Homework 4

MAE 263F Marwan Fayed

(1) t vs. δz (t) with steady-state identification (accompanied by a de-
    scription of steady-state criterion used), and helix snapshots (≥ 5) with labeled
    axes/units/time;

Figure 1 presents the vertical displacement at the tip, δz(t), when the helix is subjected to a
constant tensile load $F = F_{char}$. The motion initially displays oscillations caused by inertia and the
compliance of the structure, followed by a gradual exponential decay.
To determine when the system has reached steady state, the following condition was applied:

$$\frac{|\,\delta_z(t) - \delta_z(t-1)\,|}{|\,\delta_z(t)\,|} < 1\%$$

Using this criterion, the long-term stabilized displacement was measured to be:
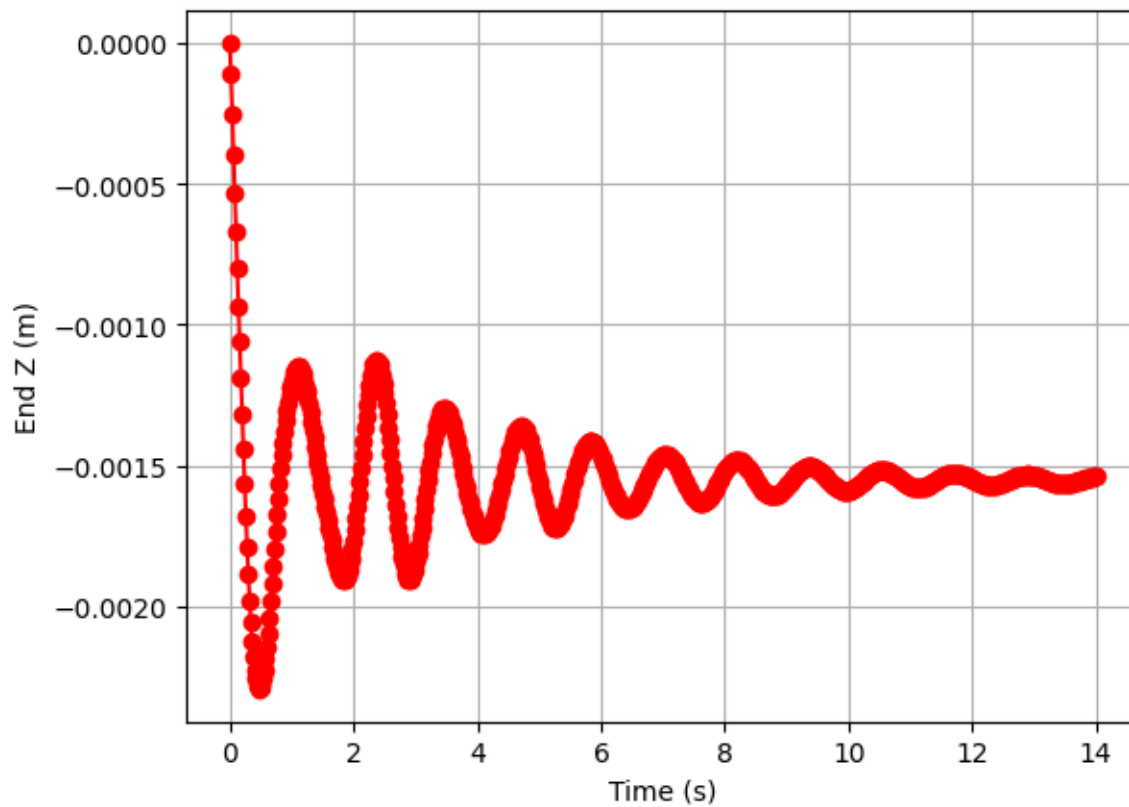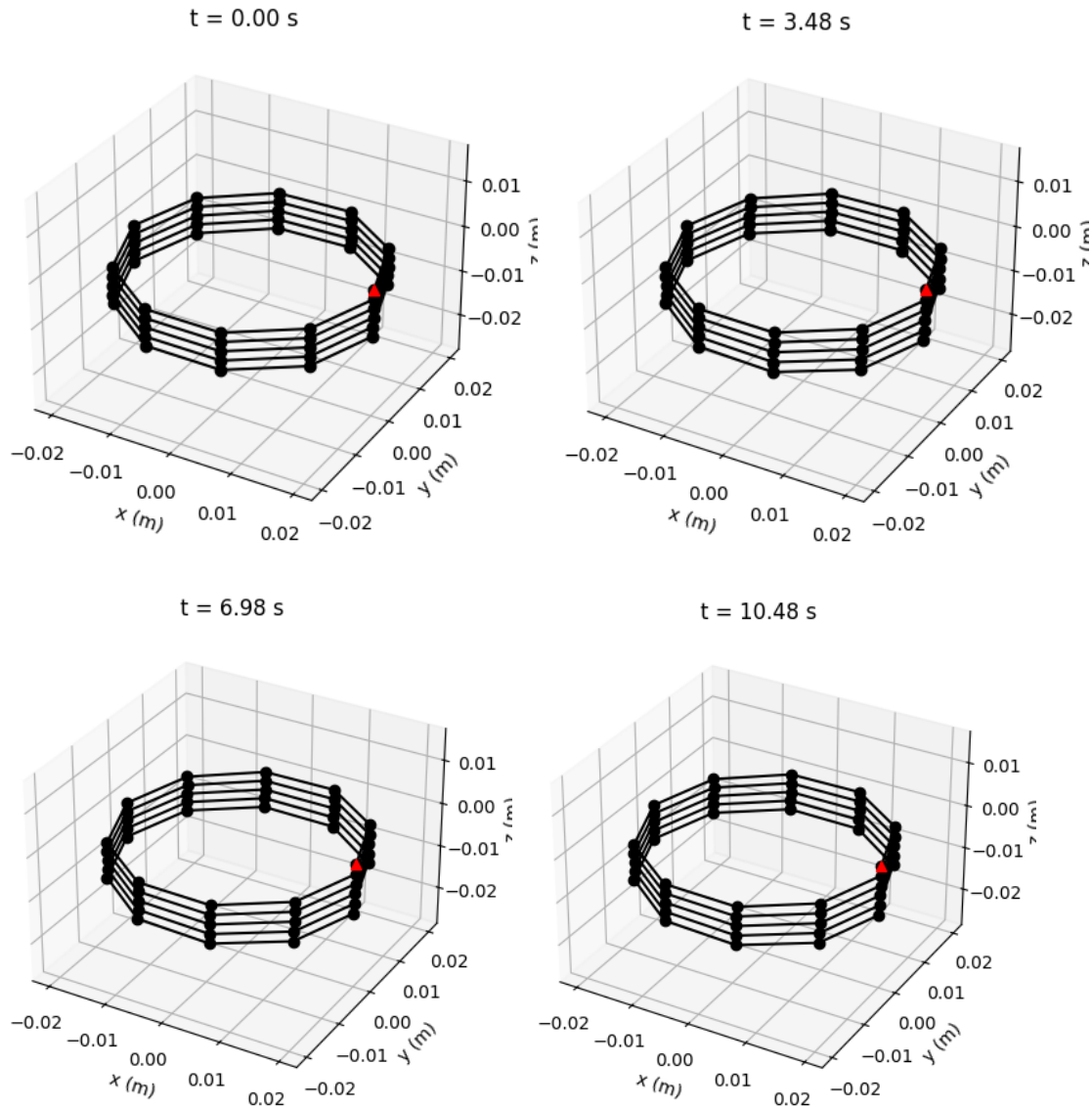$$\delta_z^* = -0.0015523549747709614 \text{ m}$$

