

Project 3

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[]: Math 128a Project 3

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1.

$$\begin{aligned}\theta_1'(t) &= w_1 \\ \theta_2'(t) &= w_2 \\ \Rightarrow \\ w_1'(t) &= \frac{-3\sin(\theta_1) - \sin(\theta_1 - 2\theta_2) - 2\sin(\theta_1 - \theta_2)(w_2^2 - w_1^2 \cos(\theta_1 - \theta_2))}{3 - \cos(2\theta_1 - 2\theta_2)} \\ w_2'(t) &= \frac{2\sin(\theta_1 - \theta_2) \cdot (2w_1^2 + 2\cos(\theta_1) + w_2^2 \cos(\theta_1 - \theta_2))}{3 - \cos(2\theta_1 - 2\theta_2)}\end{aligned}$$

```
function ydot = fpend(y)

th1 = y(1);
th2 = y(2);
w1 = y(3);
w2 = y(4);

rt1 = w1;
rt2 = w2;
rt3 = ((-3).*sin(th1) - sin(th1-2.*th2) - 2.*sin(th1-th2).*(w2.^2 + w1.^2.
    ↪ *cos(th1-th2)))./(3-cos(2.*th1-2.*th2));
rt4 = (2.*sin(th1-th2).*(2.*w1.^2 + 2.*cos(th1) + w2.^2.*cos(th1-th2)))./
    ↪ (3-cos(2.*th1-2.*th2));
ydot = [rt1, rt2, rt3, rt4];

end
```

2.

```
function [y1] = fth_rk(f, y, h)

th1 = y(1);
th2 = y(2);
w1 = y(3);
w2 = y(4);

fst = f(y);
k1_1 = fst(1);
k1_2 = fst(2);
w1_1 = fst(3);
w1_2 = fst(4);
y1 = [th1 + (k1_1.*h./2), th2 + (k1_2.*h./2), w1 + (w1_1.*h./2), w2 + (w1_2.*h./
→2)];

sd = f(y1);
k2_1 = sd(1);
k2_2 = sd(2);
w2_1 = sd(3);
w2_2 = sd(4);
y2 = [th1 + (k2_1.*h./2), th2 + (k2_2.*h./2), w1 + (w2_1.*h./2), w2 + (w2_2.*h./
→2)];

trd = f(y2);
k3_1 = trd(1);
k3_2 = trd(2);
w3_1 = trd(3);
w3_2 = trd(4);
y3 = [th1 + (h.*k3_1), th2 + (h.*k3_2), w1 + (h.*w3_1), w2 + (h.*w3_2)];

fth = f(y3);
k4_1 = fth(1);
k4_2 = fth(2);
w4_1 = fth(3);
w4_2 = fth(4);

a1 = th1 + (h./6)*(k1_1 + (2.*k2_1) + (2.*k3_1) + k4_1);
a2 = th2 + (h./6)*(k1_2 + (2.*k2_2) + (2.*k3_2) + k4_2);
b1 = w1 + (h./6)*(w1_1 + (2.*w2_1) + (2.*w3_1) + w4_1);
b2 = w2 + (h./6)*(w1_2 + (2.*w2_2) + (2.*w3_2) + w4_2);
y1 = [a1, a2, b1, b2];

end
```

