IE 400

Project

Report

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In this project, we have worked on the optimization problem for Dr. Who's basic regimen for chemotherapy sessions. There are seven drugs which are to be used for cancer patients. However, these seven drugs have an impact on the patient's Q-score value, which indicates the quality of life for the patients. A higher Q-score depicts a better We have been provided with an equation which estimates the Q-score:

$$\begin{aligned} Q(\mathbf{p},\mathbf{y},\mathbf{x}) = & -5p_1 - 0.5p_2 - 12p_3 - 8p_4 - 5p_5 - 5p_6 - 1p_7 - 3p_8 - 2p_9 \\ & - 5y_1 - 6y_2 - 4y_3 - 4y_4 - 8y_5 - 6y_6 - 7y_7 \\ & + 0.28x_1 + 0.30x_2 + 0.25x_3 + 0.17x_4 + 0.31x_5 + 0.246x_6 + 0.40x_7 \end{aligned}$$

This equation is used in all the parts of the project to predict the amount of dosage of each drug to be used. The primary focus of this project is the optimization of different drug dosages in order to maximize the Q-score for patients and minimize the cost of addition and removal of drugs in the base regimen.

Part A

In the first part of the project, we are tasked to maximize the Q-score of the patient with four drugs provide. In this part of the project, there is no specific Q-score to achieve, rather maximize the Q-score with certain drugs. A base regimen is provided to us in which the four drugs are included: Melphalan (X1), Decitabine (X3), Pentostatin (X4), and Thiotepa (X7). A unit of an acceptable chemotherapy regimen is 100 centiliters (cl) and must include one or more of these drugs. Thus, we are initially provided with 20 cl of X1, 30 cl of X3, 15 cl of X4, and 35 cl of X7.

The goal was to maximize the Q-score from the above-mentioned equation by using these drugs as the only drugs in the regimen. We could alter the dosage of these four drugs in order to maximize the Q-score. For this, we programmed a mathematical optimization solver with CPLEX in order to calculate the maximized Q-score. Moreover, to start with the program, we outlined our necessary decision variables and constraints:

Decision Variables

x_i = dosage of drug j in the regimen, and x = 1, 2, 3, 4, 5, 6, 7

p_i = (p1, p2, ..., p9) patient's characteristics which are (1,1,0,0,1,1,0,0,0)

y_i = (y1, y2, .., y7) the presence of drug 1 to 7 which is 1 if it is present and 0 if not

Objective Function:

P_value = - 5*1– 0.5 * 1 – 12*0 – 8*0 – 5*1 – 1*0 – 3*0 – 2*0 ➔ which is -15.5

Y_value = -5*1 – 6*0 – 4*1 – 4*1 – 8*0 – 6*0 – 7 *1 ➔ which is -20

Q-score = P_value + Y_value + 0.28* X1 + 0.30*X2 + 0.25*X3 + 0.17*X4 + 0.31*X5 + 0.246*X6 + 0.40*X7

max Q-score

Subject to:

The lower and upper bounds for the four drugs' dosage and equating the others to zero:

80 >= X1 >= 20

100 >= X3 >= 20

100 >= X4 >= 10

50 >= X7 >= 20

X2, X5, X6 = 0

The constraint of total maximum quantity:

X1 + X3 + X4 + X7 = 100

Feasible Solution:

Objective value/Q-score value: **-3.200003**

Values of decision variables**: [20, 20, 10, 50] for [X1, X3, X4, X7]**

It can be concluded that the best value for Q-score could be -3.2 with drugs X1, X3, X4, X7 used in the base regimen only. The best combination for these four drugs to attain the maximum Q-score value **X1 = 20 cl, X3 = 20 cl, X4 = 10 cl, X7 = 50 cl.**

Part B

Decision Variables

x_i = Amount of dosage of drug i

y_i = 1 if drug i is included, 0 otherwise

c_i = 1 if drug i is included, 0 otherwise

a_i = Amount of dosage changed of drug i

Objective Function

$$\min\ (25c_1 + 1x_1) + (50c_2 + 2x_2) + (10c_3 + 1x_3) + (25c_4 + 3x_4) + (20c_5 + 2x_5) + (30c_6 + 1x_6) + (40 + 1x_7)$$

s.t.

$$\begin{aligned} Q(\mathbf{p}, \mathbf{y}, \mathbf{x}) = &- 5p_1 - 0.5p_2 - 12p_3 - 8p_4 - 5p_5 - 5p_6 - 1p_7 - 3p_8 - 2p_9 \\ &- 5y_1 - 6y_2 - 4y_3 - 4y_4 - 8y_5 - 6y_6 - 7y_7 \\ &+ 0.28x_1 + 0.30x_2 + 0.25x_3 + 0.17x_4 + 0.31x_5 + 0.246x_6 + 0.40x_7 \end{aligned}$$

Q (p, x, y) = 25

c1 = 1 – y1

c2 = y2

c3 = 1 – y3

c4 = 1 – y4

c5 = 1 – y5

c6 = y6

c7 = 1 – y7

add_i + $remove_i$ <= 1 for i = 1,2,3,4,5,6,7

x1 + x2 + x3 + x4 + x5 + x6 + x7 = 200

20 <= x1 <= 80

10 <= x2 <= 50

20 <= x3 <= 100

10 <= x4 <= 100

10 <= x5 <= 70

10 <= x6 <= 70

20 <= x7 <= 50

Feasible Solution

After minimizing the cost, we get the following variable dosages:

x1 = 71

x2 = 0

x3 = 100

x4 = 0

x5 = 0

x6 = 0

x7 = 29

Quality of life = 25

Using these variables, the cost is minimized cost is 206

Part C

In this part, we are tasked to add further constraints to our model in part B. In this part, we primarily focus on further improvising the base regimen's content by introducing additional constraints. In B part, our focus was to reach the Q-score of 25 (for our patient number 49) by altering the base regimen's content in terms of all the drugs, that is we could administer a base regimen with all drugs included and with maximum quantity of 100 cl. Moreover, we were tasked to minimize the cost of addition/removal of those drugs along with the unit cost of change in dosage.

In part c, the additional constraints we added to our LP/IP model are as follows:

1) Melphalan and Oxaliplatin dosage:

$X1 + X2 >= 30$

$X1 + X2 <= 70$

2) Either Epirubicin included or Decitabine should be less than 25 cl:

w = { 0 if Epirubicin is included ($Y5 = 1$) , 1 otherwise ($Y5 = 0$) }

$Y3 <= w$

$X5(1-Y3) < 25$

3) If both Pentostatin and Lomoustine included then at least one of Thiotepa and Epirubicin also be included:

z = { 1 if ($Y4 + Y6 = 2$), 0 otherwise }

$Y4 + Y6 = 2z$

$Y5 + Y7 >= z$

Feasible Solution

After minimizing the cost, we get the following variable dosages:

$x1 = 68$

$x2 = 0$

$x3 = 0$

x4 = 0

x5 = 0

x6 = 90

x7 = 43

Quality of life = 25

Using these variables, the cost is minimized and it is 286.8