Thinking Probabilistically-Discrete Variables

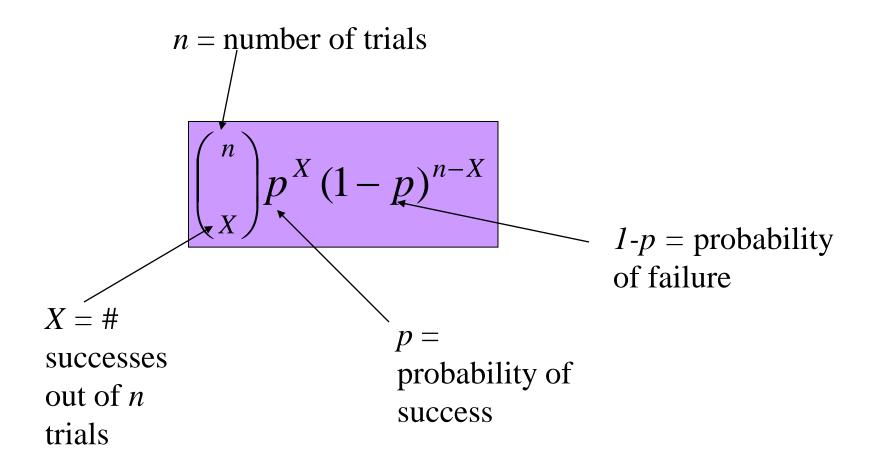
Binomial distribution, generally

- **Binomial:** Suppose that *n* independent experiments, or trials, are performed, where *n* is a fixed number, and that each experiment results in a "success" with probability *p* and a "failure" with probability *l-p*. The total number of successes, *X*, is a binomial random variable with parameters *n* and *p*.
- We write: X ~ Bin (n, p) {reads: "X is distributed binomially with parameters n and p}
- And the probability that X=r (i.e., that there are exactly r successes) is:

$$P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Binomial distribution, generally

Note the general pattern emerging \rightarrow if you have only two possible outcomes (call them 1/0 or yes/no or success/failure) in n independent trials, then the probability of exactly X "successes"=



**All probability distributions are characterized by an expected value and a variance:

If X follows a binomial distribution with parameters n and p: $X \sim Bin(n, p)$

Then:

$$\mu_{X} = E(X) = np$$

$$\sigma_{X}^{2} = Var(X) = np(1-p)$$

$$\sigma_{X} = SD(X) = \sqrt{np(1-p)}$$

$$\sigma_{X} = \frac{1}{\sqrt{np(1-p)}}$$

Note: the variance will always lie between

0*N-.25 *N

p(1-p) reaches maximum at p=.5

P(1-p)=.25

Introduction to the **Poisson Distribution**

- Poisson distribution is for counts—if events happen at a constant rate over time, the Poisson distribution gives the probability of X number of events occurring in time T.
- Poisson distribution with arrival rate equal to np approximates a Binomial distribution for n Bernoulli trials with probability p of success (with n large and p small). Importantly, the Poisson distribution is often simpler to work with because it has only one parameter instead of two for the Binomial distribution

Poisson Mean and Variance

Mean

$$\mu = \lambda$$

Variance and StandardDeviation

For a Poisson random variable, the variance and mean are the same!

$$\sigma = \sqrt{\lambda}$$

where λ = expected number of hits in a given time period