Project 4

May 4, 2020

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1.
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1.

$$w_{i+1}^{(0)} = w_i,$$

$$\Rightarrow w_{i+1}^{(k+1)} = w_{i+1}^{(k)} - \frac{w_{i+1}^{(k)} - w_i - hf(t_{i+1}, w_{i+1}^{(k)})}{1 - hf_y(t_{i+1}, w_{i+1}^{(k)})}$$

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2.

function [t,w] = backeuler(f, dfdy, a, b, alpha, N, maxiter, tol)
h = (b - a)/N;
w = zeros(N,1);
t = zeros(N,1);
w(1) = alpha;
t(1) = a;
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for i = 1 : N
    t_h = t(i) + h;
    w0 = w(i);

    this_f = @(x) x - w0 - h*f(t_h,x);
    this_df = @(x) 1 - h*dfdy(t_h,x);

    w(i+1) = newton(this_f, this_df, w0, tol, maxiter);
    #I used Newton.m in course webpage with a little change.
    t(i+1) = t_h;
end
end

function p = newton(f, df, p0, tol, maxiter)
% Solve f(p) = 0 using Newton's method.

for i = 1 : maxiter
    #just edited this part (while 1 ==> for i = 1 : maxiter from Newton.m in course webpage)
    p = p0 - f(p0)/df(p0);
    if abs(p-p0) < tol, break; end
    p0 = p;
end

3.
a)
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3

a) $h\lambda = -2.7853$

$h = 2.7853$

$$N > \frac{2000}{2.7853} = 718.0555$$

N should be larger than 718

$\Rightarrow N = 719.$

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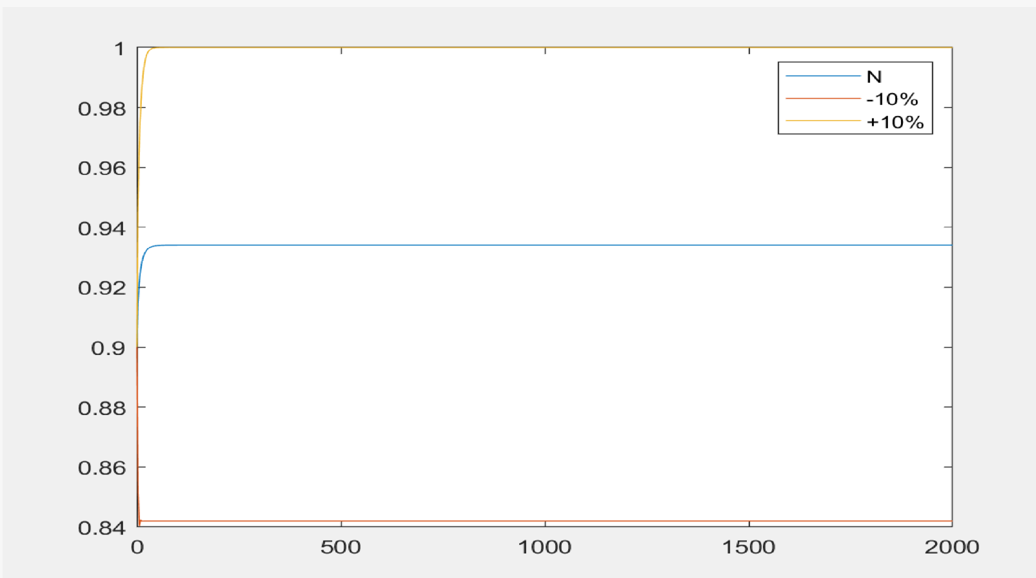
b)
f = @(t,y)(y.^2)*(1-y);
a = 0; b = 2000; alpha = 0.9;
N = 719;
N1 = 647;
N2 = 790;
[t1, w1] = rk4(f, a, b, alpha, N);
[t2, w2] = rk4(f, a, b, alpha, N1);
[t3, w3] = rk4(f, a, b, alpha, N2);
plot(t1, w1, t2, w2, t3, w3)
legend('N', '-10%', '+10%')
```



c)

c)

$$w_{i+1} = w_i + h f(t_{i+1}, w_{i+1})$$

$$y' = f(t, y) = \lambda y$$

$$\Rightarrow w_{i+1} = w_i + h\lambda w_{i+1}$$

$$(1 - h\lambda) w_{i+1} = w_i$$

$$w_{i+1} = \left(\frac{1}{1-h\lambda}\right) w_i$$

$$\Rightarrow Q(h\lambda) = \frac{1}{1-h\lambda}, \quad |Q(h\lambda)| < 1$$

$$\rightarrow hy < 0$$

$\Rightarrow$ backward Euler is A-stable.

~~[illegible]~~.

N should be larger $\frac{b-a}{h}$

$N > 0$.

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d)
f = @(t, y) y*y*(1-y);
df = @(t, y) 2*y - 3*y*y;
a = 0; b = 2000; alpha = 0.9; maxiter = 20; tol = 1e-12;
N = 1; N1 = 5; N2 = 10;
[t1,w1] = backeuler(f, df, a, b, alpha, N, maxiter, tol);
[t2,w2] = backeuler(f, df, a, b, alpha, N1, maxiter, tol);
[t3,w3] = backeuler(f, df, a, b, alpha, N2, maxiter, tol);
plot(t1, w1, t2, w2, t3, w3)
legend('1', '5', '10')
```

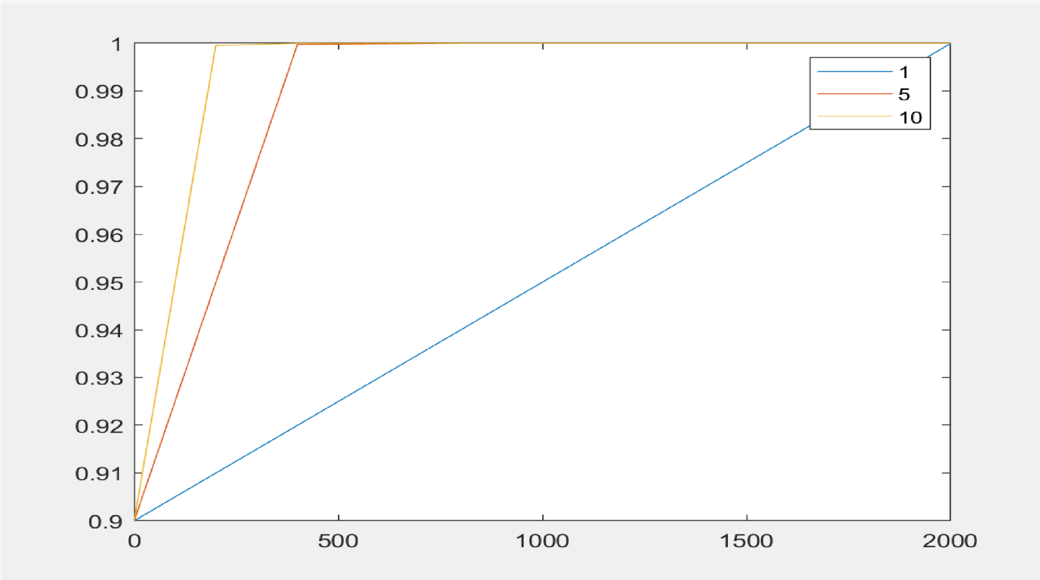


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All values with N = 1, 5, 10 close to 1 when t = 2000.
I think this method is stable.
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