Dynamics of a Bio-Inspired Tail Modeled as an Elastic Beam Across Media, Loadings, and Material Properties

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Abstract— **This project models a bio-inspired tail as a slender elastic beam to investigate its dynamic response in low Reynolds number fluids under various loadings and material properties. Using a discrete mechanics-based simulator with implicit time integration, we will map how stiffness and actuation frequency influence tail kinematics, energy behavior, and propulsive performance—bridging biomechanics and soft-robotic design.**

I. Introduction

Many biological and robotic tails act as lightweight, compliant propulsors or stabilizers, playing a key role in maneuverability, balance, and propulsion. Their performance arises from the complex coupling between internal elasticity, actuation, and the surrounding fluid environment, particularly in low Reynolds number regimes where viscous effects dominate. Understanding these coupled interactions provides insight into soft-robotic propulsion and stability mechanisms, while also connecting to the biomechanics of natural systems such as fish fins and sperm flagella.

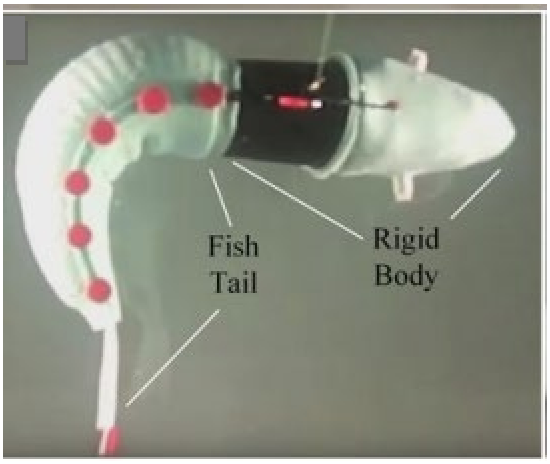

Figure 1. Classical Example of a robotic fish with an elastic tale [2].

Prior research has established several foundational approaches for modeling the dynamics of flexible tails and filaments. In the context of elastic-rod swimming theory, McMillen and Holmes [1] developed a mathematical model for anguilliform swimming that captured the interplay between internal elasticity and external fluid forces. In robotic tail systems, Saab et al. [2] reviewed a wide range of designs demonstrating how compliant tails can improve propulsion and maneuverability in both terrestrial and aquatic robots (fig. 2), though they noted that unified, mechanics-based simulation frameworks remain limited. For base-actuated flexible tails, Wang et al. [3] analyzed the dynamics of robotic fish driven by prescribed base oscillations, providing valuable boundary-condition formulations and validation data for base-driven actuation models.

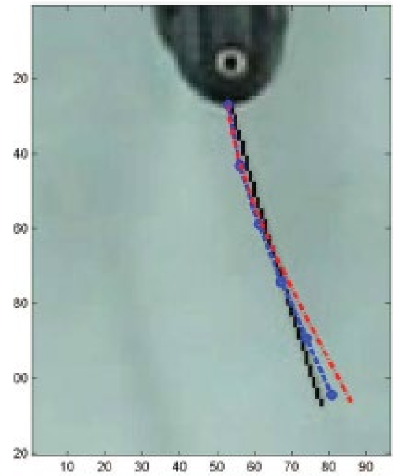

Figure 2. Comparison between experimental measurement of the time-dependent tail shape with model predictions [3].

Furthermore, studies on flagellar motion, such as Rikmenspoel's analysis of bull spermatozoa [4], have motivated the characteristic parameter regimes—including oscillation amplitude, frequency, and fluid viscosity—relevant to low Reynolds number locomotion. More recently, Nguyen et al. [5] combined dynamic modeling and experimental validation for flexible tail fins, identifying stiffness and frequency ranges that yield effective thrust and motion control.

This project models a tail as a slender elastic beam with a fixed base to investigate its dynamic response driven by curvature-based actuation and opposed by viscous drag in a low Reynolds number fluid. The surrounding environment is modeled to capture the dominant viscous interactions characteristic of microscale swimmers and flexible appendages. Building on prior studies, our work implements a discrete, mechanics-grounded simulator that combines bending and stretching energies with implicit time integration to capture the nonlinear, coupled dynamics of the flexible tail. By systematically varying material, actuation, and geometric parameters, we aim to identify performance regimes and transitions. This unified framework extends prior single-configuration analyses by merging energy-based beam modeling, curvature actuation, and viscous drag within a single computational toolchain, providing both physical insight and design guidance for bio-inspired, low-Reynolds-number robotic tails.

