Dimensions of Important Physical Quantities

Physical quantity	Dimensions	Physical quantity	Dimensions
Momentum	$M^1 L^1 T^{-1}$	Capacitance	$M^{-1} L^{-2} T^4 A^2$
Calorie	$M^1 L^2 T^{-2}$	Modulus of rigidity	$M^1 L^{-1} T^{-2}$
Latent heat capacity	$M^0 L^2 T^{-2}$	Magnetic permeability	$M^1 L^1 T^{-2} A^{-2}$
Self inductance	$M^1 L^2 T^{-2} A^{-2}$	Pressure	$M^1 L^{-1} T^{-2}$
Coefficient of thermal conductivity	$M^1 L^1 T^{-3} K^{-1}$	Planck's constant	$M^1 L^2 T^{-1}$
Power	$M^1 L^2 T^{-3}$	Solar constant	$M^1 L^0 T^{-3}$
Impulse	$M^1 L^1 T^{-1}$	Magnetic flux	$M^1 L^2 T^{-2} A^{-1}$
Hole mobility in a semi conductor	$M^{-1} L^0 T^2 A^1$	Current density	$M^0 L^{-2} T^0 A^1$
Bulk modulus of elasticity	$M^1 L^{-1} T^{-2}$	Young modulus	$M^1 L^{-1} T^{-2}$
Potential energy	$M^1 L^2 T^{-2}$	Magnetic field intensity	$M^0 L^{-1} T^0 A^1$
Gravitational constant	$M^{-1} L^3 T^{-2}$	Magnetic Induction	$M^1 T^{-2} A^{-1}$
Light year	$M^0 L^1 T^0$	Permittivity	$M^{-1} L^{-3} T^4 A^2$
Thermal resistance	$M^{-1} L^{-2} T^3 K$	Electric Field	$M^1 L^1 T^{-3} A^{-1}$
Coefficient of viscosity	$M^1 L^{-1} T^{-1}$	Resistance	$M L^2 T^{-3} A^{-2}$

Sets of Quantities having same dimensions

S.N.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass and inertia	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-2}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat capacity and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2T^{-2}K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2Rt), $\frac{V^2}{R}t$, (VIt), (PV), (RT), (mL), $(mc\,\Delta T)$	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\frac{R}{L}, \frac{1}{RC}, \frac{1}{\sqrt{LC}}$	$[M^0 L^0 T^{-1}]$
13.	$\left(\frac{\ell}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), (\sqrt{LC})$, time	$[M^0 L^0 T^1]$
14.	(VI), (I^2R), (V^2/R), Power	$[M L^2 T^{-3}]$

Some Fundamental Constants

Gravitational constant (G)	6.67×10^{-11} N m^2 kg^{-2}
Speed of light in vacuum (c)	3×10^8 ms^{-1}
Permeability of vacuum (μ_0)	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of vacuum (ε_0)	8.85×10^{-12} F m^{-1}
Planck constant (h)	6.63×10^{-34} Js
Atomic mass unit (amu)	1.66×10^{-27} kg
Energy equivalent of 1 amu	931.5 MeV
Electron rest mass (m_e)	9.1×10^{-31} kg $\equiv$ 0.511 MeV
Avogadro constant (N_A)	6.02×10^{23} mol^{-1}
Faraday constant (F)	9.648×10^4 C mol^{-1}
Stefan–Boltzmann constant (σ)	5.67×10^{-8} W m^{-2} K^{-4}
Wien constant (b)	2.89×10^{-3} mK
Rydberg constant (R_∞)	1.097×10^7 m^{-1}
Triple point for water	273.16 K (0.01°C)
Molar volume of ideal gas (NTP)	22.4 L = 22.4×10^{-3} m^3 mol^{-1}

KEY POINTS

- Trigonometric functions sinθ, cosθ, tanθ etc and their arrangements θ are dimensionless.
- Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$
- Dimensions of integrals $\left[\int y\,dx\right] = [yx]$
- We can't add or subtract two physical quantities of different dimensions.
- Independent quantities may be taken as fundamental quantities in a new system of units.

PRACTICAL PHYSICS

- **Rules for Counting Significant Figures**

  For a number greater than 1
  - All non-zero digits are significant.
  - All zeros between two non-zero digits are significant. Location of decimal does not matter.
  - If the numbe is without decimal part, then the terminal or trailing zeros are not significant.
  - Trailing zeros in the decimal part are significant.

- **For a Number Less Than 1**

  Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant.
- **Significant Figures**

  All accurately known digits in measurement plus the first uncertain digit together form significant figure.

  Ex. $0.108 \rightarrow 3SF$, $40.000 \rightarrow 5SF$,

  $1.23 \times 10^{-19} \rightarrow 3SF$, $0.0018 \rightarrow 2SF$
- **Rounding off**

  $6.87 \rightarrow 6.9$, $6.84 \rightarrow 6.8$, $6.85 \rightarrow 6.8$,

  $6.75 \rightarrow 6.8$, $6.65 \rightarrow 6.6$, $6.95 \rightarrow 7.0$
- **Order of magnitude** :

  Power of 10 required to represent a quantity

  $49 = 4.9 \times 10^1 \approx 10^1 \Rightarrow$ order of magnitude $= 1$

  $51 = 5.1 \times 10^1 \approx 10^2 \Rightarrow$ order of magnitude $= 2$

  $0.051 = 5.1 \times 10^{-2} \approx 10^{-1} \Rightarrow$ order of magnitude $= -1$
- **Propagation of combination of errors**

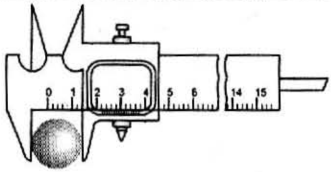
  Error in Summation and Difference : $x = a + b$ then $\Delta x = \pm(\Delta a + \Delta b)$
- **Error in Product and Division** A physical quantity X depend upon Y & Z as $X = Y^a Z^b$ then maximum possible fractional error in X.

$$\frac{\Delta X}{X} = |a|\frac{\Delta Y}{Y} + |b|\frac{\Delta Z}{Z}$$

- **Error in Power of a Quantity** : $x = \frac{a^m}{b^n}$ then $\frac{\Delta x}{x} = \pm\left[m\left(\frac{\Delta a}{a}\right) + n\left(\frac{\Delta b}{b}\right)\right]$

- **Least count** : The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count of the measuring instrument.
- **Vernier Callipers** Least count = 1MSD – 1 VSD

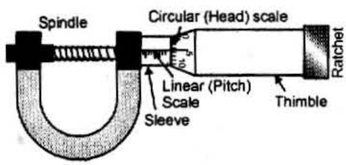
  (MSD → main scale division, VSD → Vernier scale division)



Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then least count = $1 \text{ mm} - \frac{9}{10} \text{ mm} = 0.1 \text{ mm}$
$[\because 9 \text{ MSD} = 10 \text{ VSD}]$

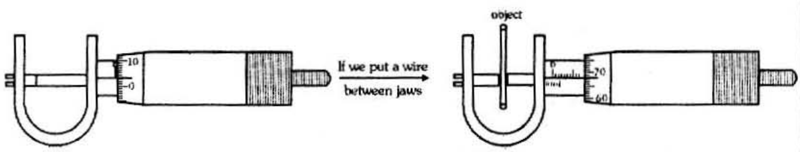
- **Screw Gauge** :

$$\text{Least count} = \frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$$



Zero Error :

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.



Excess reading = 0.07 mm (zero error)

It gives 3.07 mm

In which there is 0.07 mm excess reading which has to be subtracted.

so actual thickness = 3.07 – 0.07 = 3.00 mm

Observed (reading) — excess reading (zero error)

Actual reading = observed reading – excess reading (zero error)

Excess reading = 0.07 mm (zero error)

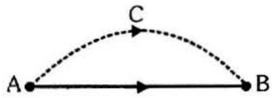
Ex. The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the humble is provided with a angular scale carrying 100 equal divisions. The least count = $\frac{1\text{mm}}{100} = 0.01 \text{ mm}$

Important Notes

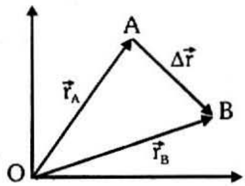
4 Kinematics

- **Distance and Displacement**

  Total length of path (ACB) covered by the particle, in definite time interval is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.



- **Displacement in terms of position vector**



From ΔOAB $\quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

- **Average velocity** $= \dfrac{\text{Displacement}}{\text{Time interval}} = \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

- **Average speed** $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

- **For uniform motion**

  Average speed $= |\text{average velocity}| = |\text{instataneous velocity}|$

- **Velocity** $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

- **Average Acceleration** $= \dfrac{\text{total change in velocity}}{\text{total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

- **Acceleration**

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

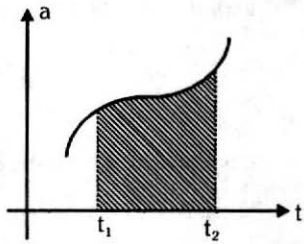
Important points about 1D motion

- Distance $\geq |\text{displacement}|$ and Average speed $\geq |\text{average velocity}|$
- If distance $> |\text{displacement}|$ this implies

  (a) atleast at one point in path, velocity is zero.

(b) The body must have retarded during the motion

- Acceleration positive indicates velocity increases and speed may increase or decrease
- Speed increase if acceleration and velocity both are positive or negative (i.e. both have same sign)
- **In 1-D motion** $a = \frac{dv}{dt} = v\frac{dv}{dx}$
- **Graphical integration in Motion analysis**

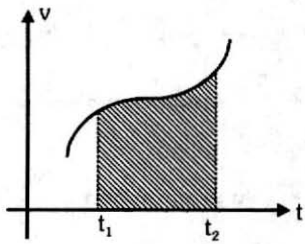


shaded area = change in velocity

- $a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$

$\Rightarrow$ Change in velocity

= Area between acceleration curve and time axis, from t_1 to t_2.



shaded area = displacement

- $v = \frac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$

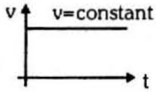
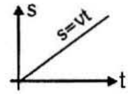
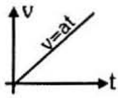
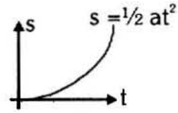
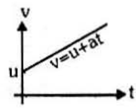
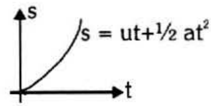
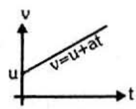
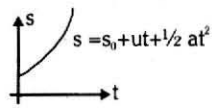
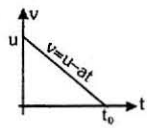
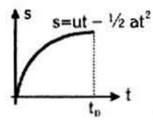
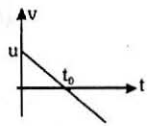
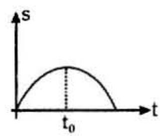
$\Rightarrow$ Change in position = displacement

= area between velocity curve and time axis, from t_1 to t_2.

Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve. $\left(v = \frac{dx}{dt}\right)$
- Slope of velocity-time curve = instantaneous acceleration $\left(a = \frac{dv}{dt}\right)$
- v-t curve area gives displacement. $\left[\Delta x = \int v\,dt\right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int a\,dt\right]$

$$\boxed{\textbf{Displacement}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Velocity}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Acceleration}}$$

Different Cases	v–t graph	s–t graph
1. Uniform motion		
2. Uniformly accelerated motion with $u = 0$ at $t = 0$		
3. Uniformly accelerated with $u \neq 0$ at $t = 0$		
4. Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

Motion with constant acceleration : Equations of motion

- *In vector form :*

$\vec{v} = \vec{u} + \vec{a}t$ $\qquad$ $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{s} = \left(\dfrac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \dfrac{1}{2}\vec{a}t^2 = \vec{v}t - \dfrac{1}{2}\vec{a}t^2$

$v^2 = u^2 + 2\vec{a}.\vec{s}$ $\qquad$ $\vec{s}_{n^{th}} = \vec{u} + \dfrac{\vec{a}}{2}(2n-1)$

[$S_{n^{th}} \rightarrow$ displacement in n^{th} second]

- *In scalar form (for one dimensional motion) :*

$v = u + at$ $\qquad$ $s = \left(\dfrac{u+v}{2}\right)t = ut + \dfrac{1}{2}at^2 = vt - \dfrac{1}{2}at^2$

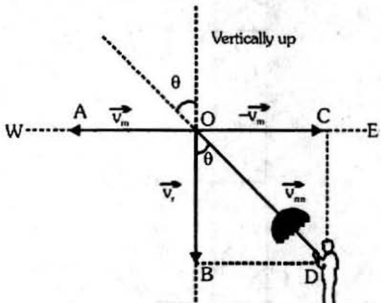
$v^2 = u^2 + 2as$ $\qquad$ $s_n = u + \dfrac{a}{2}(2n-1)$

- **RELATIVE MOTION**

There is no meaning of motion without reference or observer. If reference is not mentioned then we take the ground as a reference of motion. Generally velocity or displacement of the particle w.r.t. ground is called actual velocity or actual displacement of the body. If we describe the motion of a particle w.r.t. and object which is also moving w.r.t. ground then velocity of particle w.r.t. ground is its actual velocity $(\vec{v}_{act})$ and velocity of particle w.r.t. moving object is its relative velocity $(\vec{v}_{rel.})$ and the velocity of moving object (w.r.t. ground) is the reference velocity $(\vec{v}_{ref.})$ then $\vec{v}_{rel} = \vec{v}_{act} - \vec{v}_{ref}$

$$\boxed{\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}}$$

- **Relative velocity of Rain w.r.t. the Moving Man :** A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

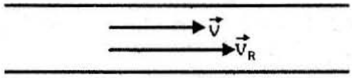
$$\therefore v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

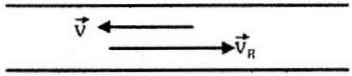
If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

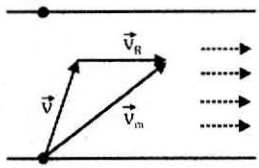
- **Swimming into the River**

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

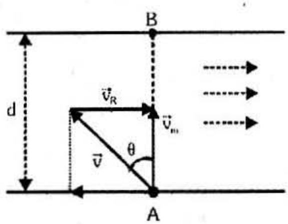
- If the swimming is in the direction of flow of water or along the downstream then 

$v_m = v + v_R$
- If the swimming is in the direction opposite to the flow of water or along the upstream then 

$v_m = v - v_R$
- If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



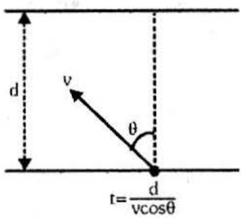
- **For shortest path :**



For minimum displacement

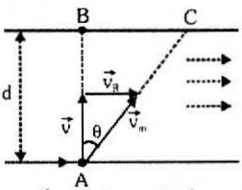
To reach at B, $v\sin\theta = v_R \Rightarrow \sin\theta = \frac{v_R}{v}$

- **Time of crossing**



Note : If $v_R > v$ then for minimum drifting $\sin\theta = \frac{v}{v_R}$

- **For minimum time**

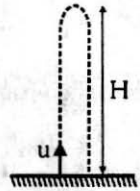


(for minimum time)

then $\boxed{t_{min} = \frac{d}{v}}$

MOTION UNDER GRAVITY

If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then



(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

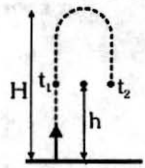
(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers** : For a freely falling body ratio of successive distance covered in equal time internval 't'

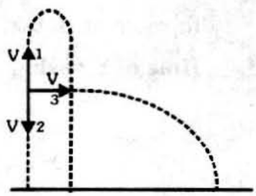
$$S_1 : S_2 : S_3 : \ldots . S_n = 1 : 3 : 5 : \ldots . : 2n-1$$

- At any point on its path the body will have same speed for upward journey and downward journey.
- If a body thrown upwards crosses a point in time t_1 & t_2 respectively then height of point $h = \frac{1}{2} g t_1 t_2$

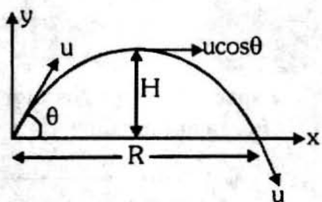


Maximum height $H = \frac{1}{8} g(t_1 + t_2)^2$

- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$



- **PROJECTILE MOTION**



▫ **Horizontal Motion**

$u\cos\theta = u_x$

$a_x = 0$

$x = u_x t = (u\cos\theta)t$

▫ **Vertical Motion :**

$v_y = u_y - gt$ where $u_y = u\sin\theta$; $y = u_y t - \frac{1}{2}gt^2 = u\sin\theta t - \frac{1}{2}gt^2$

Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

▫ **At any instant** : $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

- **For projectile motion** :

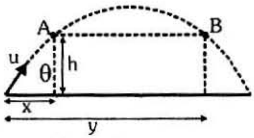
- A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then

  (a) $x + y = R$

  (b) $t_1 + t_2 = T$

  (c) $h = \frac{1}{2} g t_1 t_2$

  (d) Average velocity from A to B is $u\cos\theta$



- If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2)

- **Velocity of particle at time t** :

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

- At highest point : $v_y=0$, $v_x=u\cos\theta$

- Time of flight : $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

- Horizontal range : $R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$

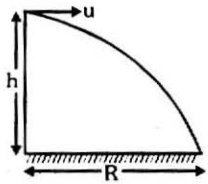
  It is same for θ and $(90° - \theta)$ and maximum for $\theta = 45°$

- Maximum height $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

- $\frac{H}{R} = \frac{1}{4}\tan\theta$

- Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

- **Horizontal projection from some height**

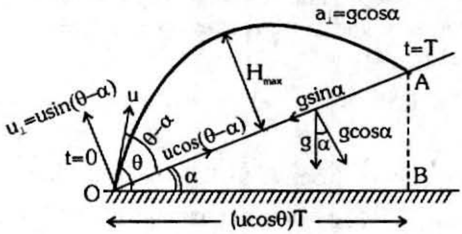


- Time of flight $T = \sqrt{\frac{2h}{g}}$

- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$

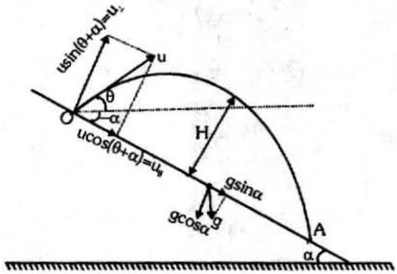
- Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

- **Projectile motion on inclined plane- up motion**



- Time of flight $T=\frac{2u_\perp}{g_\perp}=\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$
- Maximum height $H_{max}=\frac{u_\perp^2}{2g_\perp}=\frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}$
- Range on inclined plane $R=OA=\frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$
- Maximum range $R_{max}=\frac{u^2}{g(1+\sin\alpha)}$ at angle $\theta=\frac{\pi}{4}+\frac{\alpha}{2}$

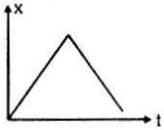
- **Projectile motion on inclined plane – down motion (put α=–α in above)**



- Time of flight $T=2t_H=\frac{2u_\perp}{a_\perp}=\frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Maximum height $H=\frac{u_\perp^2}{2a_\perp}=\frac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}$
- Range on inclined plane $R=OA=\frac{2u^2\cos\theta\sin(\theta+\alpha)}{g\cos^2\alpha}$
- Maximum range $R_{max}=\frac{u^2}{g(1-\sin\alpha)}$ at angle $\theta=\frac{\pi}{4}-\frac{\alpha}{2}$

KEY POINTS :

- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion, cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.



- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall, the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile w.r.t. another projectile is zero.

Important Notes

5 Laws of Motion and Friction

- **Force**

  A push or pull that one object exerts on another.

- **Forces in nature**

  There are four fundamental forces in nature :

  1. Gravitational force
  2. Electromagnetic force
  3. Strong nuclear force
  4. Weak force

- **Types of forces on macroscopic objects**

  (a) Field Forces or Range Forces :

  These are the forces in which contact between two objects is not necessary.

  Ex. (i) Gravitational force between two bodies.

  (ii) Electrostatic force between two charges.

  (b) Contact Forces :

  Contact forces exist only as long as the objects are touching each other.

  Ex. (i) Normal force. (ii) Frictional force

  (c) Attachment to Another Body :

  Tension (T) in a string and spring force (F = kx) comes in this group.

- **Newton's first law of motion (or Galileo's law of Inertia)**

  Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external unbalanced force to change that state.

  Inertia : Inertia is the property of the body due to which body opposes the change of it's state. Inertia of a body is measured by mass of the body.

  $$\boxed{\text{inertia} \propto \text{mass}}$$

- **Newton's second law**

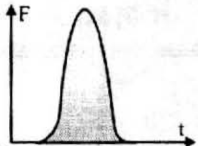
  $$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \quad \text{(Linear momentum } \vec{p} = m\vec{v}\text{)}$$

  ▫ For constant mass system $\vec{F} = m\vec{a}$

- **Momentum** : It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

  SI Unit : kg m s^{-1} **Dimensions** : $[M L T^{-1}]$

- **Impulse** : Impulse = product of force with time.



impulse = area under curve

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F} dt$

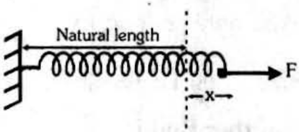
If constant force acts for an interval Δt then : Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem

Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta \vec{p}}$

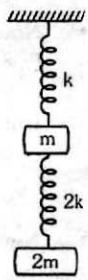
- **Newton's third law of motion** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.
- **Spring Force (According to Hooke's law) :**

In equilibrium F=kx (k is spring constant)



Note : Spring force is non impulsive in nature.

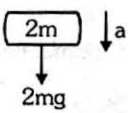
Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.



Sol. Initial stretches $x_{upper} = \dfrac{3mg}{k}$ and $x_{lower} = \dfrac{mg}{k}$

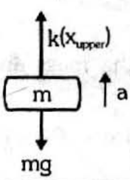
On cutting the lower spring, by virtue of non–impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring. Thus,

Lower block :



$2mg = 2ma \Rightarrow a = g$

Upper block :



$k\left(\dfrac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$

- **Motion of bodies in contact**

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in fig.(i)

$$F = (m_1 + m_2)a$$

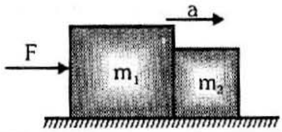


Fig.(1) : When the force F acts on mass m_1

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1: $(F - f_1) = m_1a$

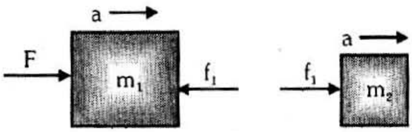


Fig. 1(a) : F.B.D. representation of action and reaction forces.

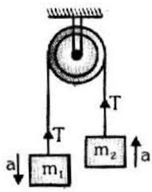
For body m_2 : $f_1 = m_2a \Rightarrow$ action of m_1 on m_2: $f_1 = \dfrac{m_2F}{m_1 + m_2}$

- **Pulley system**

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

SOME CASES OF PULLEY

Case – I



Let $m_1 > m_2$

now for mass m_1, $m_1 g - T = m_1a$

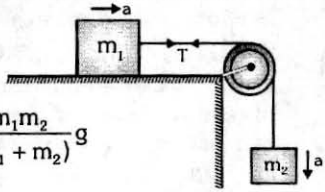
for mass m_2, $T - m_2 g = m_2 a$

$$\text{Acceleration} = a = \frac{(m_1 - m_2)}{(m_1 + m_2)}g = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = T = \frac{2m_1m_2}{(m_1 + m_2)}g = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}}g$$

$$\text{Reaction at the suspension of pulley} \quad R = 2T = \frac{4m_1m_2\,g}{(m_1 + m_2)}$$

Case – II



For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2g - T = m_2 a$

Acceleration $a = \frac{m_2g}{(m_1 + m_2)}$ and $T = \frac{m_1m_2}{(m_1 + m_2)}g$

FRAME OF REFERENCE

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

  All the fundamental laws of physics have been formulated in respect of inertial frame of reference.

- **Non–inertial frame of reference :** An accelerating frame of reference is called a non–inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

- **Pseudo force**: The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

- **Man in a Lift**

  (a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

  So apparent weight W′=Mg=Actual weight.

  (b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

  (i) Weight W=Mg downward

  (ii) Fictitious force F_0=Ma downward.

  So apparent weight $W' = W + F_0 = Mg + Ma = M(g+a)$

(c) If the lift is accelerated downward with acceleration $a<g$.

Then w.r.t. observer inside the lift fictitious force $F_0=Ma$ acts upward while weight of man $W = Mg$ always acts downward.

So apparent weight $W' = W - F_0 = Mg - Ma = M(g-a)$

Special Case :

If $a=g$ then $W'=0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c). Apparent weight $W' = M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

FRICTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.)

