ivd: an R package for individual variance detection

A holistic view of academic performance: Beyond Averages with MELSM and Spike-and-Slab

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Introduction

To get a more complete picture of academic achievement of a student or a school, it's important to look at both average performance and variability within a cluster. We adapted the mixed-effects location scale model (MELSM) using the Spike and Slab regularization technique to shrink random effects to their fixed effect. This allows us to identify clusters with unusually (in)consistent academic achievement.

Model formulation

The random effects from the scale and location can be modeled as, $\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$. The covariance matrix of the random effects can be decomposed into $\boldsymbol{\Sigma} = \boldsymbol{\tau}\boldsymbol{\Omega}\boldsymbol{\tau}'$ to specify independent priors for each element of $\boldsymbol{\tau}$ (random-effects standard deviations) and $\boldsymbol{\Omega}$ (correlation matrix among all random effects).

$\boldsymbol{\Omega}$ can be factorized via the Cholesky $\mathbf{L}$ of $\boldsymbol{\Omega} = \mathbf{L}'\mathbf{L}$. Then, $\mathbf{u}$ can be recovered multiplying $\mathbf{L}$ by the random effect standard deviations, $\boldsymbol{\tau}$, and scaling it with a standard normally distributed $\boldsymbol{z}$

An indicator variable (δ) is included in the prior to allow switching between the spike and slab throughout the MCMC sampling process. For each school j, we obtain a probabilistic measure on whether the scale random effect is to be included or not:

$$\mathbf{u}_j = \boldsymbol{\tau}\mathbf{L}\boldsymbol{\delta}_j\mathbf{z}_j$$

For a random intercept only model,

$$\alpha_{0j} = \begin{cases} \eta_0, & \text{if } \delta_j = 0, \\ \eta_0 + u_{0j}, & \text{if } \delta_j = 1 \end{cases}.$$

The posterior inclusion probabilities (PIP) can then be computed as

$$Pr(\alpha_{0j} = \eta_0 | \mathbf{Y}) = 1 - \frac{1}{S}\sum_{s=1}^{S}\delta_{js},$$

where $S = \{1, \ldots, s\}$ denotes the posterior samples and η_0 is the average within-person variability.

The methods have been implemented in the R package `ivd`:

```
devtools::install_github("consistentlybetter/ivd")
```

Illustrative example

The data comes from The Basic Education Evaluation System (Saeb) conducted by Brazil's National Institute for Educational Studies and Research (Inep). It is also available as the `saeb` dataset in the `ivd` package. The outcome variable is `math_proficiency` at the end of grade 12. Both, location and scale are modeled as a function of student and school SES.

```
out <- ivd(
  location_formula = math_proficiency ~ student_ses * school_ses + (1|school_id),
  scale_formula =  ~ student_ses * school_ses + (1|school_id),
  data = saeb,
  niter = 2000, nburnin = 7000, WAIC = TRUE, workers = 4)
```

