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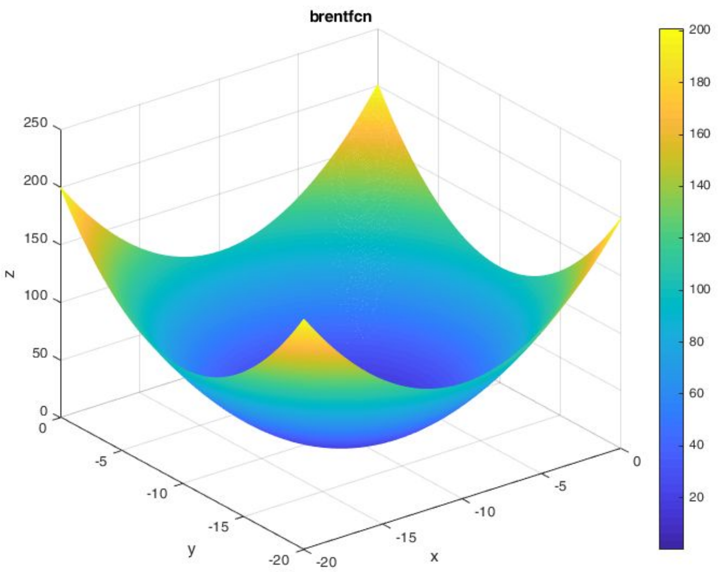
Part 1 :

Brent Function(unimodal function)

Mathematical Definition

$$f(x, y) = (x + 10)^2 + (y + 10)^2 + e^{-x^2 - y^2}$$



Description and Features

- **The function is convex.**
- **The function is defined on 2-dimensional space.**
- **The function is non-separable.**
- **The function is differentiable.**

Input Domain

The function can be defined on any input domain but it is usually evaluated on $x_i \in [-20, 0]$ for $i = 1, 2$.

Global Minima

The function has one global minimum at f (x ∗) = e − 200 located at x ∗ = (− 10 −10) .

Implementation results :

- **Gradient descent :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[-20 0]	0.001169	(x,y) =(-10.024179,-9.975821)	27	0.473804se
Alpha fixe=0.1	[15 3]	0.001217	(x,y) = (-9.969051,-9.983907)	30	0.529884sec
Alpha fixe=0.1	[-14 4]	0.001239	(x,y) = (-10.009671,-9.966150)	27	0.495761sec
Armijo method	[-7 -3]	0.000007	(x,y) = (-9.998988,-9.997638)	4	0.228159sec
Armijo method	[-20 0]	0.000049	(x,y) = (-9.995056,-10.004944)	4	0.180788sec
Armijo method	[18 1]	0.000001	(x,y)=(-9.999091,-9.999643)	5	0.350833sec

- **Quasi Newton's SR1 method :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[-20 0]	0.004197	(x,y) = (-10.045811,-9.954189)	50	1.148088sec
Alpha fixe=0.1	[-3 16]	0.003481	(x,y) =-9.984662,-9.943030	57	10.386732sec
Alpha fixe=0.1	[-4,-6]	0.004021	(x,y) =(-9.977736,-9.940629	52	1.113597sec
Armijo method	[10 -9]	0.004179	(x,y) = (-10.064567,-10.003228)	3	1.907198sec
Armijo method	[7,10]	0.000000	(x,y) = (-9.992251,-9.998339)	6	2.761418sec
Armijo method	[-4 2]	0.000012	(x,y) = (-9.998442,-9.996885	5	3.740813se

- **Quasi Newton's BFGS method :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[-15 8]	0.003892	(x,y) = (-10.016698,-9.939886	53	1.11979sec
Alpha fixe=0.1	[20 0]	0.003889	(x,y) = (-9.940839,-9.980280)	58	1.289430sec
Alpha fixe=0.1	[2 -1]	0.003826	(x,y) = (-9.950509,-9.962898)	51	1.148230sec
Armijo method	[-4,-6]	0.004021	(x,y) =(-9.977736,-9.940629	9	1.113597sec
Armijo method	[-7 8]	0.000000	(x,y) = (-10.000076,-10.000453)	8	0.486532sec
Armijo method	[1,-3]	0.000000	(x,y) = (-10.000135,-10.000156	6	0.27045sec