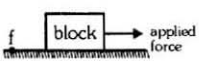
Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

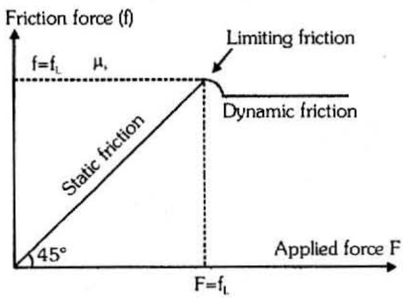
- **Cause of Friction:** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.
- **Types of friction**

Friction

- Static friction (No relative motion between objects)
- Dynamic friction (Kinetic friction) (There is relative motion between objects)

- **Graph between applied force and force of friction**



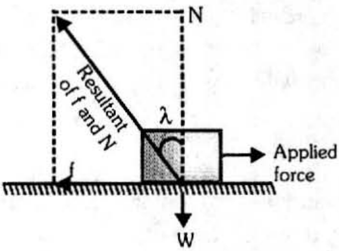


- **Static friction coefficient** $\mu_s = \frac{(f_s)_{max}}{N}$, $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied}$

  $(f_s)_{max} = \mu_s N =$ limiting friction

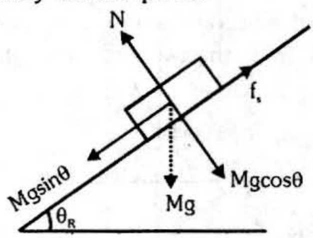
- **Sliding friction coefficient** $\mu_k = \frac{f_k}{N}$, $\vec{f}_k = -(\mu_k N)\hat{v}_{relative}$

- **Angle of Friction (λ)**



$\tan\lambda = \frac{f_S}{N} = \frac{\mu_s N}{N} = \mu_s$

- **Angle of repose** : The maximum angle of an inclined plane for which a block remains stationary on the plane.



$\tan\theta_R = \mu_s$

- For smooth surface $\theta_R = 0$

- **Dependent Motion of Connected Bodies**

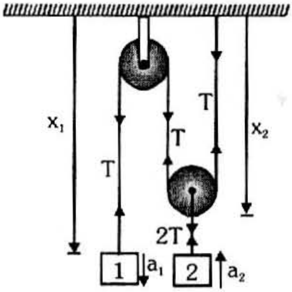
  Method I : Method of constraint equations

$$\boxed{\Sigma x_i = \text{constant} \Rightarrow \Sigma \dot{x}_i = 0 \Rightarrow \Sigma \ddot{x}_i = 0}$$

- For n moving bodies we have $x_1, x_2, \dots x_n$
- No. of constraint equations = no. of strings

Method II : Method of virtual work : The sum of scalar products of forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\boxed{\sum \vec{F}_i \cdot \delta\vec{r}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow \sum \vec{F} \cdot \vec{a}_i = 0}$$



Here $2a_2 = a_1$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling a lawn roller is easier than pushing it because pushing increases the apparent weight and hence friction.
- A moongphaliwala sells his moongphali using a weighing machine in an elevator. He gain more profit if the elevator is accelerating up because the apparent weight of an object increases in an elevator while accelerating upward.
- Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface because normal reaction is less in pulling than in pushing.

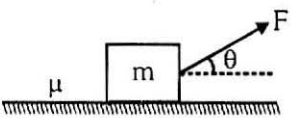


Fig. I ;

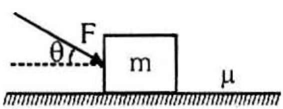


Fig. II

- While walking on ice, one should take small steps to avoid slipping. This is because smaller step increases the normal reaction and that ensure smaller friction.
- A man in a closed cabin (lift) falling freely does not experience gravity as inertial and gravitational mass have equivalence.

Important Notes

6 Work, Energy and Power

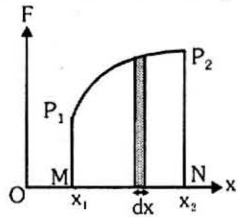
- **Work done** $W = \int dW = \int \vec{F}.d\vec{r} = \int F dr \cos\theta$

  [where θ is the angle between $\vec{F}$ & $d\vec{r}$]

  - For constant force $W = \vec{F}.\vec{d} = Fd\cos\theta$
  - For Unidirectional force

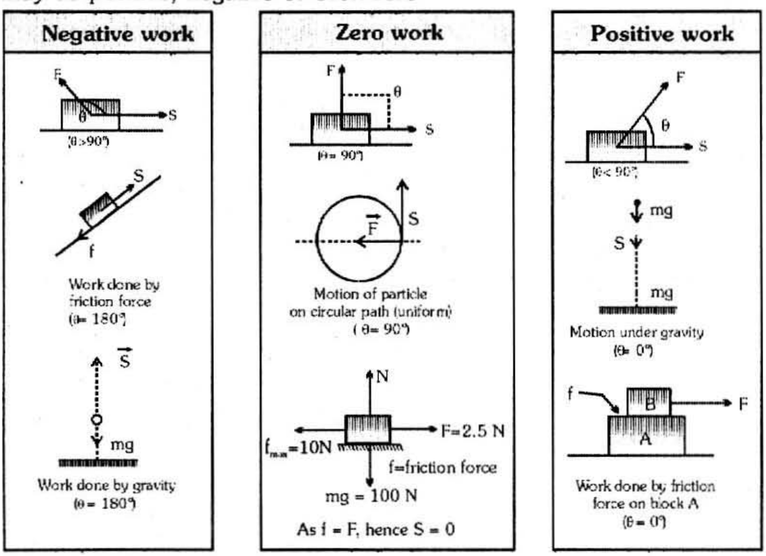
    $W = \int dW = \int F dx =$ Area between F–x curve and x-axis.

- **Calculation of work done from force–displacement graph :**



Total work done, $W = \sum_{x_1}^{x_2} dW = \sum_{x_1}^{x_2} F dx =$ Area of $P_1P_2NM = \int_{x_1}^{x_2} F dx$

- **Nature of work done** : Although work done is a scalar quantity, yet its value may be positive, negative or even zero



Conservative Forces

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc. are conservative forces
- When only a conservative force acts within a system, the kinetic energy and potential energy can change into each other. However, their sum, the mechanical energy of the system, doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then $\vec{\nabla}\times\vec{F}=\vec{0}$ (i.e. curl of $\vec{F}$ is zero)

Non-conservative Forces

- Work done depends upon path.
- Work done in a round trip is not zero.
- Force are velocity–dependent & retarding in nature e.g. friction, viscous force etc.
- Work done against a non–conservative force may be dissipated as heat energy.
- Work done is not recoverable.

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K=\frac{1}{2}mv^2=\frac{1}{2}m(\vec{v}.\vec{v})$$

- Kinetic energy is a frame dependent quantity because velocity is a frame depends.

Potential energy

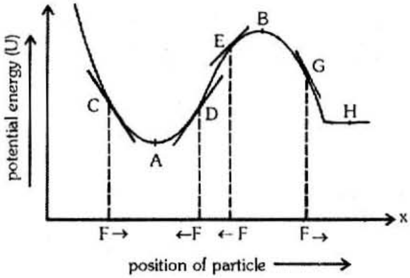
- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Relationship between conservative force field and potential energy :

$$\vec{F}=-\nabla U=-\text{grad}(U)=-\frac{\partial U}{\partial x}\hat{i}-\frac{\partial U}{\partial y}\hat{j}-\frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then

$$F=-\frac{dU}{dx}\Rightarrow U=-\int_{x_1}^{x_2}F\,dx$$

- **Potential energy curve and equilibrium**



It is a curve which shows change in potential energy with postion of a particle.

- **Stable Equilibrium :**

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium then it is said to be in stable equilibrium

At point **C** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **A** : It is the point of stable equilibrium.

$$U = U_{min} \;,\; \frac{dU}{dx} = 0 \quad \text{and} \quad \frac{d^2U}{dx^2} = \text{positive}$$

- **Unstable equilibrium :**

When a particle is slightly displaced from equilibrium and it tends to move away from equilibrium position then it is said to be in unstable equilibrium

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : It is the point of unstable equilibrium.

$$U = U_{max},\; \frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{negative}$$

- **Neutral equilibrium :**

  When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium. Point H is at neutral equilibrium $\Rightarrow U = \text{constant}$; $\frac{dU}{dx}=0,\ \frac{d^2U}{dx^2}=0$

- **Work energy theorem** : $W = \Delta KE$

  Change in kinetic energy = work done by all force

- **For conservative force** $F(x) = -\frac{dU}{dx}$

  change in potential energy $\Delta U = -\int F(x)dx$

- **Law of conservation of Mechanical energy**

  Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

  Proof : For internal conservative forces $W_{int} = -\Delta U$

  So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE$

  $\Rightarrow \Delta(KE + U) = 0 \Rightarrow KE + U = \text{constant}$

- Spring force $F=-kx$, Elastic potential energy stored in spring $U(x) = \frac{1}{2}kx^2$

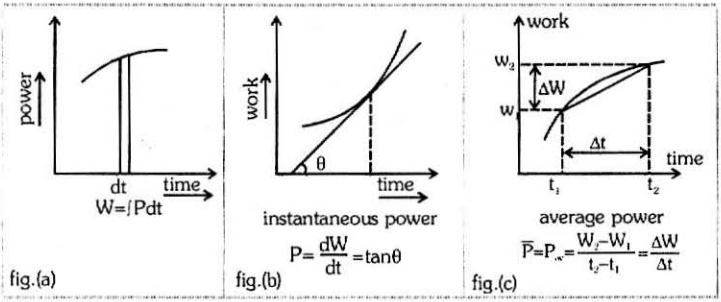
- Mass and energy are equivalent and are related by $E = mc^2$

- **Power**

  - Power is a scalar quantity with dimension $M^1L^2T^{-3}$
  - SI unit of power is J/s or watt
  - 1 horsepower = 746 watt = 550 ft–lb/sec.

- **Average power** $P_{av} = W/t$

- **Instantaneous power** $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$



- For a system of varying mass $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$
- If $\vec{v}$ = constant then $\vec{F} = \vec{v}\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$
- In rotatory motion : $P = \tau\frac{d\theta}{dt} = \tau\omega$

KEY POINTS

- A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is a conservative force.
- Work done by static friction may be positive because static friction may acts along the direction of motion of an object.

Important Notes

7 Circular Motion

- **Definition of Circular Motion**

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector :

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outward

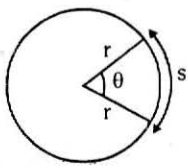
Frequency (n) :

No. of revolutions described by particle per sec. is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T) :

It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

- Angle $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$



- Average angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ (a scalar quantity)
- Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$ (a vector quantity)
- For uniform angular velocity $\omega = \frac{2\pi}{T} = 2\pi f$ or $2\pi n$

Angular displacement $\theta = \omega t$

$\omega \rightarrow$ Angular frequency n or f = frequency

- Relation between ω and v $\qquad \omega = \dfrac{v}{r}$

  - In vector form velocity $\qquad \vec{v} = \vec{\omega} \times \vec{r}$

- Acceleration $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\vec{\omega} \times \vec{r}) = \dfrac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \dfrac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$

- Tangential acceleration: $a_t = \dfrac{dv}{dt} = \alpha r$

$$\left[\vec{a}_t = \text{component of } \vec{a} \text{ along } \vec{v} = (\vec{a} \cdot \hat{v})\hat{v} = \left(\frac{dv}{dt}\right)\hat{v}\right]$$

- Centripetal acceleration : $a_C = \omega v = \dfrac{v^2}{r} = \omega^2 r$ or $\vec{a}_C = \omega^2 r(-\hat{r})$

- Magnitude of net acceleration : $a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\dfrac{v^2}{r}\right)^2 + \left(\dfrac{dv}{dt}\right)^2}$

- Maximum speed of in circular motion.

