KEY POINTS :

- Kinetic energy per unit volume $E_v = \frac{1}{2}\left(\frac{mN}{V}\right)v_{rms}^2 = \frac{3}{2}P$
- At absolute zero, the motion of all molecules of the gas stops.
- At higher temperature and low pressure or at higher temperature and low density, a real gas behaves as an ideal gas.
- **For any general process**

  (a) Internal energy change $\Delta U = nC_v dT$

  (b) Heat supplied to a gas $\Delta Q = nCdT$

  where C for any polytropic process PV^x = constant is $C = C_V + \frac{R}{1-x}$

  (c) Work done for any process $\Delta W = P\Delta V$

  It can be calculated as area under P-V curve

  (d) Work done $= \Delta Q - \Delta U = \frac{nR}{1-x}dT$

  For any polytropic process PV^x = constant

THERMODYNAMICS

- **Zeroth law of thermodynamics :** If two systems are each in thermal equilibrium with a third, they are also in thermal equilibrium with each other.

- **First law of thermodynamics :** Heat supplied (Q) to a system is equal to algebraic sum of change in internal energy (ΔU) of the system and mechanical work (W) done by the system

  $Q = W + \Delta U$ [Here $W = \int PdV$; $\Delta U = nC_v\Delta T$]

  For differential change $\boxed{\underbrace{\delta Q = \delta W}_{\text{path dependent}} + \underbrace{dU}_{\text{path independent}}}$

- **Sign Convention**

  Heat absorbed by the system $\rightarrow$positive

  Heat rejected by the system $\rightarrow$ negative

  Increase in internal energy (i.e. rise in temperature)$\rightarrow$positive

  Decrease in internal energy (i.e. fall in temperature)$\rightarrow$ negative

  Work done by the system $\rightarrow$positive

  Work done on the system $\rightarrow$negative

- **For cyclic process** $\Delta U = 0 \Rightarrow Q = W$

- **For isochoric process** V = constant $\Rightarrow P \propto T$ & $W = 0$
  $Q = \Delta U = \mu C_V \Delta T$

- **For isobaric process** P = constant $\Rightarrow V \propto T$
  $Q = \mu C_p \Delta T$, $\Delta U = \mu C_V \Delta T$
  $W = P(V_2 - V_1) = \mu R \Delta T$

- **For adiabatic process** PV^γ = constant
  or $T^\gamma P^{1-\gamma}$ = constant
  or $TV^{\gamma-1}$ = constant
  In this process Q = 0 and

  $$W = -\Delta U = \mu C_V (T_1 - T_2) = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

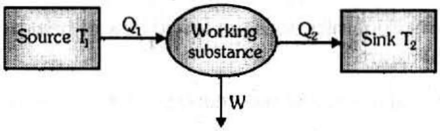
- **For Isothermal Process** T = constant
  or $\Delta T = 0 \Rightarrow PV$ = constant
  In this process $\Delta U = \mu C_V \Delta T = 0$

  $$\text{So, } Q = W = \mu RT\,\ell n\left(\frac{V_2}{V_1}\right) = \mu RT\ell n\left(\frac{P_1}{P_2}\right)$$

- **For any general polytropic process** PV^x = constant
  - Molar heat capacity $C = C_V + \dfrac{R}{1-x}$
  - Work done by gas $W = \dfrac{nR(T_1 - T_2)}{x-1} = \dfrac{(P_1V_1 - P_2V_2)}{x-1}$
  - Slope of P-V diagram

    (also known as indicator diagram at any point $\dfrac{dP}{dV} = -x\dfrac{P}{V}$)

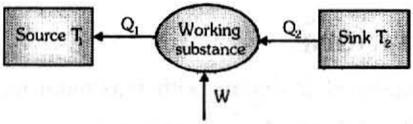
Efficiency of a cycle



$$\eta = \frac{\text{Work done by working substance}}{\text{Heat supplied}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

For carnot cycle $\quad \dfrac{Q_2}{Q_1} = \dfrac{T_2}{T_1}$ so $\eta = 1 - \dfrac{Q_2}{Q_1} = 1 - \dfrac{T_2}{T_1}$

For refrigerator



$$\text{Coefficient of performance } \beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

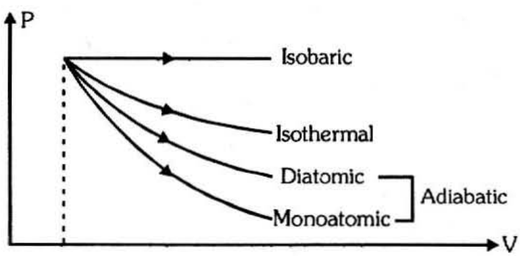
Bulk modulus of gases : $\quad B = \dfrac{\Delta P}{-\dfrac{\Delta V}{V}}$

Isothermal bulk modulus of elasticity, $B_{IT} = -V\left(\dfrac{\partial P}{\partial V}\right)_{T=\text{constant}}$

Adiabatic bulk modulus of elasticity, $B_{AD} = -\gamma V\left(\dfrac{\partial P}{\partial V}\right) \Rightarrow B_{AD} = \gamma B_{IT}$

KEY POINTS

- Work done is least for monoatomic gas (adiabatic process) in shown expansion.



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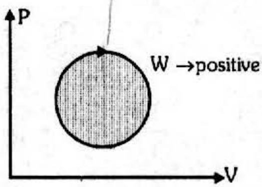
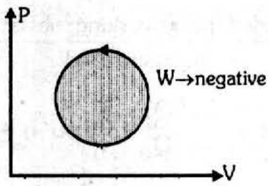
- Air quickly leaking out of a balloon becomes cooler as the leaking air undergoes adiabatic expansion.
- First law of thermodynamics does not forbid flow of heat from lower temperature to higher temperature.
- First law of thermodynamics allows many processes which actually don't happen.
- 

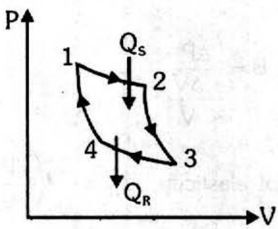


- **CARNOT ENGINE**

  It is a hypothetical engine with maximum possible efficiency

  Process 1→2 & 3→4 are isothermal

  Process 2→3 & 4→1 are adiabatic.



$$\eta = 1 - \frac{T_{3-4}}{T_{1-2}}$$

Important Notes

13 Oscillations

SIMPLE HARMONIC MOTIONS

- **Periodic Motion**

  Any motion which repeats itself after regular interval of time (i.e. time period) is called periodic motion or harmonic motion.

  Ex. (i) Motion of planets around the sun.

  (ii) Motion of the pendulum of wall clock.

- **Oscillatory Motion**

  The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.

  The fixed point about which the body oscillates is called mean position or equilibrium position.

  Ex.: (i) Vibration of the wire of 'Sitar'.

  (ii) Oscillation of the mass suspended from spring.

  Note : Every oscillatory motion is periodic but every periodic motion is not oscillatory.

- **Simple Harmonic Motion (S.H.M.)**

  Simple harmonic motion is the simplest form of vibratory or oscillatory motion.

- **Some Basic Terms in SHM**

  - **Mean Position**

    The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

  - **Restoring Force**

    The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.

    Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.

  - **Amplitude**

    The maximum (positive or negative) value of displacement of particle from mean position is defined as amplitude.

  - **Time period (T)**

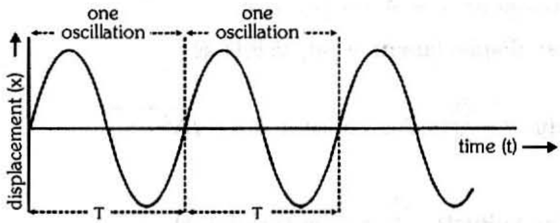
The minimum time after which the particle keeps on repeating its motion is known as time period.

The smallest time taken to complete one oscillation or vibration is also defined as time period.

It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.

- **One oscillation or One vibration**

When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



- **Frequency (n or f)**

The number of oscillations per second is defined as frequency.

It is given by $n = \frac{1}{T} = \frac{\omega}{2\pi}$

- **Phase**

Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

In the equation x = A sin (ωt + ϕ), (ωt+ϕ) is the phase of the particle.

The phase angle at time t = 0 is known as initial phase or epoch.

The difference of total phase angles of two particles executing SHM with respect to the mean position is known as phase difference.

Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e. $\Delta\phi = 2n\pi$ where n = 0, 1, 2, 3,....

Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ where n = 0, 1, 2, 3,....

- **Angular frequency (ω)** :The rate of change of phase angle of a particle with respect to time is defined as its angular frequency. $\omega = \sqrt{\frac{k}{m}}$

- **For linear SHM** $(F \propto -x)$: $F = m\frac{d^2x}{dt^2} = -kx = -m\omega^2 x$ where $\omega = \sqrt{\frac{k}{m}}$

- **For angular SHM** $(\tau \propto -\theta)$: $\tau = I\frac{d^2\theta}{dt^2} = I\alpha = -k\theta = -m\omega^2\theta$ where $\omega = \sqrt{\frac{k}{m}}$

- **Displacement** $x = A\sin(\omega t + \phi)$,

- **Angular displacement** $\theta = \theta_0 \sin(\omega t + \phi)$

- **Velocity** $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$

- **Angular velocity** $\frac{d\theta}{dt} = \theta_0\,\omega\cos(\omega t + \phi)$

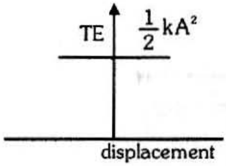
- **Acceleration** $a = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 x$

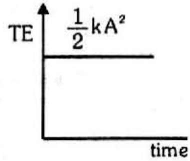
- **Angular acceleration** $\frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t + \phi) = -\omega^2\theta$

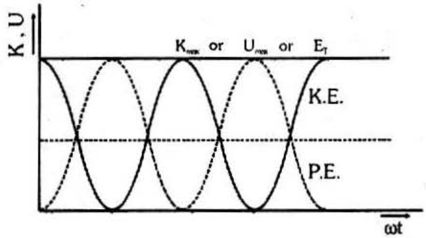
- **Kinetic energy** $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi)$

- **Potential energy** $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi)$

- **Total energy** $E = K + U = \frac{1}{2}m\omega^2 A^2 = \text{constant}$







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

- **Average energy in SHM**

(i) The time average of P.E. and K.E. over one cycle is

(a) $\langle K\rangle_t = \frac{1}{4}kA^2$ (b) $\langle PE\rangle_t = \frac{1}{4}kA^2$ (c) $\langle TE\rangle_t = \frac{1}{2}kA^2 + U_0$

(ii) The position average of P.E. and K.E. between x = – A to x=A

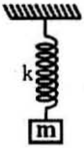
(a) $\langle K\rangle_x = \frac{1}{3}kA^2$ (b) $\langle PE\rangle_x = U_0 + \frac{1}{6}kA^2$ (c) $\langle TE\rangle_x = \frac{1}{2}kA^2 + U_0$

- **Differential equation of SHM**

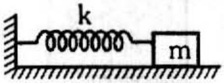
- Linear SHM: $\frac{d^2x}{dt^2} + \omega^2 x = 0$

- Angular SHM: $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

- **Spring block system**



$T = 2\pi\sqrt{\frac{m}{k}}$



$T = 2\pi\sqrt{\frac{m}{k}}$

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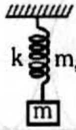
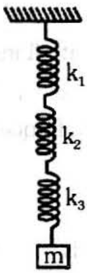
$T = 2\pi\sqrt{\frac{\mu}{k}}$ where μ=reduced mass $= \frac{m_1 m_2}{m_1 + m_2}$

- When spring mass is not negligible :



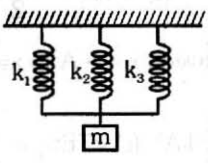
$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$

- **Series combination of springs**



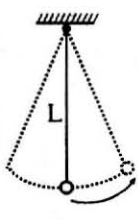
$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ where $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$

- **Parallel combination of springs**

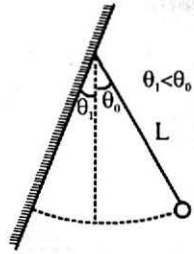


$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ where $k_{eff} = k_1 + k_2 + k_3$

- **Time period of simple pendulum**



Time period $T = 2\pi\sqrt{\frac{L}{g}}$



Time period $T=\left\{\pi+2\sin^{-1}\left(\frac{\theta_1}{\theta_0}\right)\right\}\sqrt{\frac{L}{g}}$

If length of simple pendulum is comparable to the radius of the earth R, then

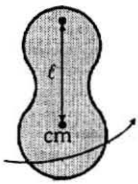
$T=2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell}+\frac{1}{R}\right)}}$. If ℓ << R then $T=2\pi\sqrt{\frac{\ell}{g}}$

If ℓ >> R then $T=2\pi\sqrt{\frac{R}{g}}\approx$ 84 minutes

- **Second pendulum**

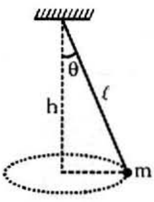
  Time period = 2 seconds, Length ≈ 1 meter (on earth's surface)

- **Time period of Physical pendulum**



$T=2\pi\sqrt{\frac{I}{mg\ell}}=2\pi\sqrt{\frac{\frac{k^2}{\ell}+\ell}{g}}$ where $I_{cm}=mk^2$

- **Time period of Conical pendulum**



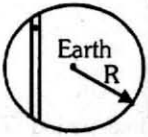
$T=2\pi\sqrt{\frac{\ell\cos\theta}{g}}=2\pi\sqrt{\frac{h}{g}}$

- **Time period of Torsional pendulum** $T=2\pi\sqrt{\frac{I}{k}}$

  where k = torsional constant of the wire

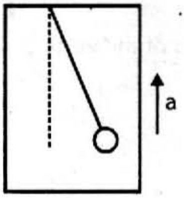
  I=moment of inertia of the body about the vertical axis

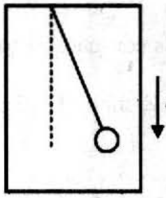
- **SHM of a particle in a tunnel inside the earth.**

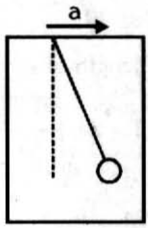


$$T = 2\pi\sqrt{\frac{R}{g}}$$

In accelerating cage







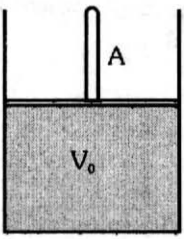
$g_{eff} = g + a$ $\quad$ $g_{eff} = g - a$ $\quad$ $g_{eff} = \sqrt{g^2 + a^2}$

