Problem Set 3

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1. Constraint 1: Any set $W_i \in W$ where W_i is also in Y, the union of all such W_i should include any atom of U exactly once. In other words, $\forall a \in U$, if $a \in Y$ then it is only in Y once. In the example given, we cannot have $Y = \{a, d, f, h\}, \{b, d, f, h\}$ as f and h are in Y twice.

   Constraint 2: For the problem to actually be satisfiable, the union of all sets $W_i \in W$ must include every $a \in U$ **at least once**. If this constraint is not met, the problem is unsatisfiable as no collection of W_i will include every atom in S. If, in the example given, W did not include a single W_i that included a, the problem would be unsolvable.

   Constraint 3: $\forall_{W_i} \in W, j \in W_i \Leftrightarrow j \in U$ - this is given via the problem prompt but is an additional constraint.

   Constraint 4: This follows from constraint one, however Y may not include any $W_i \in W$ more than once, so Y cannot include $\{a, d, f, h\}$ twice.

2. Problem 2:

   (a) `Parent(Anne, Ted)`
   (b) $\forall_{x,y}$ $\texttt{Parent}(\texttt{x},\texttt{y}) \Rightarrow \texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{Birth}(\texttt{q}))$
   (c) $\forall_{p,q}, \texttt{Living}(\texttt{p},\texttt{t}) \Leftrightarrow \texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{t}) \wedge \texttt{Earlier}(\texttt{t},\texttt{Death}(\texttt{p}))$
   (d) $\forall_{ta,tb}$ $\texttt{Earlier}(\texttt{ta},\texttt{tb}) \Rightarrow \neg\texttt{Earlier}(\texttt{tb},\texttt{ta})$ (I use implies instead of iff to account for the case where ta == tb)
   (e) $\forall_{ta,tb,tc}$ $\texttt{Earlier}(\texttt{ta},\texttt{tb}) \wedge \texttt{Earlier}(\texttt{tb},\texttt{tc}) \Rightarrow \texttt{Earlier}(\texttt{ta},\texttt{tc})$
   (f) $\forall_p$ $\exists_t$ $\texttt{s.t.Living}(\texttt{p},\texttt{t})$
   (g) $\neg\texttt{Living}(\texttt{Ted},\texttt{Born}(\texttt{Anne}))$
   (h) $\forall_p$ $\texttt{Earlier}(\texttt{Born}(\texttt{p}),\texttt{Death}(\texttt{p}))$

3. After conversion to CNF, we are left with the following clauses:

   a. `Parent(Anne, Ted)`
   b. $\neg\texttt{Parent}(\texttt{x},\texttt{y}) \vee \texttt{Earlier}(\texttt{Birth}(\texttt{x}),\texttt{Birth}(\texttt{y}))$
   c1. $\neg\texttt{Living}(\texttt{p},\texttt{t}) \vee \texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{t})$
   c2. $\neg\texttt{Living}(\texttt{p},\texttt{t}) \vee \texttt{Earlier}(\texttt{t},\texttt{Death}(\texttt{p}))$
   c3. $\neg\texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{t}) \vee \neg\texttt{Earlier}(\texttt{t},\texttt{Death}(\texttt{p})) \vee \texttt{Living}(\texttt{p},\texttt{t})$
   d. $\neg\texttt{Earlier}(\texttt{ta},\texttt{tb}) \vee \neg\texttt{Earlier}(\texttt{tb},\texttt{ta})$
   S: `Living(Ted, Born(Anne))` **- this is our negated assumption to prove.**

   The resolutions done follow below:

ID	Clauses	Substitution	Resolution
r1	a, b	$\{x \to Anne, y \to Ted\}$	`E(B(Anne), B(Ted))`
r2	r1, d	$\{ta \to B(Anne), tb \to B(Ted)\}$	$\neg\texttt{E}(\texttt{B}(\texttt{Ted}),\texttt{B}(\texttt{Anne}))$
r3	r2, c1	$\{p \to Ted, t \to B(Anne)\}$	$\neg\texttt{L}(\texttt{Ted},\texttt{B}(\texttt{Anne}))$
sol	r3, S	N/A	negation

   And we prove by contradiction that S cannot be true.

4. After conversion to CNF, we are left with the following clauses:

   c1. $\neg\texttt{Living}(\texttt{p},\texttt{t}) \vee \texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{t})$
   c2. $\neg\texttt{Living}(\texttt{p},\texttt{t}) \vee \texttt{Earlier}(\texttt{t},\texttt{Death}(\texttt{p}))$
   c3. $\neg\texttt{Earlier}(\texttt{Birth}(\texttt{p}),\texttt{t}) \vee \neg\texttt{Earlier}(\texttt{t},\texttt{Death}(\texttt{p})) \vee \texttt{Living}(\texttt{p},\texttt{t})$
   e. $\neg\texttt{Earlier}(\texttt{ta},\texttt{tb}) \vee \neg\texttt{Earlier}(\texttt{tb},\texttt{tc}) \vee \texttt{Earlier}(\texttt{ta},\texttt{tc})$
   f. $\texttt{Living}(\texttt{p},\lambda(\texttt{p}))$
   S. $\neg\texttt{Earlier}(\texttt{Born}(\texttt{p}),\texttt{Death}(\texttt{p}))$ **- this is our negated assumption to prove.**

   Again, the resolutions follow:

ID	Clauses	Substitution	Resolution
r1	c1, f	$\lambda(p) \to t$	$\texttt{E}(\texttt{B}(\texttt{p}),\texttt{t})$
r2	c2, f	$\lambda(p) \to t$	$\texttt{E}(\texttt{t},\texttt{D}(\texttt{p}))$
r3	r1, e	$\texttt{ta} \to \texttt{B}(\texttt{p}), \texttt{tb} \to \texttt{t}$	$\neg\texttt{E}(\texttt{t},\texttt{tc}) \vee \texttt{E}(\texttt{B}(\texttt{p}),\texttt{tc})$
r4	r2, r3	$\texttt{tc} \to \texttt{D}(\texttt{p})$	$\texttt{E}(\texttt{B}(\texttt{p}),\texttt{D}(\texttt{P}))$
sol	r4, S	N/A	negation

   Hence we prove by contradiction that S is not true.