

Seminar_1

January 14, 2020

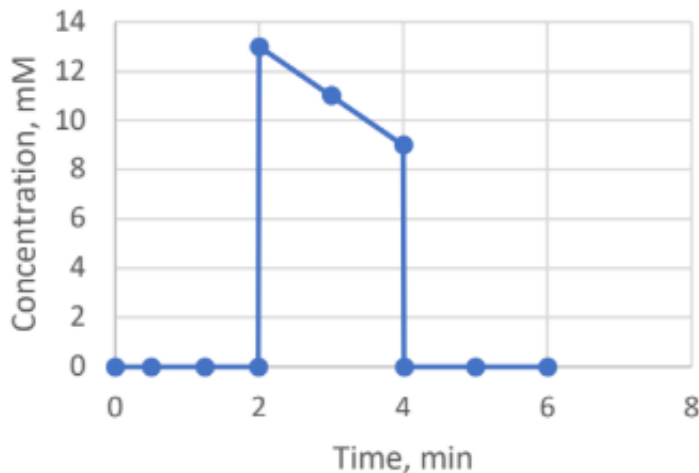
Q1. A pulse tracer input into a PFR showed the following outlet concentration:

The experimental data between 2 and 4 min were fit into the line equation $C_{out} = -2t + 17$, with C_{out} in $[mM]^*$ and t in $[min]$.

- Find the RTD function.
- Find mean residence time.
- Find variance.
- What is the fraction of material that spends in the reactor 3 minutes and longer?

- $[M]$ means $[mol/L]$ – it is a molar concentration in a fluid (gas or liquid)

Exiting tracer concentration
(pulse input)



```
[3]: import numpy as np
t=np.linspace(0.,8.,100)
C=np.zeros(len(t))

for i in range(0,len(t)):
    if 0<t[i]<2:
        C[i]=0
    elif 2<=t[i]<=4:
```

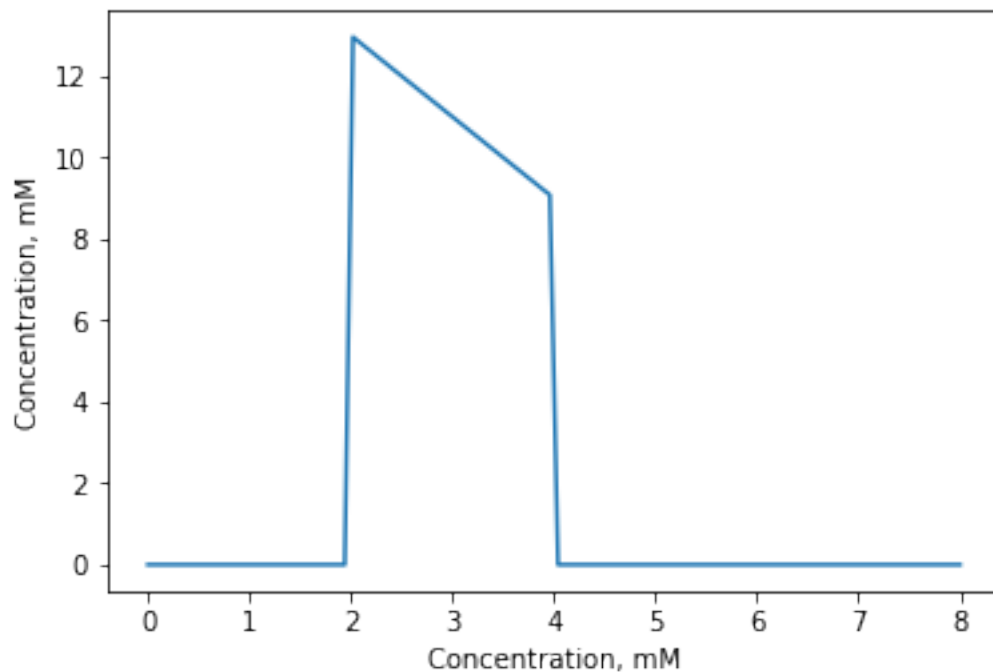
```

        C[i]=-2*t[i]+17
    else:
        C[i]=0

import matplotlib.pyplot as plt
plt.plot(t,C)
plt.ylabel('Concentration, mM')
plt.xlabel('Concentration, mM')

