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Sec : 2
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STOCHASTIC FINAL PROJECT

1 Problem & Requirements

The signal equation is defined as follows:

$$y(n) = c0x(n) + c1x(n-1) + c2x(n-2) + c3x(n-3) + v(n)$$

where: y(n) is the distorted signal. x(n) is the source signal, and v(n) is WGN v(n) $\sim N(0,\sigma^2)$
$c0 = -1$ $c1 = -0.75$ $c2 = -0.5$ $c3 = -0.25$ and $\sigma_v^2 = 0.01$

- Using provided model, build a fifth order Wiener filter.
- Apply this filter on the signal and show the output.
- Calculate mean square error of filtered signal (Source signal is provided for that).

2 Solving The equations to get C matrix

$y(n) = c0x(n) + c1x(n-1) + c2x(n-2) + c3x(n-3) + v(n)$

•At (n = 0) :

$\mathrm{R}_{yy}(0) = E[y(n)y(n)]$
$R_{yy}(0) = E[y(n)(c0x(n) + c1x(n-1) + c2x(n-2) + c3x(n-3) + v(n))]$
$= c0E[x(n)y(n)] + c1E[x(n-1)y(n)] + c2E[x(n-2)y(n)] + c3E[x(n-3)y(n)] + E[v(n)y(n)]$
$= c0E[x(n)y(n)] + c1E[x(n-1)y(n)] + c2E[x(n-2)y(n)] + c3E[x(n-3)y(n)] + E[v^2(n)]$
$R_{yy}(0) - 0.01 = c0R_{xy}(0) + c1R_{xy}(1) + c2R_{xy}(2) + c3R_{xy}(3)$............(1)

•At (n = 1) :

$\mathrm{R}_{yy}(1) = E[y(n-1)y(n)]$
$R_{yy}(1) = E[y(n-1)(c0x(n) + c1x(n-1) + c2x(n-2) + c3x(n-3) + v(n))]$
$= c0R_{xy}(1) + c1R_{xy}(0) + c2R_{xy}(-1) + c3R_{xy}(-2)$
$R_{xy}(-1) = R_{xy}(1)$
$R_{yy}(1) = (c0 + c2)R_{xy}(1) + c1R_{xy}(0) + c3R_{xy}(2)$...............(2)

•At (n = 2) :

$\mathrm{R}_{yy}(2) = E[y(n-2)y(n)]$
$R_{yy}(2) = c0R_{xy}(2) + (c1+c3)R_{xy}(1) + c2R_{xy}(0)$..............(3)

•At (n = 3) :

$\mathrm{R}_{yy}(3) = E[y(n-3)y(n)]$
$R_{yy}(3) = c0R_{xy}(3) + c1R_{xy}(2) + c2R_{xy}(1) + c3R_{xy}(0)$...............(4)

•At (n = 4) :

$\mathrm{R}_{yy}(4) = E[y(n-4)y(n)]$
$R_{yy}(4) = c0R_{xy}(4) + c1R_{xy}(3) + c2R_{xy}(2) + c3R_{xy}(1)$...............(5)

•At (n = 5) :

$\mathrm{R}_{yy}(5) = E[y(n-5)y(n)]$
$R_{yy}(5) = c0R_{xy}(5) + c1R_{xy}(4) + c2R_{xy}(3) + c3R_{xy}(2)$...............(6)

From eqn.1 to eqn.5 we get (Using Python To get Matrices):

$$R_{yy} = \begin{bmatrix} \mathrm{R}_{yy}(0) \\ \mathrm{R}_{yy}(1) \\ \mathrm{R}_{yy}(2) \\ \mathrm{R}_{yy}(3) \\ \mathrm{R}_{yy}(4) \\ \mathrm{R}_{yy}(5) \end{bmatrix} = \begin{bmatrix} 0.37022914 \\ 0.41682667 \\ 0.37804229 \\ 0.31820253 \\ 0.23307373 \\ 0.14933956 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \mathrm{R}_{yy}(0) - 0.01 \\ \mathrm{R}_{yy}(1) \\ \mathrm{R}_{yy}(2) \\ \mathrm{R}_{yy}(3) \\ \mathrm{R}_{yy}(4) \\ \mathrm{R}_{yy}(5) \end{bmatrix} = \begin{bmatrix} c0 & c1 & c2 & c3 & 0 & 0 \\ c1 & c0+c2 & c3 & 0 & 0 & 0 \\ c2 & c1+c3 & c0 & 0 & 0 & 0 \\ c3 & c2 & c1 & c0 & 0 & 0 \\ 0 & c3 & c2 & c1 & c0 & 0 \\ 0 & 0 & c3 & c2 & c1 & c0 \end{bmatrix} \begin{bmatrix} \mathrm{R}_{xy}(0) \\ \mathrm{R}_{xy}(1) \\ \mathrm{R}_{xy}(2) \\ \mathrm{R}_{xy}(3) \\ \mathrm{R}_{xy}(4) \\ \mathrm{R}_{xy}(5) \end{bmatrix} \tag{2}$$

$$
\begin{bmatrix} R_{xy}(0) \\ R_{xy}(1) \\ R_{xy}(2) \\ R_{xy}(3) \\ R_{xy}(4) \\ R_{xy}(5) \end{bmatrix} = \begin{bmatrix} c0 & c1 & c2 & c3 & 0 & 0 \\ c1 & c0+c2 & c3 & 0 & 0 & 0 \\ c2 & c1+c3 & c0 & 0 & 0 & 0 \\ c3 & c2 & c1 & c0 & 0 & 0 \\ 0 & c3 & c2 & c1 & c0 & 0 \\ 0 & 0 & c3 & c2 & c1 & c0 \end{bmatrix}^{-1} \begin{bmatrix} R_{yy}(0) - 0.01 \\ R_{yy}(1) \\ R_{yy}(2) \\ R_{yy}(3) \\ R_{yy}(4) \\ R_{yy}(5) \end{bmatrix} \quad (3)
$$

