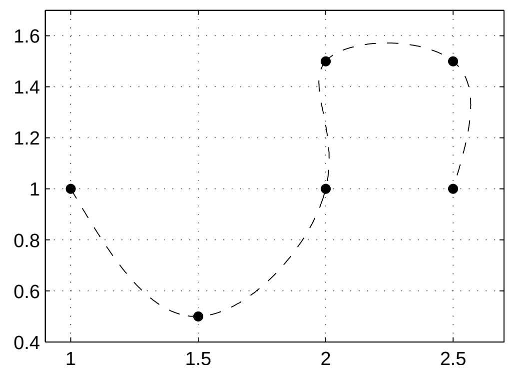
UCB Math 128A, Spring 2020: Programming Assignment 2

Due March 18

In this assignment, we will fit a parametric curve using cubic splines, solve for intersections, and integrate numerically to find the length of a curve segment. You may use the MATLAB functions on the course web page, in particular `ncspline.m`, `splineeval.m`, and `diffsplineeval.m`.

1. Consider a parameteric curve $(x(t), y(t))$, $0 \le t \le 5$ (see figure below). The position of the curve is given at 6 parameter values:

t	$x(t)$	$y(t)$
0	1.0	1.0
1	1.5	0.5
2	2.0	1.0
3	2.0	1.5
4	2.5	1.5
5	2.5	1.0



Approximate the curve by fitting *natural cubic splines* to this data, independently for $x(t)$ and $y(t)$. Plot the curve in MATLAB by

```
plot(xx,yy, x,y,'o'), axis equal, grid on
```

where `x,y` contain the given values and `xx,yy` contain the spline data evaluated for a large number of parameter values between 0 and 5.

Your report should contain your MATLAB commands, the computed a_j, b_j, c_j, d_j values for $x(t)$,$y(t)$, and a plot of the curve.

2. Use Newton's method to find the parameter values t_1 and t_2 where the curve intersects the line $y = 1.2$.

Your report should contain your MATLAB commands and the computed parameter values t_1, t_2 with 8 significant digits.

3. Compute the length of the segment of the curve above $y = 1.2$, by numerically evaluating the integral

$$L = \int_{t_1}^{t_2} \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

using the composite trapezoidal rule. Compute a series of approximations $L_{16}, L_{32}, L_{64}, L_{128}$ using $n = 16, 32, 64, 128$, respectively. Also compute a highly accurate value L using $n = 10,000$. Plot the errors $|L_n - L|$ versus $h = (t_2 - t_1)/n$ in a log-log plot and estimate the slope.

Your report should contain your MATLAB commands, the values of the 5 computed integrals, the convergence plot, and the estimated slope.