```

[3]: Text(0.5, 0, 'Concentration, mM')



[31]: t

[31]: array([0.00000000e+00, 8.00080008e-04, 1.60016002e-03, ..., 7.99839984e+00, 7.99919992e+00, 8.00000000e+00])

$$\int_0^\infty C(t)dt = 0 + \int_2^4 (-2*t + 17)dt + 0 = -2 \int_2^4 tdt + 17 \int_2^4 dt = 22$$

```

[87]: import numpy as np
import scipy.integrate as integrate

t=np.linspace(0.,8.,10000)
C=np.zeros(len(t))

for i in range(0,len(t)):
    if 0<t[i]<2:

```

```

        C[i]=0
    elif 2<=t[i]<=4:
        C[i]=-2*t[i]+17
    else:
        C[i]=0

I = integrate.cumtrapz(C, t, initial=0)
print ("{0:.3f}".format(I[len(I)-1]))

```

$\int_0^{\infty} C(t) dt = 22.003$

RTD function: $E(t) = \frac{C(t)}{\int_0^{\infty} C(t) dt}$

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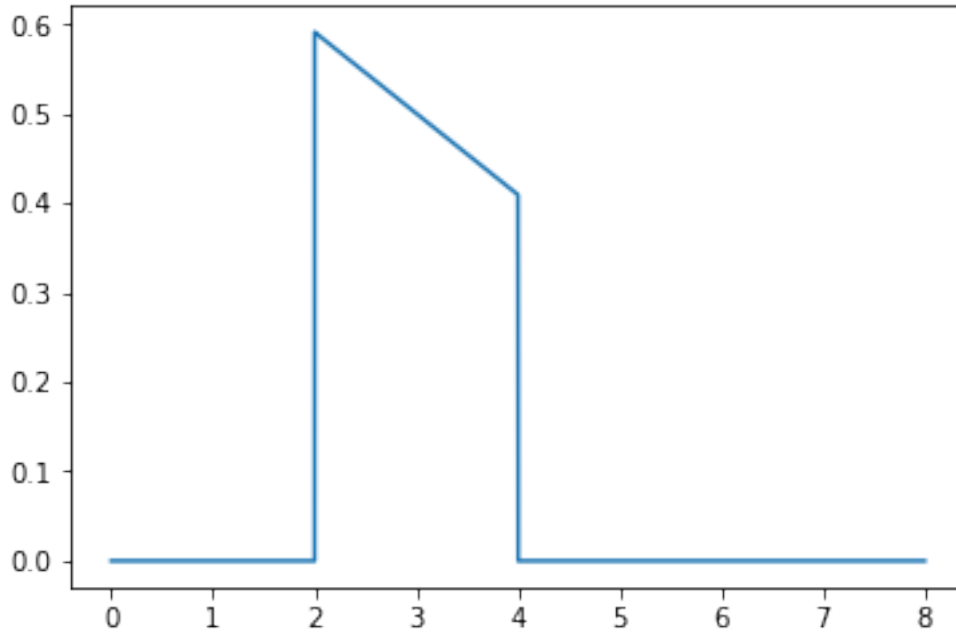
[15]: import numpy as np
import matplotlib.pyplot as plt
E=np.zeros(len(t))

for i in range(0,len(t)):
#     if 0<t[i]<2:
#         E[i]=0
#     elif 2<=t[i]<=4:
#         E[i]=(-2*t[i]+17)/22
#     else:
#         E[i]=0

    E[i]=C[i]/I[len(I)-1]
E
plt.plot(t,E)

```

[15]: [<matplotlib.lines.Line2D at 0x7f2421f83b38>]



b. Mean residence time $\bar{t} = \int_0^{\infty} tE(t)dt = 0 + \int_2^4 tE(t)dt + 0 = \int_2^4 (t \frac{-2t+17}{22})dt = -\frac{2}{22}(\frac{4^3}{3} - \frac{2^3}{3}) + \frac{17}{22}(\frac{4^2}{2} - \frac{2^2}{2}) = 2.9min$