Figure 1

Five configurations of the helix at different instants during the simulation are shown in Figures 2(a–e). The snapshots correspond to times $t = 0, 3.48, 6.98, 10.48,$ and 13.98 seconds. All spatial axes have consistent scaling. Over time, the helix extends slightly downward before gradually settling into a static configuration. Because the total deformation is small, visual changes in the coil geometry are subtle.
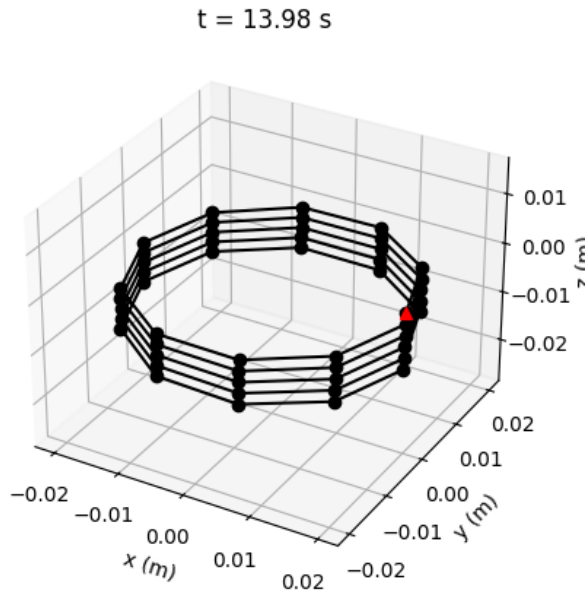
t = 13.98 s

