Firmin Doko Tchatoka, School of Economics (UoA)

ECON 7204: Problem set 3 (due date: 19 May 2022). *Return your main file and the code file in the appropriate dropbox on MyUni.*

Question 1 (IV estimation– 60 marks).

Consider the following linear IV regression model

perfect multicol? there are 2 instruments

need to regenerate?

$$y_t = \beta_0 + x_t\beta + u_t,\; t = 1, 2, \ldots, T \tag{1}$$
$$x_t = \pi_0 + z_{1t}\pi_1 + z_{2t}\pi_2 + v_t,\; t = 1, \ldots, T, \tag{2}$$

homo, auto

where $(z_{1t}, z_{2t}, u_t, v_t)' \overset{i.i.d.}{\sim} N\Big(0, \Omega\Big)$, and $\Omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.8 \\ 0 & 0 & 0.8 & 1 \end{bmatrix}$ for all t. The true values of β_0, β and π_0 are set at $\beta_0 = \beta = \pi_0 = 1$. Let $x_1 = ones(T, 1)$ (a T-dimensional column vector of ones) and define $Z = [x_1\ z_1\ z_2]$ (a $T \times 3$ matrix of instruments). The OLS and 2SLS estimators of β are then given by $\hat{\beta} = \big(x'M_{x_1}x\big)^{-1}x'M_{x_1}y$, and $\tilde{\beta} = \big(x'(M_{x_1} - M_Z)x\big)^{-1}x'(M_{x_1} - M_Z)y$, where $M_{x_1} = I - x_1(x_1'x_1)^{-1}x_1'$ and $M_Z = I - Z(Z'Z)^{-1}Z'$.

1. Is x_t exogenous in (1)-(2)? Why? ?
2. Are z_{1t} and z_{1t} valid IVs in (1)-(2)? Why? z_{1t} & z_{2t}
3. Suppose first that $\pi_1 = 5$, $\pi_2 = 10$ (strong IVs). For sample sizes $T = 50$ and $T = 500$:
   (a) Generate $r = 1, 2, \ldots R = 1000$ pseudo samples of size T for y and x using the above information. For each pseudo sample, compute the OLS and 2SLS estimators, $\hat{\beta}$ and $\tilde{\beta}$, of β; their corresponding t-statistics, t_β and $\tilde{t}_\beta$, for $H_0 : \beta = 1$, as well as the Sargan's overidentifying restrictions statistic J_S. You will form a collection of $R = 1000$ estimators; $\hat{\beta}^{(1)}, \hat{\beta}^{(2)}, \ldots, \hat{\beta}^{(R)}$ and $\tilde{\beta}^{(1)}, \tilde{\beta}^{(2)}, \ldots, \tilde{\beta}^{(R)}$, $R = 1000$ t-statistics; $t_\beta^{(1)}, t_\beta^{(2)}, \ldots, t_\beta^{(R)}$ and $\tilde{t}_\beta^{(1)}, \tilde{t}_\beta^{(2)}, \ldots, \tilde{t}_\beta^{(R)}$, and $R = 1000$ J_S-statistics; $J_S^{(1)}, J_S^{(2)}, \ldots, J_S^{(R)}$.
   Hint: For the J_S statistic, define $x_1 = ones(T, 1)$ (a T-dimensional column vector of ones) and $Z = [x_1\ z_1\ z_2] : T \times 3$. Then, J_S has the form $J_S = T g_T' W_T g_T$, where $W_T = \big(\frac{z'M_{x_1}z}{T}\big)^{-1}$, $g_T = \frac{1}{T}z'M_{x_1}(y - x\tilde{\beta})$, and $M_{x_1} = I - x_1(x_1'x_1)^{-1}x_1'$; so as $J_S \xrightarrow{d} \chi^2(1)$ when the exclusion restrictions hold.
   (b) Plot the histograms of the $R = 1000$ estimates of: $\hat{\beta}$ and $\tilde{\beta}$, t_β and $\tilde{t}_\beta$, and J_S. Comment on the results.
   (c) Compute the estimated bias as $\widehat{Bias}(\beta) = \frac{1}{R}\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta) = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^{(r)} - \beta$ and $\widetilde{Bias}(\beta) = \frac{1}{R}\sum_{r=1}^{R}(\tilde{\beta}^{(r)} - \beta) = \frac{1}{R}\sum_{r=1}^{R}\tilde{\beta}^{(r)} - \beta$. Comment on the results.