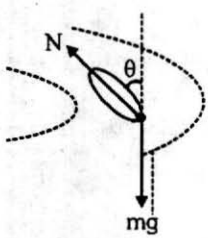
  - On unbanked road : $v_{max} = \sqrt{\mu_s Rg}$

  - On banked road : $v_{max} = \sqrt{\left(\dfrac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)Rg} = \sqrt{\tan(\theta + \phi)Rg}$

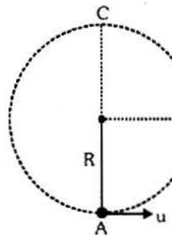
$$v_{min} = \sqrt{Rg\tan(\theta - \phi)}\,;\quad v_{min} \le v_{car} \le v_{max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking

- Bending of cyclist : $\tan\theta = \dfrac{v^2}{rg}$



- **Circular motion in vertical plane**



A. Condition to complete vertical circle $u \geq \sqrt{5gR}$

- If $u = \sqrt{5gR}$ then Tension at C is equal to 0

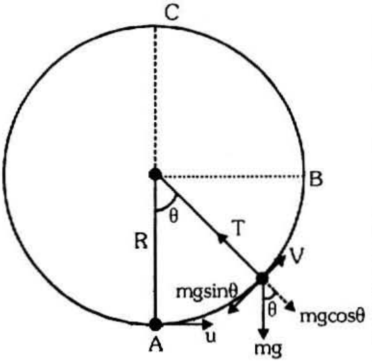
  and tension at A is equal to 6mg

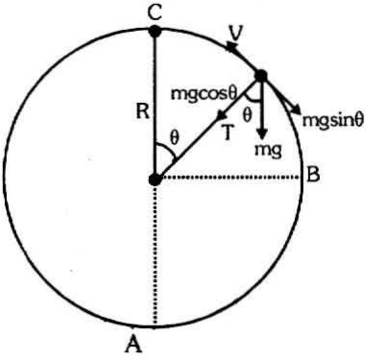
  Velocity at B: $v_B = \sqrt{3gR}$

  Velocity at C: $v_C = \sqrt{gR}$

  From A to B : $T = mg\cos\theta + \dfrac{mv^2}{R}$

  From B to C : $T = \dfrac{mv^2}{R} - mg\cos\theta$





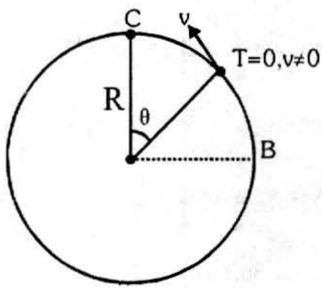
B. Condition for pendulum motion (oscillating condition)

$u \le \sqrt{2gR}$ (in between A to B)

Velocity can be zero but T never be zero between A & B.

Because T is given by $T = mg\cos\theta + \frac{mv^2}{R}$

C. Condition for leaving path : $\sqrt{2gR} < u < \sqrt{5gR}$



Particle crosses the point B but not complete the vertical circle.

Tension will be zero in between B to C & the angle where T = 0

$$\cos\theta = \frac{u^2 - 2gR}{3gR}$$

θ is from vertical line

Note : After leaving the circle, the particle will follow a parabolic path.

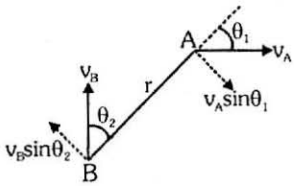
KEY POINTS

- Average angular velocity is a scalar physical quanity whereas instantaneous angular velocity is a vector physical quanity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But} \quad \vec{\theta}_1 + \vec{\theta}_2 \ne \vec{\theta}_2 + \vec{\theta}_1$$

- **Relative Angular Velocity**

  Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.



That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

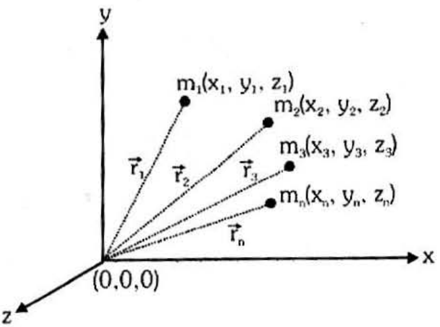
$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\qquad \therefore \omega_{AB} = \dfrac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

Important Notes

8 Collisions and Centre of Mass

- **Centre of mass :** For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated.
- **Centre of mass of system of discrete particles**



Total mass of the body : $M = m_1 + m_2 + ... + m_n$ then

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \frac{1}{M}\Sigma m_i\vec{r}_i$$

co-ordinates of centre of mass :

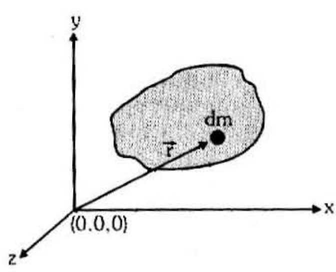
$$x_{cm} = \frac{1}{M}\Sigma m_i x_i \ , \ y_{cm} = \frac{1}{M}\Sigma m_i y_i \text{ and } z_{cm} = \frac{1}{M}\Sigma m_i z_i$$

- **Centre of mass of continuous distribution of particles**

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}dm$$

$$x_{cm} = \frac{1}{M}\int x\, dm \ , \ y_{cm} = \frac{1}{M}\int y\, dm$$

and $z_{cm} = \frac{1}{M}\int z\, dm$



x, y, z are the co-ordinate of the COM of the dm mass.

The centre of mass after removal of a part of a body

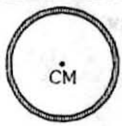
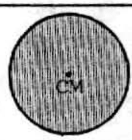
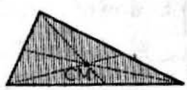
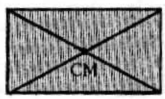
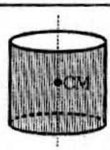
Original mass (M) – mass of the removed part (m)

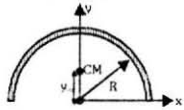
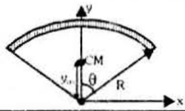
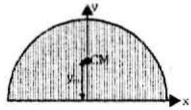
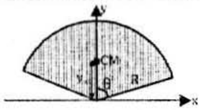
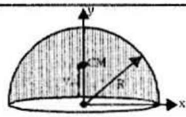
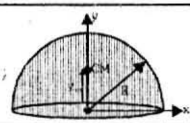
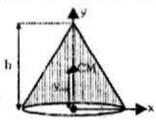
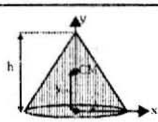
= {original mass (M)} + { – mass of the removed part (m)}

The formula changes to :

$$x_{CM} = \frac{Mx - mx'}{M - m}\ ;\ y_{CM} = \frac{My - my'}{M - m}\ ;\ y_{CM} = \frac{Mz - mz'}{M - m}$$

CENTRE OF MASS OF SOME COMMON OBJECTS

Body	Shape of body	Position of centre of mass
Uniform Ring		Centre of ring
Uniform Disc		Centre of disc
Uniform Rod		Centre of rod
Solid sphere/ hollow sphere		Centre of sphere
Triangular plane lamina		Point of intersection of the medians of the triangle i.e. centroid
Plane lamina in the form of a square or rectangle or parallelogram		Point of intersection of diagonals
Hollow/solid cylinder		Middle point of the axis of cylinder

Body	Shape of body	Position of centre of mass
Half ring		$y_{cm} = \frac{2R}{\pi}$
Segement of a ring		$y_{cm} = \frac{R\sin\theta}{\theta}$
Half disc (plate)		$y_{cm} = \frac{4R}{3\pi}$
Sector of a disc (plate)		$y_{cm} = \frac{2R\sin\theta}{3\theta}$
Hollow hemisphere		$y_{cm} = \frac{R}{2}$
Solid hemisphere		$y_{cm} = \frac{3R}{8}$
Hollow cone		$y_{cm} = \frac{h}{3}$
Solid cone		$y_{cm} = \frac{h}{4}$

MOTION OF CENTRE OF MASS

For a system of particles, velocity of centre of mass

$$\vec{v}_{CM} = \frac{d\vec{R}_{CM}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots}{m_1 + m_2 + \ldots}$$

Similarly acceleration

$$\vec{a}_{CM} = \frac{d}{dt}\left(\vec{v}_{CM}\right) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots}{m_1 + m_2 + \ldots}$$

- **Law of conservation of linear momentum**

  Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass.

  From Newton's second law $\vec{F}_{ext.} = \dfrac{d\left(M\vec{v}_{CM}\right)}{dt}$

  If $\vec{F}_{ext.} = \vec{0}$ then $M\vec{v}_{CM} = \text{constant}$

  If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

- **Impulse – Momentum theorem**

  $$\boxed{\int_{t_1}^{t_2} \vec{F}dt = \Delta\vec{p}}$$

  Impulse of a force is equal to the change of momentum

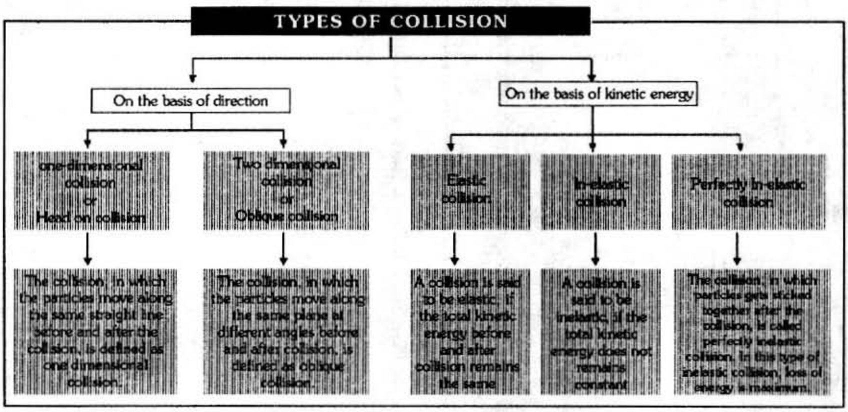
  force time graph area gives change in momentum.

Collision of bodies

The event or the process, in which two bodies either coming in contact with each other or due to mutual interaction at distance apart, affect each others motion (velocity, momentum, energy or direction of motion) is defined as a collision.

In collision

- The particles come closer before collision and after collision they either stick together or move away from each other.
- The particles need not come in contact with each other for a collision.
- The law of conservation of linear momentum is necessarily applicable in a collision, whereas the law of conservation of mechanical energy is not.



- **Coefficient of restitution (Newton's law)**

$$e = -\frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

- **Head on collision**

Before collision　　Collision　　After collision

Head on inelastic collision of two particles

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $e = \dfrac{v_2 - v_1}{u_1 - u_2}$ ...(ii)

By solving eq. (i) and (ii) :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 = \frac{m_1u_1 + m_2u_2 - m_1e(u_2 - u_1)}{m_1 + m_2}$$

Elastic Collision (e=1)

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If the mass of a body is negligible as compared to other.**

  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$ when a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A. If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remains at rest.

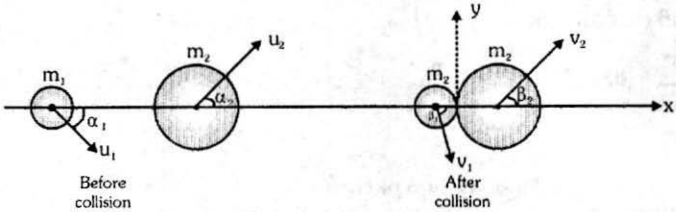
- **Loss in kinetic energy in inelastic collision**

$$\Delta K = \frac{m_1m_2}{2(m_1 + m_2)}(1 - e^2)(u_1 - u_2)^2$$

Oblique Collision

Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2 \text{ and}$$
$$m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$$



Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved.

$$m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1 \ \& \ m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$$

By using Newton's experimental law along the line of impact

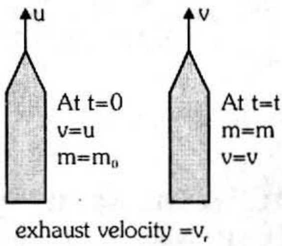
$$e = \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$$

Rocket propulsion :

Thrust force on the rocket= $v_r\left(-\frac{dm}{dt}\right)$

Velocity of rocket at any instant

$v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right)$



KEY POINTS

- Sum of mass moments about centre of mass is zero. i.e. $\sum m_i \vec{r}_{i/cm} = \vec{0}$
- A quick collision between two bodies is more violent then slow collision, even when initial and final velocities are equal because the rate of change of momentum determines that the impulsive force small or large.
- Heavy water is used as moderator in nuclear reactors as energy transfer is maximum if $m_1 \simeq m_2$
- Impulse momentum theorem is equivalent to Newton's second law of motion.
- For a system, conservation of linear momentum is equivalent to Newton's third law of motion.