Command

```
→=====

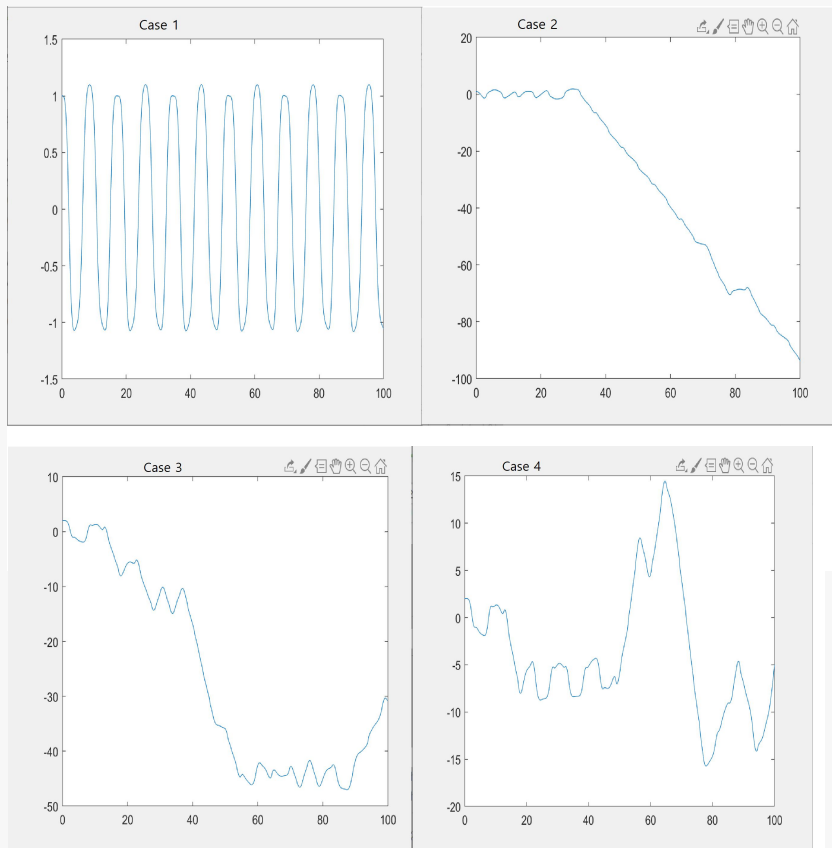
x = 0 : 0.05 : 100;

[CASE 1]
y = [1, 1, 0, 0];
y1 = [];
for t = 0 : 0.05 : 100
y = fth_rk(@fpend, y, 0.05);
y1 = [y1, y(2)];
end
plot(x, y1)

[CASE 2]
y = [pi, 1, 0, 10.^(-10)];
y1 = [];
for t = 0 : 0.05 : 100
y = fth_rk(@fpend, y, 0.05);
y1 = [y1, y(2)];
end
plot(x, y1)

[CASE 3]
y = [2, 2, 0, 0];
y1 = [];
for t = 0 : 0.05 : 100
y = fth_rk(@fpend, y, 0.05);
y1 = [y1, y(2)];
end
plot(x, y1)

[CASE 4]
y = [2, (2 + 10.^(-3)), 0, 0];
y1 = [];
for t = 0 : 0.05 : 100
y = fth_rk(@fpend, y, 0.05);
y1 = [y1, y(2)];
end
plot(x, y1)
```



3.

```
h1 = 0.05;
h2 = 0.05/(2);
h3 = 0.05/(2.^2);
h4 = 0.05/(2.^3);
h = 0.001;
h_s = [h1, h2, h3, h4];

y = [1, 1, 0, 0];
for t = 0 : h1 : 100
    y = fth_rk(@fpend, y, h1);
    s1 = y(2);
end
y = [1, 1, 0, 0];
for t = 0 : h2 : 100
```

```

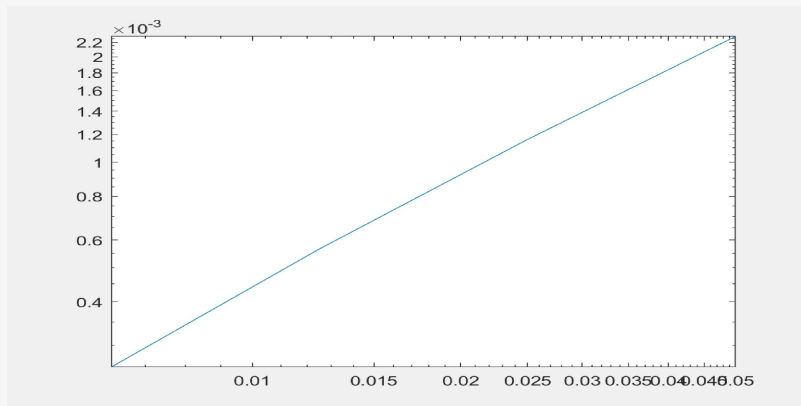
y = fth_rk(@fpend, y, h2);
s2 = y(2);
end
y = [1, 1, 0, 0];
for t = 0 : h3 : 100
y = fth_rk(@fpend, y, h3);
s3 = y(2);
end
y = [1, 1, 0, 0];
for t = 0 : h4 : 100
y = fth_rk(@fpend, y, h4);
s4 = y(2);
end
y = [1, 1, 0, 0];
for t = 0 : h : 100
y = fth_rk(@fpend, y, h);
s = y(2);
end

s_s = [s1, s2, s3, s4, s];
s_s = [-1.054407842273153   -1.053282936636117   -1.052687422198012   -1.
↪052382054995190   -1.052121764278673];

e_s = [abs(s - s1), abs(s - s2), abs(s - s3), abs(s - s4)];
e_s = [0.002286077994480    0.001161172357444    0.000565657919339    0.
↪000260290716516];
h_s = [0.050000000000000    0.025000000000000    0.012500000000000    0.
↪006250000000000];

loglog(h_s, e_s)

```



```
logh_s = log(h_s);  
loge_s = log(e_s);  
slope = (loge_s(4) - loge_s(1))/(logh_s(4) - logh_s(1));  
slope = 1.044892954061185  
The order of convergence is  $O(h)$ .
```