```
[16]: t_r=np.zeros(len(t))
      for i in range(0,len(t)):
          t_r[i]=t[i]*E[i]

      tr = integrate.trapz(t_r, t)

      print ("mean residence time ={0:.3f}".format(tr))
```

mean residence time =2.939

c. Find the variance

$$\sigma^2 = \int_0^{\infty} t^2 E(t)dt - \bar{t}^2 = \int_2^4 t^2 (\frac{-2t+17}{22})dt - 2.9^2 = \frac{-1}{11}(\frac{4^4}{4} - \frac{2^4}{4}) + \frac{17*4^3}{22*3} - \frac{17*2^3}{22*3} - 2.9^2 = 0.56min^2$$

```
[17]: v=np.zeros(len(t))

      for i in range(0,len(t)):
          v[i]=t[i]*t[i]*E[i]

      Va = integrate.trapz(v, t)
      print(Va)
      print(tr)
```

```

Var=Va-(2.9*2.9)
Var1=Va-(tr*tr)
print ('Variance of residence times =', Var1, 'min^2', Var)

```

8.969109601086757

2.9392829183001035

Variance of residence times = 0.3297255272759845 min^2 0.5591096010867567

d. Fraction of the material spends in the reactor longer than 3 min.

Use cumulative distribution function, $F(t) = \int_0^t E(t)dt$.

$1 - F(t)$?

$F(3) = \int_0^3 t dt + \frac{17}{22} \int_2^3 dt = \frac{-2}{22}(\frac{3^2}{2} - \frac{2^2}{2}) + \frac{17}{22}(3 - 2) = 0.545$

$1-F(3)=0.455$

```

[89]: y=np.zeros(len(t))

for i in range (2,len(t)+1):
    y[i-1]=integrate.trapz(E[0:i],t[0:i])

k=0

plt.plot(t,y)

for i in range(0,len(t)):
    if 2.99 < t[i] < 3.00:
        k=i

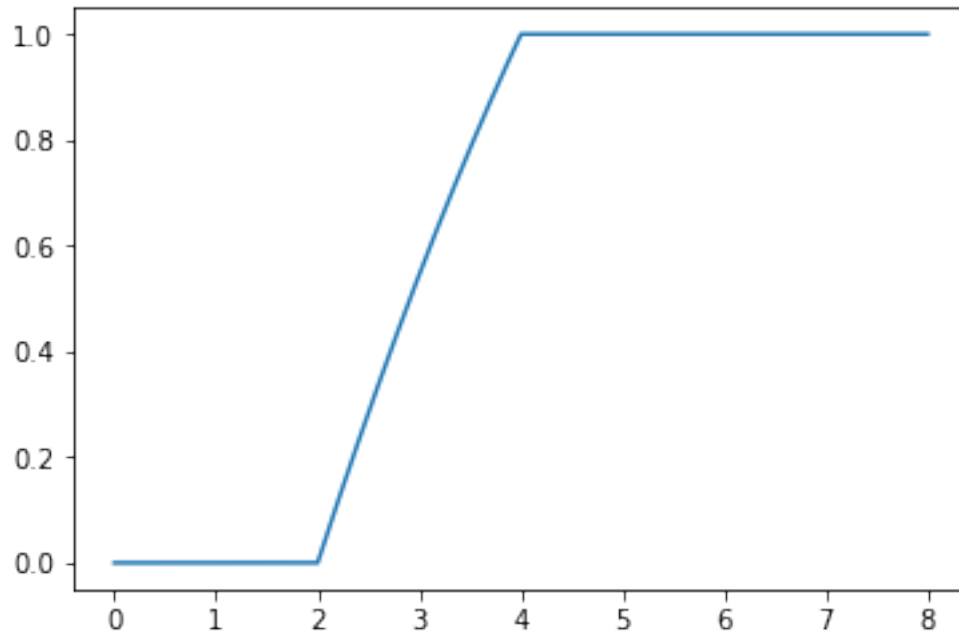
#print(k)