Important Notes

9 Rotational Motion

- **Angular velocity** $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$

- **Angular acceleration** $\vec{\alpha} = \dfrac{d\vec{\omega}}{dt} = \dfrac{d^2\vec{\theta}}{dt^2}$

- **Angular momentum** $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

- **Torque** $\vec{\tau} = \vec{r} \times \vec{F} = \dfrac{d\vec{L}}{dt}$

- **Rotational Kinetic energy** $K = \dfrac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$

- **Rotational Power** $P = \vec{\tau}.\vec{\omega}$

- **For constant angular acceleration**

$$\omega = \omega_0 + \alpha t,\ \theta = \omega_0 t + \frac{1}{2}\alpha t^2,\ \ \omega^2 = \omega_0^2 + 2\alpha\theta,\ \ \theta_n = \omega_0 + \frac{\alpha}{2}(2n - 1)$$

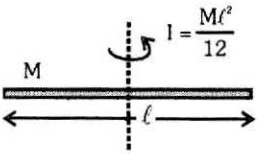
- **Moment of Inertia**
  - A tensor but for fixed axis it is a scalar
  - For discrete distribution of mass $I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots = \sum m_i r_i^2$
  - For continuous distribution of mass $I = \int dI = \int dm r^2$

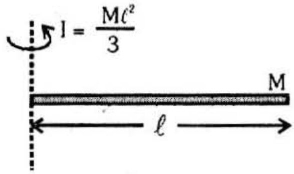
- **Radius of gyration** $k = \sqrt{\dfrac{I}{M}}$

- **Theorems regarding moment of inertia**
  - Theorem of parallel axes $I_{axis} = I_{cm} + md^2$

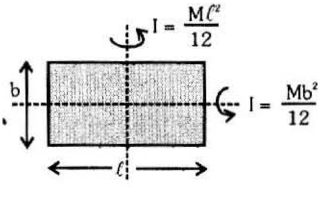
    where d is the perpendicular distance between parallel axes.
  - Theorem of perpendicular axes $I_z = I_x + I_y$

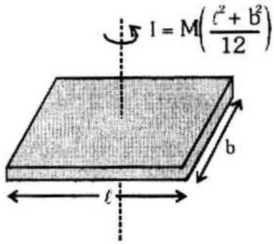
- **Rod**



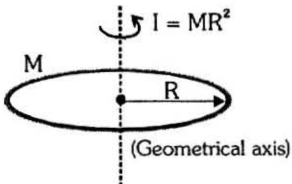


- **Rectangular Lamina**

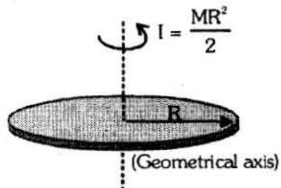




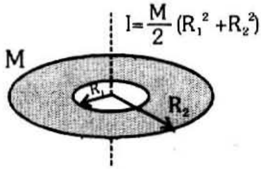
- **Ring :**



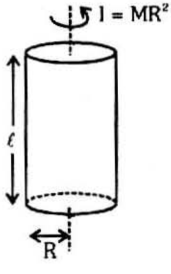
- **Disc :**

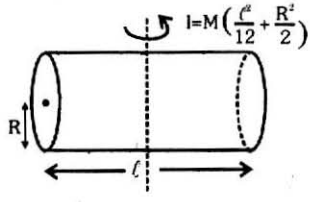


- **Circular Hollow Disk :**

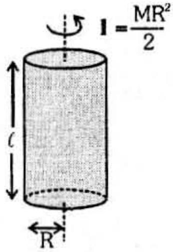


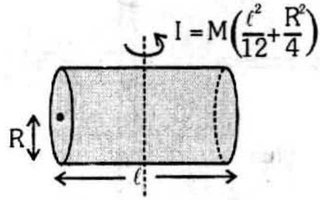
- **Hollow cylinder**



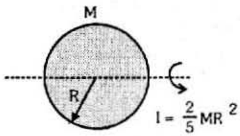


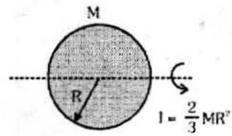
- **Solid cylinder**



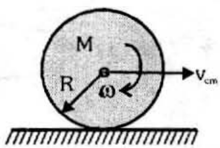


- **Solid & Hollow sphere**

Solid 

Hollow 

- **Rolling motion**



- ▫ Total kinetic energy = $\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{cm}\omega^2$
- ▫ Total angular momentum $= Mv_{CM}R + I_{cm}\omega$

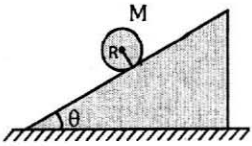
- **Pure rolling (or rolling without slipping) on stationary surface**

- ▫ Condition : $v_{cm} = R\omega$

  In accelerated motion $a_{cm} = R\alpha$
- ▫ If $v_{cm} > R\omega$ then rolling with forward slipping,
- ▫ If $v_{cm} < R\omega$ then rolling with backward slipping
- ▫ Total kinetic energy in pure rolling

$$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(Mk^2)\left(\frac{v_{cm}^2}{R^2}\right) = \frac{1}{2}Mv_{cm}^2\left(1+\frac{k^2}{R^2}\right)$$

- **Pure rolling motion on an inclined plane**



- Acceleration $a = \dfrac{g\sin\theta}{1+k^2/R^2}$
- Minimum frictional coefficient $\mu_{min} = \dfrac{\tan\theta}{1+R^2/k^2}$

- **Torque** $\vec{\tau} = I\vec{\alpha} = I\dfrac{d\vec{\omega}}{dt} = \dfrac{d(I\vec{\omega})}{dt} = \dfrac{d\vec{L}}{dt} \text{ or } \dfrac{d\vec{J}}{dt}$

- **Change in angular momentum** $\Delta\vec{L} = \vec{\tau}\Delta t$

- **Work done by a torque** $W = \int \vec{\tau}\cdot d\vec{\theta}$

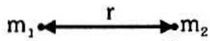
KEY POINTS

- A ladder is more apt to slip, when you are high up on it than when you just begin to climb because at the high up on a ladder the torque is large and on climbing up the torque is small.
- When a sphere is rolls on a horizontal table, it slows down and eventually stops because when the sphere rolls on the table, both the sphere and the surface deform near the contact. As a result the normal force does not pass through the centre and provide an angular deceleration.
- The spokes near the top of a rolling bicycle wheel are more blurred than those near the bottom of the wheel because the spokes near the top of wheel are moving faster than those near the bottom of the wheel.
- Instantaneous angular velocity is a vector quantity because infinitesimal angular displacement is a vector.
- The relative angular velocity between any two points of a rigid body is zero at any instant.
- All particles of a rigid body, which do not lie on an axis of rotation move on circular paths with centres at an axis of rotation.
- Instantaneous axis of rotation is stationary w.r.t. ground.
- Many greater rivers flow toward the equator. The sediment that they carry increases the time of rotation of the earth about its own axis because the angular momentum of the earth about its rotation axis is conserved.
- The hard boiled egg and raw egg can be distinguished on the basis of spinning of both.

Important Notes

10 Gravitation

- **Newton's law of gravitation**

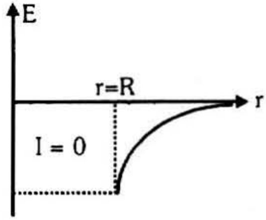


Force of attraction between two point masses $F = \frac{Gm_1m_2}{r^2}$

Directed along the line joining of point masses.

- It is a conservative force field ⇒ mechanical energy is conserved.
- It is a central force field ⇒ angular momentum is conserved.

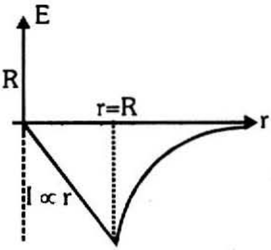
- **Gravitational field due to spherical shell**



- Outside the shell $E_g = \frac{GM}{r^2}$, where r > R
- On the surface $E_g = \frac{GM}{R^2}$, where r=R
- Inside the shell $E_g = 0$, where r<R

[Note : Direction always towards the centre of the sphere]

- **Gravitational field due to solid sphere**



- Outside the sphere $E_g = \frac{GM}{r^2}$, where r > R
- On the surface $E_g = \frac{GM}{R^2}$, where r=R
- Inside the sphere $E_g = \frac{GMr}{R^3}$, where r< R

- **Acceleration due to gravity** $g = \frac{GM}{R^2}$

- At height h $g_h = \frac{GM}{(R+h)^2}$ If h << R ; $g_h \approx g_s\left(1-\frac{2h}{R}\right)$
- At depth d $g_d = \frac{GM(R-d)}{R^3} = g_s\left(1-\frac{d}{R}\right)$
- Effect of rotation on g : $g' = g - \omega^2 R\cos^2\lambda$ where λ is angle of latitude.

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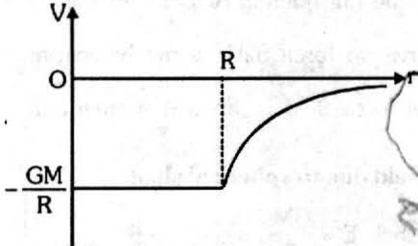
- **Gravitational potential**

  Due to a point mass at a distance $V = -\frac{GM}{r}$

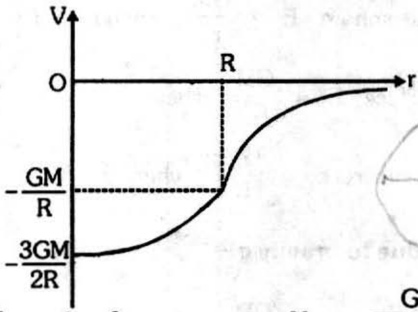
- **Gravitational potential due to spherical shell**

  - Outside the shell $V = -\frac{GM}{r}, r>R$
  - Inside/on the surface the shell $V = -\frac{GM}{R}, r \leq R$



- **Potential due to solid sphere**

  - Outside the sphere $V = -\frac{GM}{r}, r > R$
  - On the surface $V = -\frac{GM}{R}, r = R$
  - Inside the sphere $V = -\frac{GM(3R^2 - r^2)}{2R^3}, r < R$



- **Potential on the axis of a thin ring at a distance x** $V = -\frac{GM}{\sqrt{R^2 + x^2}}$

- **Escape velocity from a planet of mass M and radius R** $v_e = \sqrt{\frac{2GM}{R}}$

- **Orbital velocity of satellite** $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$

- For nearby satellite $v_0 = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$

  Here V_e = escape velocity on earth surface.

- **Time period of satellite** $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$
- **Energies of a satellite**
  - Potential energy $U = -\frac{GMm}{r}$
  - Kinetic energy $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$
  - Mechanical energy $E = U + K = -\frac{GMm}{2r}$
  - Binding energy $BE = -E = \frac{GMm}{2r}$
- **Kepler's laws**
  - Ist Law of orbitals

    Path of a planet is elliptical with the sun at a focus.
  - IInd Law of areas Areal velocity $\frac{dA}{dt}$ = constant $= \frac{L}{2m}$
  - IIIrd - Law of periods $T^2 \propto a^3$ or $T^2 \propto \left(\frac{r_{max} + r_{min}}{2}\right)^3 \propto (\text{mean radius})^3$

    For circular orbits $T^2 \propto R^3$

KEY POINTS

- At the centre of earth, a body has centre of mass, but no centre of gravity.
- The centre of mass and centre of gravity of a body coincide if gravitation field is uniform.
- You does not experience gravitational force in daily life due to objects of same size as value of G is very small.
- Moon travellers tie heavy weight at their back before landing on Moon due to smaller value of g at Moon.
- Space rockets are usually launched in equatorial line from West to East because g is minimum at equator and earth rotates from West to East about its axis.
- Angular momentum in gravitational field is conserved because gravitational force is a central force.
- Kepler's second law or constancy of areal velocity is a consequence of conservation of angular momentum.

Important Notes

11 Properties of Matter and Fluid Mechanics

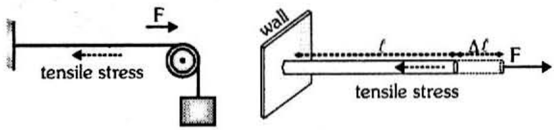
(A) ELASTICITY

STRESS = $\dfrac{\text{Internal restoring force}}{\text{Area of cross-section}} = \dfrac{F_{Res}}{A}$.

