Supplementary of
`ElecMeter`: an index for Evaluating an Election social Dissatisfaction

Maryam Shahdoust[1], Zahra Eidi[1], and Mehdi Sadeghi[2*]

[1]School of Biological Sciences, Institute For Research In Fundamental Sciences(IPM), Tehran, Iran
[2]Department of Medical Genetics,National Institute for Genetic Engineering and Biotechnology,Tehran,Iran
[*]Corresponding Author

1 Clustering Methodology

To identify patterns in the structure of dissatisfaction distributions across simulated populations, we applied hierarchical clustering using the Jensen–Shannon Divergence (JSD)[1, 2] as the distance metric. JSD provides a symmetric and bounded measure of divergence between probability distributions, making it well-suited for comparing voter dissatisfaction profiles. For each pair of distributions, we computed the JSD and constructed a distance matrix. We then performed agglomerative hierarchical clustering using the complete linkage method[3], which merges clusters based on the maximum pairwise distance. This approach emphasizes compact and well-separated clusters. The resulting dendrogram(Figure S1) was cut to form four distinct clusters, based on visual inspection and stability of the clustering structure. These clusters reflect qualitatively different patterns of voter sentiment, such as concentrated satisfaction, polarization, or extreme dissatisfaction. In addition, hierarchical clustering was applied separately to populations based on their average dissatisfaction (S) and the `ElecMeter` values. In these cases, the distance matrix was calculated using Euclidean distance between the respective values for each population. The number of clusters was again set to four to enable direct comparisons across the different clustering criteria. The dendrograms for each clustering approach are shown in the figure below.

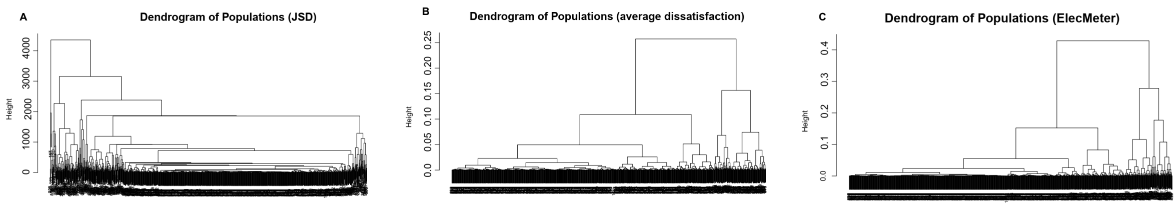

Figure S1: Dendrograms showing hierarchical clustering of simulated populations based on different measures of voter dissatisfaction. (A) Clustering using the Jensen–Shannon Divergence (JSD) between full dissatisfaction distributions. (B) Clustering based on normalized average dissatisfaction (S) using Euclidean distance. (C) Clustering based on the `ElecMeter` index using Euclidean distance. All dendrograms were cut to form four clusters, highlighting distinct patterns of voter sentiment, including concentrated satisfaction, polarization, and extreme dissatisfaction.

2 Borda Count

The Borda count is a positional voting method used to aggregate ranked preferences into a collective decision[4, 5, 6]. It assigns points to candidates based on their position in each voter's ranking. Specifically, in an election with K candidates, a candidate receives $K-1$ points for each first-place vote, $K-2$ points for each second-place vote, and so on, down to 0 points for a last-place vote.

Formally, for a voter i, let their ranked list of candidates be $c_1 \succ c_2 \succ \cdots \succ c_K$. Then, the Borda score assigned to candidate c_j by this voter is:$\mathrm{Score}_i(c_j) = K - r_{ij}$, where r_{ij} is the rank of candidate c_j in voter i's list (with 1 being the highest rank).

The total Borda score for each candidate is the sum of their scores across all voters:
$\mathrm{TotalScore}(c_j) = \sum_{i=1}^{N} \mathrm{Score}_i(c_j)$.
The candidate with the highest total score is declared the winner.

3 Sensitivity to Aggregation Rule

We evaluated alternative ways of aggregating the normalized divergences (I_1, I_2, I_3) into a single structural dissatisfaction component. In addition to the maximum operator $(I_{\max})$, we computed the arithmetic mean, the geometric mean, and the range $|\max(I_1, I_2, I_3) - \min(I_1, I_2, I_3)|$. These alternatives are denoted by I_{mean}, I_G, and I_{range}, respectively. Each variant yields values bounded within $[0, 1]$.

To assess robustness, we compared ElecMeter values across all simulated populations with different numbers of candidates ($K = 3, 4, 5, 7$), using both Pearson and Spearman correlations. All indices were constructed using Equation 11 in the main text, with $I_{\max}$ replaced by I_{mean}, I_G, or I_{range}. The resulting indices are denoted `ElecMeter_mean`, `ElecMeter_G`, and `ElecMeter_range`, respectively.

The results (Table S1) show that ElecMeter based on $I_{\max}$ is highly correlated with the mean- and geometric-based variants across all K. The range-based variant is less consistent, particularly in rank correlation for $K = 7$ ($\rho \approx 0.64$), reflecting its greater sensitivity to noise. These findings indicate that ElecMeter is robust to the choice of aggregation rule, while the use of $I_{\max}$ provides a conservative minimax approach that ensures sensitivity to extreme divergence patterns. All results in Table S1 are based on $p = 3$ in the index equation.

Table S1: Correlations between ElecMeter variants based on different aggregation rules.