print ('Fraction spends in the reactor less than 3 min =', y[k-1])
print ('Fraction spends in the reactor greater than 3 min =', 1-y[k-1])

```

Fraction spends in the reactor less than 3 min = 0.5448581920338935

Fraction spends in the reactor greater than 3 min = 0.4551418079661065



Q2. Residence time distribution in real reactors and its characteristics. *From Chapter 13, 4th Ed. Fogler*

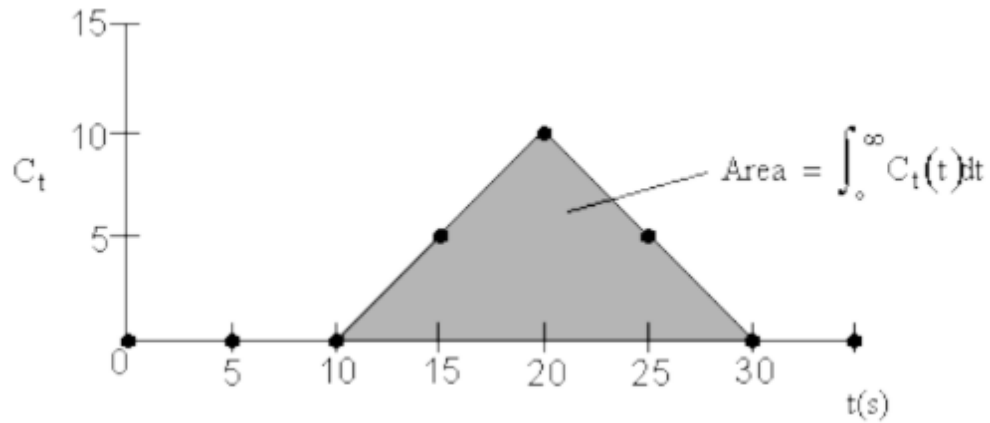
The following data were obtained from a pulse tracer test to a real flow reactor:

t(s)	0	5	10	15	20	25	30	35
C(mg/dm ³)	0	0	0	5	10	5	0	0

- Plot RTD function
- Find the fraction of material that spends between 15 and 20 seconds in the reactor
- Plot cumulative distribution function $F(t)$
- What fraction of the material spends 25 seconds or less in the reactor?
- Find mean residence time.

a. RTD function is $E(t) = \frac{C(t)}{\int_0^\infty C(t)dt}$

To find the total concentration in the denominator, we plot $C(t)$ vs. time and evaluate the area:

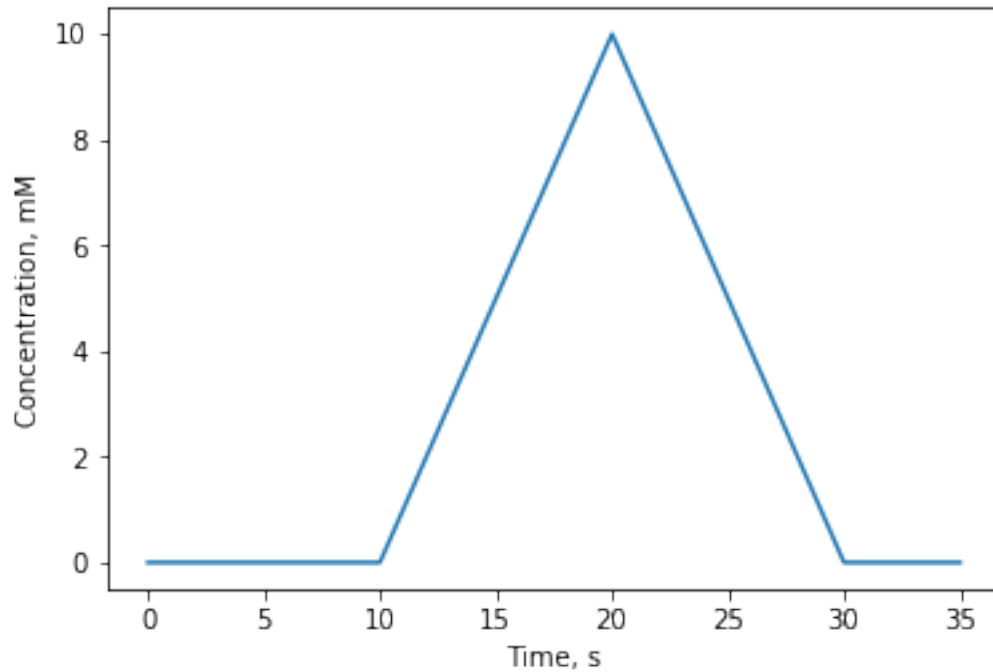