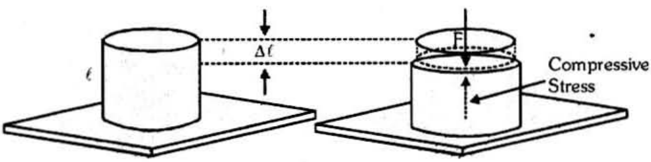
There are three types of stress :-

- **Longitudinal Stress**

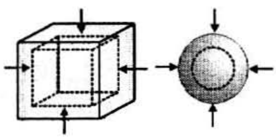
(a) Tensile Stress:



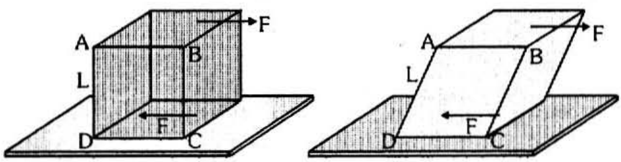
(b) Compressive Stress :



- **Volume Stress**



- **Tangential Stress or Shear Stress**

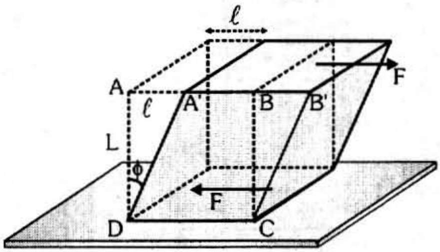


- **Strain** = $\dfrac{\text{Change in size of the body}}{\text{Original size of the body}}$

- **Longitudinal strain** = $\dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$

- **Volume strain** = $\dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

- **Shear strain** : $\tan\phi = \dfrac{\ell}{L}$ or $\phi = \dfrac{\ell}{L} = \dfrac{\text{displacement of upper face}}{\text{distance between two faces}}$



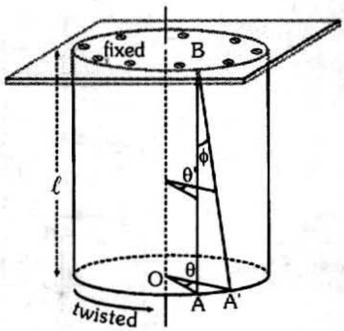
- **Relation between angle of twist (θ) & angle of shear (ϕ)**

AA' = r θ and Arc AA' = $\ell\phi$

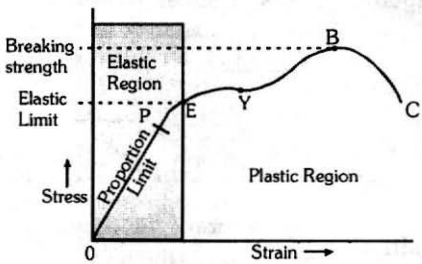
So $r\theta = \ell\phi \Rightarrow \phi = \dfrac{r\theta}{\ell}$

where θ = angle of twist,

ϕ = angle of shear



- **Stress – Strain Graph**



- **Hooke's Law** within elastic limit $\boxed{\text{Stress} \propto \text{strain}}$

- **Young's modulus of elasticity** $Y = \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \dfrac{F\ell}{A\,\Delta\ell}$

- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire $Y = \frac{(Mg/\pi r^2)}{(\ell/L)} = \frac{MgL}{\pi r^2 \ell}$

- **Increment in length due to own weight** $\Delta\ell = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$

- **Bulk modulus of elasticity** $K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{\left(\frac{-\Delta V}{V}\right)} = \frac{P}{\left(\frac{-\Delta V}{V}\right)}$

- **Bulk modulus of an ideal gas is process dependence.**
  - For isothermal process PV = constant

    $\Rightarrow PdV + VdP = 0 \Rightarrow P = \frac{-dP}{dV/V}$ So bulk modulus = P

  - For adiabatic process PV^γ = constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^\gamma dP = 0$

    $\Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \frac{-dP}{dV/V}$; So bulk modulus = γP

  - For any polytropic process PV^n= constant

    $\Rightarrow nPV^{n-1}dV + V^n dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \frac{-dP}{dV/V}$

    So bulk modulus = nP

- **Compressibility** $C = \frac{1}{\text{Bulk modulus}} = \frac{1}{K}$

- **Modulus of rigidity** $\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{(F_{\text{tangential}})/A}{\phi}$

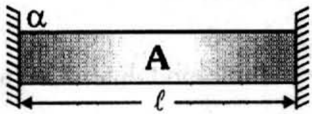
- **Poisson's ratio** $(\sigma) = \dfrac{\text{lateral strain}}{\text{Longitudinal strain}}$

- **Work done in stretching wire**

$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} :$$

$$W = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta\ell}{\ell} \times A \times \ell = \frac{1}{2} F \times \Delta\ell$$

- **Rod is rigidly fixed between walls**



  - Thermal Strain = $\alpha\Delta\theta$
  - Thermal stress = $Y\alpha\Delta\theta$
  - Thermal tension = $Y\alpha A\Delta\theta$

- **Effect of Temperature on elasticity**

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

- **Effect of Impurity on elasticity**

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

(B) HYDROSTATICS

- **Density** $= \dfrac{\text{mass}}{\text{volume}}$

- **Specific weight** $= \dfrac{\text{weight}}{\text{volume}} = \rho g$

- **Relative density** $= \dfrac{\text{density of given liquid}}{\text{density of pure water at } 4°C}$

- **Density of a Mixture of substance in the proportion of mass**

the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3....}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...}$

- **Density of a mixture of substance in the proportion of volume**

the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

- **Pressure** $= \dfrac{\text{normal force}}{\text{area}}$

- **Variation of pressure with depth**

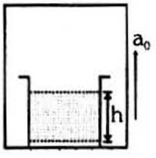
Pressure is same at two points in the same horizontal level $P_1 = P_2$

The difference of pressure between two points separated by a depth h

$$(P_2 - P_1) = h\rho g$$

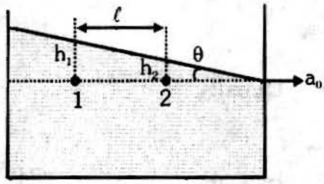
- **Pressure in case of accelerating fluid**

(i) **Liquid placed in elevator:** When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P = h\rho[g + a_0]$



(ii) Free surface of liquid in case of horizontal acceleration :

$$\tan\theta = \frac{ma_0}{mg} = \frac{a_0}{g}$$

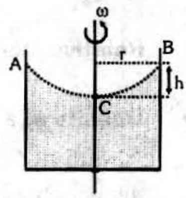


If P_1 and P_2 are pressures at point 1 & 2 then

$$P_1 - P_2 = \rho g\,(h_1 - h_2) = \rho g \ell \tan\theta = \rho \ell a_0$$

(iii) Free surface of liquid in case of rotating cylinder

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$



- **Pascal's Law**

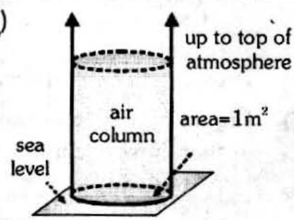
  - The pressure in a fluid at rest is same at all the points if gravity is ignored.
  - A liquid exerts equal pressures in all directions.
  - If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude. *[for ideal fluids]*

- **Types of Pressure** : Pressure is of three types

  (i) Atmospheric pressure (P_0) (ii) Gauge pressure (P_{gauge})

  (iii) Absolute pressure ($P_{abs.}$)

- **Atmospheric pressure** : Force exerted by air column on unit cross-section area of sea level called atmospheric pressure (P_o)



$$P_o = \frac{F}{A} = 101.3 \text{ kN/m}^2$$

$$\therefore P_o = 1.013 \times 10^5 \text{ N/m}^2$$

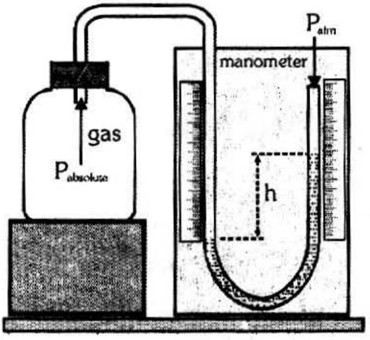
Barometer is used to measure atmospheric pressure.

Which was discovered by **Torricelli**.

Atmospheric pressure varies from place to place and at a particular place from time to time.

- **Gauge Pressure :**

  Excess Pressure ($P - P_{atm}$) measured with the help of pressure measuring instrument called Gauge pressure. $P_{gauge} = h\rho g$ or $P_{gauge} \propto h$



  Gauge pressure is always measured with help of "**manometer**"

- **Absolute Pressure :**

  Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \Rightarrow P_{abs} = P_o + h\rho g$$

  The pressure which we measure in our automobile tyres is gauge pressure.

- **Buoyant force** = Weight of displaced fluid = $V\rho g$

- **Apparent weight** = Weight – Upthrust

- **Rotatory – Equilibrium in Floatation :** for rotational equilibrium of floating body the meta–centre must always be higher than the centre of gravity of the body.

- **Relative density of body** $= \dfrac{\text{Density of body}}{\text{Density of water}}$

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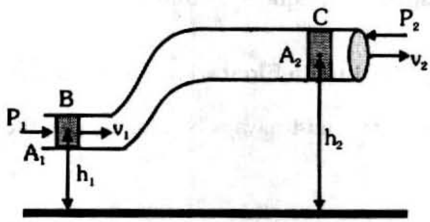
E

(C) HYDRODYNAMICS

- **Steady and Unsteady Flow :** *Steady flow* is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.
- **Streamline Flow :** In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a *streamline.*
- **Laminar and Turbulent Flow :** *Laminar flow* is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel.
- **Compressible and Incompressible Flow :** In *compressible flow* the density of fluid varies from point to point i.e. the density is not constant for the fluid whereas in *incompressible flow* the density of the fluid remains constant throughout.
- **Rotational and Irrotational Flow :** *Rotational flow* is the flow in which the fluid particles while flowing along path–lines also rotate about their own axis. In *irrotational flow* particles do not rotate about their axis.
- **Equation of continuity** $A_1v_1 = A_2v_2$ **Based** on conservation of mass
- **Bernoulli's theorem :** $\boxed{P+\frac{1}{2}\rho v^2+\rho gh = \text{constant}}$

  Based on energy conservation



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- **Kinetic Energy**

kinetic energy per unit volume $= \dfrac{\text{Kinetic Energy}}{\text{volume}} = \dfrac{1}{2}\dfrac{m}{V}v^2 = \dfrac{1}{2}\rho v^2$

- **Potential Energy**

Potential energy per unit volume $= \dfrac{\text{Potential Energy}}{\text{volume}} = \dfrac{m}{V}gh = \rho gh$

- **Pressure Energy**

Pressure energy per unit volume $= \dfrac{\text{Pressure energy}}{\text{volume}} = P$

- **For horizontal flow in venturimeter**

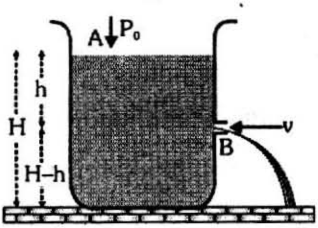
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v^2 \Rightarrow v_1 = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

- **Rate of flow :**

Volume of water flowing per second $\quad Q = A_1v_1 = A_1A_2\sqrt{\dfrac{2gh}{A_1^2 - A_2^2}}$

- **Velocity of efflux** $v = \sqrt{2gh}$

- **Horizontal range** $R = 2\sqrt{h(H-h)}$



(D) SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum surface area. This property of liquid is called *surface tension*.

Intermolecular forces

(a) Cohesive force

The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) Adhesive force

The force acting between different types of molecules or molecules of different substance is called adhesive force.

- Intermolecular forces are different from the gravitational forces and do not obey the inverse–square law
- The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.
- Molecular range depends on the nature of the substance

Properties of surface tension

- Surface tension is a scalar quantity.
- It acts tangential to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

Dependency of Surface Tension

- **On Cohesive Force :** Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

- **On Impurities :** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

  (a) On mixing detergent in water its surface tension decreases.

  (b) Surface tension of water is more than (alcohol + water) mixture.

- **On Temperature**

  On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

  Note : Surface tension of water is maximum at 4°C

- **On Contamination**

  The dust particles or lubricating materials on the liquid surface decreases its surface tension.

- **On Electrification**

  The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.

Definition of surface tension

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

- For floating needle $2T\ell \sin\theta = mg$

- **Required excess force for lift**

- Wire $F_{ex} = 2T\ell$
- Hollow disc $F_{ex} = 2\pi T (r_1 + r_2)$
- For ring $F_{ex} = 4\pi rT$
- Circular disc $F_{ex} = 2\pi rT$
- Square frame $F_{ex} = 8aT$
- Square plate $F_{ex} = 4aT$

- **Work** = surface energy = $T\Delta A$
  - Liquid drop $W = 4\pi r^2 T$
  - Soap bubble $W = 8\pi r^2 T$
- **Splitting of bigger drop into smaller droples** $R = n^{1/3} r$

  Work done= Change in surface energy $= 4\pi R^3 T\left(\frac{1}{r}-\frac{1}{R}\right) = 4\pi R^2 T\,(n^{1/3}-1)$
- **Excess pressure** $P_{ex} = P_{in} - P_{out}$
  - In liquid drop $P_{ex} = \frac{2T}{R}$
  - In soap bubble $P_{ex} = \frac{4T}{R}$

ANGLE OF CONTACT (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the *angle of contact*.