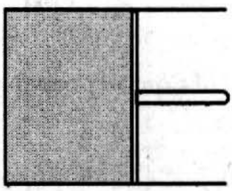
$$T = 2\pi\sqrt{\frac{\ell}{g+a}} \qquad T = 2\pi\sqrt{\frac{\ell}{g-a}} \qquad T = 2\pi\sqrt{\frac{\ell}{(g^2+a^2)^{1/2}}}$$

- **SHM of gas-piston system**

Here elastic force is developed due to bulk elasticity of the gas

$$B = \frac{\Delta P}{-\Delta V / V} \Rightarrow F = -\frac{BA^2}{V_0}x \Rightarrow T = 2\pi\sqrt{\frac{m}{BA^2 / V_0}}$$





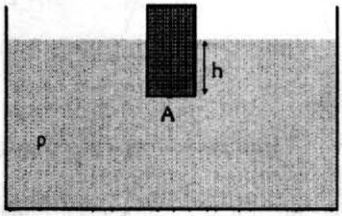
- **SHM of Floating Body**

Restoring force $\rightarrow$ Thurst

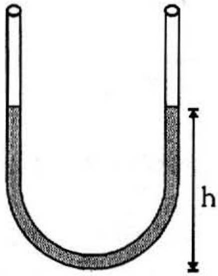
mg =ρAhg $\rightarrow$ Equilibrium

Restoring force F= $-$ (ρAg)x

$$T = 2\pi\sqrt{\frac{h}{g}}$$



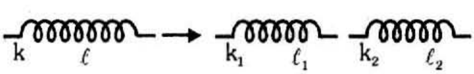
- **SHM in U-tube**



$$T = 2\pi\sqrt{\frac{h}{g}}$$

KEY POINTS

- SHM is the projection of uniform circular motion along one of the diameters of the circle.
- The periodic time of a hard spring is less as compared to that of a soft spring because the spring constant is large for hard spring.
- For a system executing SHM, the mechanical energy remains constant.
- Maximum kinetic energy of a particle in SHM may be greater than mechanical energy as potential energy of a system may be negative.
- The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
- **Spring cut into two parts :**



Here $\frac{\ell_1}{\ell_2} = \frac{m}{n}$

$$\ell_1 = \left(\frac{m}{m+n}\right)\ell, \quad \ell_2 = \left(\frac{n}{m+n}\right)\ell \qquad \text{But } k\ell = k_1\ell_1 = k_2\ell_2$$

$$\Rightarrow k_1 = \frac{(m+n)}{m}k; \quad k_2 = \frac{(m+n)}{n}k$$

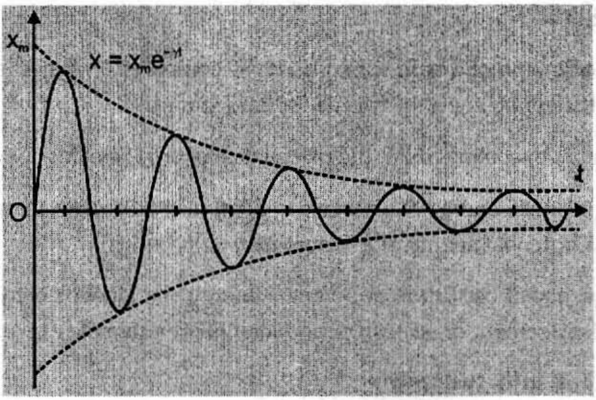
FREE, DAMPED, FORCED OSCILLATIONS AND RESONANCE

Free oscillation

- The oscillations of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations.

Damped oscillations

- The oscillations of a body whose amplitude goes on decreasing with time are defined as damped oscillations.
- In these oscillations the amplitude of oscillations decreases exponentially due to damping forces like frictional force, viscous force etc.
- If initial amplitude is X_m then amplitude after time t will be $x = x_m e^{-\gamma t}$ where γ = Damping coefficient



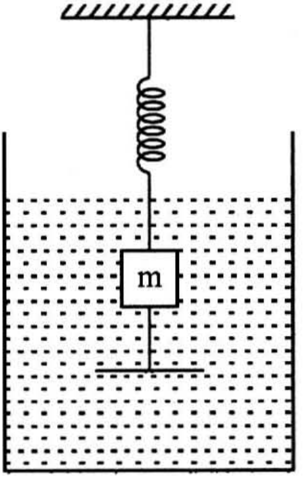
FORCED OSCILLATIONS

- The oscillations in which a body oscillates under the influence of an external periodic force (driver) are known as forced oscillations.
- The driven body does not oscillate with its natural frequency rather it oscillates with the frequency of the driver.
- The amplitude of oscillator decreases due to damping forces but on account of the energy gained from the external source (driver) it remains constant.

RESONANCE

- When the frequency of external force (driver) is equal to the natural frequency of the oscillator (driven), then this state of the driver and the driven is known as the state of resonance.

- In the state of resonance, there occurs maximum transfer of energy from the driver to the driven. Hence the amplitude of motion becomes maximum.
- In the state of resonance the frequency of the driver (ω) is known as the resonant frequency.



Damped Oscillations :

Damping force $F_d = -bv$

where v = velocity,

b = damping constant

Restoring force on block F = –kx

So net force on block

$$F_{net} = -kx - bv$$

$$\Rightarrow \quad ma = -kx - bv$$

$$\Rightarrow \quad \boxed{m\frac{d^2x}{dt^2} + kx + bv = 0}$$

It is the differential equation of damped oscillation.

Solution of this equation is given by

$$\boxed{x = A_0 e^{\left(-\frac{bt}{2m}\right)} \sin(\omega' t + \phi)}$$

where $A(t) = A_0 e^{\left(-\frac{bt}{2m}\right)}$

$$\Rightarrow \quad A(t) = A_0 e^{-\gamma t}$$

so $\boxed{\gamma = \frac{b}{2m}}$ and $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Energy in damped oscillation

$$E(t) = \frac{1}{2}kA^2(t) = \frac{1}{2}k\left[A_0 e^{-\frac{bt}{2m}}\right]^2$$

$$\Rightarrow \quad E(t) = \frac{1}{2}kA_0^2 e^{(-bt/m)} \Rightarrow \boxed{E(t) = E_0 e^{(-bt/m)}}$$

Important Notes

14 Wave Motion and Doppler's Effect

A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

CLASSIFICATION OF WAVES

- **Medium Necessity**
  - Mechanical (Elastic) waves
  - Non-mechanical waves (EM waves)
- **Propagation of energy**
  - Progressive waves
  - Stationary (standing) waves
- **Vibration of medium particle**
  - Transverse waves
  - Longitudinal waves
- **Dimension**
  - 1-D (Waves on strings)
  - 2-D (Surface waves or ripples on water)
  - 3-D (Sound or light waves)

- A mechanical wave will be transverse or longitudinal depending on the nature of medium and mode of excitation.
- In strings, mechanical waves are always transverse.
- In gases and liquids, mechanical waves are always longitudinal because fluids cannot sustain shear.
- Partially transverse waves are possible on a liquid surface because surface tension provide some rigidity on a liquid surface. These waves are called as ripples as they are combination of transverse & longitudinal.
- In solids mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation.
- In longitudinal wave motion, oscillatory motion of the medium particles produce regions of compression (high pressure) and rarefaction (low pressure).

Plane Progressive Waves

- Wave equation : $y = A\sin(\omega t - kx)$ where $k = \frac{2\pi}{\lambda}$ = wave propagation constant
- Differential equation : $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

  Wave velocity (phase velocity) $\boxed{v = \frac{dx}{dt} = \frac{\omega}{k}}$ $\because \omega t - kx = \text{constant} \Rightarrow \frac{dx}{dt} = \frac{\omega}{k}$

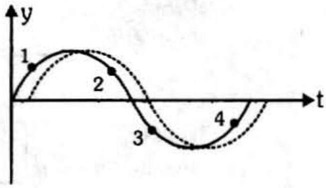
- Particle velocity $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$ $\boxed{v_p = -v \times \text{slope} = -v\left(\frac{dy}{dx}\right)}$
- Particle acceleration : $a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(\omega t - kx) = -\omega^2 y$

  For particle 1 : $v_p \downarrow$ and $a_p \downarrow$

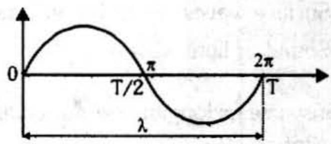
  For particle 2 : $v_p \uparrow$ and $a_p \downarrow$

  For particle 3 : $v_p \uparrow$ and $a_p \uparrow$

  For particle 4 : $v_p \downarrow$ and $a_p \uparrow$



- Relation between phase difference, path difference & time difference



$$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}}$$

Energy in Wave Motion

- $\frac{KE}{\text{volume}} = \frac{1}{2}\left(\frac{\Delta m}{\text{volume}}\right)v_p^2 = \frac{1}{2}\rho v_p^2 = \frac{1}{2}\rho\omega^2 A^2\cos^2(\omega t - kx)$
- $\frac{PE}{\text{volume}} = \frac{1}{2}\rho v^2\left(\frac{dy}{dx}\right)^2 = \frac{1}{2}\rho\omega^2 A^2\cos^2(\omega t - kx)$
- $\frac{TE}{\text{volume}} = \rho\omega^2 A^2\cos^2(\omega t - kx)$
- Pressure energy density [i.e. Average total energy / volume] $\boxed{u = \frac{1}{2}\rho\omega^2 A^2}$
- **Power** : P = (energy density) (volume/ time) $P = \left(\frac{1}{2}\rho\omega^2 A^2\right)(Sv)$

  [where S = Area of cross-section]

- **Intensity :** $I = \dfrac{\text{Power}}{\text{area of cross-section}} = \dfrac{1}{2}\rho\omega^2A^2v$
- **Speed of transverse wave on string :**

$v = \sqrt{\dfrac{T}{\mu}}$ where μ = mass/length and T = tension in the string.

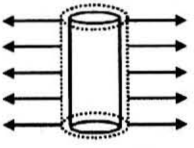
KEY POINTS

- A wave can be represented by function $y = f(kx \pm \omega t)$ because it satisfy the differential equation $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\left(\dfrac{\partial^2 v}{\partial t^2}\right)$ where $v = \dfrac{\omega}{k}$.
- A pulse whose wave function is given by $y=4 / [(2x+5t)^2+2]$ propagates in $-x$ direction as this wave function is of the form $y=f(kx+\omega t)$ which represent a wave travelling in $-x$ direction.
- Longitudinal waves can be produced in solids, liquids and gases because bulk modulus of elasticity is present in all three.

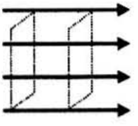
WAVE FRONT

- Spherical wave front (source $\rightarrow$point source)

- Cylindrical wave front (source $\rightarrow$linear source)



- Plane wave front (source $\rightarrow$ point / linear source at very large distance)



INTENSITY OF WAVE

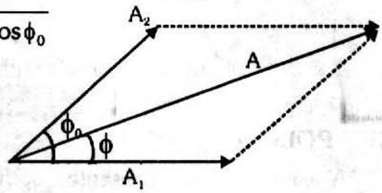
- Due to point source $I \propto \dfrac{1}{r^2}$ $\qquad y(r,t) = \dfrac{A}{r}\sin(\omega t - \vec{k}\cdot\vec{r})$
- Due to cylindrical source $I \propto \dfrac{1}{r}$ $\qquad y(r,t) = \dfrac{A}{\sqrt{r}}\sin(\omega t - \vec{k}\cdot\vec{r})$
- Due to plane source I = constant $\qquad y(r,t) = A\sin(\omega t - \vec{k}\cdot\vec{r})$

INTERFERENCE OF WAVES

$y_1 = A_1 \sin(\omega t - kx)$, $\quad y_2 = A_2 \sin(\omega t - kx + \phi_0)$

$y = y_1 + y_2 = A \sin(\omega t - kx + \phi)$

where $\quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi_0}$



and $\quad \tan\phi = \dfrac{A_2 \sin\phi_0}{A_1 + A_2\cos\phi_0}$

As $\quad I \propto A^2$

So $\quad \boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_0}$

- For constructive interference [Maximum intensity]

  $\phi_0 = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, ...$

  $$\boxed{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2}$$

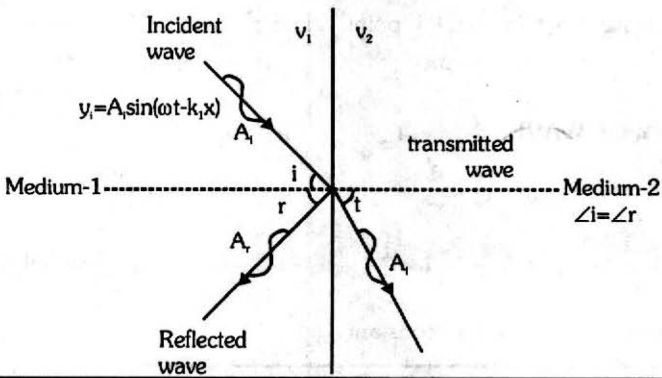
- For destructive interference [Minimum Intensity]

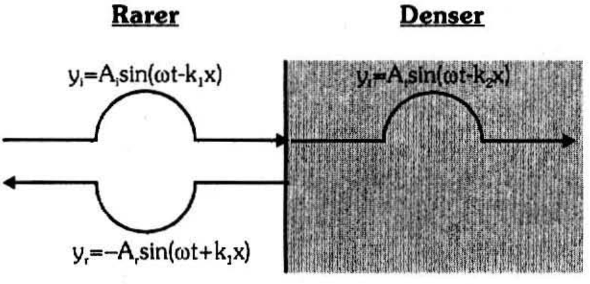
  $\phi_0 = (2n+1)\pi$ or path difference $= (2n+1)\dfrac{\lambda}{2}$

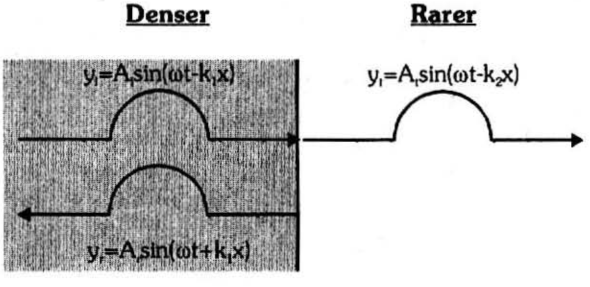
  where $n = 0, 1, 2, 3, ...$ $\quad \boxed{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$

- Degree of hearing $= \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$

Reflection and Refraction (transmission) of waves



- The frequency of the wave remain unchanged.
- Amplitude of reflected wave→ $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)A_i$
- Amplitude of transmitted wave → $A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$
- If $v_2 > v_1$ i.e. medium-2 is rarer
  $A_r > 0 \Rightarrow$ no phase change in reflected wave
- If $v_2 < v_1$ i.e. medium-1 is rarer
  $A_r < 0 \Rightarrow$ There is a phase change of π in reflected wave
- As A_t is always positive whatever be v_1 & v_2 the phase of transmitted wave always remains unchanged.
- In case of reflection from a denser medium or rigid support or fixed end, there is inversion of reflected wave i.e. phase difference of π between reflected and incident wave.
- the transmitted wave is never inverted.
- 

- 

Beats :

When two sound waves of nearly equal (but not exactly equal) frequencies travel in same direction, at a given point due to their super position, intensity alternatively increases and decreases periodically. This periodic waxing and waning of sound at a given position is called beats.