```
[90]: t2=np.linspace(0.,35,1000)
      C2=np.zeros(len(t2))

      for i in range(0,len(t2)):
          if 0<t2[i]<10:
              C2[i]=0
          elif 10<=t2[i]<20:
              C2[i]=t2[i]-10
          elif 20<=t2[i]<=30:
              C2[i]=30-t2[i]
          else:
              C2[i]=0

      import matplotlib.pyplot as plt
      plt.plot(t2,C2)
      plt.ylabel('Concentration, mM')
      plt.xlabel('Time, s')
```

```
[90]: Text(0.5, 0, 'Time, s')
```



```
[61]: I2 = integrate.cumtrapz(C2, t2, initial=0)
print ("The total concentration in denominator ={0:.3f}".format(I[len(I)-1]),
      ↳ 'mg.s/dm^3')
```

The total concentration in denominator =99.990 mg.s/dm³

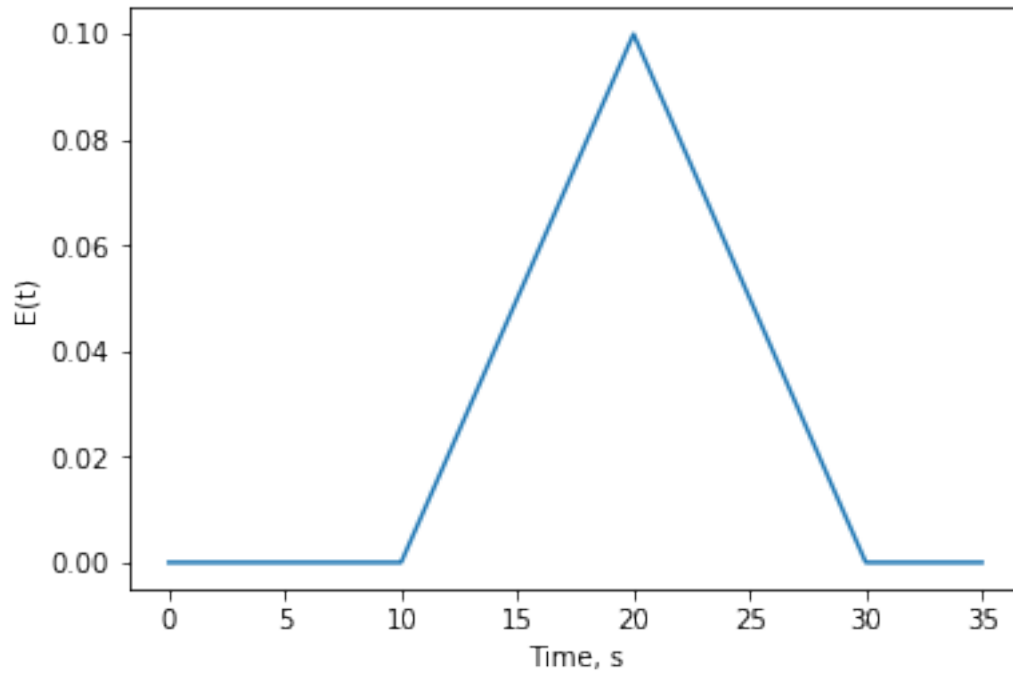
RTD function is $E(t) = \frac{C(t)}{\int_0^\infty C(t) dt}$

