Winter 2022 MTH 261 Mini Test 1

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Question 1

Consider the system of equations bellow:

$$\begin{aligned} x - 7y \quad\quad + 6t &= 5 \\ z - 2t &= -3 \\ -x + 7y - 4z + 2t &= 7 \end{aligned}$$

(a) Write the system of equations as a vector equation.
(b) Write the system as a matrix equation A**x**=**b**
(c) Solve the system of equations using linear algebra techniques. Indicate your solution in parametric vector form.

Part (a)

$$\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} + \begin{bmatrix} -7y \\ 0 \\ 7y \end{bmatrix} + \begin{bmatrix} 0 \\ z \\ -4z \end{bmatrix} + \begin{bmatrix} 6t \\ -2t \\ 2t \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 7 \end{bmatrix} \tag{1}$$

$$x \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + y \begin{bmatrix} -7 \\ 0 \\ 7 \end{bmatrix} + z \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + t \begin{bmatrix} 6 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 7 \end{bmatrix} \tag{2}$$

Part (b)

$$\begin{bmatrix} 1 & -7 & 0 & 6 \\ 0 & 0 & 1 & -2 \\ -1 & 7 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 7 \end{bmatrix} \tag{3}$$

Part (c)

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \tag{4}$$

$$\sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix} \tag{5}$$

$$\sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow RREF \tag{6}$$

$$\therefore x - 7y + 6t = 5 \rightarrow x = 5 + 7y - 6t$$

$$y = y$$

$$z - 2t = -3 \rightarrow z = -3 + 2t$$

$$t = t$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 5 + 7y - 6t \\ y \\ -3 + 2t \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7y \\ y \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6t \\ 0 \\ 2t \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + y \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 6 \\ 0 \\ 2 \\ 1 \end{bmatrix} \quad (7)$$

Question 2

$$Let\ \mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ -8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ -11 \\ -6 \end{bmatrix}.$$

$$Is\ \mathbf{b} \in \text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}?$$

$$\begin{bmatrix} 1 & 0 & -2 \\ -2 & 5 & 0 \\ 2 & 5 & -8 \end{bmatrix} \tag{8}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 \\ -2 & 5 & 0 \\ 0 & 10 & -8 \end{bmatrix} \tag{9}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & -4 \\ 0 & 10 & -8 \end{bmatrix} \tag{10}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow REF \tag{11}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow RREF \tag{12}$$

$$\mathbf{u} = -2; \mathbf{v} = -4/5; \mathbf{w} = \mathbf{w}$$

$$\mathbf{b} = \begin{bmatrix} -1 \\ -11 \\ -6 \end{bmatrix} \neq \begin{bmatrix} -2 \\ -4/5 \\ \mathbf{w} \end{bmatrix} \tag{13}$$

Since the equation above is a false statement, the following can be determined:

$$\therefore \mathbf{b} \notin \text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\} \tag{14}$$

Question 3

$$\textit{Let } \mathbf{u} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \textit{ and } \mathbf{v} = \begin{bmatrix} -1 \\ -6 \end{bmatrix}.$$

Draw a Cartesian plane to draw the following graphs on.
Graph **u**, **v**, 2**u**, -**v**, 2**u** - **v**.

$$\mathbf{u} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}; \mathbf{v} = \begin{bmatrix} -1 \\ -6 \end{bmatrix}; 2\mathbf{u} = 2\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}; -\mathbf{v} = -1\begin{bmatrix} -1 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$2\mathbf{u} - \mathbf{v} = 2\begin{bmatrix} -3 \\ 2 \end{bmatrix} - \begin{bmatrix} -1 \\ -6 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ -6 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}$$

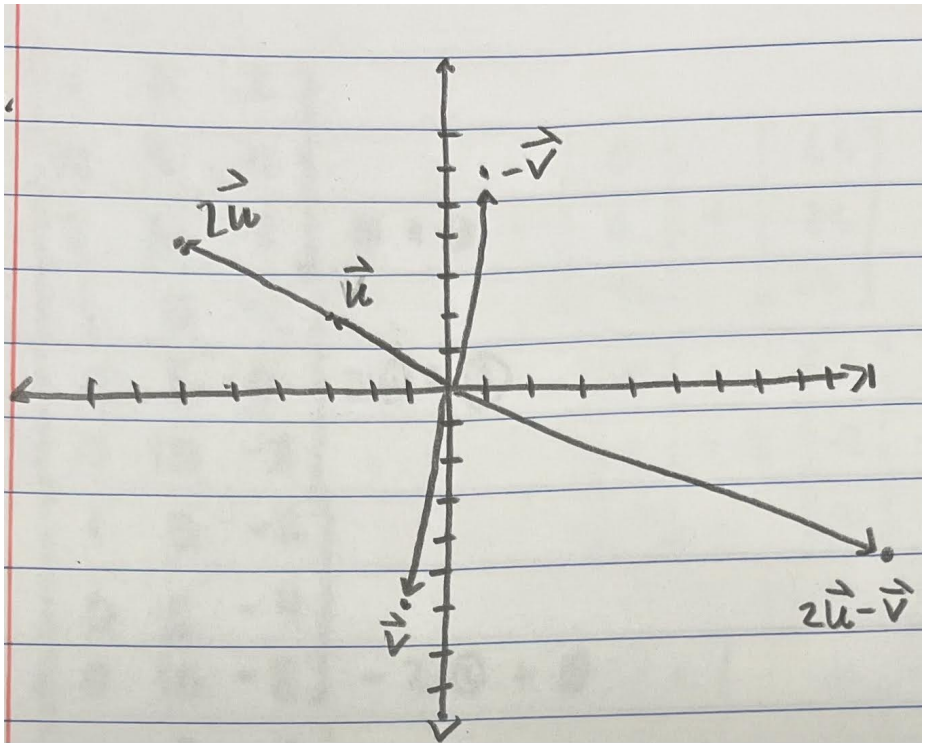

Figure 1: Graph of vectors