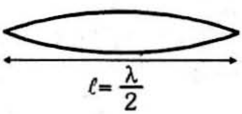
Beat frequency = difference of frequencies of two interfering waves

Stationary waves or standing waves : When two waves of same frequency and amplitude travel in opposite direction at same speed, their superstition gives rise to a new type of wave, called stationary waves or standing waves. Formation of standing wave is possible only in bounded medium.

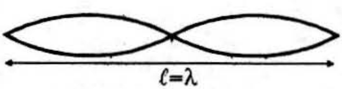
- Let two waves are $y_1 = A\sin(\omega t - kx)$; $y_2 = A\sin(\omega t + kx)$ by principle of superposition $y = y_1 + y_2 = 2A\cos kx \sin\omega t$ ← Equation of stationary wave
- As this equation satisfies the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, it represent a wave.
- Its amplitude is not constant but varies periodically with position.
- **Nodes** → amplitude is minimum : $\cos kx = 0 \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$
- **Antinodes** → amplitude is maximum : $\cos kx = 1 \Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$
- The nodes divide the medium into segments (loops). All the particles in a segment vibrate in same phase but in opposite phase with the particles in the adjacent segment.
- As nodes are permanently at rest, so no energy can be transmitted across them, i.e. energy of one region (segment) is confined in that region.

Transverse stationary waves in stretched string

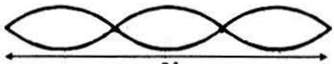
- [Fixed at both ends] [fixed end →Node & free end→Antinode]



Fundamental or first harmonic $f = \frac{v}{2\ell}$

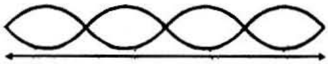


second harmonic first overtone $f = \frac{2v}{2\ell}$



$\ell=\frac{3\lambda}{2}$

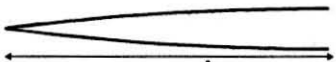
third harmonic
second overtone $f=\frac{3v}{2\ell}$



$\ell=2\lambda$

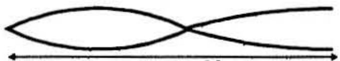
fourth harmonic
third overtone $f=\frac{4v}{2\ell}$

- **Fixed at one end**



$\ell=\frac{\lambda}{4}$

Fundamental $f=\frac{v}{4\ell}$

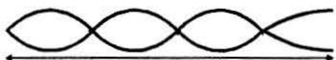


$\ell=\frac{3\lambda}{4}$

third harmonic
first overtone $f=\frac{3v}{4\ell}$

$\ell=\frac{5\lambda}{4}$

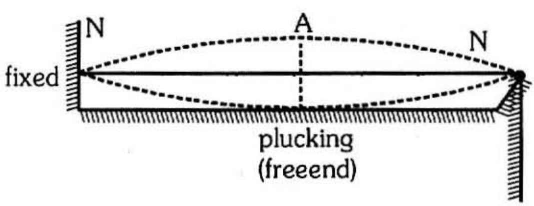
fifth harmonic
second overtone $f=\frac{5v}{4\ell}$



$\ell=\frac{7\lambda}{4}$

seventh harmonic
third overtone $f=\frac{7v}{4\ell}$

Sonometer

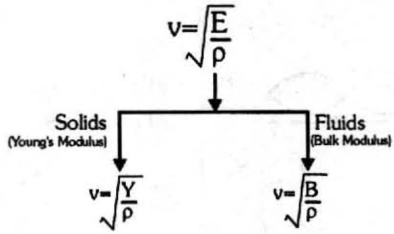


$$f_n=\frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$$

[p : number of loops]

Sound Waves

Velocity of sound in a medium of elasticity E and density ρ is



- **Newton's formula** : Sound propagation is isothermal $B = P \Rightarrow v = \sqrt{\frac{P}{\rho}}$
- **Laplace correction** : Sound propagation is adiabatic $B = \gamma P \Rightarrow v = \sqrt{\frac{\gamma P}{\rho}}$

KEY POINTS

- With rise in temperature, velocity of sound in a gas increases as $v = \sqrt{\frac{\gamma RT}{M_w}}$
- With rise in humidity velocity of sound increases due to presence of water in air.
- Pressure has no effect on velocity of sound in a gas as long as temperature remains constant.

Displacement and pressure wave

A sound wave can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure wave).

Displacement wave $\quad y=A\sin(\omega t-kx)$

Pressure wave $\quad p = p_0\cos(\omega t-kx)$

where $\quad p_0 = ABk = \rho A v\omega$

Note : As sound-sensors (e.g., ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.

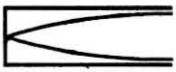
KEYPOINTS

- The pressure wave is 90° out of phase w.r.t. displacement wave, i.e. displacement will be maximum when pressure is minimum and vice-versa.
- Intensity in terms of pressure amplitude $I = \frac{p_0^2}{2\rho v}$

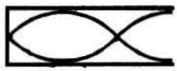
Vibrations of organ pipes

Stationary longitudinal waves closed end → displacement node, open end→ displacement antinode

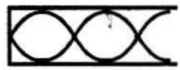
- **Closed end organ pipe**



$\ell = \frac{\lambda}{4} \Rightarrow f = \frac{v}{4\ell}$



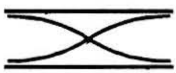
$\ell = \frac{3\lambda}{4} \Rightarrow f = \frac{3v}{4\ell}$



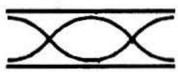
$\ell = \frac{5\lambda}{4} \Rightarrow f = \frac{5v}{4\ell}$

- Only odd harmonics are present
- Maximum possible wavelength = 4ℓ
- Frequency of mth overtone $= (2m+1)\frac{v}{4\ell}$

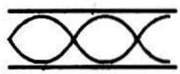
- **Open end organ pipe**



$\ell = \frac{\lambda}{2} \Rightarrow f = \frac{v}{2\ell}$



$\ell = \lambda \Rightarrow f = \frac{2v}{2\ell}$



$\ell = \frac{3\lambda}{2} \Rightarrow f = \frac{3v}{2\ell}$

- All harmonics are present
- Maximum possible wavelength is 2ℓ.
- Frequency of mth overtone $= (m+1)\frac{v}{2\ell}$

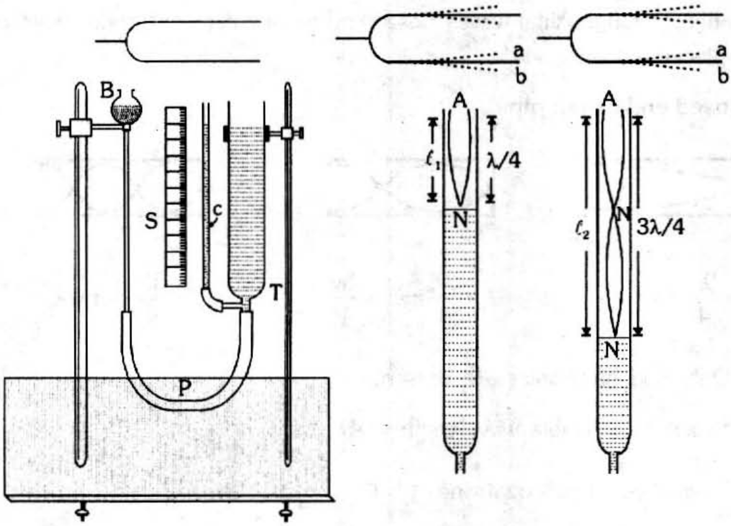
- **End correction** :

Due to finite momentum of air molecules in organ pipes reflection takes place not exactly at open end but some what above it, so antinode is not formed exactly at free end but slightly above it.

In closed organ pipe $f_1 = \frac{v}{4(\ell + e)}$ where e = 0.6 R (R=radius of pipe)

In open organ pipe $f_1 = \frac{v}{2(\ell + 2e)}$

- **Resonance Tube**



Wavelength $\lambda = 2(\ell_2 - \ell_1)$

End correction $e = \dfrac{\ell_2 - 3\ell_1}{2}$

Intensity of sound in decibels

Sound level, $SL = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

Where I_0 = threshold of human ear = 10^{-12} W/m^2

Characteristics of sound

- Loudness → Sensation received by the ear due to intensity of sound.
- Pitch → Sensation received by the ear due to frequency of sound.
- Quality (or Timbre)→ Sensation received by the ear due to waveform of sound.

Doppler's effect in sound :

A stationary source emits wave fronts that propagate with constant velocity with constant separation between them and a stationary observer encounters them at regular constant intervals at which they were emitted by the source.

A moving observer will encounter more or lesser number of wavefronts depending on whether he is approaching or receding the source.

A source in motion will emit different wave front at different places and therefore alter wavelength i.e. separation between the wavefronts.

The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect.

Source $\bullet \longrightarrow v_s$ $\sim\sim\sim\rightarrow$ $v_0 \longleftarrow \bullet$ observer

n   Sound Wave

Observed frequency $n' = \dfrac{\text{speed of sound wave w.r.t. observer}}{\text{observed wavelength}}$

$$n' = \frac{v+v_0}{\left(\dfrac{v-v_s}{n}\right)} = \left(\frac{v+v_0}{v-v_s}\right)n$$

If v_0, v_s <<<v then $n' \approx \left(1+\dfrac{v+v_s}{v}\right)n$

- Mach Number$=\dfrac{\text{speed of source}}{\text{speed of sound}}$

- **Doppler's effect in light :**

Case I :   **Observer** $\bullet$ O   **Light Source** $\overset{\longleftarrow}{v} \bullet$ S

$$\left.\begin{array}{ll} \text{Frequency} & \nu' = \left(\sqrt{\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\nu \approx \left(1+\dfrac{v}{c}\right)\nu \\ \text{Wavelength} & \lambda' = \left(\sqrt{\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\lambda \approx \left(1-\dfrac{v}{c}\right)\lambda \end{array}\right\}\text{Violet Shift}$$

Case II :   **Observer** $\bullet$ O   **Light Source** $\bullet \overset{\longrightarrow}{v}$ S

$$\left.\begin{array}{ll} \text{Frequency} & \nu' = \left(\sqrt{\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\nu \approx \left(1-\dfrac{v}{c}\right)\nu \\ \text{Wavelength} & \lambda' = \left(\sqrt{\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\lambda \approx \left(1+\dfrac{v}{c}\right)\lambda \end{array}\right\}\text{Red Shift}$$

Important Notes

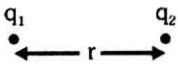
15 Electrostatics

Electric Charge

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

Coulomb's law :

Force between two charges $\vec{F} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$ ϵ_r = dielectric constant



NOTE : The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are extended, induction may change the charge distribution.

Principle Of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots\ldots.$$

NOTE : The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity or Electric Field Strength (Vector Quantity)

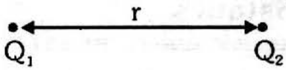
In the surrounding region of a charge there exist a physical property due to which other charge experiences a force. The direction of electric field is direction of force experienced by a positively charged particle and the magnitude of the field (electric field intensity) is the force experienced by a unit charge.

$$\vec{E} = \frac{\vec{F}}{q} \text{ unit is N/C or V/m.}$$

- **Electric field intensity due to charge Q**

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} \hat{r}$$

Null point for two charges :



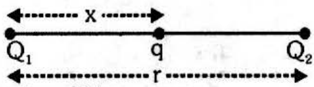
If $|Q_1| > |Q_2|$

$\Rightarrow$ Null point near Q_2

$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}$$ $x \rightarrow$ distance of null point from Q_1 charge

(+) for like charges
(–) for unlike charges

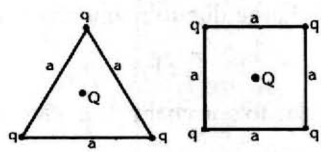
❑ Equilibrium of three point charges



(i) Two charges must be of like nature.
(ii) Third charge should be of unlike nature.

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \quad \text{and} \quad q = \frac{-Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

❑ Equilibrium of symmetric geometrical point charged system

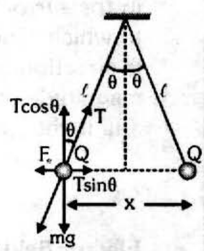


Value of Q at centre for which system to be in state of equilibrium

(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$ (ii) For square $Q = \frac{-q(2\sqrt{2}+1)}{4}$

❑ Equilibrium of suspended point charge system

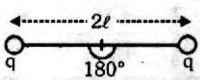
For equilibrium position



$$T\cos\theta = mg \ \& \ T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$

- If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$)



$$T = F_e = \frac{kq^2}{4\ell^2}$$

- **Electric potential difference** $\Delta V = \dfrac{\text{work}}{\text{charge}} = W/q$

- **Electric potential** $V_P = -\int_{\infty}^{P} \vec{E}.\vec{dr}$

  It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point

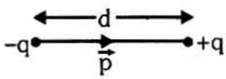
  ❑ For point charge : $V = K\dfrac{q}{r}$ ❑ For several point charges : $V = K\sum \dfrac{q_i}{r_i}$

- **Relation between $\vec{E}$ & V**

  $\vec{E} = -\text{grad } V = -\nabla V$, $E = -\dfrac{\partial V}{\partial r}$; $\vec{E} = -\dfrac{\partial V}{\partial x}\hat{i} - \dfrac{\partial V}{\partial y}\hat{j} - \dfrac{\partial V}{\partial z}\hat{k}$, $V = \int -\vec{E}.\vec{dr}$

- **Electric potential energy of two charges :** $U = \dfrac{1}{4\pi \in_0}\dfrac{q_1 q_2}{r}$

- **Electric dipole**



  ❑ Electric dipole moment p = qd

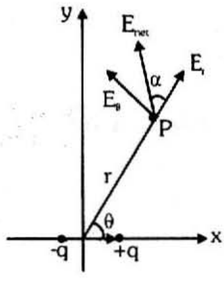
  ❑ Torque on dipole placed in uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$

  ❑ Work done in rotating dipole placed in uniform electric field

  $$W = \int \tau d\theta = \int_{\theta_0}^{\theta} PE\sin\theta d\theta = pE(\cos\theta_0 - \cos\theta)$$

  ❑ Potential energy of dipole placed in an uniform field $U = -\vec{p}.\vec{E}$

  ❑ At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.



