Dynamics of a Bio-Inspired Tail Modeled as an Elastic Beam Across Media, Loadings, and Material Properties

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Abstract*— *This project models a bio-inspired tail as a slender elastic beam to investigate its dynamic response in low Reynolds number fluids under various loadings and material properties. Using a discrete mechanics-based simulator with implicit time integration, we will map how stiffness and actuation frequency influence tail kinematics, energy behavior, and propulsive performance—bridging biomechanics and soft-robotic design.

I. Introduction

Many biological and robotic systems rely on compliant, lightweight tails or flexible appendages to generate propulsion, maneuver, or stabilize their motion. Natural swimmers such as eels, fish, and spermatozoa achieve locomotion through the coordinated propagation of curvature waves along slender filaments. Their efficiency arises from the nonlinear interplay between internal elasticity, active bending patterns, and the surrounding viscous fluid environment—particularly in low Reynolds number regimes where viscous drag dominates inertial forces. Robotic implementations mirror these strategies: compliant tails have been shown to enhance agility, energy efficiency, and robustness in both terrestrial and aquatic robots.

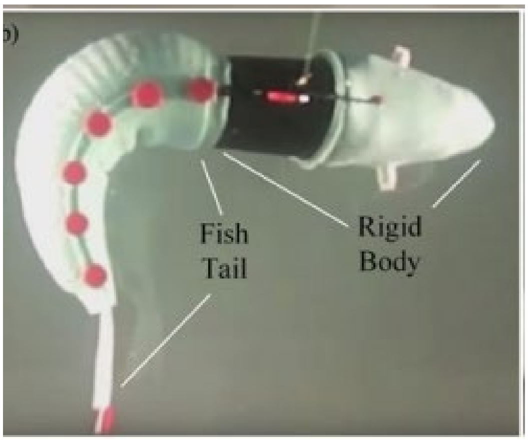

Figure 1. Classical Example of a robotic fish with an elastic tale [2].

Robotic implementations mirror these strategies: compliant tails have been shown to enhance agility, energy efficiency, and robustness in both terrestrial and aquatic robots. Prior research has established essential modeling foundations. McMillen and Holmes [1] developed an elastic-rod framework for anguilliform swimming, capturing how distributed bending interacts with fluid forces. Saab et al. [2] reviewed robotic tail designs, highlighting the benefits of compliant appendages while noting the need for unified, mechanics-based simulation tools. For base-driven artificial fish, Wang et al. [3] analyzed the dynamics of oscillatory actuation and provided validation benchmarks.

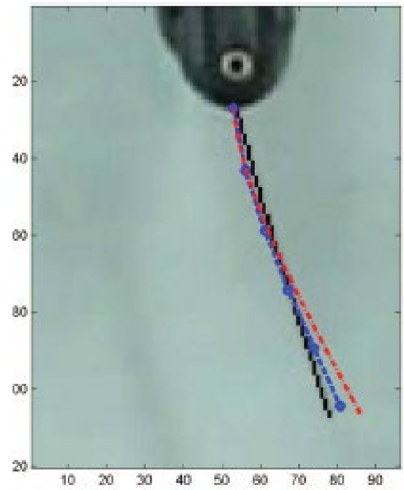

Figure 2. Comparison between experimental measurement of the time-dependent tail shape with model predictions [3].

Classical studies of flagellar motion, such as Rikmenspoel's work on bull spermatozoa [4], identified characteristic amplitude and frequency regimes for low-Re locomotion. Nguyen et al. [5] extended these insights through dynamic modeling of flexible fins with experimental verification.

Building on this prior work, the present project develops a mechanics-grounded simulation framework for a curvature- distributed Stokes drag. The resulting nonlinear equations of motion are solved using fully implicit time integration to ensure numerical stability under large deformations. By systematically varying stiffness, actuation frequency, material parameters, and damping, the simulation aims to identify dynamic patterns, energy behavior, and performance regimes relevant to soft-robotic swimmers and biological filaments.

This midterm report documents the current progress, including a complete implementation of the simulation environment, early performance observations, and a detailed plan for the second half of the project.

II. OBJECTIVES

The overarching objective of this project is to create a unified computational framework capable of:

-Simulating curvature-driven bending across a slender tail with heterogeneous mechanical properties.

-Capturing coupled stretching–bending behavior within a nonlinear elastic-rod formulation.

-Modeling viscous drag at low Reynolds number, appropriate for biological swimmers.

-Achieving stable simulations using a fully implicit time integrator.

-Supporting future extensions, including hydrodynamic force models, cross-media transitions, and adaptive stiffness tuning.

The framework will be used to map how actuation frequency, stiffness gradients, and viscous damping influence tail kinematics, amplitude growth, phase delay, and energy evolution.

III. BACKGROUND AND LITERATURE REVIEW

A. Elastic Rod Modeling

Slender biological and robotic structures are commonly modeled using Cosserat-rod theory or variational rod formulations, such as the Discrete Elastic Rod (DER) method introduced by Bergou et al. These models capture stretching, bending, and torsion with high fidelity and are widely used in biomechanics, graphics, and soft-robotics research.

B. Curvature-Driven Propulsion

Biological swimmers use traveling curvature waves to produce thrust. Lighthill's elongated-body theory and later works by Tytell and Lauder established the governing relationships between wave amplitude, wavelength, and fluid interactions. This idea is central to soft robotic fish, dielectric elastomer swimmers, and magnetically driven artificial flagella.

C. Robotics and Soft Actuation

Robotic tails and flexible propulsion systems often rely on heterogeneous stiffness distributions or internal actuation mechanisms. Saab et al. highlighted the gap between biological smooth curvature propagation and robotic discrete actuation. Modeling a continuously deformable, curvature-programmed system remains a key challenge.

