

Problem Set 3

Mark Xavier (xaviem01)

April 1, 2019

1. Constraint 1: Any set $W_i \in W$ where W_i is also in Y , the union of all such W_i should include any atom of U exactly once. In other words, $\forall a \in U$, if $a \in Y$ then it is only in Y once. In the example given, we cannot have $Y = \{a, d, f, h\}, \{b, d, f, h\}$ as f and h are in Y twice.

Constraint 2: For the problem to actually be satisfiable, the union of all sets $W_i \in W$ must include every $a \in U$ **at least once**. If this constraint is not met, the problem is unsatisfiable as no collection of W_i will include every atom in S . If, in the example given, W did not include a single W_i that included a , the problem would be unsolvable.

Constraint 3: $\forall W_i \in W, j \in W_i \Leftrightarrow j \in U$ - this is given via the problem prompt but is an additional constraint.

2. Problem 2:

- (a) `Parent(Arne, Ted)`
- (b) $\forall_{x,y} \text{Parent}(x,y) \Rightarrow \text{Earlier}(\text{Birth}(p), \text{Birth}(q))$
- (c) $\forall_{p,q} \text{Living}(p,t) \Leftrightarrow \text{Earlier}(\text{Birth}(p), t) \wedge \text{Earlier}(t, \text{Death}(p))$
- (d) $\forall_{ta,tb} \text{Earlier}(ta,tb) \Rightarrow \neg \text{Earlier}(tb,ta)$ (I use implies instead of iff to account for the case where $ta == tb$)
- (e) $\forall_{ta,tb,tc} \text{Earlier}(ta,tb) \wedge \text{Earlier}(tb,tc) \Rightarrow \text{Earlier}(ta,tc)$
- (f) $\forall_p \exists_t \text{s.t. Living}(p,t)$
- (g) $\neg \text{Living}(\text{Ted}, \text{Born}(\text{Anne}))$
- (h) $\forall_p \text{Earlier}(\text{Born}(p), \text{Death}(p))$

3. After conversion to CNF, we are left with the following clauses:

- a. `Parent(Arne, Ted)`
- b. $\neg \text{Parent}(x,y) \vee \text{Earlier}(\text{Birth}(x), \text{Birth}(y))$
- c1. $\neg \text{Living}(p,t) \vee \text{Earlier}(\text{Birth}(p), t)$
- c2. $\neg \text{Living}(p,t) \vee \text{Earlier}(t, \text{Death}(p))$
- c3. $\neg \text{Earlier}(\text{Birth}(p), t) \vee \neg \text{Earlier}(t, \text{Death}(p)) \vee \text{Living}(p,t)$
- d. $\neg \text{Earlier}(ta,tb) \vee \neg \text{Earlier}(tb,ta)$

Now assume `Living(Ted, Born(Arne))`. Given (a), we know `Earlier(Birth(Arne), Birth(Ted))`. Then we know from (c1) that for `Living(Ted, Born(Arne))` we need `Earlier(Birth(Ted), Birth(Arne))`.

This leads to `Earlier(Birth(Arne), Birth(Ted))` from (a), but also `Earlier(Birth(Ted), Birth(Arne))` from (c1), and per (d), we have a contradiction. Therefore we prove by contradiction that $\neg \text{Living}(\text{Ted}, \text{Born}(\text{Anne}))$.

4. After conversion to CNF, we are left with the following clauses:

- c1. $\neg \text{Living}(p,t) \vee \text{Earlier}(\text{Birth}(p), t)$
- c2. $\neg \text{Living}(p,t) \vee \text{Earlier}(t, \text{Death}(p))$
- c3. $\neg \text{Earlier}(\text{Birth}(p), t) \vee \neg \text{Earlier}(t, \text{Death}(p)) \vee \text{Living}(p,t)$
- e. $\neg \text{Earlier}(ta,tb) \vee \neg \text{Earlier}(tb,tc) \vee \text{Earlier}(ta,tc)$
- f. `Living(p, λ(p))`

Now assume $\neg \text{Earlier}(\text{Born}(p), \text{Death}(p))$. Then from (f), we have that p is living at time $\lambda(p)$. From (c1), we have `Earlier(Birth(p), λ(p))` and (c2) gives `Earlier(λ(p), Death(p))`. However, at (e), we get that `Earlier(Birth(p), λ(p)) ∧ Earlier(λ(p), Death(p))` leads to `Earlier(Birth(p), Death(p))`. However, this is in direct contradiction to our starting assumption, meaning our assumption was wrong and (h) is proven.