(d) Compute the rejection frequencies of the t-test and J_S-test at 5% level, i.e., the quantity $Re_{OLS} = \frac{1}{R}\sum_{r=1}^{R} \mathbb{1}[|t_{\beta}^{(r)}| > 1.96]$, $Re_{2SLS} = \frac{1}{R}\sum_{r=1}^{R} \mathbb{1}[|\tilde{t}_{\beta}^{(r)}| > 1.96]$, $Re_{J_S} = \frac{1}{R}\sum_{r=1}^{R} \mathbb{1}[J_S^{(r)} > 3.84]$, where $\mathbb{1}[C] = 1$ if condition C holds and $\mathbb{1}[C] = 0$ otherwise. Comment on the results.

4. Suppose that $\pi_1 = 0.01$ and $\pi_2 = 10$ (i.e., z_1 is weak but z_2 is strong). For sample sizes $T = 50$ and $T = 500$, repeat (a)-(d) of question 3. Conclude.

5. Suppose that $\pi_1 = \pi_2 = 0.01$ (both z_1 and z_2 are weak). For sample sizes $T = 50$ and $T = 500$, repeat (a)-(d) of question 3. Conclude.

6. Suppose now that $\pi_1 = \pi_2 = 0$ (both z_1 and z_2 are irrelevant). For sample sizes $T = 50$ and $T = 500$, repeat (a)-(d) of question 3. Conclude.

QUESTION 2 (GMM estimation– 40 marks).

Consider the following population moment condition resulting from the consumption based asset pricing model:

$\theta = \begin{bmatrix} \gamma \\ \delta \end{bmatrix}$

$g(x_{1t}, x_{2t}, z_t, \theta) = f_t(\theta) z_t$
$f_t(\theta) = \delta x_{1t}^{\gamma-1} x_{2t} - 1$

$$\boxed{\mathbb{E}[g(x_{1t}, x_{2t}, z_t, \theta)] = 0, \quad t = 1, \ldots, T,} \tag{3}$$

where $\theta = (\gamma, \delta)'$, $g(x_{1t}, x_{2t}, z_t, \theta) = f_t(\theta) \times z_t$, $f_t(\theta) = \delta x_{1t}^{\gamma-1} x_{2t} - 1$, $x_{1t} = c_{t+1}/c_t$, $x_{2t} = r_{t+1}/p_t$, $z_t = (1, x_{1,t-1}, x_{1,t-2}, x_{2,t-1}, x_{2,t-2})'$, c_t is aggregate per capita consumption, and r_{t+1}, p_t are respectively the gain in period $t+1$ and the price in period t on a **NYSE index**. The data is monthly and the sample period is 1960.1-1997.12. You are tasked to estimate $\theta = (\gamma, \delta)'$ using alternative GMM methods. The files *cbapmewrdata.dat* and *cbapmewrinstr.dat* contain data on (x_{1t}, x_{2t}) and z_t, respectively.

1. *Efficient Two-Step GMM Estimation*

   (a) Write a code that computes the efficient two-step GMM estimates of γ and δ, as well as their resulting standard error estimates.

   (b) Using the above data sets and your code, find the efficient two-step GMM estimates of γ and δ, and their standard error estimates. Comment on the results.

   (c) Are the instruments in z satisfy the exclusion restrictions? Justify.

2. *Iterative GMM Estimation*

   (a) Write a code that computes the iterative GMM estimates of γ and δ, as well as their resulting standard error estimates.

   (b) Using the above data sets and your code, find the efficient iterative GMM estimates of γ and δ, and their standard error estimates (*NB: initialize θ at the efficient two-step GMM estimate of the previous question*). Compare your results with the efficient two-step GMM ones.

$f_t(\theta) = \delta \left(\frac{c_{t+1}}{c_t}\right)^{\gamma-1} \left(\frac{r_{t+1}}{p_t}\right) - 1$

(c) Are the instruments in z satisfy the exclusion restrictions? Why?

3. Conclude.

$$g(x_{1t}, x_{2t}, z_t, \theta) = \left(\delta\, x_{1t}^{\gamma-1} x_{2t} - 1\right) \times z_t$$

$$\theta = \begin{bmatrix} \gamma \\ \delta \end{bmatrix}$$

$$x_{1t} = \frac{c_{t+1}}{c_t}$$

$$x_{2t} = \frac{r_{t+1}}{p_t}$$

$$z_t = \left(1, x_{1,t-1}, x_{1,t-1}, x_{2,t-1}, x_{2,t-2}\right)'$$