The angle of contact depends the nature of the solid and liquid in contact.

- **Angle of contact** $\theta < 90° \Rightarrow$ concave shape, Liquid rise up

  Angle of contact $\theta > 90° \Rightarrow$ convex shape, Liquid falls

  Angle of contact $\theta = 90° \Rightarrow$ plane shape, Liquid neither rise nor falls
- **Effect of Temperature on angle of contact**

  On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decrease. So on increasing temperature, θ_c decreases.
- **Effect of Impurities on angle of contact**

  (a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

  (b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

- **Effect of Water Proofing Agent**

  Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

- **Capillary rise** $h = \frac{2T\cos\theta}{r\rho g}$

  - Zurin's law $h \propto \frac{1}{r}$

  - Jeager's method $T = \frac{rg}{2}(H\rho - hd)$

  - The height 'h' is measured from the bottom of the meniscus. However, there exist some liquid above this line also. If correction of this is applied

    then the formula will be $T = \frac{r\rho g\left[h + \frac{1}{3}r\right]}{2\cos\theta}$

- When two soap bubbles are in contact then radius of curvature of the common surface $r = \frac{r_1 r_2}{r_1 - r_2}(r_1 > r_2)$

- When two soap bubbles are combining to form a new bubble then radius of new bubble $r = \sqrt{r_1^2 + r_2^2}$

- Force required to separate two plates $F = \frac{2AT}{d}$

(E) VISCOSITY

- **Newton's law of viscosity** $F = \eta A \frac{\Delta v_x}{\Delta y}$

  - **SI UNITS :** $\frac{N \times s}{m^2}$ or deca poise
  - **CGS UNITS :** dyne–s/cm^2 or poise (1 decapoise = 10 poise)

- **Dependency of viscosity of fluids**

On Temperature of Fluid

(a) Since cohesive forces decrease with increase in temperature as increase in K.E.. Therefore with the rise in temperature, the viscosity of liquids decreases.

(b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

On Pressure of Fluid

(a) The viscosity of liquids increases with the increase of pressure.

(b) The viscosity of gases is practically independent of pressure.

On Nature of Fluid

- **Poiseuillel's formula** $Q = \frac{dV}{dt} = \frac{\pi p r^4}{8\eta L}$

- **Viscous force** $F_v = 6\pi\eta r v$

- **Terminal velocity** $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow v_T \propto r^2$

- **Reynolds number** $R_e = \frac{\rho v d}{\eta}$

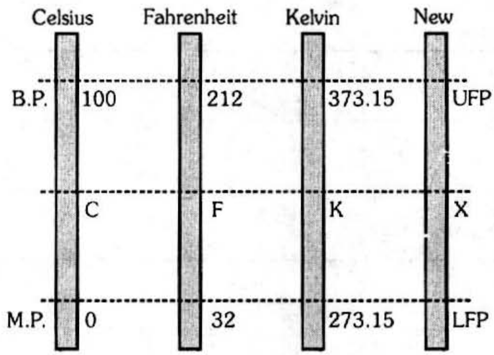
$R_e < 1000$ laminar flow, $R_e > 2000$ turbulent flow

Important Notes

12 Thermal Physics

TEMPERATURE SCALES AND THERMAL EXPANSION

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Kelvin	K	273.15 K	373.15 K	100



$$\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{K-273.15}{373.15-273.15}=\frac{X-LFP}{UFP-LFP}$$

$$\Rightarrow \frac{\Delta C}{100}=\frac{\Delta F}{180}=\frac{\Delta K}{100}=\frac{\Delta X}{UFP-LFP}$$

- **Old thermometry**

  $\frac{\theta-0}{100-0}=\frac{X-X_0}{X_{100}-X_0}$ [two fixed points – ice & steam points]

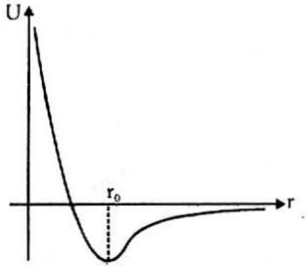
  where X is thermometric property i.e. length, resistance etc.

- **Modern thermometry** $\frac{T-0}{273.16-0}=\frac{X}{X_{tr}}$

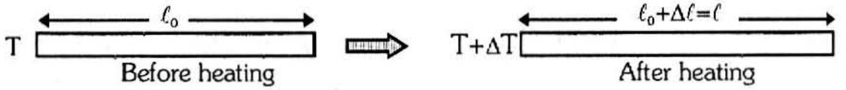
  [Only one reference point – triple point of water is chosen]

THERMAL EXPANSION

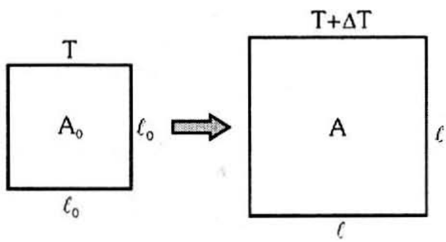
It is due to asymmetry in potential energy curve.



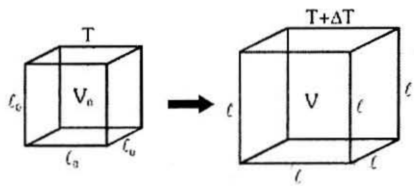
In solids $\rightarrow$ Linear expansion $\ell = \ell_0\,(1+\alpha\Delta T)$



In solids $\rightarrow$ Areal expansion $A = A_0\,(1+\beta\Delta T)$



In solids, liquids and gases $\rightarrow$ volume expansion $V=V_0(1+\gamma\Delta T)$



[For isotropic solids : $\alpha : \beta : \gamma = 1 : 2 : 3$]

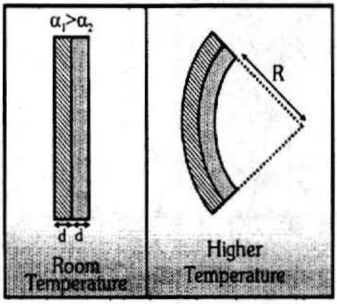
Thermal expansion of an isotropic object may be imagined as a photographic enlargement.

For anisotropic materials $\beta_{xy} = \alpha_x + \alpha_y$ and $\gamma = \alpha_x + \alpha_y + \alpha_z$

If α is variable : $\Delta\ell = \int_{T_1}^{T_2} \ell_0 \alpha\, dT$

Application of Thermal expansion in solids

I. Bi-metallic strip *(used as thermostat or auto-cut in electric heating circuits)*

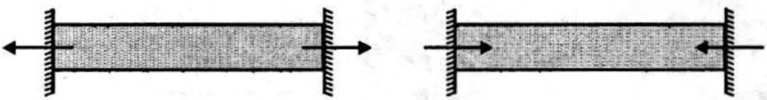


II. Simple pendulum : $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \ell^{1/2} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$

Fractional change in time period $= \frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

III. Scale reading : Due to linear expansion / contraction, scale reading will be lesser / more than actual value.
If temperature ↑ then actual value = scale reading $(1+\alpha\Delta\theta)$

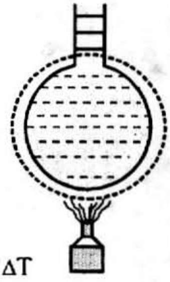
IV. Thermal Stress



Thermal strain $= \frac{\Delta \ell}{\ell} = \alpha\Delta\theta$

As Young's modulus $Y = \frac{F/A}{\Delta\ell/\ell}$; So thermal stress $= YA\alpha\Delta\theta$

Thermal expansion in liquids (Only volume expansion)



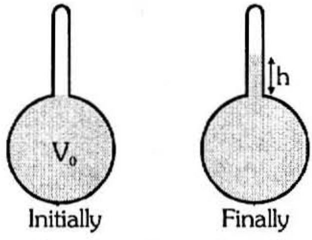
$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Initial volume} \times \text{Temperature rise}}$$

$$\gamma_r = \frac{\text{real increse in volume}}{\text{initial volume} \times \text{temperature rise}}$$

$\gamma_r = \gamma_a + \gamma_{vessel}$

Change in volume of liquid w.r.t. vessel $\Delta V = V_0(\gamma_r - 3\alpha)\Delta T$

Expansion in enclosed volume



[$\alpha \rightarrow$ vessel; $\gamma_L \rightarrow$ liquid]

Increase in height of liquid level in tube when bulb was initially completely filled.

$$h = \frac{\text{apparent change in volume of liquid}}{\text{area of tube}} = \frac{V_0(\gamma_L - 3\alpha)\Delta T}{A_0(1+2\alpha)\Delta T}$$

Anomalous expansion of water :

In the range 0°C to 4 °C water contract on heating and expands on cooling.

At 4°C $\rightarrow$ density is maximum.

Aquatic life is able to survive in very cold countries as the lake bottom remains unfrozen at the temperature around 4°C.

Thermal expansion of gases :

- Coefficient of volume expansion $\gamma_V = \dfrac{\Delta V}{V_0 \Delta T} = \dfrac{1}{T}$

[PV = nRT at constant pressure $V \propto T \Rightarrow \dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$]

- Coefficient of pressure expansion $\gamma_P = \dfrac{\Delta P}{P_0 \Delta T} = \dfrac{1}{T}$

KEY POINTS :

- Liquids usually expand more than solids because the intermolecular forces in liquids are weaker than in solids.
- Rubber contract on heating because in rubber as temperature increases, the amplitude of transverse vibrations increases more than the amplitude of longitudinal vibrations.
- Water expands both when heated or cooled from 4°C because volume of water at 4°C is minimum.
- In cold countries, water pipes sometimes burst, because water expands on freezing.

CALORIMETRY

$$\boxed{1\ \text{cal} = 4.186\ \text{J} ; 4.2\ \text{J}}$$

- **Thermal capacity of a body** = $\frac{Q}{\Delta T}$

  Amount of heat required to raise the temp. of a given body by 1°C (or 1K).

- **Specific heat capacity** = $\frac{Q}{m\Delta T}$ (m = mass)

  Amount of heat required to raise the temperature of unit mass of a body through 1°C (or 1K)

- **Molar heat capacity**= $\frac{Q}{n\Delta T}$ (n=number of moles)

- **Water equivalent :** If thermal capacity of a body is expressed in terms of mass of water, it is called water equivalent. Water equivalent of a body is the mass of water which when given same amount of heat as to the body, changes the temperature of water through same range as that of the body.

  Therefore water equivalent of a body is the quantity of water, whose heat capacity is the same as the heat capacity of the body.

  Water equivalent of the body,

  $$W = \text{mass of body} \times \left(\frac{\text{specific heat of body}}{\text{specific heat of water}}\right)$$

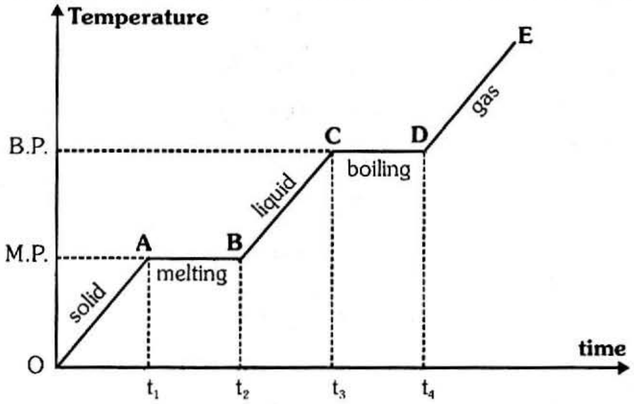
  Unit of water equivalent is g or kg.

- **Latent Heat (Hidden heat) :** The amount of heat that has to supplied to (or removed from) a body for its complete change of state (from solid to liquid, liquid to gas etc) is called latent heat of the body. Remember that phase transformation is an isothermal (i.e. temperature = constant) change.

- **Principle of calorimetry** : Heat lost = heat gained

  For temperature change $Q = ms\Delta T$, For phase change $Q = mL$

- **Heating curve :** If to a given mass (m) of a solid, heat is supplied at constant rate (Q) and a graph is plotted between temperature and time, the graph is called heating curve.

