Solving Linear and Integer Programs

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Outline

❑ **Linear Programming:** *Bob Bixby*

    ❑ Example and introduction to basic LP, including duality

    ❑ Primal and dual simplex algorithms

    ❑ Computational progress in linear programming

    ❑ Implementing the dual simplex algorithm

❑ **Mixed-Integer Programming:** *Ed Rothberg*

An Example

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Diet Problem[*]

Bob wants to plan a nutritious diet, but he is on a limited budget, so he wants to spend as little money as possible. His nutritional requirements are as follows:

1. 2000 kcal

2. 55 g protein

3. 800 mg calcium

* From Linear Programming, by Vašek Chvátal

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Diet Problem

Nutritional values

Bob is considering the following foods:

Food	Serving Size	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving
Oatmeal	28 g	110	4	2	$0.30
Chicken	100 g	205	32	12	$2.40
Eggs	2 large	160	13	54	$1.30
Whole milk	237 cc	160	8	285	$0.90
Cherry pie	170 g	420	4	22	$0.20
Pork and beans	260 g	260	14	80	$1.90

Diet Problem

Variables

We can represent the number of servings of each type of food in the diet by the variables:

x_1 servings of oatmeal

x_2 servings of chicken

x_3 servings of eggs

x_4 servings of milk

x_5 servings of cherry pie

x_6 servings of pork and beans

Nutritional values

Bob is considering the following foods:

Food	Serving Size	Energy (kcal)	Protein (g)	Calcium (mg)	Price per serving	
Oatmeal	28 g	110	4	2	$0.30	x_1
Chicken	100 g	205	32	12	$2.40	x_2
Eggs	2 large	160	13	54	$1.30	x_3
Whole milk	237 cc	160	8	285	$0.90	x_4
Cherry pie	170 g	420	4	22	$2.00	x_5
Pork and beans	260 g	260	14	80	$1.90	x_6

KCAL constraint:

$$110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 \geq 2000$$
$$(110x_1 = \text{kcals in oatmeal})$$

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LP formulation

Minimize **Cost**

$$0.3x_1 + 2.40x_2 + 1.30x_3 + 0.90x_4 + 2.0x_5 + 1.9x_6$$

subject to: **Nutritional requirements**

$$110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 \geq 2000$$
$$4x_1 + 32x_2 + 13x_3 + 8x_4 + 4x_5 + 14x_6 \geq 55$$
$$2x_1 + 12x_2 + 54x_3 + 285x_4 + 22x_5 + 80x_6 \geq 800$$

Bounds

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

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Diet Problem

Solution

When we solve the preceding LP (using CPLEX, of course) we get a solution value of $6.71, which is achieved with the following menu:

14.24 servings of oatmeal

0 servings of chicken

0 servings of eggs

2.71 servings of milk

0 servings of cherry pie

0 servings of pork and beans

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Some Basic Theory

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