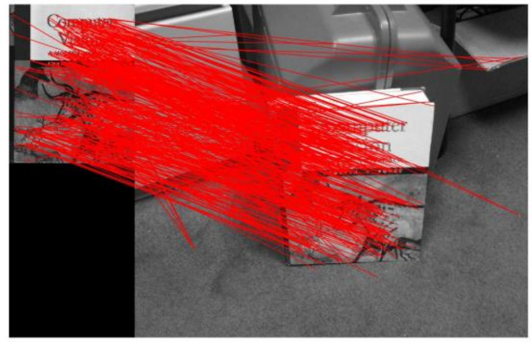
By using BRIEF descriptor, I test on two sets of pictures. The matching results are shown as below:

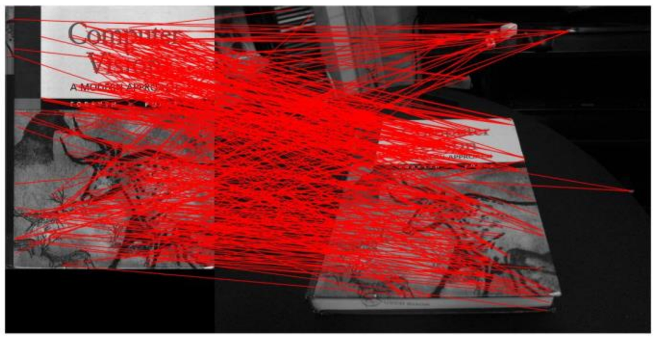
Also, when doing the match we should remove those interest points who are too close to the image boundary. The example is demonstrated below:



Then I compare the image of one CV text book with itself in different scenarios.

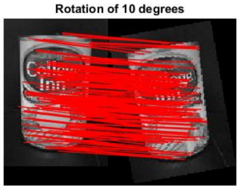




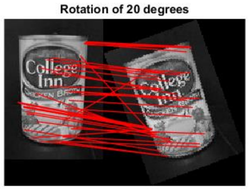


According to these experiments, I observe that the BRIEF descriptor matching shows decent result with scaling, which means the results are not sensitive to the distance between camera to the object or the angle of the camera. But rotation can extensively damage the matching. Below is some experiment I did on rotation through the function 'briefRotTest.m'. It shows that even a slight rotation may cause the entire system to crash.

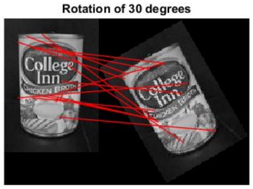
Rotation of 10 degrees

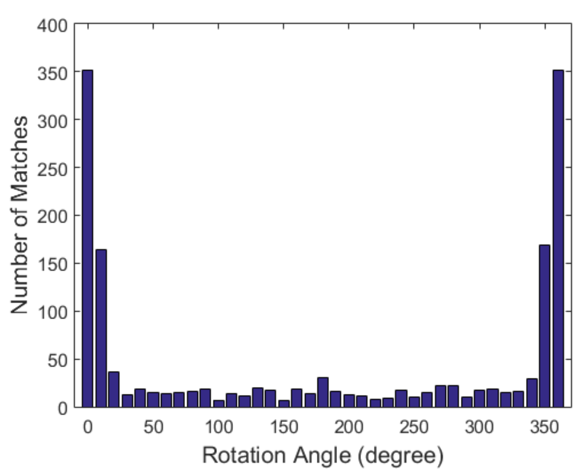


Rotation of 20 degrees



Rotation of 30 degrees





Q3.1

(a) $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $and \begin{cases} x' = {x_1'}/{x_3'} \\ y' = {x_2'}/{x_3'} \end{cases}$ $so\ we\ have \begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Therefore $A = \begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \end{bmatrix}$. But this is just for one pixel, the actual *A* has 2N rows.

(b) There are 9 elements in *h*.

(c) Since *H* has 8 degrees of freedom and each correspondence provides 2. So at least 4 different correspondence will be needed to determine *H*.

(d) The problem is $\arg\min_H \|Ah\| = \arg\min_H h^T A^T A h$ and $A^T A$ is a Hermitian matrix.

According to Rayleigh quotient theorem, $\frac{h^T A^T A h}{h^T h} \geq \lambda_{min}$ where λ_{min} is the minimum eigen value of $A^T A$. Since *h* has only 8 degrees of freedom which means we can treat $h^T h$ as a fixed value. So *h* should be the corresponding eigen vector of the minimum eigen value of $A^T A$.