Problem Set 3

Mark Xavier (xaviem01)

March 31, 2019

1. Constraint 1: Any set $W_i \in W$ where W_i is also in Y, the union of all such W_i should include any atom of U exactly once. In other words, $\forall a \in U$, if $a \in Y$ then it is only in Y once.

   Constraint 2: For the problem to actually be satisfiable, the union of all sets $W_i \in W$ must include every $a \in U$ **at least once**. If this constraint is not met, the problem is unsatisfiable as no collection of W_i will include every atom in S.

2. Problem 2:

   (a) `Parent(Anne, Ted)`

   (b) $\forall_{x,y}$ $\texttt{Parent(x, y)} \Rightarrow \texttt{Earlier(Birth(p), Birth(q))}$

   (c) $\forall_{p,q}, \texttt{Living(p, t)} \Leftrightarrow \texttt{Earlier(Birth(p), t)} \wedge \texttt{Earlier(t, Death(p))}$

   (d) $\forall_{ta,tb}$ $\texttt{Earlier(ta, tb)} \Rightarrow \neg\texttt{Earlier(tb, ta)}$ (I use implies instead of iff to account for the case where ta == tb)

   (e) $\forall_{ta,tb,tc}$ $\texttt{Earlier(ta, tb)} \wedge \texttt{Earlier(tb, tc)} \Rightarrow \texttt{Earlier(ta, tc)}$

   (f) $\forall_p$ $\exists_t$ $\texttt{s.t.Living(p, t)}$

   (g) $\neg\texttt{Living(Ted, Born(Anne))}$

   (h) $\forall_p$ $\texttt{Earlier(Born(p), Death(p))}$

3. After conversion to CNF, we are left with the following clauses:

   a. `Parent(Anne, Ted)`
   b. $\neg\texttt{Parent(x, y)} \vee \texttt{Earlier(Birth(x), Birth(y))}$
   c1. $\neg\texttt{Living(p, t)} \vee \texttt{Earlier(Birth(p), t)}$
   c2. $\neg\texttt{Living(p, t)} \vee \texttt{Earlier(t, Death(p))}$
   c3. $\neg\texttt{Earlier(Birth(p), t)} \vee \neg\texttt{Earlier(t, Death(p))} \vee \texttt{Living(p, t)}$
   d. $\neg\texttt{Earlier(ta, tb)} \vee \neg\texttt{Earlier(tb, ta)}$

   Now assume `Living(Ted, Born(Anne))`. Given (a), we know `Earlier(Birth(Anne), Birth(Ted))`. Then we know from (c1) that for `Living(Ted, Born(Anne)) we need Earlier(Birth(Ted), Birth(Anne))`.
   This leads to `Earlier(Birth(Anne), Birth(Ted))` from (a), but also `Earlier(Birth(Ted), Birth(Anne))` from (c1), and per (d), we have a contradiction. Therefore we prove by contradiction that ¬ `Living(Ted, Born(Anne))`.

4. After conversion to CNF, we are left with the following clauses:

   c1. $\neg\texttt{Living(p, t)} \vee \texttt{Earlier(Birth(p), t)}$
   c2. $\neg\texttt{Living(p, t)} \vee \texttt{Earlier(t, Death(p))}$
   c3. $\neg\texttt{Earlier(Birth(p), t)} \vee \neg\texttt{Earlier(t, Death(p))} \vee \texttt{Living(p, t)}$
   e. $\neg\texttt{Earlier(ta, tb)} \vee \neg\texttt{Earlier(tb, tc)} \vee \texttt{Earlier(ta, tc)}$
   f. $\texttt{Living(p}, \lambda\texttt{(p))}$

   Now assume $\neg\texttt{Earlier(Born(p), Death(p))}$. Then from (f), we have that p is living at time $\lambda(p)$. From (c1), we have `Earlier(Birth(p),` $\lambda(p)$`)` and (c2) gives `Earlier(`$\lambda(p)$`, Death(p))`. However, at (e), we get that `Earlier(Birth(p),` $\lambda(p)$`)` $\wedge$ `Earlier(`$\lambda(p)$`, Death(p))` leads to `Earlier(Birth(p), Death(p))`. However, this is in direct contradiction to our starting assumption, meaning our assumption was wrong and (h) is proven.