  • Potential $V = \dfrac{1}{4\pi \in_0}\dfrac{p\cos\theta}{r^2}$

  • Electric field $E = \dfrac{1}{4\pi \in_0}\dfrac{p\sqrt{1+3\cos^2\theta}}{r^3}$

  • Direction $\tan\alpha = \dfrac{E_\theta}{E_r} = \dfrac{1}{2}\tan\theta$

  ❑ Electric field at axial point (or End-on) $\vec{E} = \dfrac{1}{4\pi \in_0}\dfrac{2\vec{p}}{r^3}$ of dipole

  ❑ Electric field at equatorial position (Broad-on) of dipole $\vec{E} = \dfrac{1}{4\pi \in_0}\dfrac{(-\vec{p})}{r^3}$

Equipotential Surface And Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where E = 0, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

Mutual Potential Energy Or Interaction Energy

"The work to be done to integrate the charge system".

For 2 particle system $U_{mutual} = \frac{q_1q_2}{4\pi\varepsilon_0 r}$

For 3 particle system $U_{mutual} = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_2q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_3q_1}{4\pi\varepsilon_0 r_{31}}$

For n particles there will be $\frac{n(n-1)}{2}$ terms.

Total energy of a system = $U_{self} + U_{mutual}$

Electric flux : $\phi = \int \vec{E}.d\vec{s}$

(i) For uniform electric field; $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ = angle between $\vec{E}$ & area vector $(\vec{A})$. Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(ii) If $\vec{E}$ is not uniform throughout the area A, then $\phi = \int \vec{E}.d\vec{A}$

Gauss's Law : $\oint \vec{E}.d\vec{s} = \frac{\sum q}{\varepsilon_0}$ (Applicable only to closed surface)

Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$.

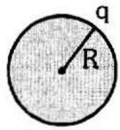
$\phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}$ where q_{en} = net charge enclosed by the closed surface.

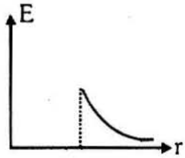
ϕ does not depend on the

(i) Shape and size of the closed surface

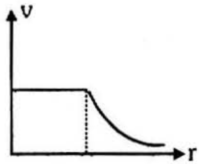
(ii) The charges located outside the closed surface.

- **For a conducting sphere**



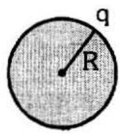


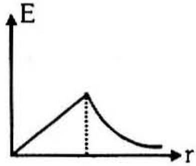
❒ For $r \geq R : E = \dfrac{1}{4\pi \in_0}\dfrac{q}{r^2}, V = \dfrac{1}{4\pi \in_0}\dfrac{q}{r}$



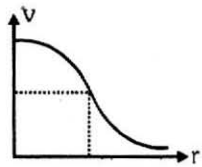
❒ For $r < R$: $E = 0, V = \dfrac{1}{4\pi \in_0}\dfrac{q}{R}$

- **For a non-conducting sphere**





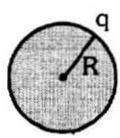
❒ For $r \geq R : E = \dfrac{1}{4\pi \in_0}\dfrac{q}{r^2}, V = \dfrac{1}{4\pi \in_0}\dfrac{q}{r}$



❒ For $r < R : E = \dfrac{1}{4\pi \in_0}\dfrac{qr}{R^3}, \ V = \dfrac{1}{4\pi \in_0}\dfrac{q(3R^2 - r^2)}{2R^3}$

$$V_C = V_{max} = \frac{3}{2}\frac{Kq}{R} = 1.5V_{Surface}$$

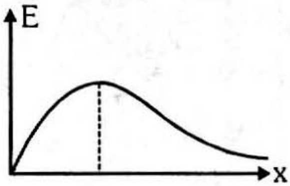
- **For a conducting/non conducting spherical shell**

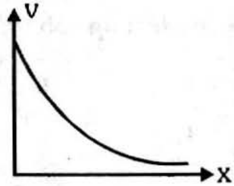


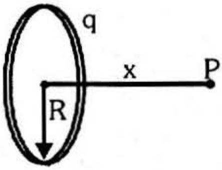
❒ For $r \geq R : E = \dfrac{1}{4\pi \in_0}\dfrac{q}{r^2}, V = \dfrac{1}{4\pi \in_0}\dfrac{q}{r}$

❒ For $r < R : E = 0, V = \dfrac{1}{4\pi \in_0}\dfrac{q}{R}$

- **For a charged circular ring**

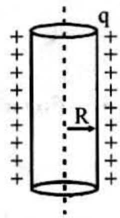






$$E_P = \frac{1}{4\pi \in_0} \frac{qx}{(x^2+R^2)^{3/2}}, \; V_P = \frac{1}{4\pi \in_0} \frac{q}{(x^2+R^2)^{1/2}}$$

Electric field will be maximum at $x = \pm \frac{R}{\sqrt{2}}$



- **For a charged long conducting cylinder**

  - For $r \geq R : E = \frac{q}{2\pi \in_0 r}$
  - For $r < R : E = 0$

- **Electric field intensity at a point near a charged conductor** $E = \frac{\sigma}{\in_0}$

- **Mechanical pressure on a charged conductor**

$$P = \frac{\sigma^2}{2\in_0}$$

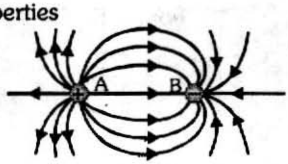
- **For non-conducting long sheet of surface charge density σ** $E = \frac{\sigma}{2\in_0}$

- **For conducting long sheet of surface charge density** $E = \frac{\sigma}{\in_0}$

- **Energy density in electric field** $u_E = \frac{1}{2}\in_0 E^2$

Electric lines of force

Electric lines of electrostatic field have following properties



$q_A > q_B$

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_0)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_0.

(v) Lines of force ends or starts normally at the surface of a conductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity.

KEY POINTS

- Electric field is always perpendicular to a conducting surface (or any equipotential surface).

  No tangential component of electric field on such surfaces.

- When a conductor is charged, the charge resides only on the surface.

- Charge density at convex sharp points on a conductor is greater. Lesser is radius of curvature at a convex part, greater is the charge density.

- For a conductor of any shape E (just outside)= $\dfrac{\sigma}{\varepsilon_0}$

- Potential difference between two points in an electric field does not depend on the path between them.

- Potential at a point due to positive charge is positive & due to negative charge is negative.

- Positive charge flows from higher to lower (i.e. in the direction of electric field) potential and negative charge from lower to higher (i.e. opposite to the electric field) potential.

- When $\vec{p} \parallel \vec{E}$ the dipole is in stable equilibrium

- When $\vec{p} \parallel (-\vec{E})$ the dipole is in unstable equilibrium

- When a charged isolated conducting sphere is connected to an uncharged small conducting sphere then potential become same on both sphere and redistribution of charge take place.

- Self potential energy of a charged conducting spherical shell $= \frac{KQ^2}{2R}$.

- Self potential energy of an insulating uniformly charged sphere= $\frac{3KQ^2}{5R}$

- A spherically symmetric charge {i.e ρ depends only on r} behaves as if its charge is concentrated at its centre (for outside points).

- **Dielectric strength of material :** The minimum electric field required to ionize the medium or the maximum electric field which the medium can bear without breaking down.

- The particles such as photon or neutrino which have no (rest) mass can never has a charge because charge cannot exist without mass.

- Electric charge is invariant because value of electric charge does not depend on frame of reference.

- A spherical body behaves like a point charge for outside points because a finite charged body may behave like a point charge if it produces an inverse square field.

- Any arbitrary displacement of charges inside a shell does not introduce any change in the electrostatic field of the outer space because a closed conducting shell divides the entire space into the inner and outer parts which are completely independent of one another in respect of electric fields.

- A charged particle is free to move in an electric field. It may or may not move along an electric line of force because initial conditions affect the motion of charged particle.

- Electrostatic experiments do not work well in humid days because water is a good conductor of electricity.

- A metallic shield in form of a hollow conducting shell may be built to block an electric field because in a hollow conducting shell, the electric field is zero at every point.

16 Capacitance and Capacitor

CAPACITOR & CAPACITANCE

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them $C = \frac{Q}{V}$

The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

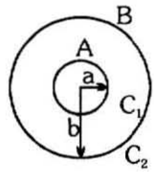
CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

$C = 4\pi \epsilon_0 \epsilon_r R$ in a medium $C = 4\pi \epsilon_0 R$ in air

* This sphere is at infinite distance from all the conductors .

SPHERICAL CAPACITOR :

It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2. We have



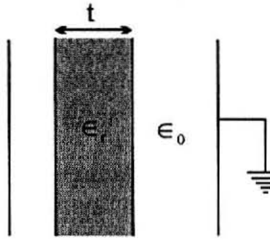
$C_1 = \frac{4\pi \epsilon_0 ab}{b-a}$ and $C_2 = 4\pi \epsilon_0 b$

PARALLEL PLATE CAPACITOR :

(i) UNIFORM DI-ELECTRIC MEDIUM :If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a parallel plate capacitor & its capacitance is given by, $C = \frac{\epsilon_0 \epsilon_r A}{d}$ in a medium; $C = \frac{\epsilon_0 A}{d}$ with air as medium

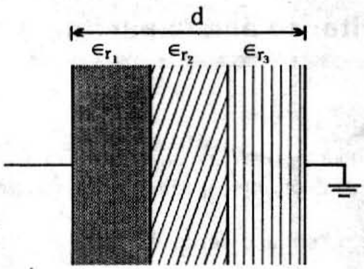
This result is only valid when the electric field between plates of capacitor is constant.

(ii) MEDIUM PARTLY AIR : $C = \frac{\epsilon_0 A}{d - \left(t - \frac{t}{\epsilon_r}\right)}$



When a di-electric slab of thickness t & relative permittivity ϵ_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \frac{t}{\epsilon_r}\right)$ irrespective of the position of the di-electric slab .

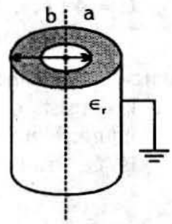
(iii) COMPOSITE MEDIUM :



$$C=\frac{\epsilon_0 A}{\frac{t_1}{\epsilon_{r_1}}+\frac{t_2}{\epsilon_{r_2}}+\frac{t_3}{\epsilon_{r_3}}}$$

CYLINDRICAL CAPACITOR :

It consists of two co-axial cylinders of radii a & b, the outer conductor is earthed. The di-electric constant of the medium filled in the space between the cylinders is ϵ_r.

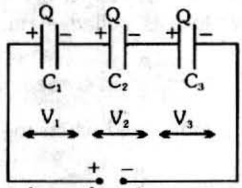


The capacitance per unit length is $C=\dfrac{2\pi\epsilon_0\epsilon_r}{\ell n\left(\frac{b}{a}\right)}$

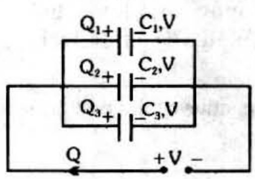
COMBINATION OF CAPACITORS :

(i) CAPACITORS IN SERIES :In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+.....+\frac{1}{C_n}$$



(ii) CAPACITORS IN PARALLEL : When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be in parallel connection. The capacitors have the same potential difference, V but the charge on each one is different (if the capacitors are unequal). $C_{eq.} = C_1 + C_2 + C_3 + ... + C_n$.



ENERGY STORED IN A CHARGED CAPACITOR :

Capacitance C, charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor .

HEAT PRODUCED IN SWITCHING IN CAPACITIVE CIRCUIT :

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as –

Heat = Work done by battery – Energy absorbed by capacitor.

- **Work done by battary to charge a capacitor** $W = CV^2 = QV = \frac{Q^2}{C}$

SHARING OF CHARGES :

When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers becomes equal. The common potential (V) after sharing of charges;

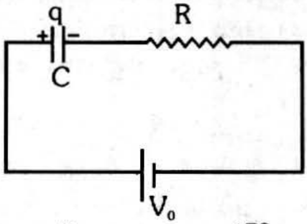
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

charges after sharing $q_1 = C_1V$ & $q_2 = C_2V$. In this process energy is lost in the connecting wire as heat.

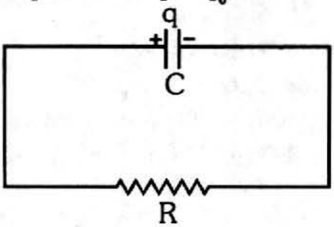
This loss of energy is $U_{initial} - U_{final} = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$.

- **Attractive force between capacitor plate** $F = \left(\frac{\sigma}{2\epsilon_0}\right)(\sigma A) = \frac{Q^2}{2\epsilon_0 A}$

- **Charging of a capacitor** : $q = q_0\left(1 - e^{-t/RC}\right)$ where $q_0 = CV_0$



- **Discharging of a capacitor : $q = q_0 e^{-t/RC}$**



KEY POINTS

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.
- The energy of an uncharged condenser = 0.
- The capacitance of a capacitor depends only on its size & geometry & the dielectric between the conducting surface. (i.e. independent of the conductor, whether it is copper, silver, gold etc)
- The two adjacent conductors carrying same charge can be at different potential because the conductors may have different sizes and means difference capacitance.
- When a capacitor is charged by a battery, both the plates received charge equal in magnitude, no matter sizes of plates are identical or not because the charge distribution on the plates of a capacitor is in accordance with charge conservation principle.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of a grounded object is taken to be zero because capacitance of the earth is very large.

Important Notes

17 Current Electricity and Heating Effects of current

ELECTRIC CURRENT :

Electric charges in motion constitute an electric current. Any medium having practically free electric charges (i.e. free to migrate) is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state. Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors.

ELECTRIC CURRENT IN A CONDUCTOR :

In absence of potential difference across a conductor, no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) flow in a definite direction which constitutes a net current in it . These electrons are not accelerated by electric field in the conductor produced by potential difference across the conductor. They move with a constant drift velocity . The direction of current is along the flow of positive charge (or opposite to flow of negative charge). $i = nv_deA$, where v_d = drift velocity .

ELECTRIC CURRENT AND CURRENT DENSITY

The strength of the current i is the rate at which the electric charges are flowing. If a charge Q coulomb passes through a given cross section of the conductor in t second the current I through the conductor is given by

$$I = \frac{Q\,\text{coulomb}}{t\,\text{second}} = \text{ampere}$$

Ampere is the unit of current. If i is not constant then $i = \frac{dq}{dt}$, where dq is net charge transported at a section in time dt. In a current carrying conductor we can define a vector which gives the direction as current per unit normal, cross sectional area & is known as current density.

$$\text{Thus } \vec{J} = \frac{I}{S}\hat{n} \quad \text{or} \quad I = \vec{J}\cdot\vec{S}$$

Where $\hat{n}$ is the unit vector in the direction of the flow of current.