D. Technical Novelty of the Present Work

This project extends prior work by:

-Implementing explicit natural curvature actuation rather than base forcing.

-Incorporating heterogeneous bending stiffness to emulate anatomical variation.

-Embedding actuation and drag into a fully implicit integration scheme for stability.

-Providing a generalizable platform for wave-driven appendages in viscous environments.

IV. METHODS AND MODELING FRAMEWORK

A. Elastic Rod Modeling

The tail is modeled as a 2D discrete elastic rod composed of N=9nodes (18 DOFs). Each segment has rest length

$$\Delta L = \frac{L}{N-1}$$

The generalized coordinate vector

$$q = [x_1, y_1, \ldots, x_N, y_N]^T$$

stores the positions of all nodes.

B. Elastic Energy Terms

Stretching Energy:

Each segment behaves as an axial spring:

$$E_s = \frac{EA}{2}(l - l_0)^2,$$

Gradients and Hessians of the stretching energy are implemented analytically for efficient Newton iteration.

Bending Energy with Natural Curvature:

Local bending energy at node k is:

$$E_b = \frac{EI}{2}\ell_k(\kappa_k - \kappa_0(s,t))^2.$$

where:

- κ_k is discrete curvature,
- κ0(s,t)is the prescribed natural curvature,
- EI is the bending modulus.

A heterogeneous stiffness distribution is used:

$$EI_{\text{eff}}(s) = EI(1 + 4\xi(s)^2), \xi = s/L.$$

increasing stiffness toward the distal end.

Full analytic gradients and Hessians are implemented.

C. Curvature-Based Actuation

The tail is actuated by a traveling curvature wave:
$\kappa_0(s,t) = \kappa_{\text{amp}}\ \xi^2\ \sin(\omega t - k_{\text{wave}} s)$,

where:

- amplitude $\kappa_{\text{amp}} = 1.5\ \text{m}^{-1}$
- frequency $f = 2$ Hz
- wave number $k_{\text{wave}} = 2\pi/L$

The envelope ξ^2 mimics biologically realistic amplitude growth along the tail length.

D. Curvature-Based Actuation

To represent motion in a viscous medium (e.g., water at low Reynolds number), node-wise Stokes drag is applied:

$$F_v = -C\frac{(q - q_{\text{old}})}{dt}.$$

Gravity is disabled to emulate neutrally buoyant behavior.

E. Implicit Time Integration

The equations of motion are solved using Newton–Raphson iteration.

residual:

$$f = F_{\text{inertia}} - F_{\text{elastic}} - F_v - W.$$

Jacobian:

$$J = J_{\text{inertia}} - J_{\text{elastic}} - J_v.$$

Solve:

$$\Delta q = J^{-1} f.$$

This yields stability at small time steps ($dt = 0.005$ s) even under large curvature.

V. Progress and Preliminary Results

A. Curvature-Based Actuation

All major components of the simulation framework are now fully implemented, including discrete geometry generation, stretching and bending energy formulation, analytic gradients and Hessians, curvature-based actuation, viscous drag modeling, and a fully implicit Newton–Raphson time integrator. Plotting and animation tools, along with tip and mid-node tracking, have also been incorporated. Together, these elements produce smooth, tail-like traveling waves throughout the simulation.

B. Observed Tail Motion

The prescribed curvature wave generates a clean and coherent traveling deformation along the tail. Mid-tail nodes oscillate with moderate amplitude, while the tip exhibits the largest displacement—a behavior consistent with biological systems and classical beam-theory predictions. The spatial envelope ξ^2 appropriately modulates the amplitude along the length, and the resulting curvature propagation remains stable, periodic, and symmetric. Overall, the motion closely resembles the kinematics of flexible fish tails and flagellar filaments.

C. Stability and Numerical Behavior

The implicit solver consistently converges within three to six Newton iterations per time step, indicating strong numerical stability. Increasing viscous damping suppresses high-frequency oscillations and generates smoother waveforms, whereas reducing damping leads to richer, more oscillatory dynamics. Additionally, raising the heterogeneous stiffness toward the tip diminishes local curvature amplitude, confirming the correct implementation of the spatially varying bending modulus EI(s).

D. Visual Output

The simulation currently generates animated tail motion, real-time deformation plots, and time histories of both the mid-node and tip-node displacements. It also produces curvature distribution snapshots that illustrate wave propagation along the rod. These visualizations will form the foundation for the final report figures and the project presentation video.

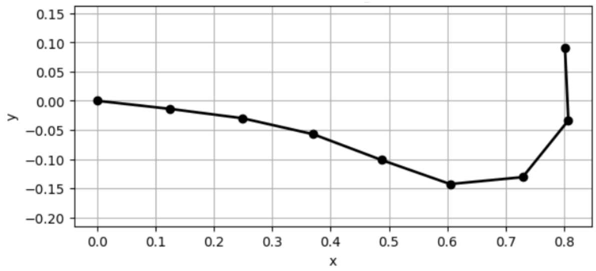

Figure 3. A snapshot of the current animated tail.

VI. PLANNED NEXT STEPS

A. Multi-Media Behavior:

- Introduce drag coefficients for air, water, viscous gels.
- Simulate transitions between media.

B. Parameter Sweeps:

- Vary:
  - bending modulus EI, taper ratio
  - curvature amplitude and frequency
  - viscosity & damping
- Observe changes in amplitude growth, phase delay, stability, and energy cost.

C. Hydrodynamic Force Models (Stretch Goal):

- Implement Resistive Force Theory (RFT) or slender-body approximations.
- Estimate thrust generation and forward swimming efficiency.

D. Final Report Development

- Produce refined figures, animations, and energy plots.
- Add biological comparisons and full citations.
- Provide quantitative metrics (e.g., amplitude ratios, power input, stability maps).

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