II. PROPOSED APPROACH

A. Model

The tail is modeled as a planar slender elastic beam of length L and circular cross-section with area $A=\pi r_0^2$ and moment of inertia $I=\pi r_0^4/4$. The beam is discretized into N_{nodes} and $N_{edges}=N_{nodes}-1$ segments. The mechanical response is governed by discrete stretching and bending energies, and the internal elastic forces are obtained as the gradients of the total energy, consistent with the discrete beam and rod formulations covered in the course. Each node carries a lumped mass, and the equations of motion are solved implicitly in time using Newton–Raphson iteration on the residual that includes inertia, elastic forces, damping, and external viscous drag.

To assess numerical stability and energy consistency, we will compare implicit Euler and Newmark–β time integrators ($\beta=0.25$, $\gamma=0.5$) in undamped tests before introducing physical damping. The Newmark–β scheme will allow evaluation of energy preservation and dissipation behavior under curvature-based actuation.

B. Actuation and Fluid Interaction

The primary actuation mechanism is prescribed curvature change along the beam to mimic biologically inspired bending patterns, replacing direct base actuation or point loading. The surrounding environment is treated as a low Reynolds number viscous fluid, where drag forces dominate inertial effects. The distributed viscous drag is represented as a linear damping term per node, $Fd=1/2Cd\rho A|V|V$, suitable for low-Re flow regimes. This force is incorporated explicitly in the force balance while maintaining a banded Jacobian structure for efficient implicit solves.

C. Parameter Sweeps

Systematic parameter sweeps will be conducted over the mechanical and geometric properties of the tail, including elastic modulus E, radius (which sets I), density ρ, and structural damping coefficients. Actuation parameters such as curvature amplitude and frequency will also be varied. The outputs will include tip displacement, curvature envelope, phase delay, input–output gain, and total energy response, enabling identification of dynamic regimes and scaling behavior.

D. Validation and Comparison

Model predictions will be compared qualitatively and quantitatively against established results in the literature. The dynamic patterns will be benchmarked against base-actuated tail dynamics from Wang et al. [3], flexible fin data from Nguyen et al. [5], and flagellar motion characteristics reported by Rikmenspoel [4]. The energy behavior and temporal stability will be validated by reproducing the expected contrast between the Newmark–β and implicit Euler schemes in a no-damping case, following the numerical methods discussed in the course.

E. Deliverables

The final deliverable will be a minimal MATLAB or Python implementation of the discrete elastic beam simulator, incorporating curvature-based actuation and viscous drag modeling. Parameter sweep scripts and visualization tools will be included to analyze deformation modes, energy evolution, and response metrics across varying material and actuation conditions.

III. ANTICIPATED CONTRIBUTIONS

This project will contribute a unified, mechanics-based simulation framework for modeling bio-inspired tails actuated through curvature changes in viscous, low-Reynolds-number environments. The work will extend classical beam and rod formulations by integrating curvature-driven actuation with distributed viscous drag into a stable implicit time-stepping scheme. Systematic parameter studies will reveal how material stiffness, geometry, and actuation frequency influence deformation patterns, energy behavior, and stability. The resulting simulations will provide both physical intuition and quantitative design guidance for soft-robotic swimmers and compliant stabilization appendages. In addition, the comparison between implicit Euler and Newmark–β integrators will offer insight into energy preservation and numerical performance for highly damped, nonlinear elastic systems.

IV. HOW IT GOES BEYOND CLASS EXAMPLES

This project extends the class material by applying discrete beam and rod formulations to a biologically inspired, dynamically actuated system operating in a viscous, low-Reynolds-number environment. While in-class examples focus on single loading cases or undamped elastic systems, this work incorporates curvature-based actuation and distributed viscous drag within an implicit time-stepping framework. The implementation emphasizes both physical realism and numerical stability, contrasting implicit Euler and Newmark–β integration schemes in highly damped regimes. By conducting systematic parameter sweeps and analyzing deformation and energy responses, the project bridges theoretical mechanics with soft-robotic applications, advancing the class concepts toward a unified and biologically relevant simulation tool.

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