Figure 2

2) F vs. δ∗

z with zero-intercept best-fit line; (3) k vs. Gd4/(8N D3) with the slope-1 reference.

F (N)	$(\delta_z^*) (m)$
1.989e-07	1.5598e-05
5.336e-07	4.1841e-05
1.431e-06	1.1222e-04
3.840e-06	3.0091e-04
1.030e-05	8.0603e-04
2.764e-05	2.1522e-03
7.414e-05	5.6766e-03
1.989e-04	1.4248e-02

Because the system behaves as a linear spring for small deformations, the data were fit to the form:

$$F = k\,\delta_z^*$$

The resulting stiffness value is:

$$k = 0.0127605356877123 \text{ N/m}$$

Figure 3 displays the force–displacement data together with the linear best-fit passing through the origin. The excellent agreement confirms that the system responds linearly within the simulated loading range.
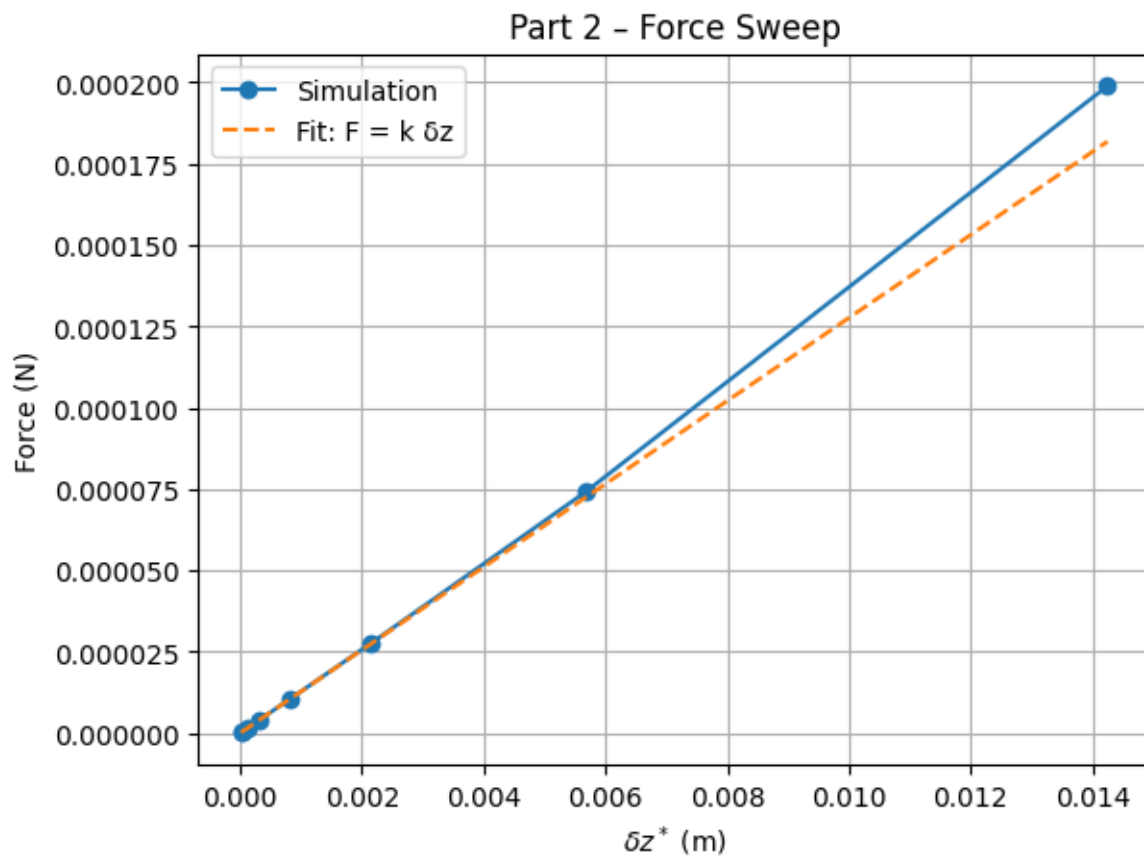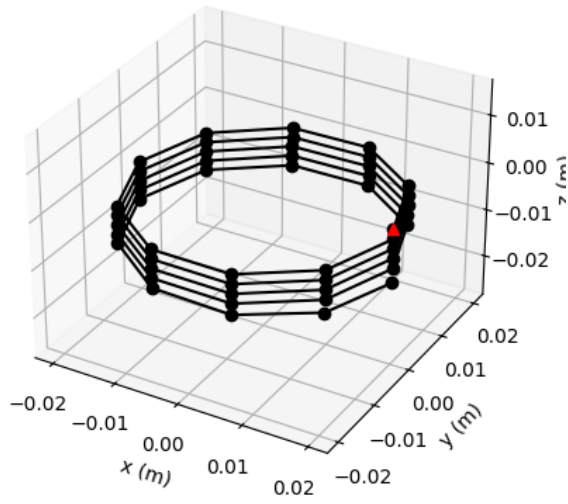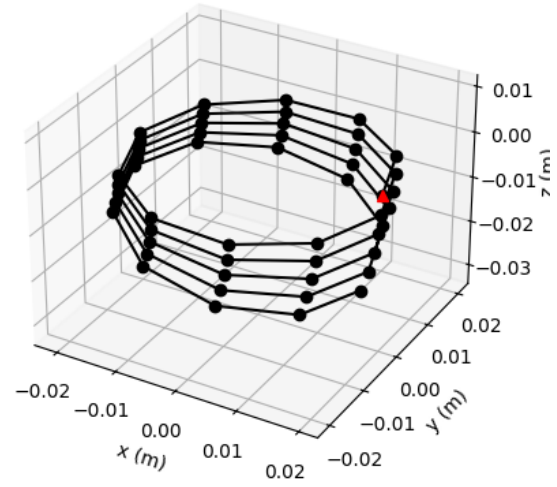