For random J or S, we use $I = \int \vec{J}\cdot\vec{ds}$

RELATION IN J, E AND v_D :

In conductors drift velocity of electrons is proportional to the electric field inside the conductor as; $v_d = \mu E$

where μ is the mobility of electrons

current density is given as $J = \frac{I}{A} = ne\,v_d = ne(\mu E) = \sigma E$

where $\sigma = ne\mu$ is called conductivity of material and we can also write

$\rho = \frac{1}{\sigma} \rightarrow$ resistivity of material.

Thus $E = \rho J$. It is called as differential form of Ohm's Law.

SOURCES OF POTENTIAL DIFFERENCE & ELECTROMOTIVE FORCE :

Dry cells , secondary cells , generator and thermo couple are the devices used for producing potential difference in an electric circuit. The potential difference between the two terminals of a source when no energy is drawn from it, is called the " **Electromotive force**" or " **EMF** " of the source. The unit of potential difference is volt.

$$1 \text{ volt} = 1 \text{ Ampere} \times 1 \text{ Ohm}.$$

ELECTRICAL RESISTANCE :

The property of a substance which opposes the flow of electric current through it, is termed as electrical resistance. Electrical resistance depends on the size, geometery, temperature and internal structure of the conductor.

LAW OF RESISTANCE :

The resistance R offered by a conductor depends on the following factors :

$R \propto \ell$ (length of the conductor) ; $R \propto \frac{1}{A}$ (cross section area of the conductor)

at a given temperature $R = \rho \frac{\ell}{A}$ .Where ρ is the resistivity of the material of the conductor at the given temperature . It is also known as **specific resistance** of the material & it depends upon nature of conductor.

DEPENDENCE OF RESISTANCE ON TEMPERATURE :

The resistance of most conductors and all pure metals increases with temperature , but there are a few in which resistance decreases with temperature. If R_0 & R be the resistance of a conductor at 0°C and θ°C , then it is found that $R = R_0(1 + \alpha\theta)$.

Here we assume that the dimensions of resistance do not change with temperature if expansion coefficient of material is considerable. Then instead of resistance we use same property for resistivity as $\rho = \rho_0(1 + \alpha\theta)$. *The materials for which resistance decreases with temperature, the temperature coefficient of resistance is negative.*

Where α is called the temperature co-efficient of resistance . The unit of α is K^{-1} or $°C^{-1}$. Reciprocal of resistivity is called conductivity and reciprocal of resistance is called conductance (G) . S.I. unit of G is mho.

OHM'S LAW :

Ohm's law is the most fundamental law of all the laws in electricity . It says that the current through the cross section of the conductor is proportional to the applied potential difference under the given physical condition . V = RI . Ohm's law is applicable to only metalic conductors .

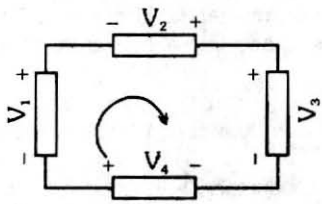
KRICHHOFF'S LAW'S :

I - Law (Junction law or Nodal Analysis) : This law is based on law of conservation of charge. It states that " The algebric sum of the currents meeting at a point is zero " or total currents entering a junction equals total current leaving the junction.

$$\Sigma I_{in} = \Sigma I_{out}$$

It is also known as KCL (Kirchhoff's current law) .

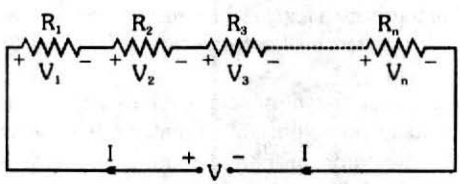
II - Law (Loop analysis) : The algebric sum of all the voltages in closed circuit is zero. Σ IR + Σ EMF = 0 in a closed loop . The closed loop can be traversed in any direction . While traversing a loop if higher potential point is entered, put a + ve sign in expression or if lower potential point is entered put a negative sign .



$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or non-linear).
It is also known as **KVL (Kirchhoff's voltage law)** .

COMBINATION OF RESISTANCES :

A number of resistances can be connected and all the complited combinations can be reduced to two different types, namely series and parallel .



(i) RESISTANCE IN SERIES :

When the resistances are connected end to end then they are said to be in series . The current through each resistor is same . The effective resistance appearing across the battery;

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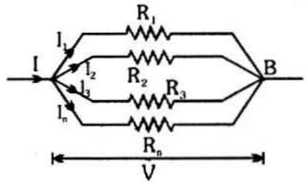
$$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots + R_n \quad \text{and}$$
$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots + V_n \,.$$

The voltage across a resistor is proportional to the resistance

$$V_1 = \frac{R_1}{R_1 + R_2 + \ldots\ldots + R_n} V; \; V_2 = \frac{R_2}{R_1 + R_2 + \ldots\ldots + R_n} V; \text{ etc.}$$

(ii) RESISTANCE IN PARALLEL :

A parallel circuit of resistors is one, in which the same voltage is applied across all the components in a parallel grouping of resistors $R_1, R_2, R_3, \ldots\ldots, R_n$.



Conclusions :

(a) Potential difference across each resistor is same .

(b) $I = I_1 + I_2 + I_3 + \ldots\ldots I_n$.

(c) Effective resistance (R) then $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots\ldots + \dfrac{1}{R_n}$.

(d) Current in different resistors is inversally proportional to the resistance .

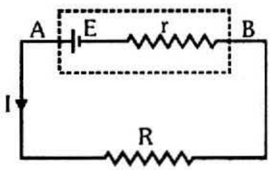
$$I_1 : I_2 : \ldots\ldots I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \ldots\ldots : \frac{1}{R_n} \,.$$

$$I_1 = \frac{G_1}{G_1 + G_2 + \ldots\ldots + G_n} I, \; I_2 = \frac{G_2}{G_1 + G_2 + \ldots\ldots + G_n} I, \text{ etc.}$$

where $G = \dfrac{I}{R}$ = Conductance of a resistor .

EMF OF A CELL & ITS INTERNAL RESISTANCE :

If a cell of emf E and internal resistance r be connected with a resistance R the total resistance of the circuit is $(R + r)$.



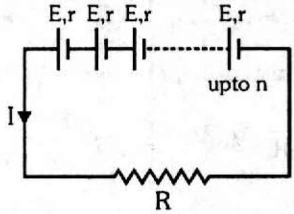
$$I = \frac{E}{R+r} \;; V_{AB} = \frac{E}{R+r}$$

where V_{AB} = Terminal voltage of the battery .

If $r \to 0$, cell is Ideal & $V \to E$ & $r = R\left(\dfrac{E}{V} - 1\right)$

GROUPING OF CELLS :

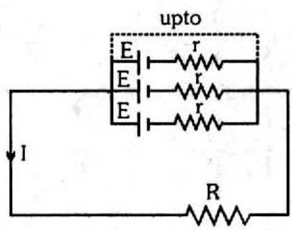
(i) CELLS IN SERIES : Let there be n cells each of emf E , arranged in series.Let r be the internal resistance of each cell. The total emf = n E . Current in the circuit $I = \dfrac{nE}{R+nr}$. If nr << R then $I = \dfrac{nE}{R}$ →Series combination should be used



If $nr >> R$ then $I = \frac{E}{r} \longrightarrow$ Series combination should not be used .

(ii) CELLS IN PARALLEL : If m cells each of emf E & internal resistance r be connected in parallel and if this combination be connected to an external resistance then the emf of the circuit = E.

Internal resistance of the circuit $= \frac{r}{m}$.



$$I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$

If $mR << r$ then $I = \frac{mE}{r} \longrightarrow$ Parallel combination should be used .

If $mR >> r$ then $I = \frac{E}{R} \longrightarrow$ Parallel combination should not be used .

(iii) CELLS IN MULTIPLE ARC :

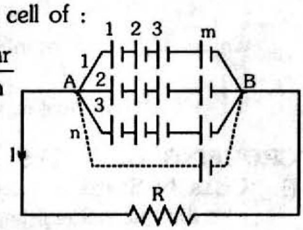
mn = number of identical cells .

n = number of rows

m = number of cells in each rows .

The combination of cells is equivalent to single cell of :

(a) emf = mE & (b) internal resistance $= \frac{mr}{n}$



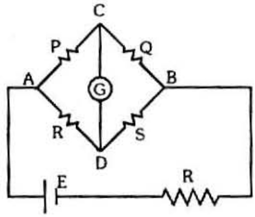
Current $I = \frac{mE}{R + \frac{mr}{n}}$.

For maximum current

$nR = mr$ or $R = \frac{mr}{n}$ so $I_{max} = \frac{nE}{2r} = \frac{mE}{2R}$

For a cell to deliver maximum power across the load internal resistance = load resistance

WHEAT STONE NETWORK :



When current through the galvanometer is zero

(null point or balance point) $\frac{P}{Q}=\frac{R}{S}$.

When

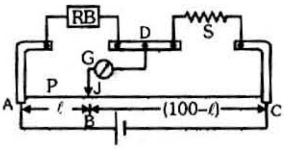
$PS > QR, V_C < V_D$ & $PS < QR, V_C > V_D$

or $PS = QR \Rightarrow$ products of opposite arms are equal. Potential difference between C & D at null point is zero . The null point is not affected by resistance of G & E. It is not affected even if the positions of G & E are interchanged.

$$I_{CD} \propto (QR - PS)$$

- **Metre Bridge**

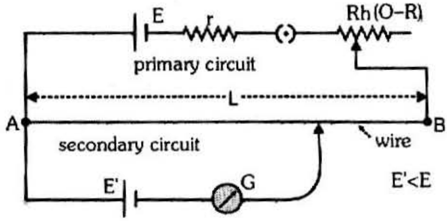
At balance condition : $\frac{P}{Q}=\frac{R}{S} \Rightarrow \frac{r\ell}{r(100-\ell)}=\frac{R}{S} \Rightarrow S=\frac{(100-\ell)}{\ell}R$



POTENTIOMETER :

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire . Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this . The potentiometer equation is $\frac{E_1}{E_2}=\frac{\ell_1}{\ell_2}$.

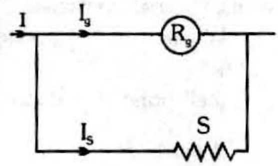
Circuits of potentiometer :



$$x=\frac{V}{L}=\frac{\text{current} \times \text{resitance of potentiometer wire}}{\text{length of potentiometer wire}}=I\left(\frac{R}{L}\right)$$

Ammeter :

It is a modified form of suspended coil galvanometer, it is used to measure current . A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter .



$$S = \frac{I_g R_g}{I - I_g}$$

where

R_g = galvanometer resistance

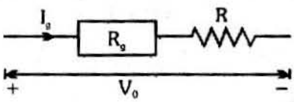
I_g = Maximum current that can flow through the galvanometer .

I = Maximum current that can be measured using the given ammeter .

An ideal ammeter has zero resistance.

Voltmeter :

A high resistance is put in series with galvanometer. It is used to measure potential difference.



$$I_g = \frac{V_o}{R_g + R}\ ; \quad R \to \infty\ , \quad \text{Ideal voltmeter}$$

Electrical Power :

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by $P = VI$, where V = potential difference across device & I = current. If the current enters the higher potential point of the device then power is consumed by it (i.e. acts as load) . If the current enters the lower potential point then the device supplies power (i.e. acts as source).

Power consumed by a resistor $P = I^2R = VI = \dfrac{V^2}{R}$

Heating Effect Of Electric Current :

When a current is passed through a resistor energy is wasted in over coming the resistances of the wire . This energy is converted into heat

$$W = VIt = I^2 Rt = \frac{V^2}{R} t$$

Joules Law Of Electrical Heating :

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by :

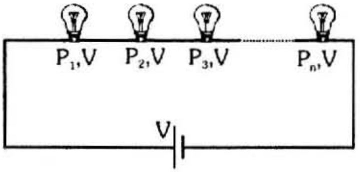
$$H = I^2 RT \text{ joule} = \frac{I^2 RT}{4.2} \text{ calories.}$$

If current is variable passing through the conductor then we use for heat produced in resistance in time 0 to T is: $H = \int_0^T I^2 R\,dt$

UNIT OF ELECTRICAL ENERGY CONSUMPTION :

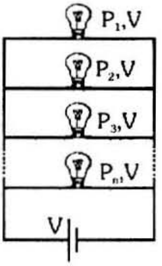
1 unit of electrical energy = kilowatt hour = 1 kWh = 3.6×10^6 joules.

- ***Series combination of Bulbs***



$$\frac{1}{P_{total}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \ldots$$

- **Parallel combination of Bulbs**



$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

KEY POINTS

- A current flows through a conductor only when there is an electric field within the conductor because the drift velocity of electrons is directly proportional to the applied electric field.
- Electric field outside the conducting wire which carries a constant current is zero because net charge on a current carrying conductor is zero.
- A metal has a resistance and gets often heated by flow of current because when free electrons drift through a metal, they make occasional collisions with the lattice. These collisions are inelastic and transfer energy to the lattice as internal energy.
- Ohm's law holds only for small current in metallic wire, not for high currents because resistance increased with increase in temperature.
- Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small.
- An ammeter is always connected in series whereas a voltmeter is connected in parallel because an ammeter is a low-resistance galvanometer while a voltmeter is a high-resistance galvanometer.
- Current is passed through a metallic wires, heating it red, when cold water is poured over half of the portion, rest of the portion becomes more hot because resistance decreases due to decrease in temperature so current through wire increases.

Important Notes

18 Magnetic Effect of Current and Magnetism

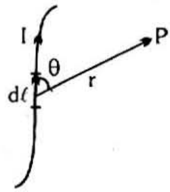
A static charge produces only electric field and only electric field can exert a force on it. A moving charge produces both electric field and magnetic field and both electric field and magnetic field can exert force on it. A current carrying conductor produces only magnetic field and only magnetic field can exert a force on it.

Magnetic charge (i.e. current), produces a magnetic field. It can not produce electric field as net charge on a current carrying conductor is zero. A magnetic field is detected by its action on current carrying conductors (or moving charges) and magnetic needles (compass). The vector quantity $\vec{B}$ known as MAGNETIC INDUCTION is introduced to characterise a magnetic field. It is a vector quantity which may be defined in terms of the force it produces on electric currents. Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of lines per unit area crossing a small area perpendicular to the direction of the induction bring numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred to as the **magnetic flux** linked with that area. For this reason $\vec{B}$ is also called **magnetic flux density**.

MAGNETIC INDUCTION PRODUCED BY A CURRENT (BIOT-SAVART LAW):

The magnetic induction dB produced by an element $d\ell$ carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{I\,d\ell\sin\theta}{r^2} \Rightarrow \overrightarrow{dB} = \frac{\mu_0\,\mu_r}{4\pi}\frac{I\left(d\vec{\ell}\times\vec{r}\right)}{r^3}$$



here the quantity $Id\ell$ is called as current element strength.