- **Newton method :**

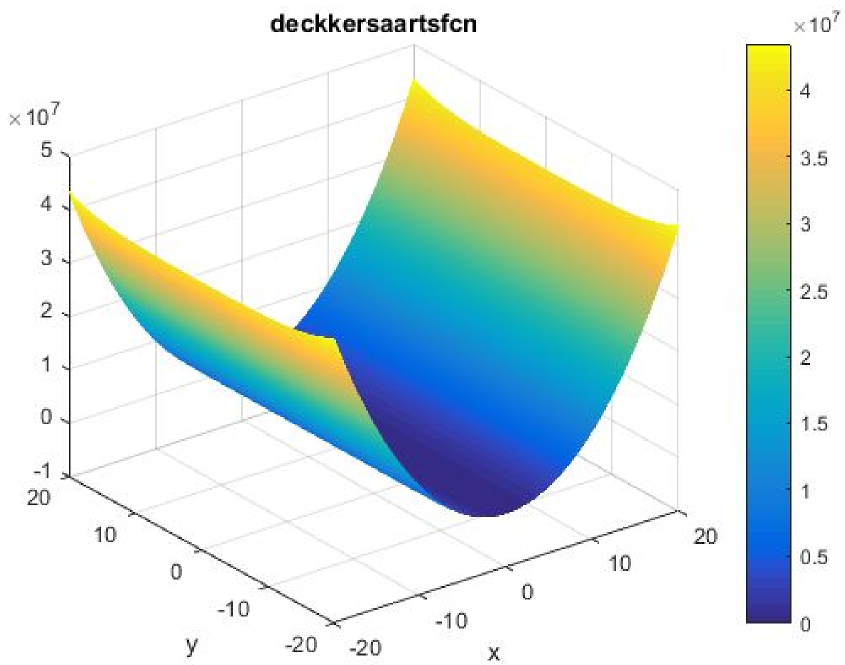
Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.01	[-13 -15]	0.000000	(x,y) = (-10.000000,-10.000000)	2	0.054112sec
Alpha fixe=0.01	[-9 -7]	0.000000	(x,y) = (-10.000000,-10.000000)	2	0.054118sec
Alpha fixe=0.01	[4 7]	0.000000	(x,y) = (-10.000000,-10.000000)	2	0.056107sec
Armijo method	[18,-3]	0.004021	(x,y) = (-10.000000,-10.000000)	2	0.058244sec
Armijo method	[-11 5	0.000000	(x,y) = (-10.000000,-10.000000)	2	0.055486sec
Armijo method	[-19,-18]	0.000000	(x,y) = (-10.000000,-10.000000)	2	0.057368sec

Deckkers-Aarts Function

(Multimodal function)

Mathematical Definition

$$f(x, y) = 10^5 x^2 + y^2 - (x^2 + y^2)^2 + 10^{-5}(x^2 + y^2)^4$$



Description and Features

- **The function is continuous.**
- **The function is not convex.**
- **The function is defined on 2-dimensional space.**
- **The function is multimodal.**
- **The function is differentiable.**
- **The function is non-separable.**

Input Domain

The function can be defined on any input domain but it is usually evaluated on $x_i \in [-20, 20]$ for $i = 1, \ldots, n$.

Global Minima

The global minima $f(x^*) = -24771.09375$ are located at $x^* = (0, \pm 15)$.

Implementation results :

- **Gradient descent :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.000005	[-7,12]	-24776.515670	(x,y) =(0.000000,14.943887)	458	6.770482sec
Alpha fixe=0.000002	[-17,-1]	-24776.511461	(x,y) = (-0.000000,-14.943146)	1277	18.643948sec
Armijo method	[-19 -3]	-24776.504466	(x,y) = (0.000070,-14.947853)	66	15.630679se
Armijo method	[-6 -8]	-24776.482137	(x,y) = (-0.000119,14.949530)	28	7.001603sec
Armijo method	[12 18]	-24776.455229	(x,y) = (-0.000226,14.950816)	61	14.934029sec

- **Quasi Newton's SR1 method :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[-9 13]	-24776.482137	(x,y) = (-0.000045,14.949530)	100	7.001603sec
Alpha fixe=0.1	[-17,-1]	-24776.455229	(x,y) = (-0.000000,-14.943146)	33	5.001603sec
Armijo method	[-9 13]	-24776.518154	(x,y) = (-0.000043,14.945137)	12	0.970856sec
Armijo method	[-16 4]	-24776.518294	(x,y) = (-0.000010,14.945259)	23	1.642936sec
Armijo method	[-13,-16]	-24776.518329	(x,y) = (0.000011,-14.945116)	15	1.172717sec

- **Newton method :**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[2 -19]	-24776.518342	(x,y) = (0.000000,-14.945112)	6	0.126326sec
Alpha fixe=0.1	[15 13]	-24776.518342	(x,y) = (0.000000,14.945112)	8	0.236287se
Alpha fixe=0.01	[9 -17]	-24776.518342	(x,y) = (0.000000,-14.945112)	7	-24776.518342

Armijo method	[7 18]	-24776.518342	(x,y) = (0.000000,14.945112)	6	0.124345se
Armijo method	[-7 -6]	-24776.518342	(x,y) = (-0.000000,-14.945112)	24	4.394500sec
Armijo method	[-10,-18]	-24776.518342	(x,y) = (0.000000,-14.945112)	29	0.777209sec

- **Quasi Newton's BFGS method :ptrobleeem alpha fixe**

Step size	X0	f(x*)	x*	Number of iteration	time
Alpha fixe=0.1	[-15 8]	-24776.518342	(x,y) = (0.000001,-14.945109)	90	1.11979sec
Alpha fixe=0.1	[20 0]	-24776.518337	(x,y) = (0.000011,-14.945116)	88	1.289430sec
Armijo method	[-12 16]	-24776.518337	(x,y) = (0.000006,14.945106)	13	1.120746sec
Armijo method	[-4 -10]	-24776.51834	(x,y) = (0.000001,-14.945109)	12	1.043241sec
Armijo method	[-1 18]	-24776.518337	(x,y) = (0.000000,-14.945169)	14	1.043241sec

Part 2 :

studied function :f(x)=x^3 - 6*x^2 + 9*x

- **Bisection method :**

init_point	opt_x	opt_y	f(x*)	time
[0.8 2]	10.99999	0.999219	f(x*)=0.004689	0.175170sec

- **Newton's method :**

init_point	opt_x	opt_y	f(x*)	time
[0.8 2]	11.000000	0.99986	f(x*) = 0.000820	0.050424sec

- **Regula falsi method :**

init_point	opt_x	opt_y	f(x*)	time
[10 0]	11.000000	1.000137	f(x) = -0.000820	0.102560sec
[0.8 2]	11.000000	0.999608	f(x)=0.002352	0.295949sec