K	Comparison	Pearson r	Spearman ρ
3	Max vs. Geometric mean	0.991	0.978
	Max vs. Arithmetic mean	0.994	0.979
	Max vs. Range	0.996	0.981
4	Max vs. Geometric mean	0.991	0.954
	Max vs. Arithmetic mean	0.995	0.964
	Max vs. Range	0.994	0.934
5	Max vs. Geometric mean	0.986	0.945
	Max vs. Arithmetic mean	0.990	0.957
	Max vs. Range	0.986	0.861
7	Max vs. Geometric mean	0.920	0.919
	Max vs. Arithmetic mean	0.938	0.931
	Max vs. Range	0.836	0.640

"Max" denotes $I_{\max}$; "Geometric mean" and "Arithmetic mean" refer to the geometric and arithmetic means of (I_1, I_2, I_3); "Range" is the absolute difference between the maximum and minimum of (I_1, I_2, I_3). Values are rounded from analysis output.

4 Summary statistics for clusters for `ElecMeter` with different p

Table S2: Summary statistics (Median, Q1, Q3, IQR) for clusters for `ElecMeter` with $p = 2, 3, 4$

p	clusters	Median	Q1	Q3	IQR
2	1	0.764	0.721	0.777	0.056
2	2	0.247	0.152	0.409	0.257
2	3	0.570	0.539	0.587	0.048
2	4	0.468	0.365	0.533	0.168
3	1	0.804	0.762	0.816	0.053
3	2	0.265	0.161	0.426	0.264
3	3	0.596	0.565	0.615	0.050
3	4	0.488	0.382	0.559	0.177
4	1	0.835	0.794	0.846	0.052
4	2	0.277	0.169	0.437	0.268
4	3	0.616	0.585	0.637	0.052
4	4	0.504	0.396	0.579	0.184

5 Boxplots of the `ElecMeter` index across clusters based on normalized average dissatisfaction (S)

To examine the distribution of `ElecMeter` values across clusters, we applied hierarchical clustering using the normalized average dissatisfaction (S).

Since the `ElecMeter` index includes a tunable power parameter p, selecting an appropriate value is important for balancing sensitivity to dissatisfaction extremes with robustness to distributional variation. To guide this choice, we computed `ElecMeter` values for $p \in \{2, 3, 4\}$ and examined their distributions across the JSD-based clusters (results shown in the main manuscript). Based on those results, we selected $p = 3$ as the primary parameter setting.

To validate this choice, we further examined the distribution of `ElecMeter` values across clusters obtained from S-based hierarchical clustering (Figure S2). Across all three parameter settings, the relative ordering of median `ElecMeter` values among clusters was consistent, with Cluster 1 showing the highest values and Cluster 4 the lowest. The separation between intermediate clusters was similar for $p = 3$ and $p = 4$, with $p = 3$ providing the best balance between discrimination of structural patterns and interpretability. Therefore, $p = 3$ was selected as the default setting for subsequent analyses.

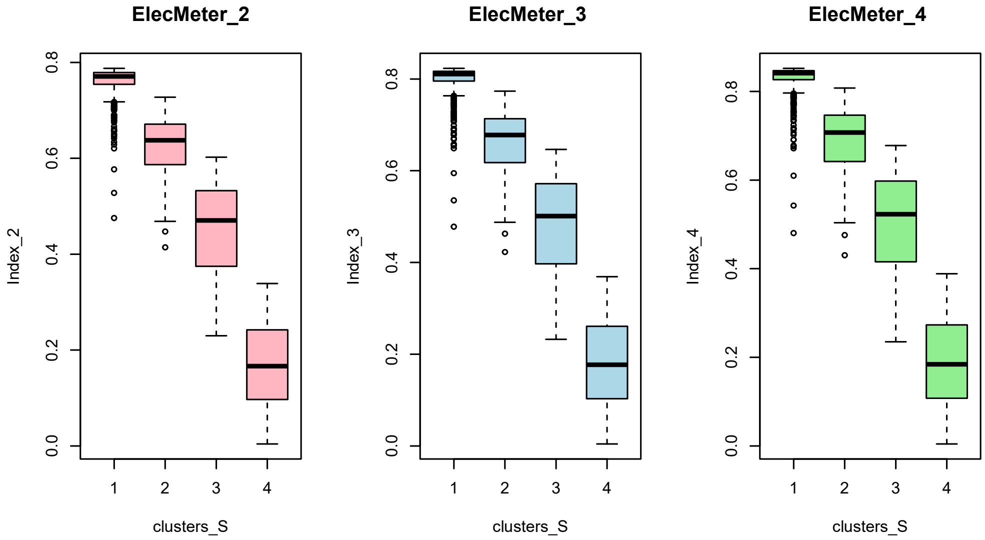

Figure S2: Boxplots of the `ElecMeter` index across the four identified clusters based on the normalized average dissatisfaction, computed using different values of the power parameter $p \in \{2, 3, 4\}$, each normalized by $2^{1/p}$. The index reveals distinct patterns of dissatisfaction corresponding to the cluster structure, with higher values indicating more widespread or extreme voter dissatisfaction.

References

[1] María Luisa Menéndez, Julio Angel Pardo, Leandro Pardo, and María del C Pardo. The jensen-shannon divergence. *Journal of the Franklin Institute*, 334(2):307–318, 1997.

[2] Solomon Kullback. Kullback-leibler divergence. *Tech. Rep.*, 1951.

[3] Richard Arnold Johnson, Dean W Wichern, et al. Applied multivariate statistical analysis. 2002.

[4] Peter Emerson. The original borda count and partial voting. *Social Choice and Welfare*, 40(2):353–358, 2013.

[5] Peter C Fishburn and William V Gehrlein. Borda's rule, positional voting, and condorcet's simple majority principle. *Public Choice*, pages 79–88, 1976.

[6] Duncan Black. Partial justification of the borda count. *Public Choice*, pages 1–15, 1976.