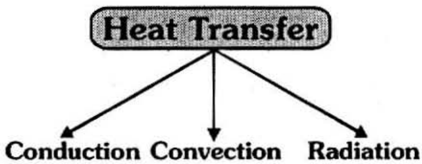


Specific heat (or thermal capacity) $\propto \dfrac{1}{\text{slope of curve}}$

Latent heat $\propto$ length of horizontal line.

KEY POINTS

- Specific heat of a body may be greater than its thermal capacity as mass of the body may be less than unity.
- The steam at 100°C causes more severe burn to human body than the water at 100°C because steam has greater internal energy than water due to latent heat of vaporization.
- Heat is energy in transit which is transferred from hot body to cold body.
- One calorie is the amount of heat required to raise the temperature of one gram of water through 1°C (more precisely from 14.5 °C to 15.5°C).
- Clausins & clapeyron equation (effect of pressure on boiling point of liquids & melting point of solids related with latent heat) $\dfrac{dP}{dT} = \dfrac{L}{T(V_2 - V_1)}$



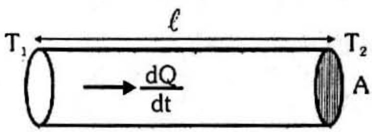
In conduction, heat is transferred from one point to another without the actual motion of heated particles.

In the process of convection, the heated particles of matter actually move.

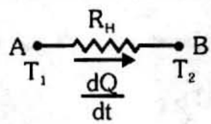
In radiation, intervening medium is not affected and heat is transferred without any material medium.

Conduction	Convection	Radiation
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer with out any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solid body (in mercury also)	Heat transfer in fluids (Liquid + gas)	All
Slow process	Slow process	Fast process (3×10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)

THERMAL CONDUCTION



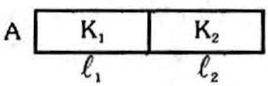
Electrical equivalent



Rate of heat flow $\frac{dQ}{dt} = -KA\frac{dT}{dx}$ or $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{\ell}$

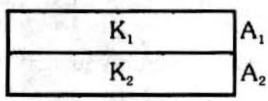
Thermal resistance $R_H = \frac{\ell}{KA}$

Rods in series



$$K_{eq} = \frac{\Sigma \ell}{\Sigma \ell / K}$$

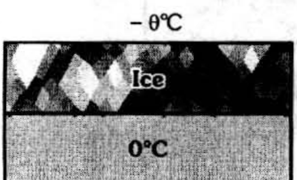
Rods in parallel



$$K_{eq} = \frac{\Sigma KA}{\Sigma A}$$

Growth of Ice on Ponds

Time taken by ice to grow a thickness from x_1 to x_2 : $t = \frac{\rho L}{2K\theta}(x_2^2 - x_1^2)$

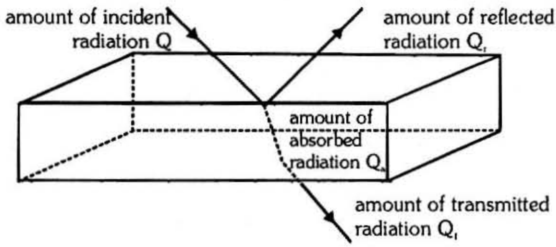


[K = thermal conductivity of ice, ρ = density of ice]

RADIATION

- **Spectral, emissive, absorptive and transmittive power of a given body surface** : Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts.

  (a) Reflection (b) Absorption (c) Transmission



From energy conservation

$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1 \Rightarrow r + a + t = 1$$

- Reflective Coefficient : $r = \frac{Q_r}{Q}$
- Absorptive Coefficient : $a = \frac{Q_a}{Q}$
- Transmittive Coefficient : $t = \frac{Q_t}{Q}$

$r = 1$ and $a = 0$, $t = 0$ $\Rightarrow$ Perfect reflector

$a = 1$ and $r = 0$, $t = 0$ $\Rightarrow$ Ideal absorber (ideal black body)

$t = 1$ and $a = 0$, $r = 0$ $\Rightarrow$ Perfect transmitter (daithermanons)

$$\text{Reflection power (r)} = \left[\frac{Q_r}{Q} \times 100\right]\%$$

$$\text{Absorption power (a)} = \left[\frac{Q_a}{Q} \times 100\right]\%$$

$$\text{Transmission power (t)} = \left[\frac{Q_t}{Q} \times 100\right]\%$$

- **Stefan's Boltzmann law :**
  Radiated energy emitted by a perfect black body per unit area/sec $E = \sigma T^4$
  For a general body $E = \sigma e_r T^4$ [where $0 \le e_r \le 1$]

- **Prevost's theory of heat exchange :** A body is simultaneously emitting radiations to its surrounding and absorbing radiations from the surroundings.
  If surrounding has temperature T_0 then $E_{net} = e_r \sigma (T^4 - T_0^4)$
- **Kirchhoff's law :** The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power

of a perfectly black body at that temperature.

$$\frac{e}{a}=\frac{E}{A}=\frac{E}{1}\Rightarrow\frac{e}{a}=E\Rightarrow e\propto a$$

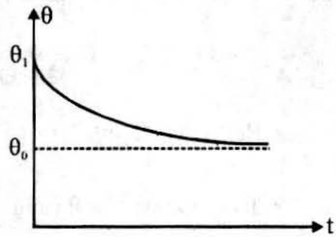
Therefore a good absorber is a good emitter.

- **Perfectly Black Body :** A body which absorbs all the radiations incident on it is called a perfectly black body.

- **Absorptive Power (a) :** Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time.
  For ideal black body, absorptive power =1

- **Emissive power(e) :** For a given surface it is defined as the radiant energy emitted per second per unit area of the surface.

- **Newton's law of cooling:**
  If temperature difference is small

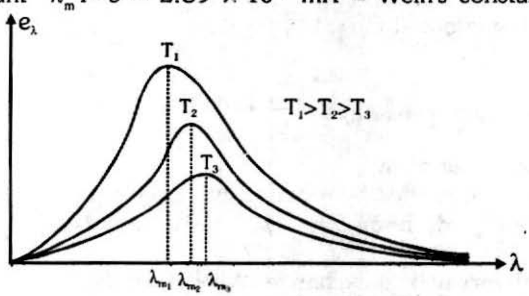
  Rate of cooling $\frac{d\theta}{dt}\propto(\theta-\theta_0)$

  $\Rightarrow \theta=\theta_0+(\theta_1-\theta_0)e^{-kt}$

  [where k = constant]

  when a body cools from θ_1 to θ_2 in time 't' in a surrounding of temperature θ_0 then $\frac{\theta_1-\theta_2}{t}=k\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$ [where k= constant]

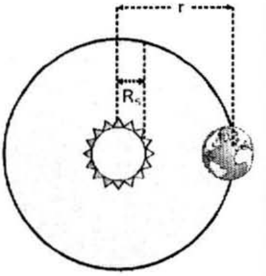
- **Wien's Displacement Law :** Product of the wavelength λ_m of most intense radiation emitted by a black body and absolute temperature of the black body is a constant $\lambda_m T=b = 2.89\times10^{-3}$ mK = Wein's constant



Area under $e_\lambda-\lambda$ graph$=\int_0^\infty e_\lambda d\lambda=e=\sigma T^4$

Solar constant

The Sun emits radiant energy continuously in space of which an insignificant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.



$$S=\frac{P}{4\pi r^2}=\frac{4\pi R_S^2\sigma T^4}{4\pi r^2}=\left(\frac{R_S}{r}\right)^2 T^4$$

where R_S = radius of sun

r = average distance between sun and earth.

Note :- $S = 2$ cal cm^{-2}min$=1.4$ kWm^{-2}

T = temperature of sun ≈ 5800 K

KEY POINTS :

- Stainless steel cooking pans are preferred with extra copper bottom because thermal conductivity of copper is more than steel.
- Two layers of cloth of same thickness provide warmer covering than a single layer of cloth of double the thickness because air (which is better insulator of heat) is trapped between them.
- Animals curl into a ball when they feel very cold to reduce the surface area of the body.
- Water cannot be boiled inside a satellite by convection because in weightlessness conditions, natural movement of heated fluid is not possible.
- Metals have high thermal conductivity because metals have free electrons.

KINETIC THEORY OF GASES

It related the macroscopic properties of gases to the microscopic properties of gas molecules.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speed is maximum.

Assumptions regarding collision:

- The gas molecules keep colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

Kinetic interpretation of pressure : $PV = \frac{1}{3}mNv_{rms}^2$

[m = mass of a molecule, N = no. of molecules]

Ideal gas equation $PV = \mu RT \Rightarrow P = \frac{\mu RT}{V} = \frac{\mu N_A kT}{V} = \left(\frac{N}{V}\right)kT = nkT$

Gas laws

- **Boyle's law** :For a given mass at constant temperature. $V \propto \frac{1}{P}$
- **Charles' law** : For a given mass at constant pressure $V \propto T$
- **Gay-Lussac's law** For a given mass at constant volume $P \propto T$
- **Avogadro's law**:If P,V & T are same then no. of molecules $N_1=N_2$
- **Graham's law** : At constant P and T, Rate of diffusion $\propto \frac{1}{\sqrt{\rho}}$
- **Dalton's law** : $P = P_1 + P_2 + ...$
  Total pressure =Sum of partial pressures

Real gas equation [Vander Waal's equation] $\left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$

where a & b are vander waal's constant and depend on the nature of gas.

Critical temperature $T_c = \frac{8a}{27Rb}$

The maximum temperature below which a gas can be liquefied by pressure alone.

Critical volume $V_c = 3b$

Critical pressure $P_c = \frac{a}{27b^2}$

Note :- For a real gas $\frac{P_C V_C}{RT_C} = \frac{3}{8}$

Different speeds of molecules

$$v_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} \qquad v_{mp} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} \qquad v_{av} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}}$$

Kinetic Interpretation of Temperature :

Temperature of an ideal gas is proportional to the average KE of molecules,

$$PV = \frac{1}{3}mNV^2_{rms} \text{ \& } PV = \mu RT \Rightarrow \frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$$

Degree of Freedom (F) :

Number of minimum coordinates required to specify the dynamical state of a system.

For monoatomic gas (He, Ar etc) f=3 (only translational)

For diatomic gas (H_2, O_2 etc) f=5 (3 translational + 2 rotational)

At higher temperature, diatomic molecules have two degree of freedom due to vibrational motion (one for KE + one for PE)

At higher temperature diatomic gas has f=7

Maxwell's Law of equipartition of energy:

Kinetic energy associated with each degree of freedom of particles of an ideal gas is equal to $\frac{1}{2}kT$

- Average KE of a particle having f degree of freedom= $\frac{f}{2}kT$
- Translational KE of a molecule = $\frac{3}{2}kT$
- Translational KE of a mole = $\frac{3}{2}RT$
- Internal energy of an ideal gas: $U = \frac{f}{2}\mu RT$

Specific heats (C_p and C_V) :

- Molar specific heat of a gas $C = \frac{dQ}{\mu dT}$
- $C_V = \left(\frac{dQ}{\mu dT}\right)_{V=constant} = \frac{dU}{\mu dT}$
- $C_P = \left(\frac{dQ}{\mu dT}\right)_{dP=0} = C_V + R$ ← Mayer's equation

Atomicity	Translational	Rotational	Total (f)	$\gamma = \frac{C_P}{C_V}$	$C_V = \frac{f}{2}R$	$C_P = C_V + R$
Monoatomic [He, Ar, Ne.].	3	0	3	$\frac{5}{3} = 1.67$	$\frac{3}{2}R$	$\frac{5}{2}R$
Diatomic [H_2, N_2..]	3	2	5	$\frac{7}{5} = 1.4$	$\frac{5}{2}R$	$\frac{7}{2}R$
Triatomic (Linear CO_2)	3	2	5	$\frac{7}{5} = 1.4$	$\frac{5}{2}R$	$\frac{7}{2}R$
Triatomic Non-linear-NH_3) & Polyatomic	3	3	6	$\frac{4}{3} = 1.33$	3R	4R

Mean free path :

Average distance between two consecutive collisions $\lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}$

where d = diameter of molecule, n = molecular density $= \frac{N}{V}$

For mixture of non-reacting gases

Molecular weight $M_{W_{mix}} = \frac{\mu_1 M_{W_1} + \mu_2 M_{W_2} + ...}{\mu_1 + \mu_2 + ...}$

Specific heat at constant V $C_{V_{mix}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2} + ...}{\mu_1 + \mu_2 + ...}$

Specific heat at constant P $C_{P_{mix}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2} + ...}{\mu_1 + \mu_2 + ...}$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2} + ...}{\mu_1 C_{V_1} + \mu_2 C_{V_2} + ...}$$