UCB Math 128A, Spring 2020: Programming Assignment 1

Due February 26

In this assignment, you will write and use two MATLAB functions for solving $f(x) = 0$.

1. Implement a MATLAB function `findzero` of the form

   ```
   function p = findzero(f, a, b, tol)
   ```

   which implements the following variant of the bisection and the secant method:

   - Initialize $w = 1$
   - Iterate for at most 100 times:
     1. Compute $p = a + \dfrac{wf(a)(a-b)}{f(b) - wf(a)}$
     2. Output $a, b, p, f(p)$ using `fprintf`
     3. If $f(p)f(b) > 0$, set $w = 1/2$, otherwise set $w = 1$ and $a = b$
     4. Set $b = p$
     5. Terminate if $|b - a| <$ tol or if $|f(p)| <$ tol

   You can use the function `bisection_table` on the course web page as a starting point.

2. Test your function `findzero` by solving $f(x) = \cos x - x$ with $a = 0$, $b = 1$, and tol $= 10^{-10}$. Include the printed table in your report, and comment on the apparent order of convergence.

3. Implement a MATLAB function `findmanyzeros` of the form

   ```
   function p = findmanyzeros(f, a, b, n, tol)
   ```

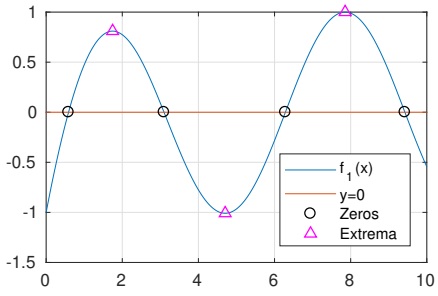
   which finds zeros in the interval $[a, b]$ using the following strategy:

   1. Compute $n + 1$ equidistant points x_k, $k = 0, \ldots, n$, between a and b
   2. For $k = 1, \ldots, n$, if $f(x_k)$ and $f(x_{k-1})$ have different signs, compute a zero using `findzero`
   3. The output vector p should contain all the computed zeros

4. Consider the functions

   $$f_1(x) = \sin x - e^{-x}$$
   $$f_2(x) = \frac{\sin(x^2)}{10 + x^2} - \frac{1}{50} e^{-x/10}$$

   Run your function `findmanyzeros` for these functions and their derivatives, on the interval $[0, 10]$ with $n = 50$ points and tol $= 10^{-10}$. Plot the functions with the computed zeros and the local extrema as in the example to the right. Give a brief comment about the results.



Reporting requirements: The GSIs will *not* run any submitted MATLAB codes. Prepare a report showing your MATLAB codes, your comments, and other requested information, such as the plots and the tables with the output from `findzero` in Problem 2 (no tables needed for Problem 4).