```
[84]: import numpy as np
import matplotlib.pyplot as plt
E2=np.zeros(len(t2))

for i in range(0,len(t2)):

    E2[i]=C2[i]/I2[len(I2)-1]
E2
plt.plot(t2,E2)
plt.ylabel('E(t)')
plt.xlabel('Time, s')
```

```
[84]: Text(0.5, 0, 'Time, s')
```

b. $\int_{15}^{20} E(t) dt$

```
[83]: y2=np.zeros(len(t2))

for i in range (2,len(t2)+1):
    y2[i-1]=integrate.trapz(E2[0:i],t2[0:i])

k2=0
k22=0

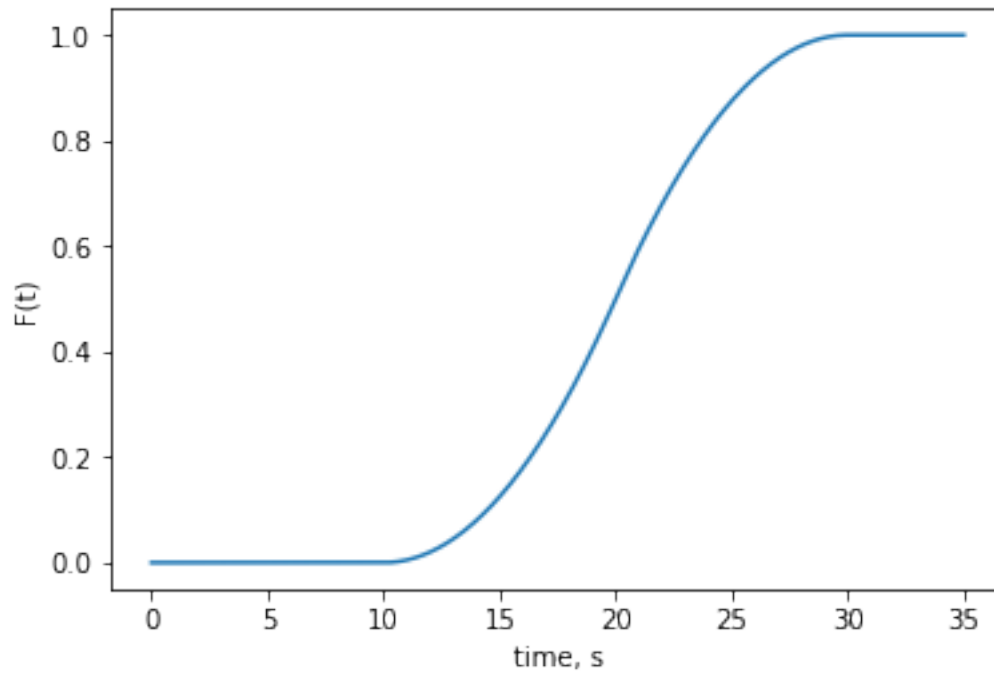
plt.plot(t2,y2)
plt.ylabel('F(t)')
plt.xlabel('Time, s')

for i in range(0,len(t2)):
    if 14.84 < t2[i] < 15.00:
        k2=i
    if 19.79 < t2[i] < 20.00:
        k22=i

#print(k2)
#print(k22)
#print(y2[k2-1])
#print(y2[k22-1])
```

```
print ("Fraction spends in reactor between 15-20 mins ={0:.3f}".
      →format(y2[k22-1]-y2[k2-1]))
```

Fraction spends in the reactor between 15-20 mins =0.371



```
[80]: k3=0
      k33=0

      for i in range(0,len(t2)):
          if 9.98 < t2[i] < 10.00:
              k3=i
          if 14.99 < t2[i] < 15.00:
              k33=i

      #print(k3)
      #print(k33)
      #print(y2[k3-1])
      #print(y2[k33-1])
      #print(y2[k33-1]-y2[k3-1])
      print ("Fraction spends in the reactor between 10-15 mins ={0:.3f}".
            →format(y2[k33-1]-y2[k3-1]))
```

Fraction spends in the reactor between 10-15 mins =0.123