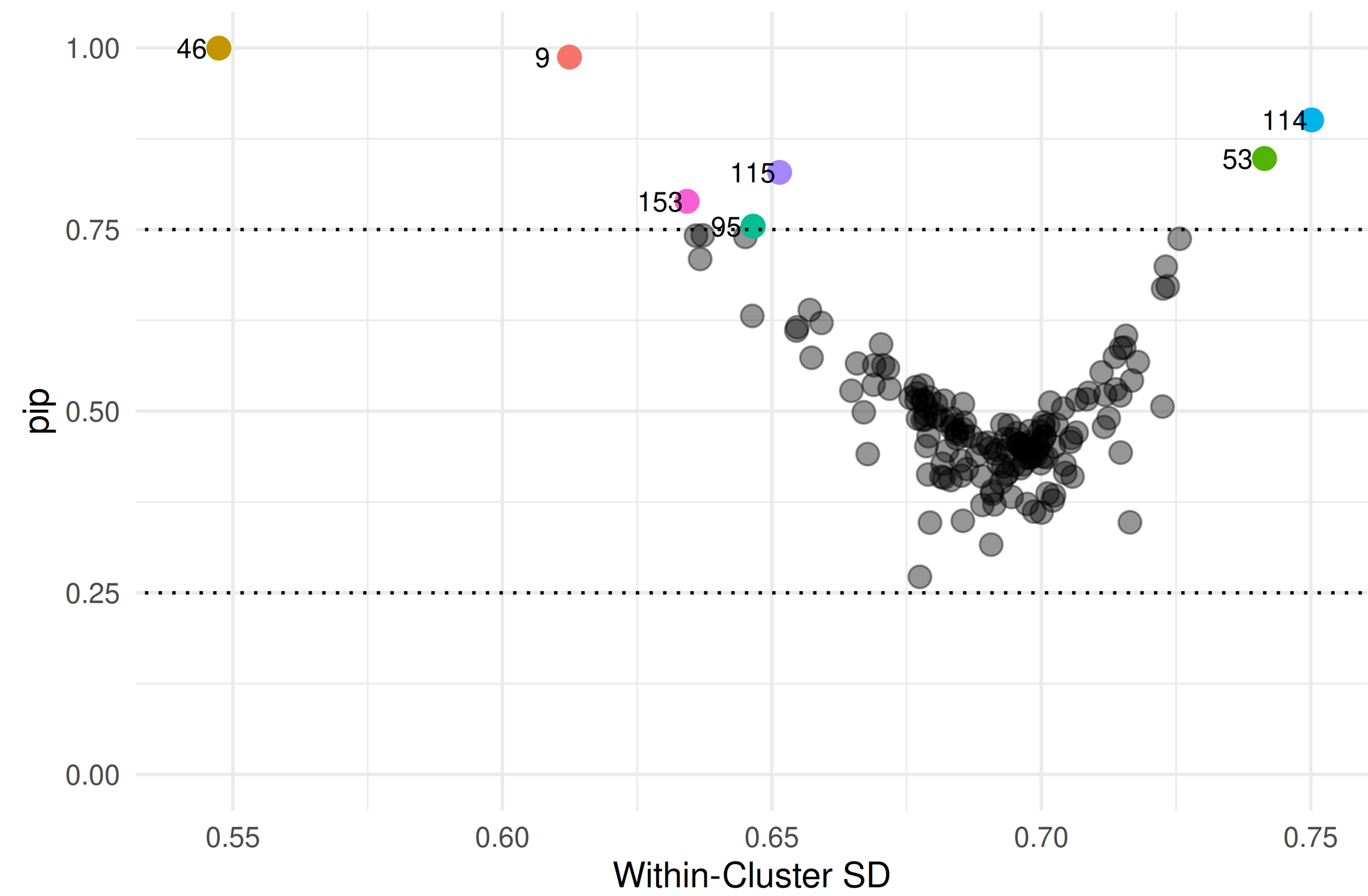

Figure 1: Funnel plot of posterior inclusion probabilities plotted against school's standard deviations. Higher PIP values express more evidence for the random effect of a given school to be included in the model.

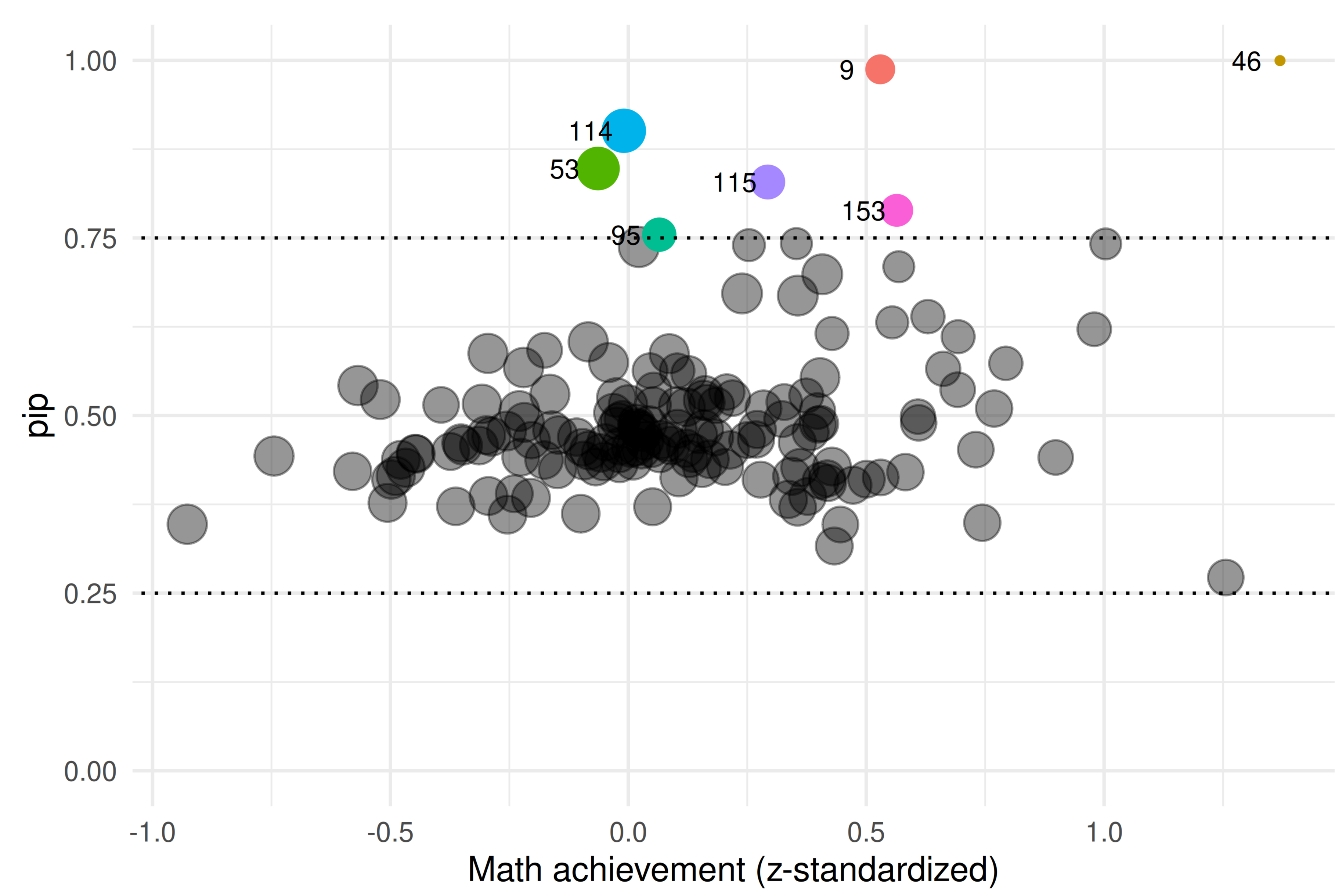

Figure 2: Posterior inclusion probabilities plotted against school's average math achievement in standardized units. Point size represents within-school standard deviation.

References

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