Problem Set 2

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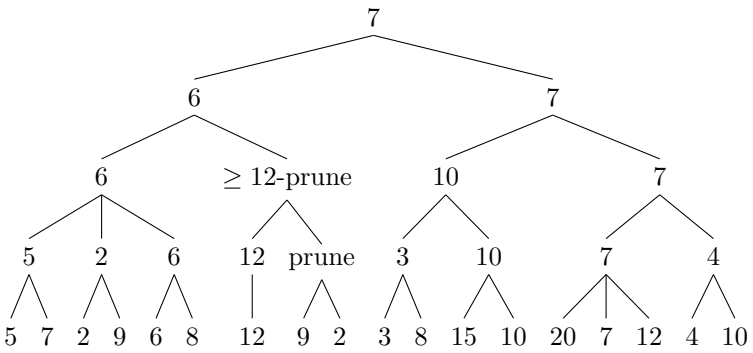
1. Alpha-Beta Pruning - assuming we work left to right and we prune if we decide that a sub-tree is not worth reviewing:



I only encountered one area that could be pruned (the node ≥ 12) - in this case the best move for MAX is to take the right side tree and get a score of 7 as opposed to 6.

2. Convert the following to clausal form:
   2.1. $C \Rightarrow (A \Leftrightarrow B)$
   2.2. $(\neg C \vee E) \Rightarrow B$
   2.3. $D \Rightarrow \neg B$
   2.4. $(A \wedge D) \Rightarrow \neg E$
   2.5. $C \vee D \vee E$
   2.6. $E \Rightarrow D$

   Taking these one at a time (**NOTE: I use the equivalency symbol $\equiv$ between steps**):

   2.1. $C \Rightarrow (A \Leftrightarrow B) \equiv C \Rightarrow ((A \Rightarrow B) \wedge (B \Rightarrow A)) \equiv \neg C \vee ((\neg A \vee B) \wedge (\neg B \vee A)) \equiv (C \vee (\neg A \vee B)) \wedge (C \vee (\neg B \vee A))$

   Which breaks to:

   $C \vee \neg A \vee B$
   $C \vee \neg B \vee A$

   2.2. $(\neg C \vee E) \Rightarrow B \equiv \neg(\neg C \vee E) \vee B \equiv (\neg\neg C \wedge \neg E) \vee B \equiv (C \wedge \neg E) \vee B \equiv (C \vee B) \wedge (\neg E \vee B)$

   Which breaks into:

   $C \vee B$
   $\neg E \vee B$

   2.3. $D \Rightarrow \neg B \equiv \neg D \vee \neg B$, which is in clausal form.

   2.4. $(A \wedge D) \Rightarrow \neg E \equiv \neg(A \wedge D) \vee \neg E \equiv \neg A \vee \neg D \vee \neg E$, which is in clausal form.

   2.5. $C \vee D \vee E$ is already in clausal form.

   2.6. $E \Rightarrow D \equiv \neg E \vee D$, which is in clausal form.

3. Davis-Putnam Algorithm on sentences in problem 2:

3.1. This case is easy, given:

$C \vee \neg A \vee B$
$C \vee \neg B \vee A$

The first run of the loop finds the pure-literal C, and since it is not a negated C it assigns the value TRUE to C, and then deletes all clauses containing C from the set of clauses S, which deletes all clauses. Then in the second run of the loop, since $S = \emptyset$, we assign TRUE or FALSE to B (if we always assign TRUE first then both are assigned TRUE), and we return C =TRUE, A =TRUE, and B =TRUE.

3.2. Again, similar to problem 1, here we encounter the pure literal B (technically C and E are also pure literals, but alphabetically we'd hit B first), we assign B =TRUE, then delete both clauses. In the next iteration of the loop we exit since $S = \emptyset$, and assign B, C, and E all TRUE.

3.3. Depending on how this algorithm is written, this can go two ways. Either $\neg B$ and $\neg D$ are both viewed as pure literals, B is assigned FALSE, and then we end as we did in the previous two responses (with D=TRUE), or we assume there are no "easy" cases and we try random assignment. In the latter case, we assign B to TRUE then delete B from the clause, then we find that the remaining clause only contains the literal $\neg D$, so we run "obviousassign()" and assign D=FALSE, making the clause TRUE, then we run a third iteration, find that S is empty, and return B=TRUE and D=FALSE (this assumes we check atoms in alphabetical order).

3.4. Just like in the previous answer, either $\neg A$ is considered a pure literal and is assigned FALSE, which makes the statement true and exits after the second iteration sees $S = \emptyset$, or A and D are assigned TRUE and removed from the clause, then $\neg E$ is found to be the only symbol remaining and E is appropriately assigned FALSE, making the entire statement true and returning.

3.5. Similar to 3 and 4, C may be considered a pure literal off the bat and assigned TRUE (assigning TRUE to the other symbols as well) or we assume no "easy" cases and C is assigned TRUE arbitrarily. On the second iteration of the loop, due to the propogation step, we find that S is empty (as C=TRUE makes the clause itself TRUE), and we assign C=TRUE and the others as TRUE as well.

3.6. Another case where we may terminate early finding that $\neg E$ or D are pure literals (if done alphabetically we assign D=TRUE first, then E is arbitrarily assigned TRUE or FALSE) or we may arbitrarily assign the symbol D TRUE, then propogate which will find the statement itself true and remove it, leaving S empty in the second iteration and returning D=TRUE and E=[TRUE or FALSE].