μ = permeability of the medium $= \mu_0\,\mu_r$

μ_0 = permeability of free space

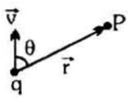
μ_r = relative permeability of the medium (Dimensionless quantity).

Unit of μ_0 & μ is NA^{-2} or Hm^{-1};

$\mu_0 = 4\pi\times10^{-7}\ Hm^{-1}$

MAGNETIC INDUCTION DUE TO A MOVING CHARGE

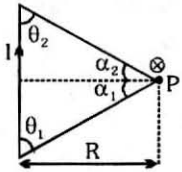
$$dB_P = \frac{\mu_0 qv\sin\theta}{4\pi r^2}$$



In vector form it can be written as $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$

Magnetic Induction Due To A Current Carrying Straight Conductor

- **Magnetic induction due to a current carrying straight wire**

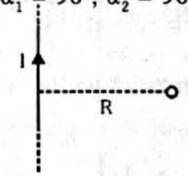


$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

If the wire is very long $\theta_1 \cong \theta_2 \cong 0°$ then , $B = \frac{\mu_0 I}{2\pi R}$

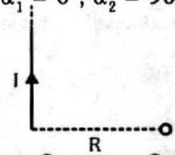
- **Magnetic induction due to a infinitely long wire** $B = \frac{\mu_0 I}{2\pi R} \otimes$

$\alpha_1 = 90°; \alpha_2 = 90°$



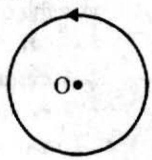
Magnetic Induction due to semi infinite straight conductor $B = \frac{\mu_0 I}{4\pi R} \otimes$

$\alpha_1 = 0°; \alpha_2 = 90°$



Magnetic Field Due To A Flat Circular Coil Carrying A Current :

(i) At its centre $B = \frac{\mu_0 NI}{2R} \odot$



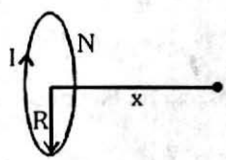
where

N = total number of turns in the coil

I = current in the coil

R = Radius of the coil

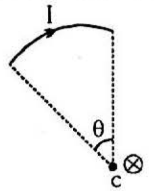
(ii) On the axis $B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$



Where x = distance of the point from the centre .

It is maximum at the centre $B_C = \frac{\mu_0 NI}{2R}$

(iii) **MAGNETIC INDUCTION DUE TO FLAT CIRCULAR ARC :** $B = \frac{\mu_0 I\theta}{4\pi R}$



- **Magnetic field due to infinite long solid cylindrical conductor of radius R**

  ❑ For $r \geq R : B = \frac{\mu_0 I}{2\pi r}$ ❑ For $r < R : B = \frac{\mu_0 Ir}{2\pi R^2}$

MAGNETIC INDUCTION DUE TO SOLENOID

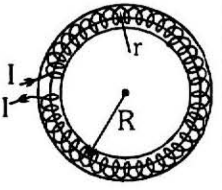
$B = \mu_0 nI$, direction along axis.

where $n \rightarrow$ number of turns per meter; $I \rightarrow$ current

MAGNETIC INDUCTION DUE TO TOROID : $B = \mu_0 nI$

where $n = \frac{N}{2\pi R}$ (no. of turns per m)

N = total turns $R >> r$



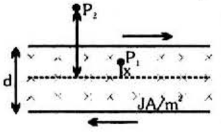
MAGNETIC INDUCTION DUE TO CURRENT CARRYING SHEET

$B = \frac{1}{2}\mu_0 I$ where I = Linear current density (A/m)

MAGNETIC INDUCTION DUE TO THICK SHEET

At point P_2 $B_{out} = \frac{1}{2}\mu_0 Jd$

At point P_1 $B_{in} = \mu_0 Jx$



GILBERT'S MAGNETISM (EARTH'S MAGNETIC FIELD) :

(a) The line of earth's magnetic induction lies in a vertical plane coinciding with the magnetic North - South direction at that place. This plane is called the MAGNETIC MERIDIAN. Earth's magnetic axis is slightly inclined to the geometric axis of earth and this angle varies from 10.5^0 to 20^0. The Earth's Magnetic poles are opposite to the geometric poles i.e. at earth's north pole, its magnetic south pole is situated and vice versa.

(b) On the magnetic meridian plane, the magnetic induction vector of the earth at any point, generally inclined to the horizontal at an angle called the **MAGNETIC DIP** at that place, such that $\vec{B}$ = total magnetic induction of the earth at that point.

$\vec{B}_v$ = the vertical component of $\vec{B}$ in the magnetic meridian plane = B sin θ

$\vec{B}_H$ = the horizontal component of $\vec{B}$ in the magnetic meridian plane=B cos θ.

$$\frac{B_v}{B_H} = \tan\theta$$

(c) At a given place on the surface of the earth, the magnetic meridian and the geographic meridian may not coincide. The angle between them is called "**DECLINATION AT THAT PLACE**"

AMPERES LAW

$$\oint \vec{B}\cdot d\vec{\ell} = \mu\Sigma I$$ where Σ I = algebraic sum of all the currents.

MOTION OF A CHARGE IN UNIFORM MAGNETIC FIELD :

(a) When $\vec{v}$ is | | to $\vec{B}$: Motion will be in a straight line and $\vec{F} = 0$

(b) When $\vec{v}$ is $\perp$ to $\vec{B}$: Motion will be in circular path with radius

$R = \frac{mv}{qB}$ and angular velocity $\omega = \frac{qB}{m}$ and F = qvB.

(c) When $\vec{v}$ is at $\angle\theta$ to $\vec{B}$: Motion will be helical with radius

$R_k = \frac{mv\sin\theta}{qB}$ and pitch $P_H = \frac{2\pi mv\cos\theta}{qB}$ and F = qvBsinθ.

LORENTZ FORCE :

An electric charge 'q' moving with a velocity $\vec{v}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F}=q\vec{v}\times\vec{B}$. There fore, if the charge moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E}+q\vec{v}\times\vec{B}$

This force is called the **LORENTZ FORCE**

MOTION OF CHARGE IN COMBINED ELECTRIC FIELD & MAGNETIC FIELD

- When $\vec{v} \parallel \vec{B}$ & $\vec{v} \parallel \vec{E}$, motion will be uniformly accelerated in straight line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$
  So the particle will be either speeding up or speeding down
- When $\vec{v} \parallel \vec{B}$ & $\vec{v} \perp \vec{E}$, motion will be uniformly accelerated in a parabolic path
- When $\vec{v} \perp \vec{B}$ & $\vec{v} \perp \vec{E}$, the particle may more undeflected & undeivated with same uniform speed if $v = \frac{E}{B}$ (This is called as velocity selector condition)

MAGNETIC FORCE ON A STRAIGHT CURRENT CARRYING WIRE : $\vec{F} = I(\vec{L} \times \vec{B})$

I = current in the straight conductor

$\vec{L}$ = length of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction. (Uniform throughout the length of conductor)

Note : In general force is $\vec{F} = \int I(d\vec{\ell} \times \vec{B})$

MAGNETIC INTERACTION FORCE BETWEEN TWO PARALLEL LONG STRAIGHT CURRENTS :

When two long straight linear conductors are parallel and carry a current in each , they magnetically interact with each other, one experiences a force. This force is of:

(i) Repulsion if the currents are anti-parallel (i.e. in opposite direction) or

(ii) Attraction if the currents are parallel (i.e. in the same direction)

This force per unit length on either conductor is given by $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors

MAGNETIC TORQUE ON A CLOSED CURRENT CIRCUIT :

When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field , it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B} = BINA\sin\theta$

where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field.

$\vec{M}$ = magnetic moment of the current circuit = $IN\vec{A}$

Note : This expression can be used only if $\vec{B}$ is uniform otherwise calculus will be used.

MOVING COIL GALVANOMETER :

It consists of a plane coil of many turns suspended in a radial magnetic feild. When a current is passed in the coil it experiences a torque which produces a twist in the suspension.

This deflection is directly proportional to the torque $\therefore NIAB = K\theta$

$I = \left(\frac{K}{NAB}\right)\theta$; K = elastic torsional constant of the suspension

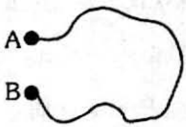
$I = C\theta$ $\quad C = \frac{K}{NAB}$ = Galvanometer Constant

FORCE EXPERIENCED BY A MAGNETIC DIPOLE IN A NON-UNIFORM MAGNETIC FIELD :

$$|\vec{F}| = \left|M\frac{\partial B}{\partial r}\right|$$

where M = Magnetic dipole moment.

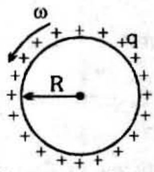
FORCE ON A RANDOM SHAPED CONDUCTOR IN A UNIFORM MAGNETIC FIELD



- Magnetic force on a closed loop in a uniform $\vec{B}$ is zero
- Force experienced by a wire of any shape is equivalent to force on a wire joining points A & B in a uniform magnetic field.

MAGNETIC MOMENT OF A ROTATING CHARGE:

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ & its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



NOTE: The ratio of magnetic moment to angular momentum of a uniform rotating object which is charged uniformly is always a constant. Irrespective of the shape of conductor M/L = q/2m

- **Magnetic dipole**
  - Magnetic moment $M = m \times 2\ell$ where m = pole strength of the magnet
  - Magnetic field at axial point (or End-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
  - Magnetic field at equatorial position (Broad-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{\left(-\vec{M}\right)}{r^3}$
  - At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.

Magnetic Potential $V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$

Magnetic field $B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$

- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M} \times \vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M}\cdot\vec{B}$

- **Intensity of magnetisation** $I = M/V$
- **Magnetic induction** $B = \mu H = \mu_0(H + I)$
- **Magnetic permeability** $\mu = \frac{B}{H}$
- **Magnetic susceptibility** $\chi_m = \frac{I}{H} = \mu - 1$
- **Curie law**
  - For paramagnetic materials $\chi_m \propto \frac{1}{T}$
- **Curie Wiess law**
  - For Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$

    Where T_C = curie temperature

KEY POINTS

- A charged particle moves perpendicular to magnetic field. Its kinetic energy will remain constant but momentum changes because magnetic force acts perpendicular to velocity of particle.
- If a unit north pole rotates around a current carrying wire then work has to be done because magnetic field produced by current is always non-conservative in nature.
- In a conductor, free electrons keep on moving but no magnetic force acts on a conductor in a magnetic field because in a conductor, the average thermal velocity of electrons is zero.
- Magnetic force between two charges is generally much smaller than the electric force between them because speeds of charges are much smaller than the free space speed of light.

Note : $\frac{F_{magnetic}}{F_{electric}} = \frac{v^2}{c^2}$

Important Notes

19 Electromagnetic Induction

MAGNETIC FLUX :

$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$ for uniform $\vec{B}$.

$\phi = \int \vec{B}\cdot d\vec{A}$ for non uniform $\vec{B}$.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION :

(i) An induced emf is setup whenever the magnetic flux linking that circuit changes.

(ii) The magnitude of the induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit, $\varepsilon \propto \frac{d\varphi}{dt}$.

LENZ'S LAWS :

The direction of an induced emf is always such as to oppose the cause producing it .

LAW OF EMI : $e = -\frac{d\varphi}{dt}$.

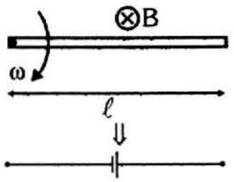
The negative sign indicates that the induced emf opposes the change of the flux.

EMF INDUCED IN A STRAIGHT CONDUCTOR IN UNIFORM MAGNETIC FIELD :

$$E = BLv\sin\theta$$

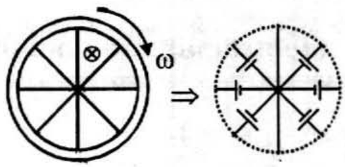
where B = flux density
L = length of the conductor
v = velocity of the conductor
θ = angle between direction of motion of conductor & B .

EMF INDUCED IN A ROD ROTATING PERPENDICULAR TO MAGNETIC FIELD



$$E = \frac{1}{2}B\omega\ell^2$$

For a wheel rotating in a earth magnetic field effective emf induced between the periphery & centre = $\frac{1}{2}B\omega\ell^2$



COIL ROTATION IN MAGNETIC FIELD SUCH THAT AXIS OF ROTATION IS PERPENDICULAR TO THE MAGNETIC FIELD :

Instantaneous induced emf. $\omega \sin E \omega t = E_0 \sin \omega t$

where N = number of turns in the coil

A = area of one turn

B = magnetic induction

ω = uniform angular velocity of the coil

E_0 = maximum induced emf

SELF INDUCTION & SELF INDUCTANCE :

When a current flowing through a coil is changed the flux linking with its own winding changes & due to the change in linking flux with the coil an emf is induced which is known as self induced emf & this phenomenon is known as self induction . This induced emf opposes the causes of Induction. The property of the coil or the circuit due to which it opposes any change of the current coil or the circuit is known as **SELF - INDUCTANCE** . It's unit is Henry.

Coefficient of Self inductance $L = \frac{\varphi_s}{i}$ or $\phi_s = Li$

i = current in the circuit .

ϕ_s = magnetic flux linked with the circuit due to the current i .

L depends only on ; **(i)** shape of the loop & **(ii)** medium

self induced emf $e_s = \frac{d\varphi_s}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$ (if L is constant)

Combination of inductors

- Series combination $L = L_1 + L_2 + \ldots\ldots\ldots$, i same, V in ratio of inductance, U in ratio of inductance, ϕ in ratio of inductance

- Parallel combination $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots\ldots$, V same, i in inverse ratio of inductance, U in inverse ratio of inductance, ϕ same

MUTUAL INDUCTION :

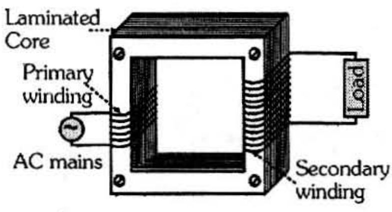
If two electric circuits are such that the magnetic field due to a current in one is partly or wholly linked with the other, the two coils are said to be electromagnetically coupled circuits . Then any change of current in one produces a change of magnetic flux in the other & the later opposes the change by inducing an emf within itself . This phenomenon is called **Mutual Induction** & the induced emf in the later circuit due to a change of current in the former is called **Mutually Induced EMF** . The circuit in which the current is changed, is called the primary & the other circuit in which the emf is induced is called the secondary. The co-efficient of mutual induction (mutual inductance) between two electromagnetically coupled circuit is the magnetic flux linked with the secondary per unit current in the primary.

