Thinking Probabilistically- Discrete Variables

Binomial distribution, generally

- **Binomial:** Suppose that *n* independent experiments, or trials, are performed, where *n* is a fixed number, and that each experiment results in a "success" with probability *p* and a "failure" with probability *1-p*. The total number of successes, *X*, is a binomial random variable with parameters *n* and *p*.
- We write: ***X ~ Bin (n, p)*** *{reads: "X is distributed binomially with parameters n and p}*
- And the probability that *X=r* (i.e., that there are exactly *r* successes) is:

$$\boxed{P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}}$$

Binomial distribution, generally

Note the general pattern emerging → if you have only two possible outcomes (call them 1/0 or yes/no or success/failure) in *n* independent trials, then the probability of exactly *X* "successes"=

n = number of trials

$$\binom{n}{X} p^X (1-p)^{n-X}$$

X = # successes out of *n* trials

p = probability of success

1-p = probability of failure

****All probability distributions are characterized by an expected value and a variance:**

<u>If</u> *X* follows a binomial distribution with parameters *n* and *p*: ***X ~ Bin (n, p)***

<u>Then:</u>

$\mu_X = E(X) = np$

$\sigma_X^2 = Var(X) = np(1-p)$

$\sigma_X = SD(X) = \sqrt{np(1-p)}$

Note: the variance will always lie between

0*N-.25 *N

p(1-p) reaches maximum at p=.5

P(1-p)=.25

Introduction to the **Poisson Distribution**

- Poisson distribution is for counts—if events happen at a constant rate over time, the Poisson distribution gives the probability of X number of events occurring in time T.
- Poisson distribution with arrival rate equal to np approximates a Binomial distribution for n Bernoulli trials with probability p of success (with n large and p small). Importantly, the Poisson distribution is often simpler to work with because it has only one parameter instead of two for the Binomial distribution

Poisson Mean and Variance

- Mean

$$\boxed{\mu = \lambda}$$

- Variance and Standard Deviation

$$\boxed{\sigma^2 = \lambda}$$

$$\boxed{\sigma = \sqrt{\lambda}}$$

For a Poisson random variable, the variance and mean are the same!

where λ = expected number of hits in a given time period