Figure 3

Below, an example of the spring deformation for the largest force $10F_{char}$ is shown:

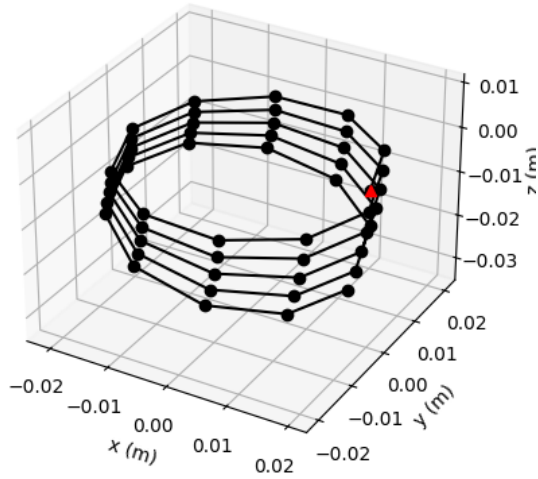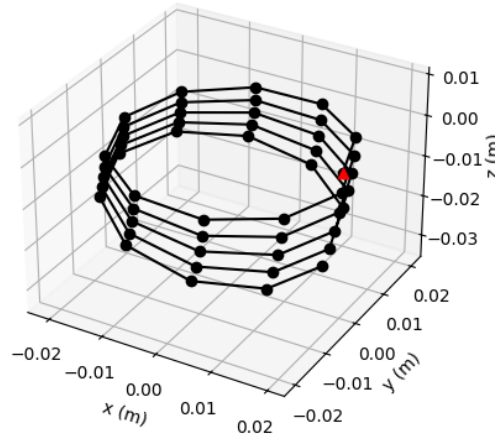F = 1.989e-04 N, t = 0.00 s

