1 Commutative Algebra

1.1 *

(ネーター) 環 A とその上の有限生成射影加群 M で、各極大イデアル $\mathfrak{m}$ において $M_{\mathfrak{m}}$ が階数 1 の自由加群となるようなものを探せ。

Find a (Noetherian) ring A and a finite projective A-module M such that $M_{\mathfrak{m}}$ is free of rank 1 over $A_{\mathfrak{m}}$ for every maximal ideal $\mathfrak{m}$ of A.

(Comment)

$X = \operatorname{Spec} A$ 上の可逆層 $\mathcal{L}$ で、$\mathcal{O}_X$ と同型でないものを探すことと同値。

This is equivalent to finding an invertible sheaf $\mathcal{L}$ over $X = \operatorname{Spec} A$ that is not isomorphic to $\mathcal{O}_X$.

See also: https://stacks.math.columbia.edu/tag/02AC 109.40.3.

1.2 *

2 Algebraic Geometry

2.1 *

When S is a graded ring of dimension d, what is the relation between d and $\dim \operatorname{Proj} S$?

3 Set Theory

3.1 *