Mutual inductance = $M = \frac{\varphi_m}{I_p} = \frac{\text{flux linked with secondary}}{\text{current in the primary}}$

mutually induced emf : $E_m = \frac{d\varphi_m}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$ (If M is constant)

M depends on (1) geometry of loops (2) medium (3) orientation & distance of loops .

- **If two coils of self inductance L_1 and L_2 are wound over each other, the mutual inductance** $M = K\sqrt{L_1L_2}$ where K is called coupling constant.
- **For two coils wound in same direction and connected in series**
  $L = L_1 + L_2 + 2M$
- **For two coils wound in opposite direction and connected in series**
  $$L = L_1 + L_2 - 2M$$
- **For two coils in parallel** $L = \frac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M}$
- **Transformer**

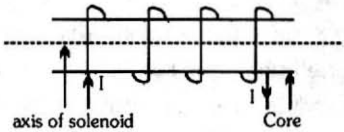


$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

❑ For ideal transformer $\frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$ ❑ Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

SOLENOID :

There is a uniform magnetic field along the axis of the solenoid



$B = \mu ni$ (ideal : length >> diameter)

where μ = magnetic permeability of the core material

n = number of turns in the solenoid per unit length

i = current in the solenoid

Self inductance of a solenoid $L = \mu_0 n^2 A l$

A = area of cross section of solenoid .

SUPER CONDUCTION LOOP IN MAGNETIC FIELD :

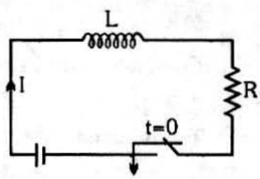
$R = 0$;$\varepsilon = 0$. Therefore ϕ_{total} = constant. Thus in a superconducting loop flux never changes. (or it opposes 100%)

(i) **ENERGY STORED IN AN INDUCTOR** : $W = \frac{1}{2} LI^2$.

(ii) Energy of interaction of two loops $U = I_1\phi_2 = I_2\phi_1 = MI_1I_2$
where M is mutual inductance

GROWTH OF A CURRENT IN AN L – R CIRCUIT :

$I = \frac{E}{R}\,(1 - e^{-Rt/L})$. [If initial current = 0]



$\frac{L}{R}$ = time constant of the circuit.

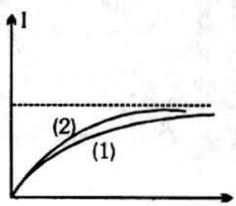
$I_0 = \frac{E}{R}$.

(i) L behaves as open circuit at t = 0 [If $i = 0$]

(ii) L behaves as short circuit at $t = \infty$ always.

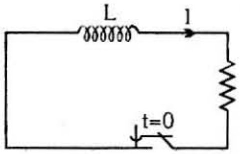
Curve (1) $\longrightarrow \frac{L}{R}$ Large

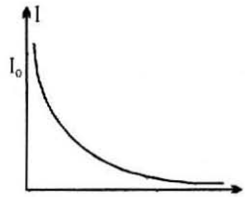
Curve (2) $\longrightarrow \frac{L}{R}$ Small



DECAY OF CURRENT :

Initial current through the inductor = I_0 ; Current at any instant $i = I_0 e^{-Rt/L}$





KEY POINTS

- An emf is induced in a closed loop where magnetic flux is varied. The induced electric field is not conservative field because for induced electric field, the line integral $\oint \vec{E}.d\vec{\ell}$ around a closed path is non-zero.
- Acceleration of a magnet falling through a long solenoid decrease because the induced current produced in a circuit always flows in such direction that it opposes the change or the cause that produces it.
- The mutual inductance of two coils is doubled if the self inductance of the primary and secondary coil is doubled because mutual inductance $M \propto \sqrt{L_1 L_2}$.
- The possibility of an electric bulb fusing is higher at the time of switching ON and OFF because inductive effects produce a surge at the time of switch-off and switch-on.
- Motional emf : If a conductor is moved in a magnetic field then motional emf will be $E = B\ell_{eff}v$

  Here $v \perp \ell_{eff}$ & $v \perp B$ & $B \perp \ell_{eff}$

  $\ell_{eff} \rightarrow$ effective length between the end points of conductor which is perpendicular to the velocity.

Important Notes

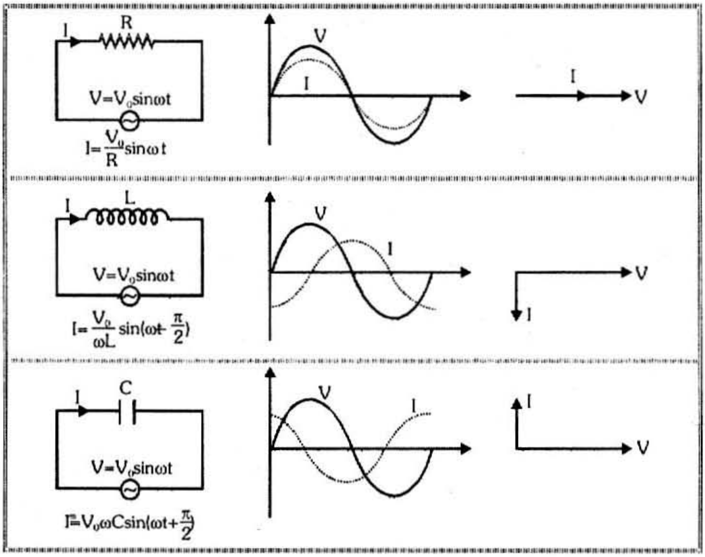
20 Alternating Current and EM Waves

- **Average value** $I_{av} = \dfrac{\int_0^T I\,dt}{\int_0^T dt} = \dfrac{1}{T}\int_0^T I\,dt$ • **RMS value** $I_{rms} = \sqrt{\dfrac{\int_0^T I^2 dt}{\int_0^T dt}}$

- For sinusoidal voltage $V = V_0 \sin\omega t$: $V_{av} = \dfrac{2V_0}{\pi}$ & $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

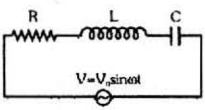
  For sinusoidal current $I = I_0 \sin(\omega t+\phi)$: $I_{av} = \dfrac{2I_0}{\pi}$ & $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

- **AC Circuits**

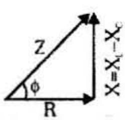


- **Impedance :** $Z = \sqrt{R^2 + X^2}$ where X = reactance

- **Series LCR Circuit**

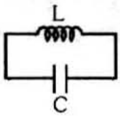


$Z = \sqrt{R^2 + (X_L - X_C)^2}$



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

- **Power Factor** = $\cos\phi = R/Z$ At resonance : $X_L = X_C \Rightarrow Z = R,\ V = V_R$

- **LC Oscillation** 

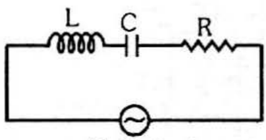
$q = q_0 \sin(\omega t + \theta),\ I = I_0\cos(\omega t + \theta)$ $\boxed{I_0 = q_0\omega}$

Energy = $\frac{1}{2}LI^2 + \frac{q^2}{2C} = \frac{q_0^2}{2C} = \frac{1}{2}LI_0^2$ = constant

Comparison with SHM $q \rightarrow x,\ I \rightarrow v,\ L \rightarrow m,\ C \rightarrow \frac{1}{K}$

Comparison of Damped Mechanical & electrical systems

- **(I) Series LCR circuit :**



$V = V_0 \sin\omega t$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}\cos\omega t$$

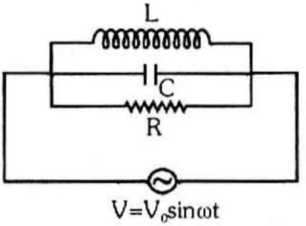
compare with mechanical damped system equation

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

where b = damping coefficient.

Mechanical system	Electrical systems (series RLC)
Displacement (x)	Charge (q)
Driving force (F)	Driving voltage (V)
Kinetic energy $\left(\frac{1}{2}mv^2\right)$	Electromagnetic energy of moving charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 = \frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Energy of static charge $\frac{q^2}{2C}$
mass (m)	L
Power P =Fv	Power P=VI
Damping (b)	Resistance (R)
Spring constant	1/C

- **(II) Parallel LCR circuit :** In this case



$$I = I_L + I_C + I_R = \frac{\phi}{L} + \frac{d}{dt}C\left(\frac{d\phi}{dt}\right) + \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \frac{d^2\phi}{dt^2} + \frac{1}{RC}\frac{d\phi}{dt} + \frac{1}{LC}\phi = \frac{V_0}{ZC}\sin\omega t$$

Displacement **(x)**	$\Longleftrightarrow$	Flux linkage **(ϕ)**
Velocity $\left(\frac{dx}{dt}\right)$	$\Longleftrightarrow$	Voltage $\left(\frac{d\phi}{dt}\right)$
Mass **(m)**	$\Longleftrightarrow$	Capacitance **(C)**
Spring constant **(k)**	$\Longleftrightarrow$	Reciprocal Industance **(1/L)**
Damping coefficient **(b)**	$\Longleftrightarrow$	Reciprocal resistance **(1/R)**
Driving force **(F)**	$\Longleftrightarrow$	Current **(i)**

- **Properties of EM Waves**
  - The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is travelling. Thus the em wave is a transverse wave.
  - EM waves carry momentum and energy.
  - EM wave travel through vacuum with the speed of light c, where

    $$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$
  - The instantaneous magnitude of $\vec{E}$ and $\vec{B}$ in an EM wave are related by the expression $\frac{E}{B} = c$
  - The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.
- **Poynting Vector** : The rate of flow of energy crossing a unit area by electro-magnetic radiation is given by poynting vector $\vec{S}$ where $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$
- **Displacement current :** In a region of space in which there is a changing electric field, there is a displacement current defined as $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ where ε_0 is the permittivity of free space and $\phi_E = \int \vec{E} \cdot d\vec{S}$ is the electric flux.

- **Maxwell's Equations**

$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$ [Gauss law for electricity]

$\oint \vec{B} \cdot d\vec{S} = 0$ [Gauss law for magnetism]

$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt}$ [Faraday's law]

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left[I_C + \epsilon_0 \frac{d\phi_E}{dt} \right]$ [Ampere's law with Maxwell's correction]

KEY POINTS

- An alternating current of frequency 50 Hz becomes zero, 100 times in one second because alternating current changes direction and becomes zero twice in a cycle.
- An alternating current cannot be used to conduct electrolysis because the ions due to their inertia, cannot follow the changing electric field.
- Average value of AC is always defined over half cycle because average value of AC over a complete cycle is always zero.
- AC current flows on the periphery of wire instead of flowing through total volume of wire. This known as skin effect.

Important Notes

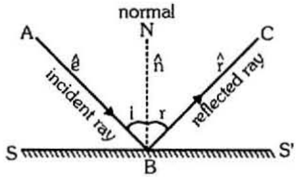
21 Ray Optics and Optical Instruments

REFLECTION

LAWS OF REFLECTION :

The incident ray (AB), the reflected ray (BC) and normal (NB) to the surface (SS') of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also plane of reflection).

The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal $\boxed{\angle i = \angle r}$



In vector form $\boxed{\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}}$

OBJECT :

Real : Point from which rays actually diverge.

Virtual: Point towards which rays appear to converge

IMAGE :

Image is decided by reflected or refracted rays only. The point image for a mirror is that point towards which the rays reflected from the mirror, actually converge (real image).

OR

From which the reflected rays appear to diverge (virtual image) .

CHARACTERISTICS OF REFLECTION BY A PLANE MIRROR :

The size of the image is the same as that of the object.

For a real object the image is virtual and for a virtual object the image is real.

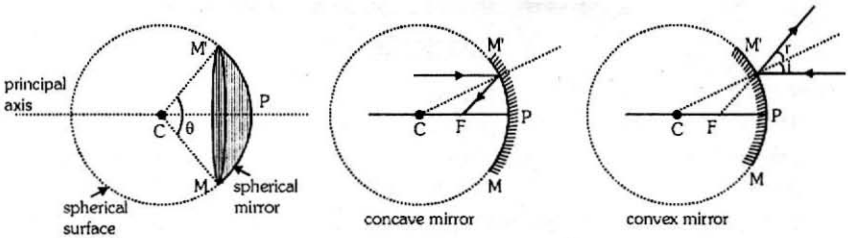
For a fixed incident light ray, if the mirror be rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.

Number of images (n) in inclined mirror Find $\frac{360}{\theta} = m$

- If m even, then n = m – 1, for all positions of object.
- If m odd , then n = m, If object not on bisector
  and n = m – 1, If object at bisector

- If m fraction then n = nearest even number

SPHERICAL MIRRORS :



PARAXIAL RAYS :

Rays which forms very small angle with axis are called paraxial rays. All formulae are valid for paraxial ray only.

SIGN CONVENTION :

- We follow cartesian co-ordinate system convention according to which the pole of the mirror is the origin.
- The direction of the incident rays is considered as positive x-axis. Vertically up is positive y-axis.
- All distance are measured from pole.

Note : According to above convention radius of curvature and focus of concave mirror is negative and of convex mirror is positive.

MIRROR FORMULA : $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$.

f = x- coordinate of focus   u = x-coordinate of object

v = x-coordinate of image   **Note :** Valid only for paraxial rays.

TRANSVERSE MAGNIFICATION : $m=\frac{h_2}{h_1}=-\frac{v}{u}$

h_2 = y co-ordinate of image h_1 = y co-ordinate of the object

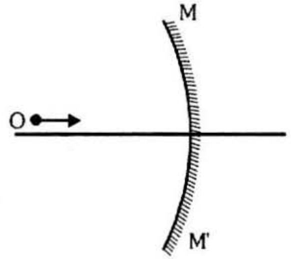
(both perpendicular to the principal axis of mirror)

Longitudinal magnification : m_2 $m_2 = \frac{\text{Length of image}}{\text{Length of object}}$

for small objects $m_2 = -m_t^2$ m_t = transverse magnification.

VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)

Velocity component along axis (Longitudinal velocity)



When an object is coming from infinite towards the focus of concave mirror

$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\quad \therefore -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \Rightarrow \vec{v}_{IM}=-\frac{v^2}{u^2}\vec{v}_{ox}=-m^2\vec{v}_{OM}$

- $v_{IM}=\frac{dv}{dt}$ = velocity of image with respect to mirror
- $v_{OM}=\frac{du}{dt}$ = velocity of object with respect to mirror.

NEWTON'S FORMULA :

Applicable to a pair of real object and real image position only . They are called conjugate positions or foci, X_1, X_2 are the distance along the principal axis of the real object and real image respectively from the principal focus

$$\boxed{X_1X_2=f^2}$$

OPTICAL POWER : Optical power of a mirror (in Diopters) = $-\frac{1}{f}$

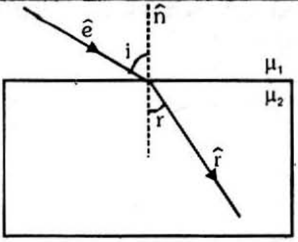
where f = focal length (in meters) with sign .

REFRACTION -PLANE SURFACE

LAWS OF REFRACTION (AT ANY REFRACTING SURFACE)

- **Laws of Refraction**

(i) Incident ray, refracted ray and normal always lie in the same plane .



In vector form $\boxed{(\hat{e}\times\hat{n}).\hat{r}=0}$

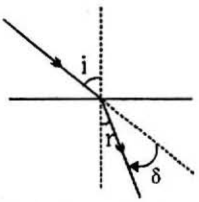
(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

Snell's law :

$\boxed{\dfrac{\text{Sin i}}{\text{Sin r}} = {}_1n_2 = \dfrac{n_2}{n_1} = \dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2}}$ In vector form $\boxed{\mu_1|\hat{\mathbf{e}}\times\hat{\mathbf{n}}| = \mu_2|\hat{\mathbf{r}}\times\hat{\mathbf{n}}|}$

Note : Frequency of light does not change during refraction .

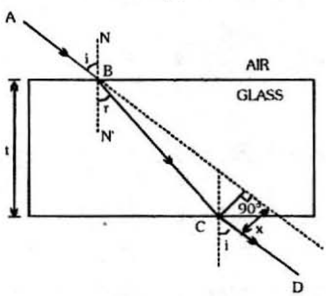
DEVIATION OF A RAY DUE TO REFRACTION



$\boxed{\text{angle of deviation, } \delta = i - r}$

REFRACTION THROUGH A PARALLEL SLAB :

Emerged ray is parallel to the incident ray, if medium is same on both sides.

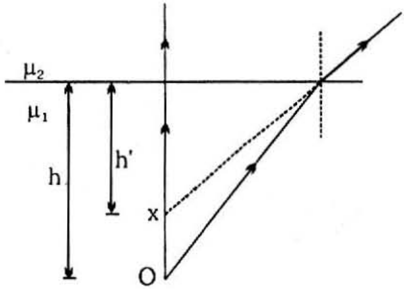


Lateral shift $x = \dfrac{t\sin(i-r)}{\cos r}$; t = thickness of slab

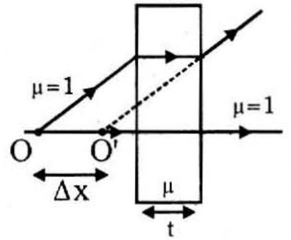
Note : Emerged ray will not be parallel to the incident ray if the medium on both the sides are different.

APPARENT DEPTH OF SUBMERGED OBJECT : (h′ < h)

$\mu_1 > \mu_2$



For near normal incidence $h' = \frac{\mu_2}{\mu_1} h$

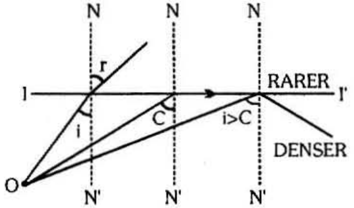


Δx = Apparent shift = $t\left(1-\frac{1}{\mu}\right)$

* always in direction of incidence ray.

Note : h and h' are always measured from surface.

CRITICAL ANGLE & TOTAL INTERNAL REFLECTION (TIR)



Conditions of TIR

- Ray is going from denser to rarer medium
- Angle of incidence should be greater than the critical angle (i > C) .

Critical angle $C = \sin^{-1}\frac{\mu_R}{\mu_D} = \sin^{-1}\frac{v_D}{v_R} = \sin^{-1}\frac{\lambda_D}{\lambda_R}$

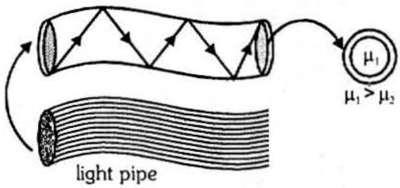
Some Illustrations of Total Internal Reflection

- **Sparkling of diamond**

  The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so C= 24°. Now the cutting of diamond are such that i > C. So TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.

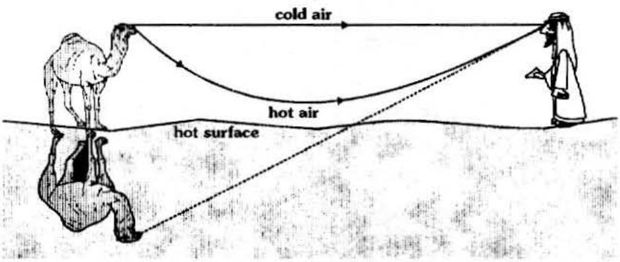
- **Optical Fibre**

  In it light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings (cladding)

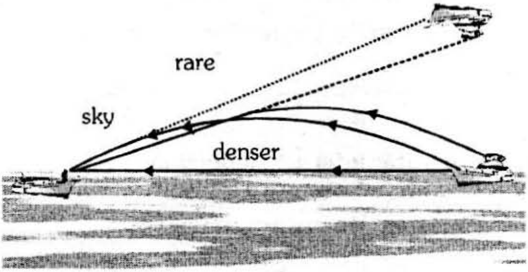


- **Mirage and looming**

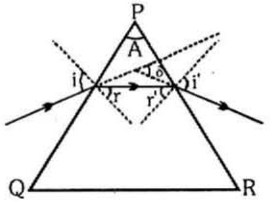
  Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_C$ so that TIR will take place and we see the image of an object along with the object as shown in figure.



Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here μ decreases with height and so the image of an object is formed in air if (i>C) as shown in figure.

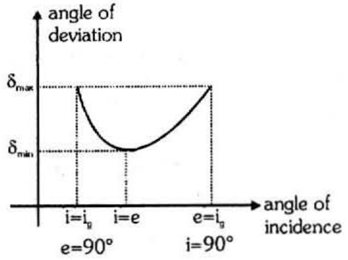


REFRACTION THROUGH PRISM :



- $\delta = (i + i') - (r + r')$ • $r + r' = A$

- Variation of δ versus i

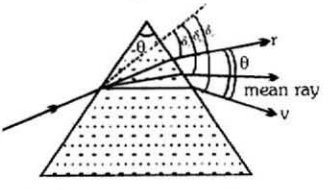


- There is one and only one angle of incidence for which the angle of deviation is minimum. When $\delta = \delta_m$ then $i = i'$ & $r = r'$, the ray passes symetrically about the prism, & then $n = \dfrac{\sin\left[\dfrac{A+\delta m}{2}\right]}{\sin\left[\dfrac{A}{2}\right]}$, where n = absolute R.I. of glass.

  Note : When the prism is dipped in a medium then n = R.I. of glass w.r.t. medium .

- For a thin prism (A ≤10°) ; $\delta=(n-1)A$
- **Dispersion Of Light** : The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called ***Dispersion of Light.***
- **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion. $\theta = \delta_v - \delta_r$.
- Dispersive power (ω) of the medium of the material of prism .

$$\omega = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$



For small angled prism ($A \le 10°$); $\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n-1}$; $n = \frac{n_v + n_R}{2}$

n_v, n_R & n are R. I. of material for violet, red & yellow colours respectively .

- **Combination Of Two Prisms :**

  Achromatic Combination : It is used for deviation without dispersion . Condition for this $(n_v - n_r) A = -(n'_v - n'_r) A'$.

  Net mean deviation $= \left[\frac{n_v + n_R}{2} - 1\right]A - \left[\frac{n'_v + n'_R}{2} - 1\right]A'$ or $\omega\delta + \omega'\delta' = 0$ where

  ω, ω' are dispersive powers for the two prisms & δ , δ' are the mean deviation.

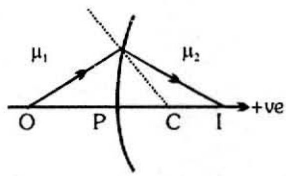
  Direct Vision Combination : It is used for producing disperion without deviation

  condition for this $\left[\frac{n_v + n_R}{2} - 1\right]A = -\left[\frac{n'_v + n'_R}{2} - 1\right]A'$

  Net angle of dispersion $= (n_v - n_r) A - (n_v' - n_r') A'$.

REFRACTION AT SPHERICAL SURFACE

(a) $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



v, u & R are to be kept with sign as

v = PI

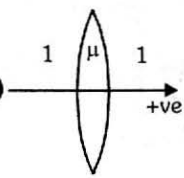
u = –PO

R = PC

(Note : Radius is with sign)

(b) $m = \frac{\mu_1 v}{\mu_2 u}$

Lens Formula :

(a) 

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ **(b)** $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ **(c)** $m = \frac{v}{u}$

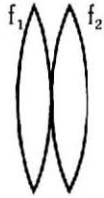
Power of Lenses

Reciprocal of focal length in meter is known as power of lens.

SI unit : dioptre (D) **Power of lens :** $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$ dioptre

Combination of Lenses

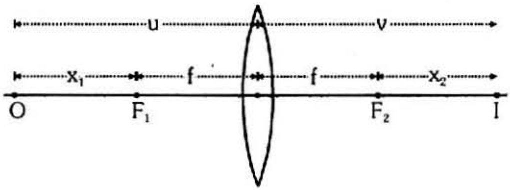
Two thin lens are placed in contact to each other



Power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention when solve numericals

Newton's Formula

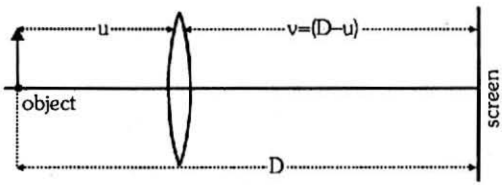


$\boxed{f = \sqrt{x_1 x_2}}$

x_1 = distance of object from focus; x_2 = distance of image from focus.

Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart.



(i) For D<4f: u will be imaginary hence physically no position of lens is possible

(ii) **For D = 4f :** $u = \frac{D}{2} = 2f$ so only one position of lens is possible and since

$v = D - u = 4f - 2f = u = 2f$

(iii) **For D > 4f :** $u_1 = \frac{D - \sqrt{D(D-4f)}}{2}$ and $u_2 = \frac{D + \sqrt{D(D-4f)}}{2}$

So there are two positions of lens for which real image will be formed on the screen.

For two positions of the lens distances of object and image are interchangeable.

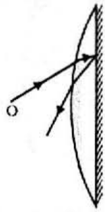
so $u_1 = \frac{D - x}{2} = v_2$ and $v_1 = \frac{D + x}{2} = u_2$

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{D+x}{D-x} \text{ and } m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{D-x}{D+x}$$

$$\text{Now } m_1 \times m_2 = \frac{D+x}{D-x} \times \frac{D-x}{D+x} \Rightarrow \frac{I_1 I_2}{O^2} = 1 \Rightarrow \boxed{O = \sqrt{I_1 I_2}}$$

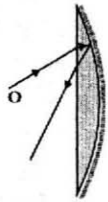
- **Silvering of one surface of lens** [use $P_{eq} = 2P_l + P_m$]

❒ When plane surface is silvered



$$F = \frac{R}{2(\mu - 1)}$$

❒ When convex surface is silvered



$$F = \frac{R}{2\mu}$$

OPTICAL INSTRUMENTS

- **For Simple microscope**
  - ❒ Magnifying power when image is formed at D : MP = 1 + D/f
  - ❒ When image is formed at infinity MP = D/f

- **For Compound microscope** : $MP = -\frac{v_0}{u_0}\left(\frac{D}{u_e}\right)$

❒ Magnifying power when final image is formed at D, $MP = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

❒ Tube length $L = v_0 + |u_e|$

❒ When final image is formed at infinity $MP = -\frac{v_0}{u_0} \times \frac{D}{f_e}$ and $L = v_0 + f_e$

- **Astronomical Telescope :** $MP = -\frac{f_0}{u_e}$

  - Magnifying power when final image is formed at D: $MP = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$
  - Tube length : $L = f_0 + |u_e|$
  - When final image is formed at infinity : $MP = -\frac{f_0}{f_e}$ and $L = f_0 + f_e$

- **For terrestrial telescope** : $MP = \frac{f_0}{f_e}$ and $L = f_0 + f_e + 4f$

- **For Galilean telescope** : $MP = \frac{f_0}{f_e}$ & $L = f_0 - f_e$

- **Lens camera** : Time of exposure $\propto \frac{1}{(\text{aperture})^2}$, f – number = $\frac{\text{focal length}}{\text{aperture}}$

- **For myopia or short-sightedness or near sightedness** $\frac{1}{\text{F.P.}} - \frac{1}{\text{object}} = \frac{1}{f} = P$

  $f = -\text{F.P.}$

- **For long - sightedness or hypermetropia** $\frac{1}{\text{N.P.}} - \frac{1}{\text{object}} = \frac{1}{f} = P$

- **Limit of resolution for microscope** $= \frac{1.22\lambda}{2a\sin\theta} = \frac{1}{\text{resolving power}}$

- **Limit of resolution for telescope** $= \frac{1.22\lambda}{a} = \frac{1}{\text{resolving power}}$

KEY POINTS :

- For observing traffic at our back we prefer to use a convex mirror because a convex mirror has a more larger field of view than a plane mirror or concave mirror.
- A ray incident along normal to a mirror retraces its path because in reflection angle of incidence is always equal to angle of reflection.
- Images formed by mirrors do not show chromatic aberration because focal length of mirror is independent of wavelength of light and refractive index of medium.

- Light from an object falls on a concave mirror forming a real image of the object. If both the object and mirror are immersed in water, there is no change in the position of image because the formation of image by reflection does not depend on surrounding medium, there is no change in position of image provided it is also formed in water.
- The images formed by total internal reflections are much brighter than those formed by mirrors and lenses because there is no loss of intensity in total internal reflection.
- A fish inside a pond will see a person outside taller than he is actually because light bend away from the normal as it enters water from air.
- A fish in water at depth h sees the whole outside world in horizontal circle of radius.

$$r = h \tan\theta_C = \frac{h}{\sqrt{\mu^2 - 1}}$$

- Just before setting, the Sun may appear to be elliptical due to refraction because refraction of light ray through the atmosphere may cause different magnification in mutually perpendicular directions.
- A lens have two principal focal lengths which may differ because light can fall on either surface of the lens. The two principal focal lengths differ when medium on two sides have different refractive indices.
- A convex lens behaves as a concave lens when placed in a medium of refractive index greater than the refractive index of its material because light in that case will travel through the convex lens from denser to rarer medium. It will bend away from the normal, i.e., the convex lens would diverge the rays.
- If lower half of a lens is covered with a black paper, the full image of the object is formed because every portion of lens forms the full image of the object but sharpness of image decrease.
- Sun glasses have zero power even though their surfaces are curved because both the surfaces of the Sun glasses are curved in the same direction with same radii.