F = 1.989e-04 N, t = 3.48 s

F = 1.989e-04 N, t = 6.98 s

F = 1.989e-04 N, t = 10.48 s

F = 1.989e-04 N, t = 13.98 s

Figure 4

Time history for F = 1.989e-04 N
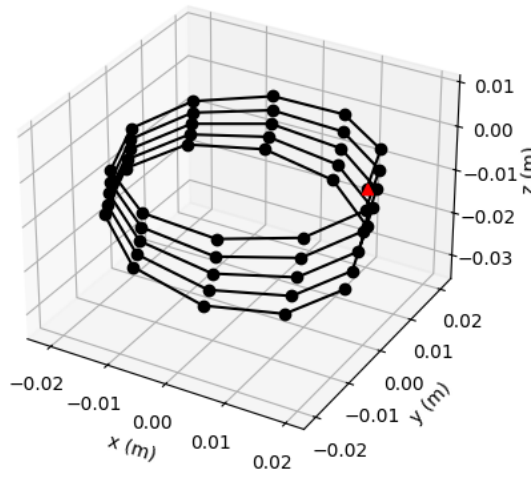

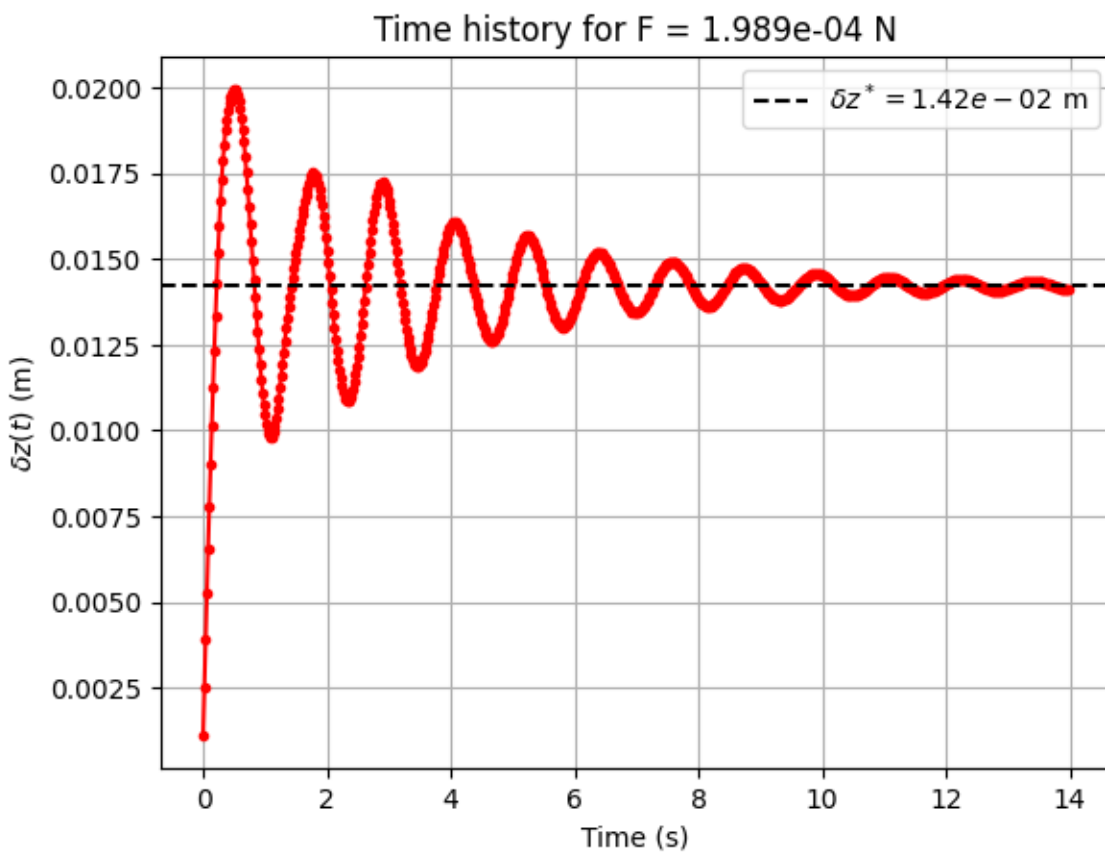Figure 5

3) k vs. Gd4/(8N D3) with the slope-1 reference.

The coil diameter D was varied from 0.01 m to 0.05 m, and the resulting stiffness values computed by the DER model are listed below:

D (m)	k (N/m)
0.010	1.074e+00
0.0144	3.998e-01
0.0189	1.947e-01
0.0233	1.099e-01
0.0278	6.860e-02
0.0322	4.616e-02
0.0367	3.293e-02
0.0411	2.458e-02
0.0456	1.902e-02
0.0500	1.512e-02

The classical close-coil torsional-spring approximation predicts:

$$k_{\text{text}} = \frac{Gd^4}{8ND^3}$$

Figure 6 compares the DER-computed stiffness with the classical expression $Gd^4/(8ND^3)$. The data reveal:

- A strong linear correlation,

- The correct physical scaling: larger D → softer spring,

- A tendency for the DER results to fall below the theoretical slope-1 line for larger diameters.

This deviation is expected because the standard formula assumes:

- negligible pitch,

- small deformations,

- thin wire assumptions,

- loading dominated purely by torsion.

The actual 3D rod experiences bending, pitch effects, and nonuniform strain distributions, all of which reduce the stiffness relative to the idealized prediction. Hence the qualitative agreement is correct and the quantitative differences are consistent with theoretical expectations.
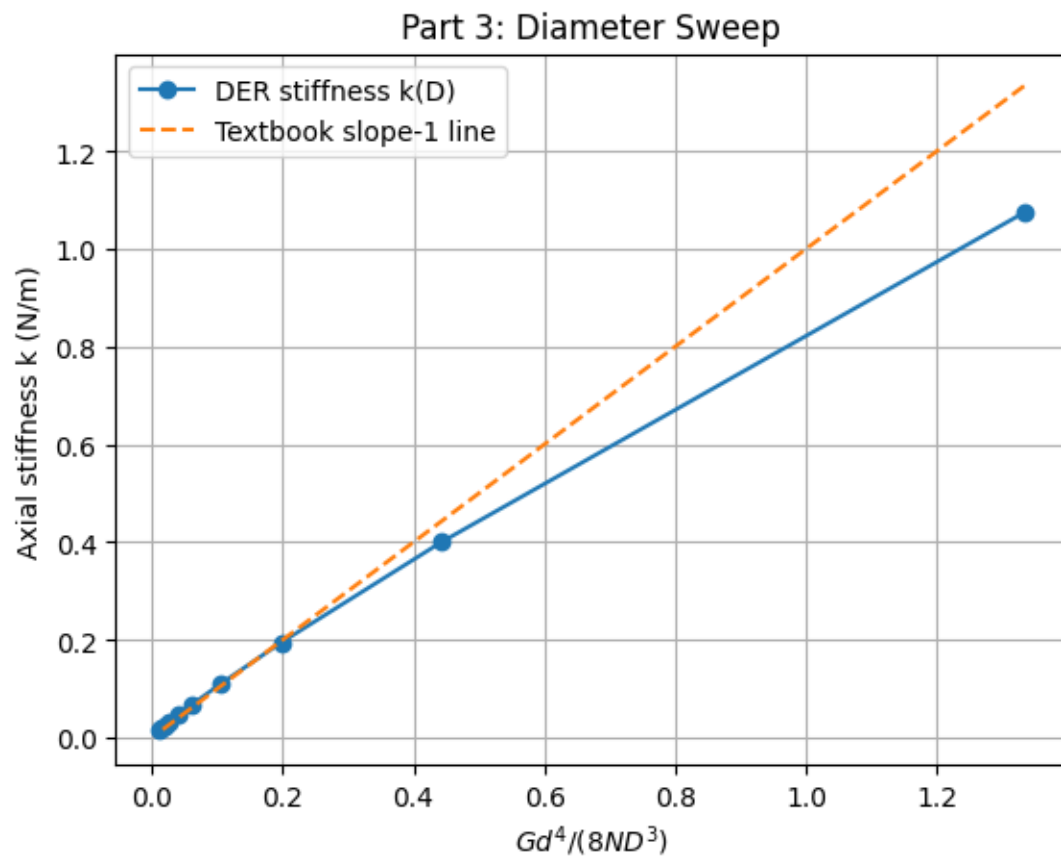

Figure 6