After substitute with c0,c1,c2,c3 Values:

$$
C = \begin{bmatrix} -1 & -0.75 & -0.5 & -0.25 & 0 & 0 \\ -0.75 & -1+-0.5 & c3 & 0 & 0 & 0 \\ -0.5 & -0.75+c3 & -1 & 0 & 0 & 0 \\ c3 & -0.5 & -0.75 & -1 & 0 & 0 \\ 0 & c3 & -0.5 & -0.75 & -1 & 0 \\ 0 & 0 & c3 & -0.5 & -0.75 & -1 \end{bmatrix}
$$

3 Results(using Python calculations)

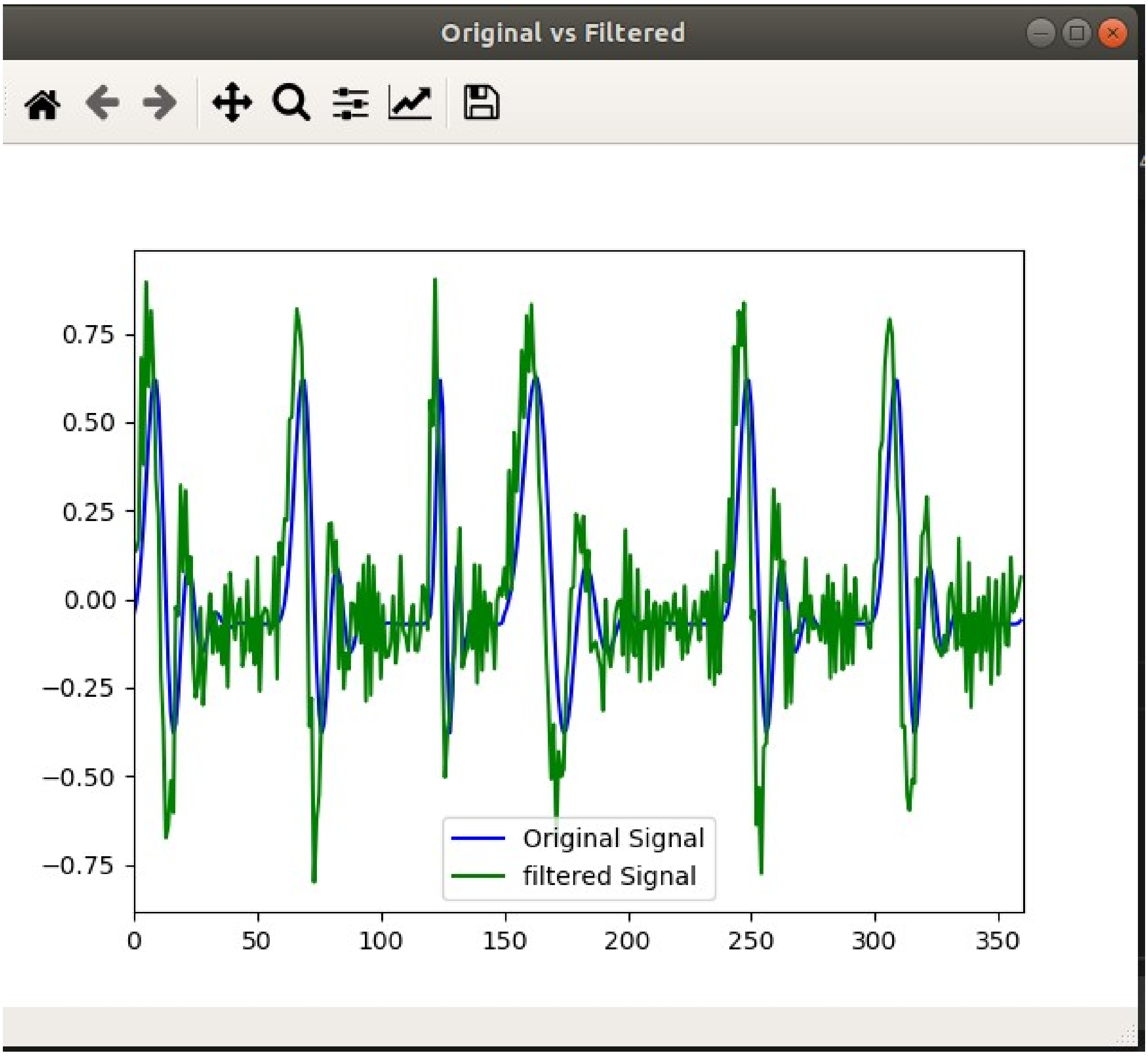
$$
\begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \end{bmatrix} = \begin{bmatrix} -0.60992753 \\ 0.43250195 \\ -0.28395866 \\ -0.02426748 \\ -0.05485965 \\ 0.21663751 \end{bmatrix} \quad (4)
$$

Calculate mean squared error of the filtered signal:

$$
\hat{x}(n) = y(n) * h(n)
$$

Then we get MSE = 0.0540871

3.1 Original VS Filtered Signals



3.2 Filtered VS Distorted Signals