```
[81]: k4=0
k44=0

for i in range(0,len(t2)):
    if 9.98 < t2[i] < 10.00:
        k4=i
    if 19.96 < t2[i] < 20.00:
        k44=i

#print(k4)
#print(k44)
#print(y2[k4-1])
#print(y2[k44-1])
#print(y2[k44-1]-y2[k4-1])
print ("Fraction spends in the reactor between 10-20 mins ={0:.3f}".
    ↳format(y2[k44-1]-y2[k4-1]))
```

Fraction spends in the reactor between 10-20 mins =0.494

```
[82]: k5=0
k55=0

for i in range(0,len(t2)):
    if 9.98 < t2[i] < 10.00:
        k5=i
    if 24.97 < t2[i] < 25.00:
        k55=i

#print(k5)
#print(k55)
#print(y2[k5-1])
#print(y2[k55-1])
#print(y2[k55-1]-y2[k5-1])
print ("Fraction spends in the reactor between 10-25 mins ={0:.3f}".
    ↳format(y2[k55-1]-y2[k5-1]))
```

Fraction spends in the reactor between 10-25 mins =0.872

e. mean residence time. $\bar{t} = \int_0^{\infty} t.E(t)dt$

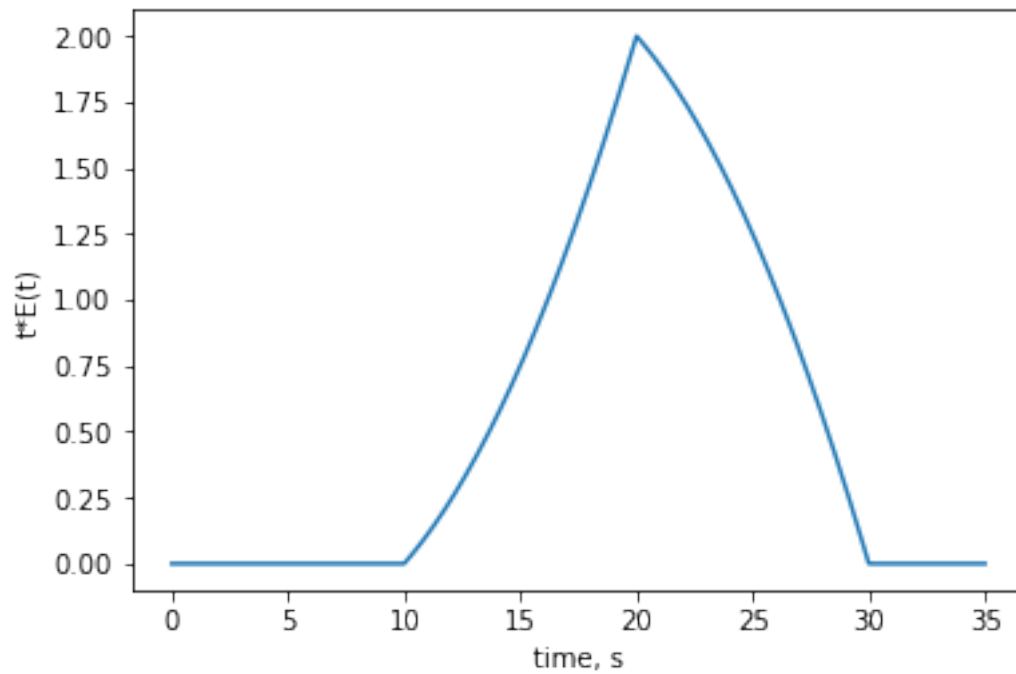
```
[74]: t_r2=np.zeros(len(t2))
for i in range(0,len(t2)):
    t_r2[i]=t2[i]*E2[i]

tr2 = integrate.trapz(t_r2, t2)
```

```
print ("mean residence time ={0:.3f}".format(tr2), 's')
plt.plot(t2,t_r2)
plt.ylabel('t*E(t)')
plt.xlabel('time, s')
```

mean residence time =20.000 s

[74]: Text(0.5, 0, 'time, s')



[]: