# Brain Signal processing & Applications

## BME 473/ELE 573

Lecture 3

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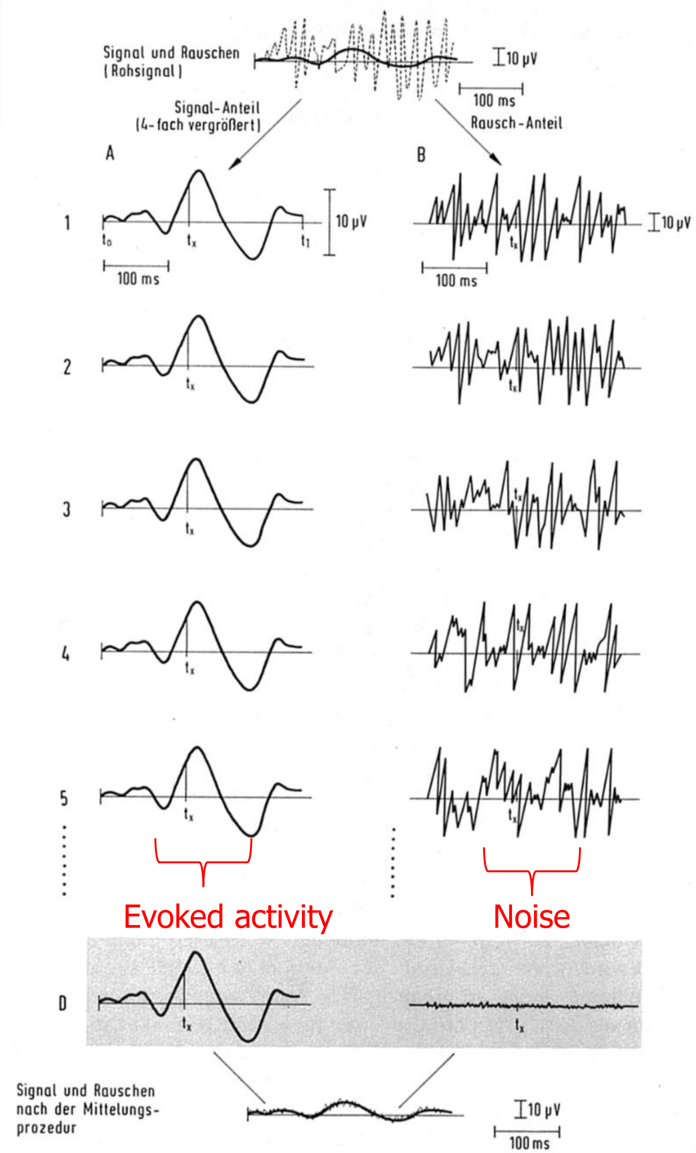
# Event-Related Potential (ERP)

## ➢ Evoked Potential (EP)

- Time-locked electrical response of a neural system to an event (i.e., sensory, cognitive, and motor)

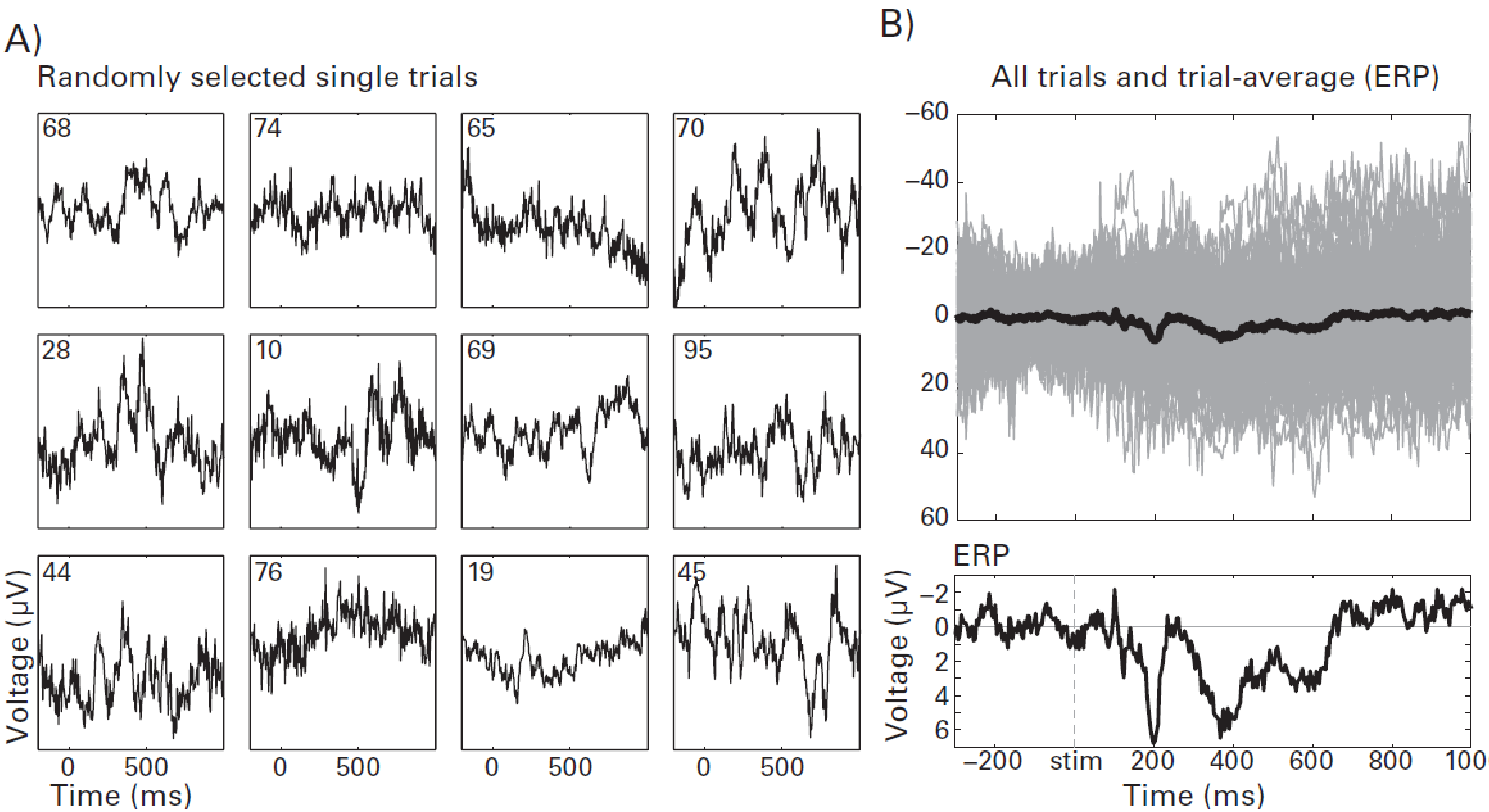
## ➢ Event-related Potential (ERP)

- Generalized EP time locked to a stimulus, response, or informational event (e.g., missing stimulus in series of stimuli)




Schandry 1989

# Event-Related Potential (ERP)



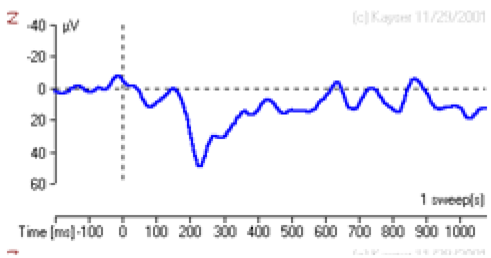

Panel A shows single-trial EEG traces from 12 randomly selected trials (number inside plot indicates trial number). Data are from electrode FCz. Panel B shows 99 single trials in gray and their average — the ERP — in black. Panel C shows the same ERP with focused *y* -axis scaling.

# ➢ Examples of Averaged ERP:

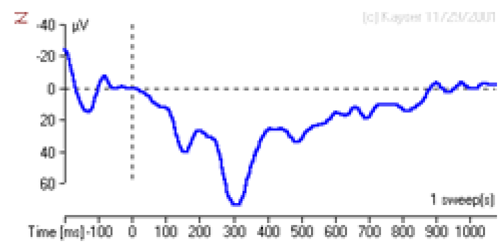
Auditory Oddball ERP

*(average)*

Nontargets




Targets




*Kayser (2001) from Psychophysiology Lab website* (http://psychophysiology.cpmc.columbia.edu)

# ➢ Applications of ERPs:

1) Pathology: slowing or distortion of EP
2) Information processing (incl. perception, cognition)

## ➢ Filtering ERPs:

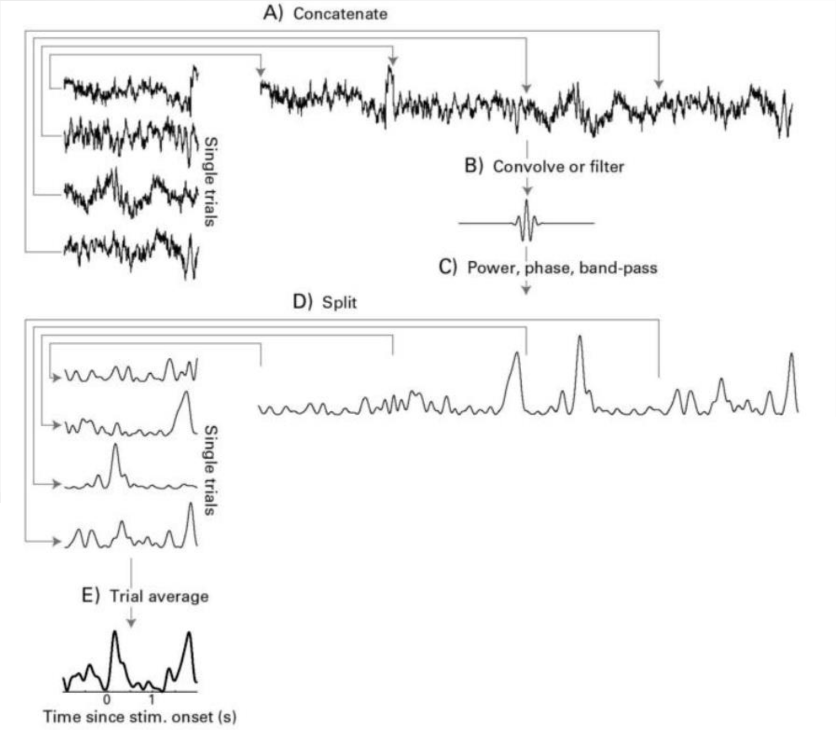
- ✓ Time-domain signal averaging over trials is itself a low-pass filter.
  - ✓ non-phase-locked activity does not survive time-domain averaging, and frequencies above around 15 Hz tend to be non-phase-locked.

- ✓ Averaging ERPs across multiple subjects provides further low-pass filtering
  - ✓ brief neural events are likely to have some temporal jitter across subjects, and thus the average will be smoothed in time.

- ✓ Nonetheless, it is common to apply additional low-pass filters when computing ERPs although not necessary when there are many trials.
  - ✓ Filtering the ERP minimizes residual high-frequency fluctuations, makes the ERPs look smoother.

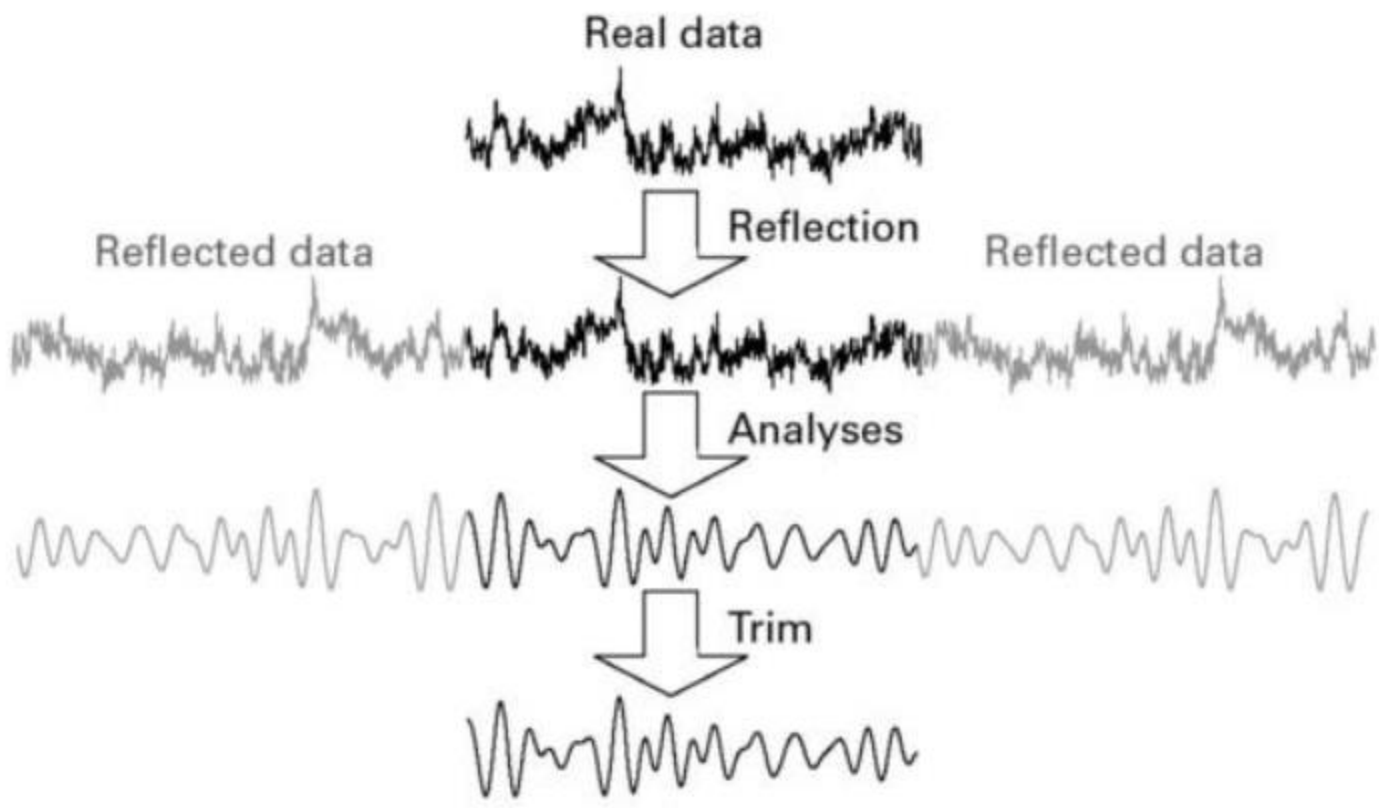
# Filtering each trial vs. filtering concatenated trials:

- It is faster to concatenate all trials into one long time series, filter that concatenated time series, and then cut out the separate trials, compared to filtering each trial separately.
- The main concern of trial concatenation is edge artifacts.
  - If the trials are not too short and the period of interest is not located over the whole trial (there is a sufficient buffer zone), then the trials can be concatenated before applying the filter. This can reduce the computational time.
  - If you have sufficient buffer time at the start and end of trials, edge artifacts will subside by the time period you will analyze.
- You should avoid filtering concatenated trials if you have short epochs right around the time period in which you want to analyze.
- Recall: Sharp transitions at the boundaries can generate edge artifact.

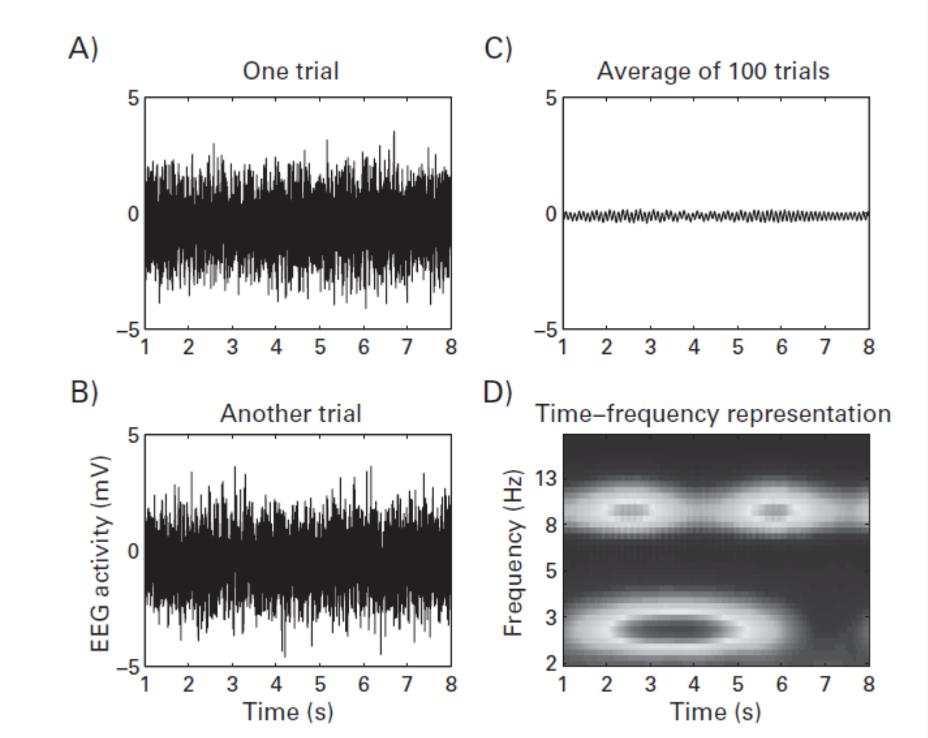


# Temporal Filtering

✓ If the trials are too short to avoid the edge artifacts, it’s better to reflect the data.



# Time-Frequency Analysis




Simulated data showing that complex and multifrequency information contained in EEG data may have no representation in the ERP, if that information is non-phase-locked. One hundred trials were simulated; panels A and B show example trials. Panel C shows the ERP of those 100 trials, and panel D shows the time-frequency power.

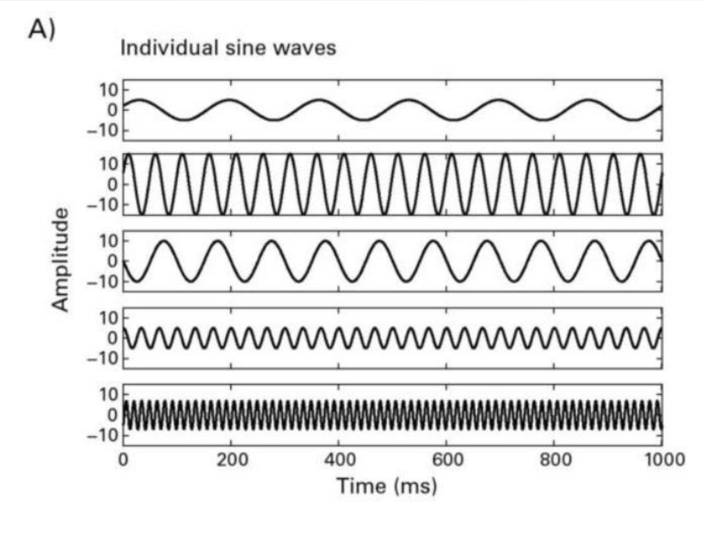
✓However, conceptualizing and analyzing EEG data as a multidimensional signal that contains frequency as a prominent dimension provide many opportunities to link EEG data to experiment manipulations, ongoing subject behavior, patient groups, and other neurophysiological processes.

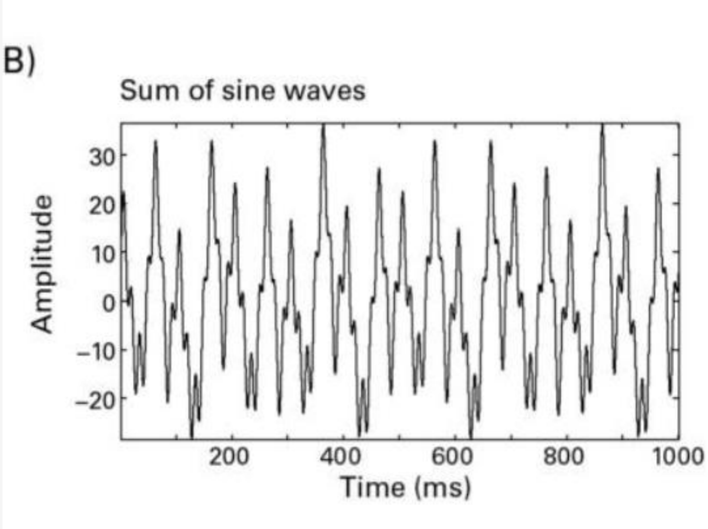
# Time-Frequency Analysis

- **Reminder:**

➢ **Fourier Transform:**

✓ Any signal or waveform could be made up just by adding together a series of pure tones (sine waves) with appropriate amplitude and phase.

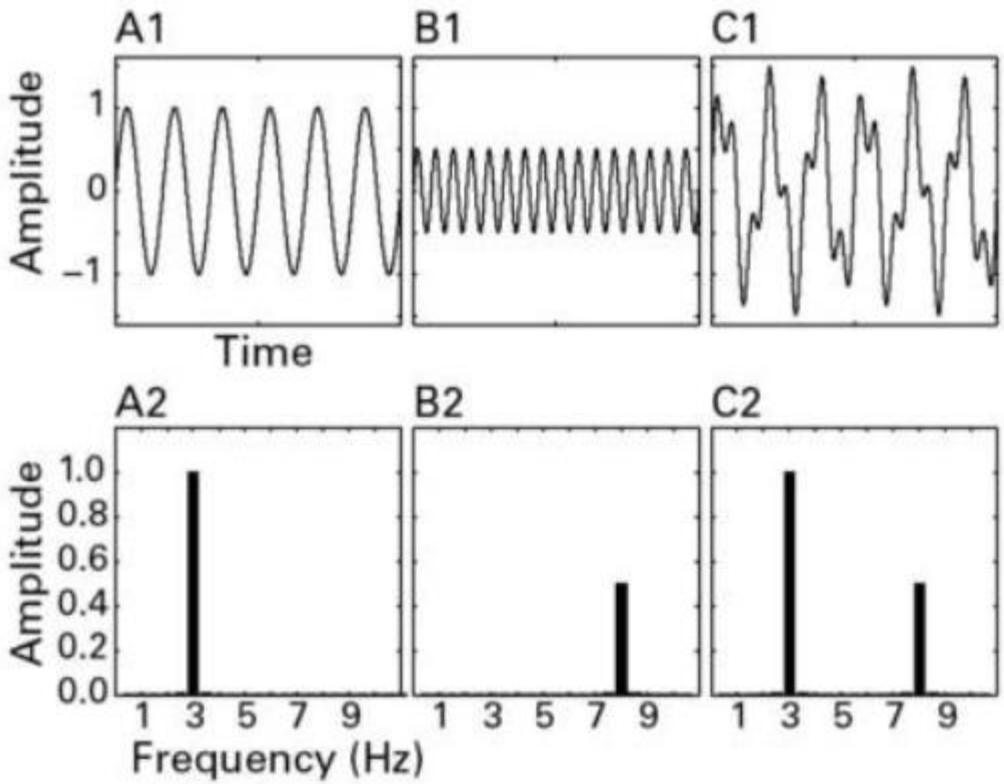



➢ **Purpose of Fourier transform in EEG:**

✓ Its opposite problem; we already have the time series and we want to know with which sine waves with which frequencies, amplitudes, and phases we can reconstruct the signal.

# Time-Frequency Analysis



# Frequency and Time-Frequency Domain Analysis

- ✓ Two disadvantage of Fourier transform
  - ✓ It does not show the time evolution of the power
  - ✓ It assumes the signal is stationary

## ➢ **Stationarity and the Fourier transform:**

- ✓ One assumption in Fourier transform is that the data would be stationary. In real EEG this assumption is violated specifically during a cognitive task.
- ✓ Violation of stationarity can decrease the peakiness of the results.
- ✓ Using temporarily localized frequency decomposition methods such as short time FFT (STFT) can overcome this concern.
- ✓ In an EEG data, a segment of few hundred millisecond is assumed to be stationary.

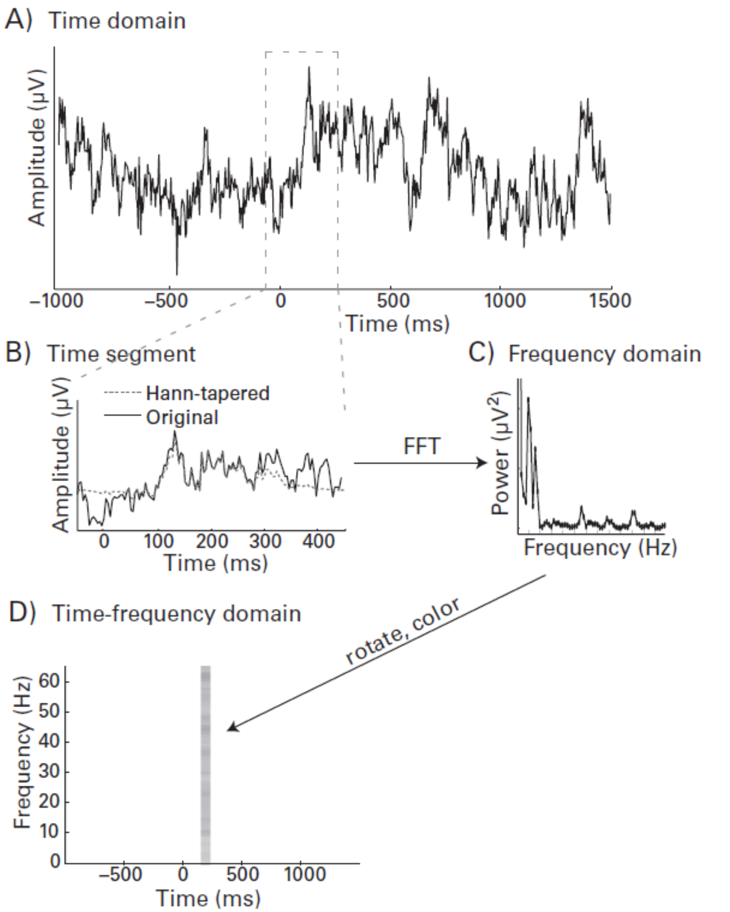
# Time-Frequency Analysis

## ➢ Short-Time FFT

✓ Uses the FFT to extract the frequency structure of brief segments of data (time windows) rather than the entire time series.

### Overview of the short-time FFT method

- ✓ From the time series data (panel A), a small segment of the data, comprising a few hundred milliseconds, is taken (panel B).
- ✓ A windowing taper is applied to that segment to minimize the possibility of edge artifacts, and then the Fourier transform of the tapered time series is taken (panel C).
- ✓ The power spectrum of that segment is then placed into a time-frequency space with the frequencies corresponding to that of the FFT and the time point corresponding to the center time point of the time segment from panel B.



# Mathematical Preliminaries

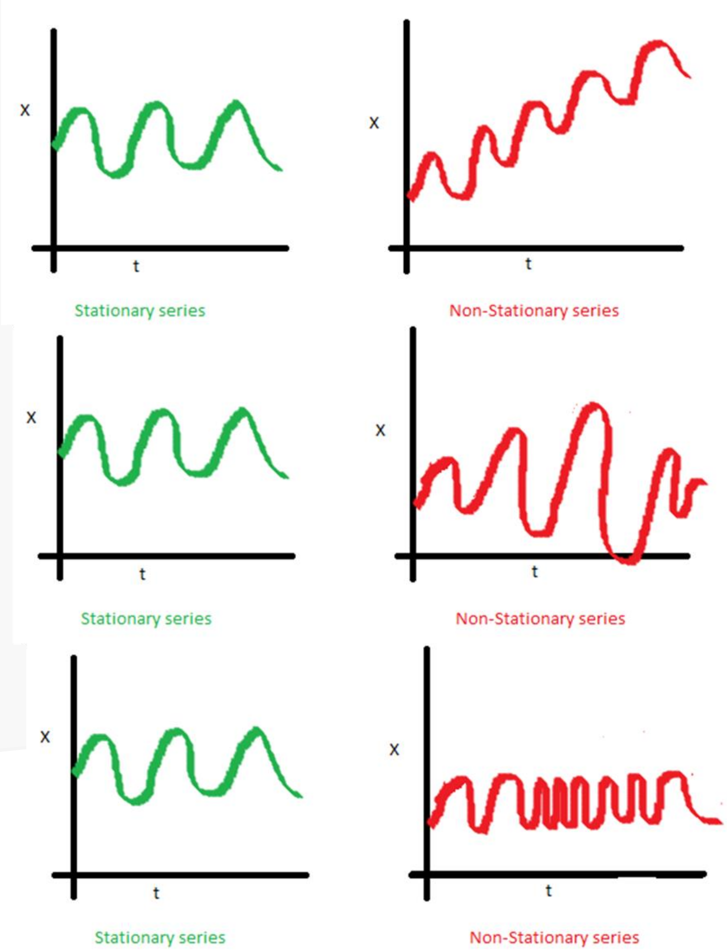
## ➢ Stationary Process:

✓ Consider a stochastic process $\{X_j(t)\}$ defined at times $t_1, t_2, \ldots, t_k$. This process is said to be strict-sense stationary (SSS) when its joint probability distribution is invariant when the signal is shifted over time.

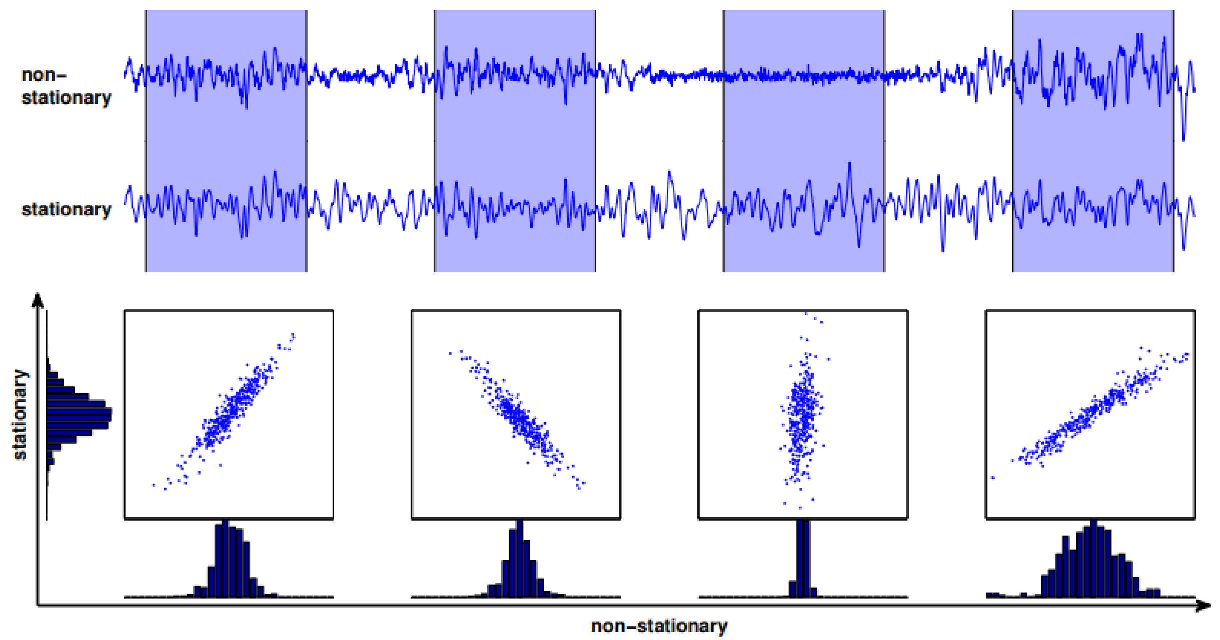
✓ A weak consideration of strict stationarity is wide-sense stationarity (WSS). WSS process are requires to satisfy the following properties:

1- Mean function $m_x(t)$ is constant.

2- The autocorrelation function is constant.



# Mathematical Preliminaries

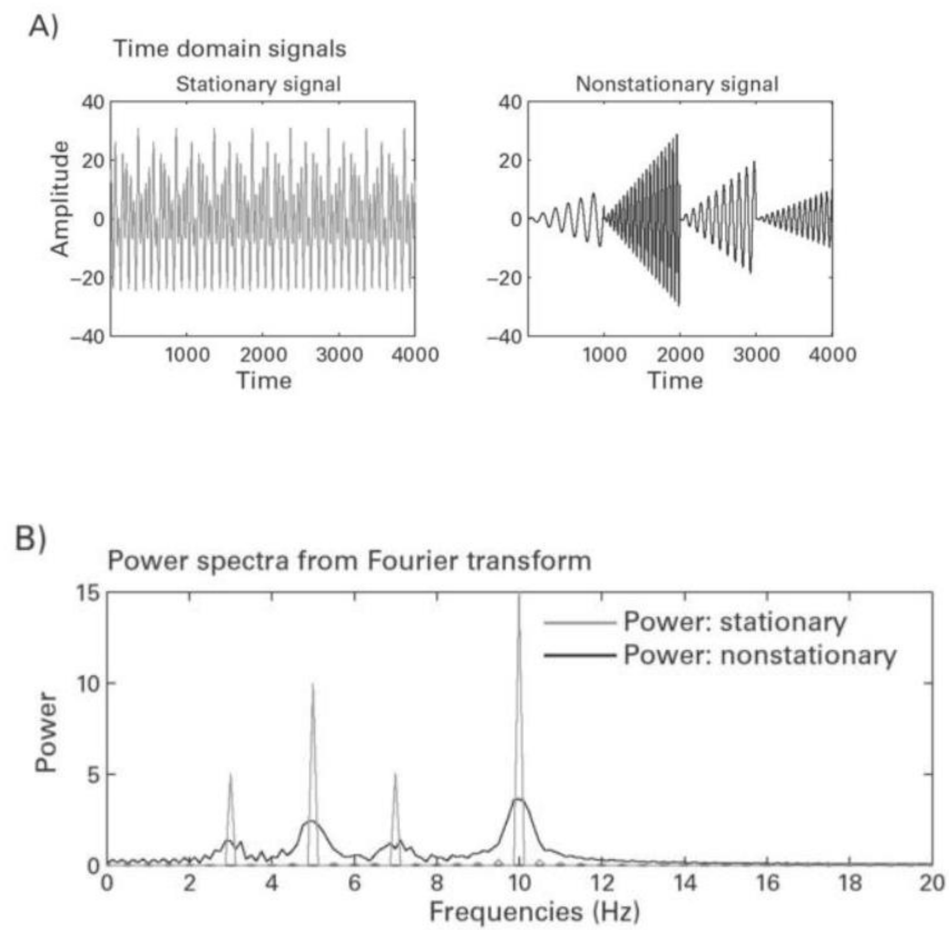



Non-stationary and stationary source with time-variable covariance illustrated by amplitude scatter plots for four epochs.

Paul von Bunau et al., 2010

# Time-Frequency Analysis

- Illustration of power spectra for two stationary and non-stationary signals:



# Time-Frequency Analysis

## ❑ Reminder:

➢ **Convolution in time domain:**

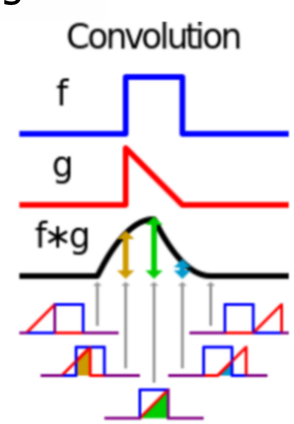
✓ Convolution in time domain is an extension of dot product, in which the dot product is computed repeatedly over time.

✓ In EEG signals when we use convolution, one vector is being considered as signal and the other one as kernel.

- Flip the kernel
- Lineup the rightmost point of the kernel with leftmost point of the signal
- Zero pad (to make the length of two vectors the same)
- Perform dot product
- Move forward till the leftmost point of kernel touches the rightmost point of the signal

$$(f * g)[n] = \sum_{m=-M}^{M} f[n-m]g[m]$$

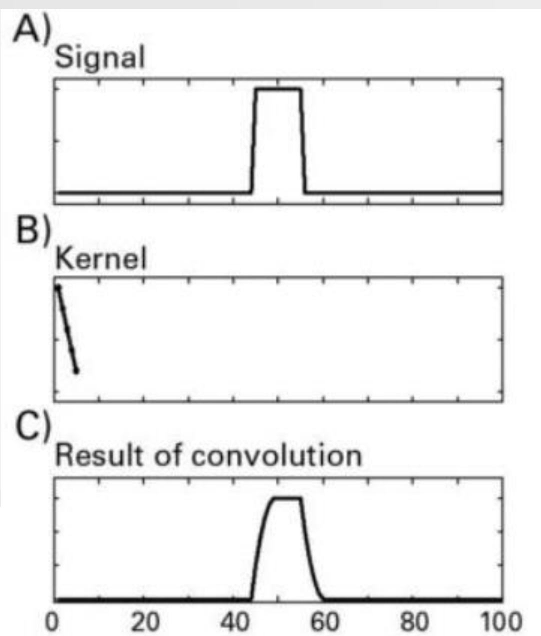
Where $n$ is the time point and m corresponds to the elements in the equal length vectors f and g and M is the length of the vectors



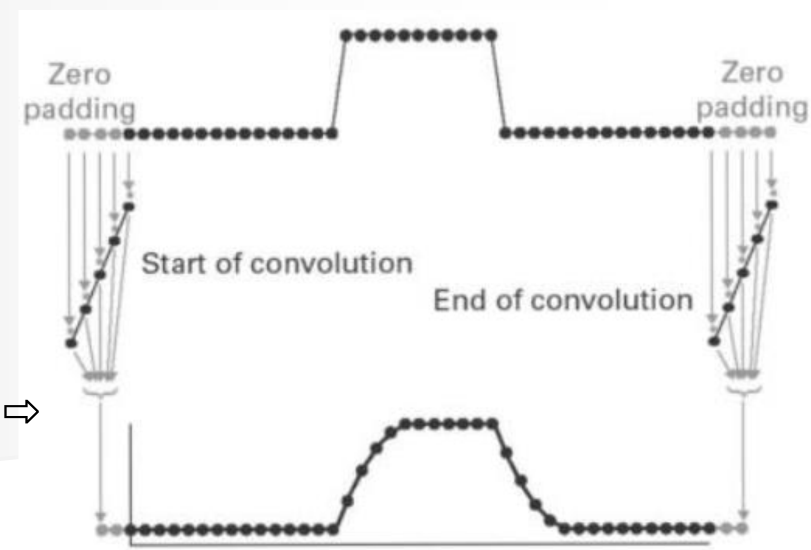

https://en.wikipedia.org/wiki/Convolution

# Time-Frequency Analysis

- Length of convolution is length(signal) + length(kernel) -1
- Each point in the kernel is multiplied by each corresponding point in the signal, the multiplications are summed, and that value-the dot product-is placed in a position corresponding to the center of the kernel.




$signal * Kernel$ ⇨



# Time-Frequency Analysis

- ✓ Example: Find the convolution of two sequences x[n] and h[n] given as below:
- ✓ X[k]=[3 1 2]; h[k]=[3 2 1]

- ❑ Hint: The value of k starts from (– length of h + 1) and continues till (length of h + length of x – 1). Here k starts from -3 + 1 = -2 and continues till 3 + 3 – 1 = 5

# Time-Frequency Analysis

- **Reminder:** Interchangeability of time domain convolution and frequency domain multiplication.

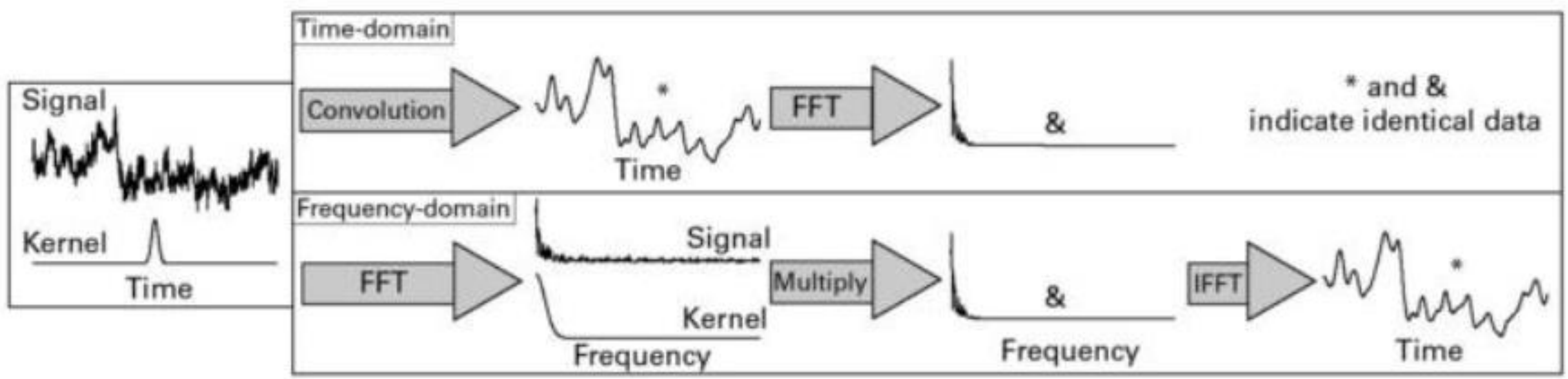
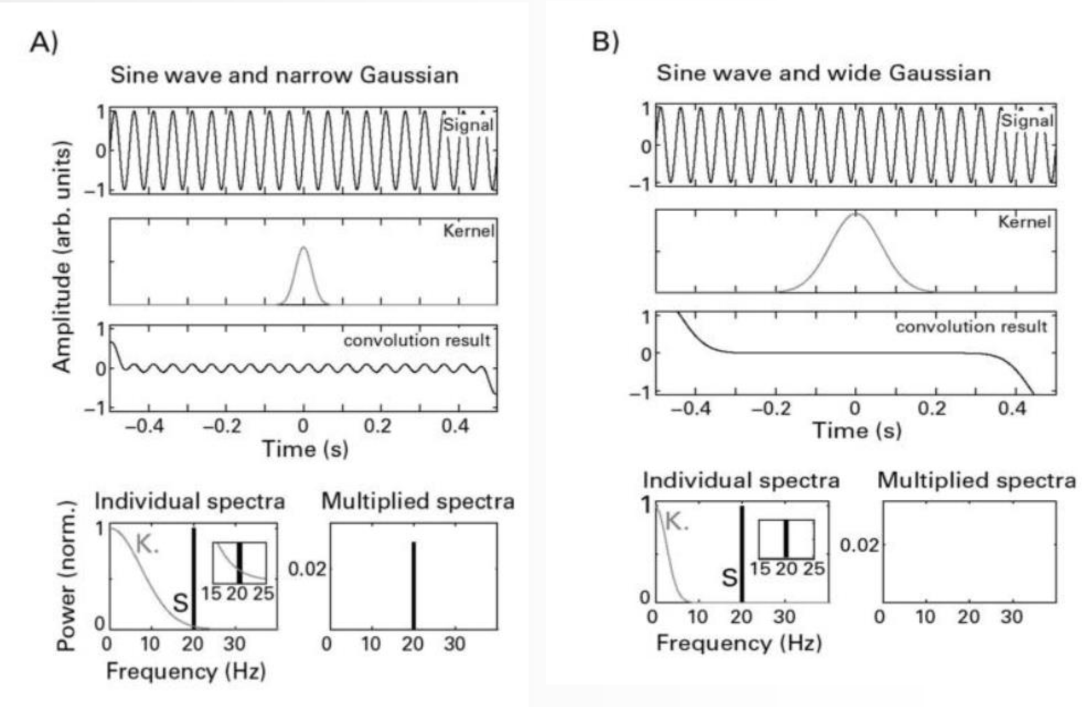



Illustration of the convolution theorem and the interchangeability of time-domain convolution and frequency-domain multiplication.

# Time-Frequency Analysis

- ✓ There should be overlap between the frequency response of the kernel and the frequency response of the signal. Otherwise, the resulting convolution frequency response would be zero.

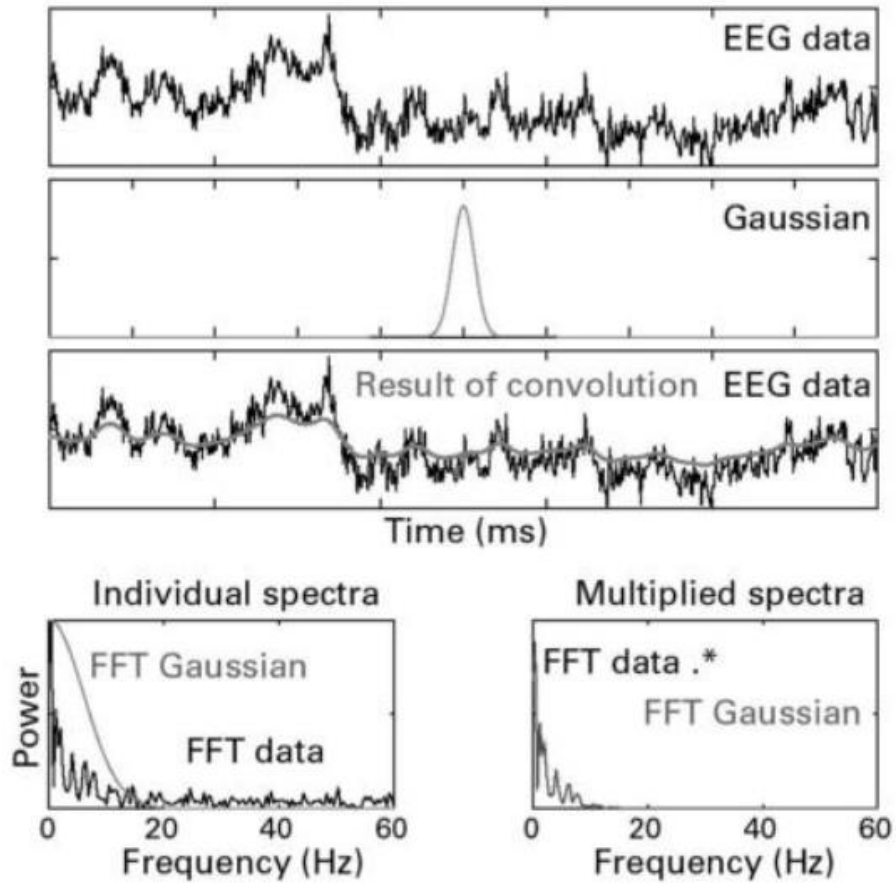



- ✓ The convolution acts as a filter such that the frequency profile of the signal is passed through the frequency profile of the kernel.

- ✓ This is the base of wavelet that you pass the signal through **sets of filters** (wavelets), and the results of convolution is the frequency-band intersection between EEG and the wavelet.

# Time-Frequency Analysis

- ✓ Thus, the Gaussian kernel is a low-pass filter



# Time-Frequency Analysis

- **Purpose of Convolution for EEG Data Analysis:**
- In EEG data analysis, convolution is used to isolate the frequency-band specific activity.
- In wavelet transform, this is done by convolving wavelets with the EEG data.
- As the wavelet (convolution kernel) is dragged along the EEG data, it determines when and to what extend the EEG data contain features that look like the wavelet.
- If convolution is repeating on the same data using wavelets with different frequencies, time-frequency maps can be formed.

# Time-Frequency Analysis

- **Wavelet Transform (WT):**
- WT is a class of time-frequency decomposition technique that is conceptually similar to short time FFT.
- WT is a scalar product of the signal with a translated and dilated version of a locally confined function (called mother wavelet or basis function).
- This scalar product tells us to what degree the shape of the signal is similar to the mother wavelet.
- Wavelet of a function depends on two parameters, "b" for translation (shift) and "a" for dilation (scale). The parameter b shifts the wavelet so that local information around time t = b is contained in the transformed function and adjusts the window length.

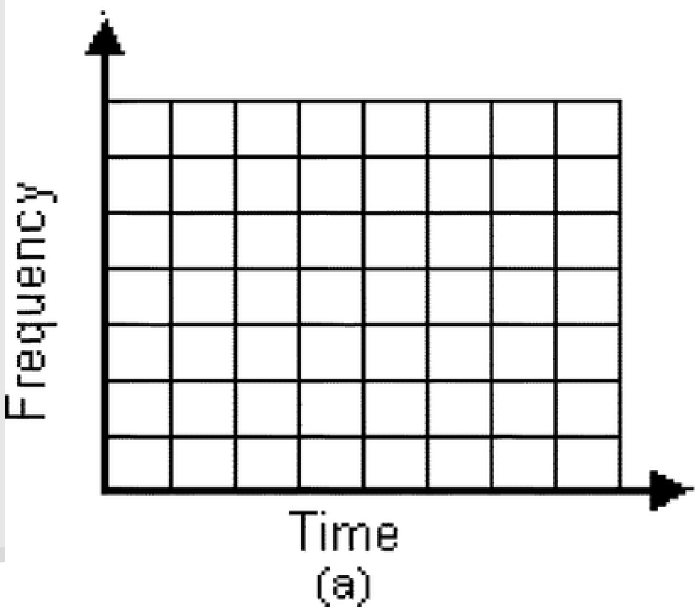
$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

- Wavelet analyses utilize a different time window length for each frequency, with the longest windows applied to the lowest frequencies and the shortest windows applied to the highest frequencies.

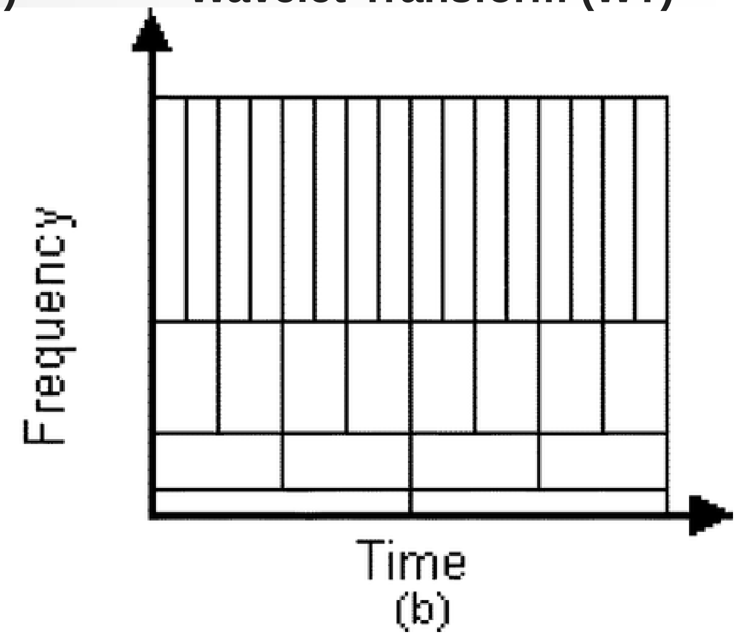
# Time-Frequency Analysis

- Wavelet analyses utilize a different time window length for each frequency, with the longest windows applied to the lowest frequencies and the shortest windows applied to the highest frequencies.
- One of the pitfalls of the STFT is that it has a fixed resolution.

**Short Time Fourier Transform (STFT)**




**Wavelet Transform (WT)**





Ganesan, R., Das, T. K., Sikder, A. K., & Kumar, A. (2003). Wavelet-based identification of delamination defect in CMP (Cu-low k) using nonstationary acoustic emission signal. *IEEE Transactions on Semiconductor Manufacturing*, *16*(4), 677-685.

# Time-Frequency Analysis

- ✓ Basis function (mother wavelet) can be selected depending on the application.
- ➢ **Can any function be a mother wavelet?**
- ✓ The mother wavelet should be oscillatory:

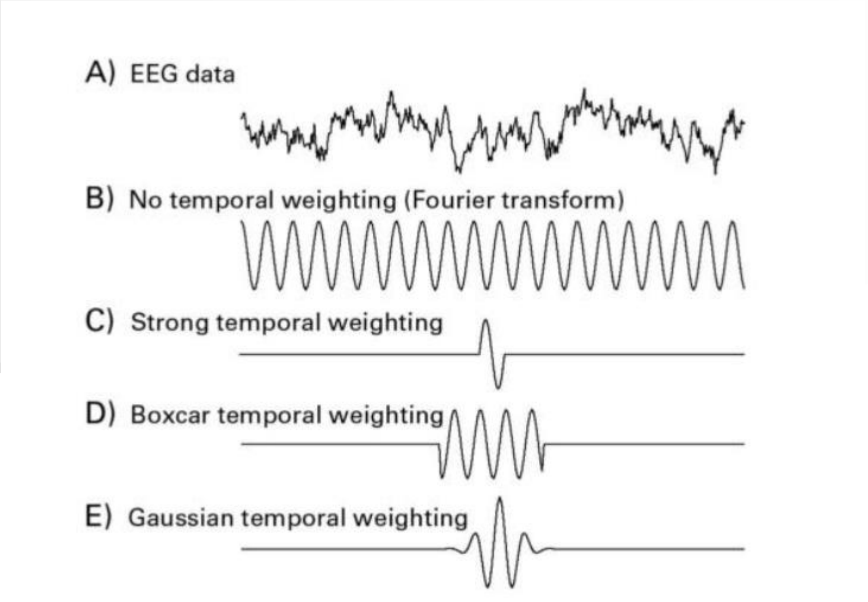
$$\int_{-\infty}^{\infty} \Psi(t)\,\mathrm{d}\,t = 0$$

- ✓ The mother wavelet should have finite energy:

$$\int_{-\infty}^{\infty} |\Psi(t)|^2 \mathrm{d}\,t \leq \infty$$

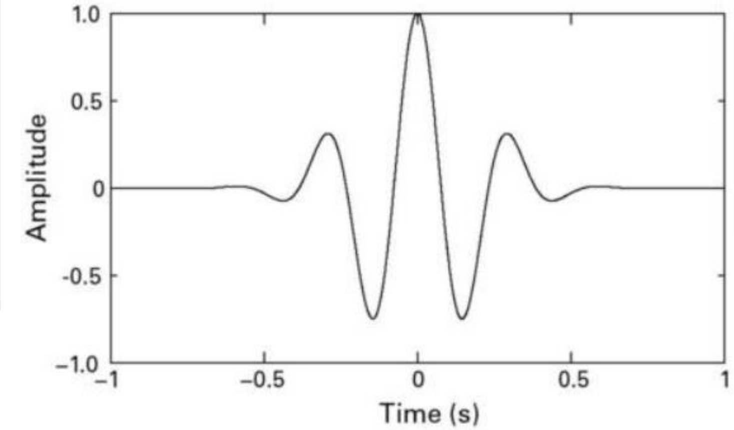
# Time-Frequency Analysis

- **What type of Kernel for Wavelet?!**
- ✓ (B) Sine wave without tapering: it does not have any temporal localization, thus, it reflects the frequency specific information but over the entire time (like Fourier transform).
- ✓ (C) One cycle sine wave: it provides good temporal precision but at the expense of frequency precision.
- ✓ (D) Uniform boxcar tapering: the temporal precision decreased and it has potential artifacts due to sharp edges.
- ✓ (E) Gaussian tapering (also known as Morlet wavelet): It provides the balance between time precision and frequency precision without introducing the edge artifacts.

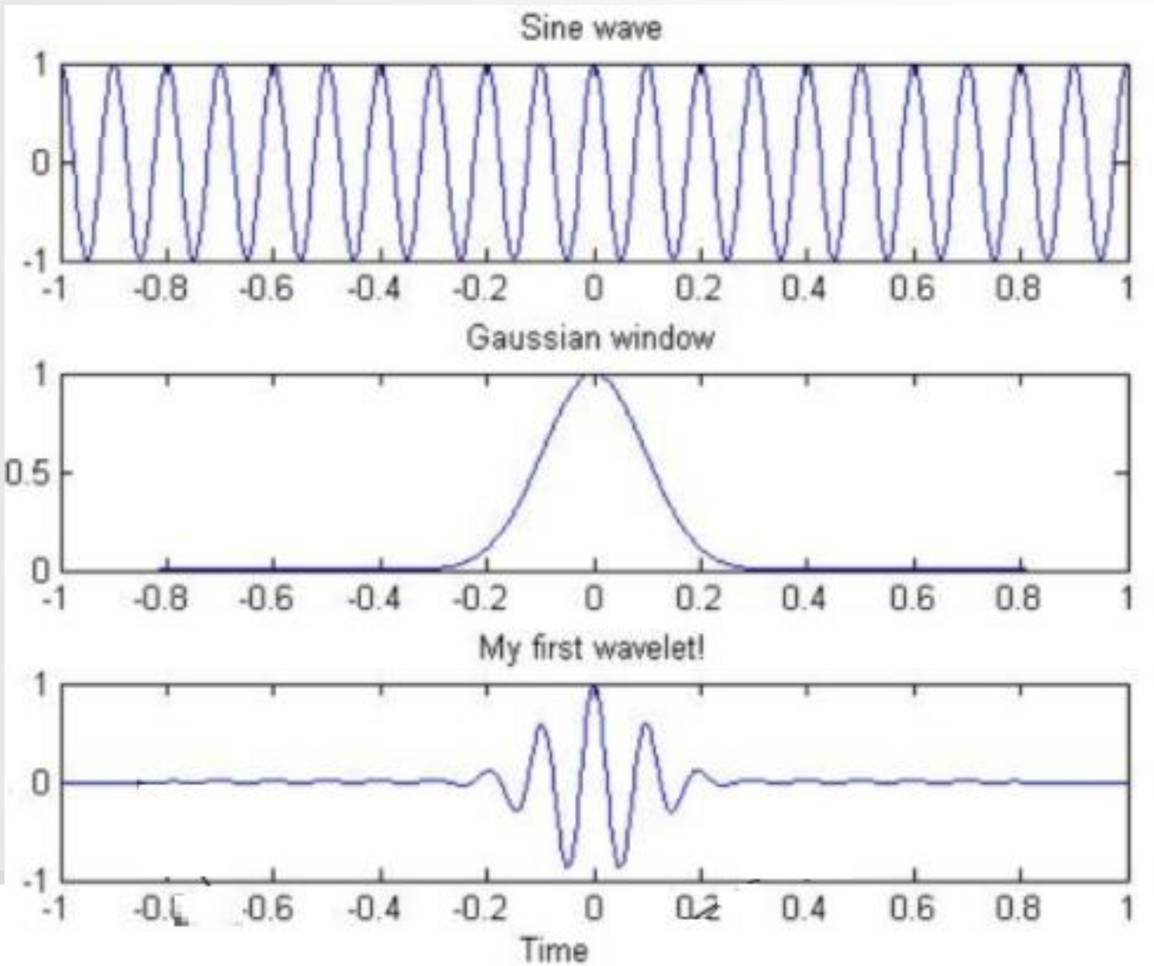


# Time-Frequency Analysis

- **Morlet Wavelets and Wavelet Convolution:**
- ✓ Jean Morlet, is a French geophysicist, and so “Morlet” can either be pronounced as “more let” or “more lay”.
- ✓ A sine wave that is “Windowed” (i.e., multiplied) by a Gaussian kernel
- ✓ The time-frequency analysis of EEG using Morlet wavelet, is that the data is convolved by Morlet wavelet at different frequencies.



# Time-Frequency Analysis




$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

$a$ = height of the curve's peak

$b$ = the position of the center of the peak

$c$ = the standard deviation

$f(x)$ = function of x

$e$ = Euler's number

$x$ = integer

# Time-Frequency Analysis

## How to Make a Morlet Wavelet :

- To make a Morlet wavelet create a sine wave, create a Gaussian wave, and multiply them together.

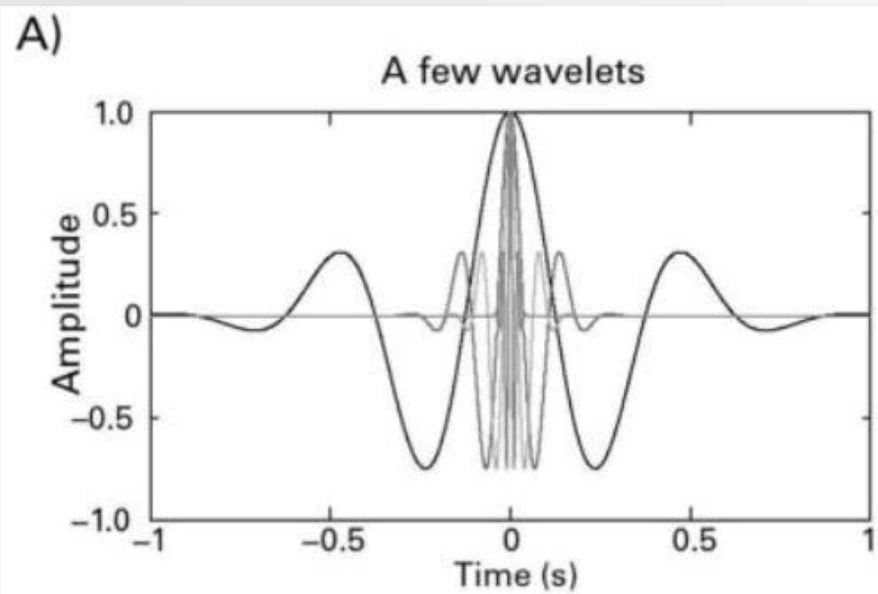
They should have the same number of time points and sampling rate (this also needs to be the same sampling rate as the EEG data).

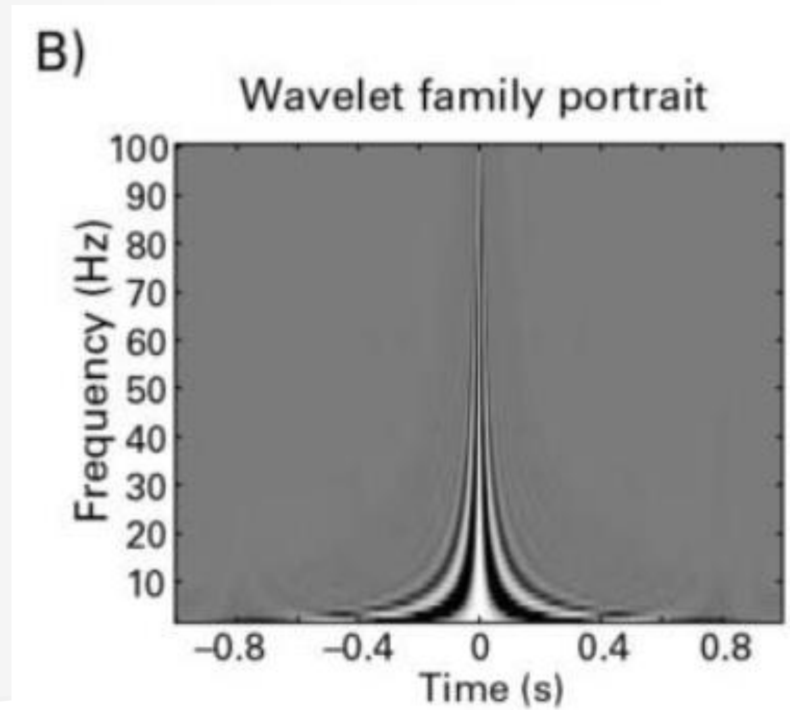
- The frequency of the wavelet is the frequency of the sine waveform. The frequency of a Morlet is called center (or peak) frequency.
- In practice we should have several cycle of the activity in the epoch in order to be able to see the frequency of the activity (for example if we want to see 1 Hz activity we need to have ~3-4sec data).
  - Don't use frequencies that are too big for your epoch
- For the higher frequencies we can have smaller epochs.
- The frequencies of the wavelet can not be above the Nyquist rate.

$$GaussWin = ae^{-(t-m)^2/(2s^2)} \qquad s = \frac{n}{2\pi f}$$

Where a is the amplitude (height of the Gaussian), m is the x-axis offset (which can be set to zero for the EEG analysis), s is the standard deviation or the width of the Gaussian, t is the time, n is number of wavelet cycles, and f is the frequency in hertz.

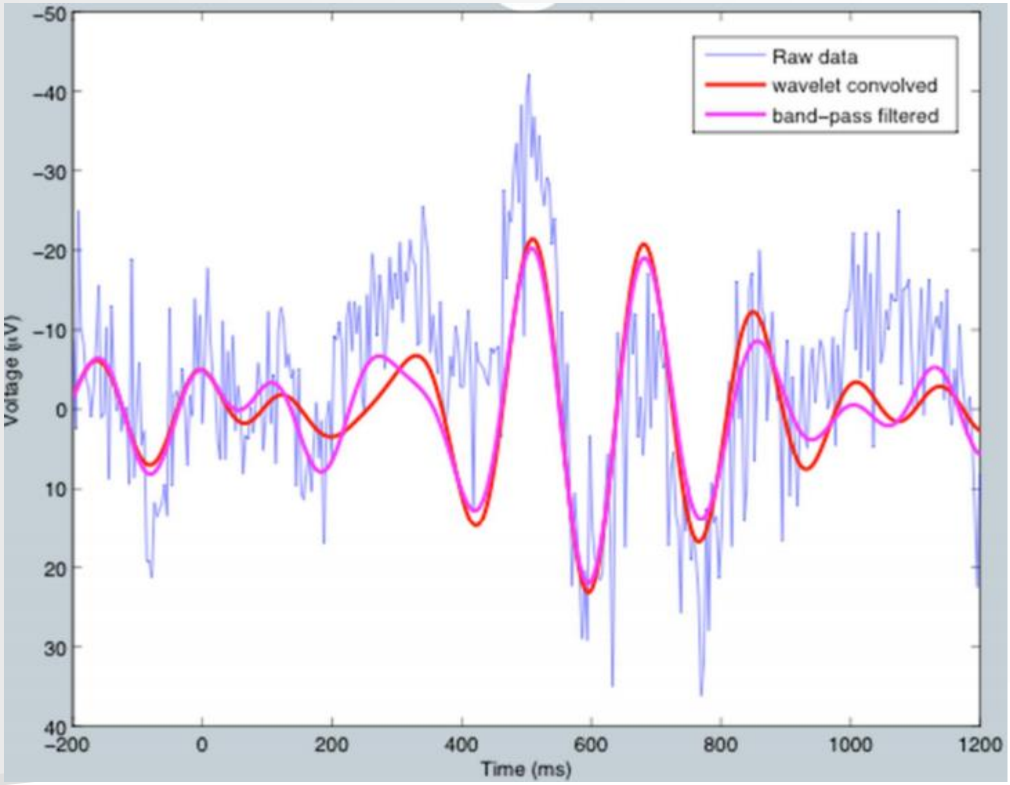
# Time-Frequency Analysis



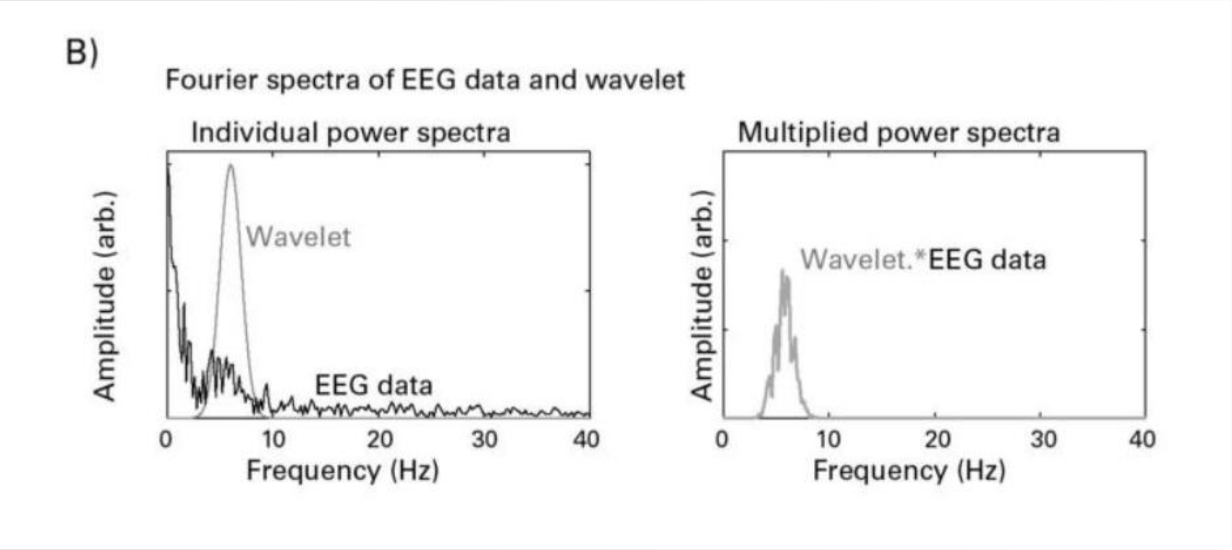
# Time-Frequency Analysis

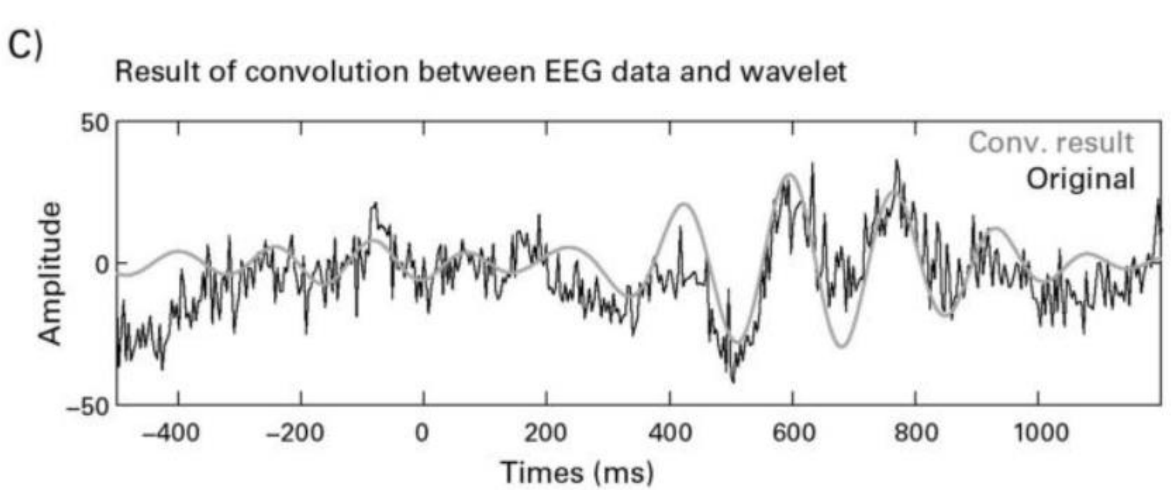
- **Wavelet Convolution as a bandpass filter!**




One trial of EEG data plotted before any filtering (blue), after being bandpass filtered between 4 and 8 Hz (pink line), and after being convolved with a 6-Hz Morlet wavelet (red line).

# Time-Frequency Analysis



# Time-Frequency Analysis

- **Limitations of real-values Morelet wavelet:**

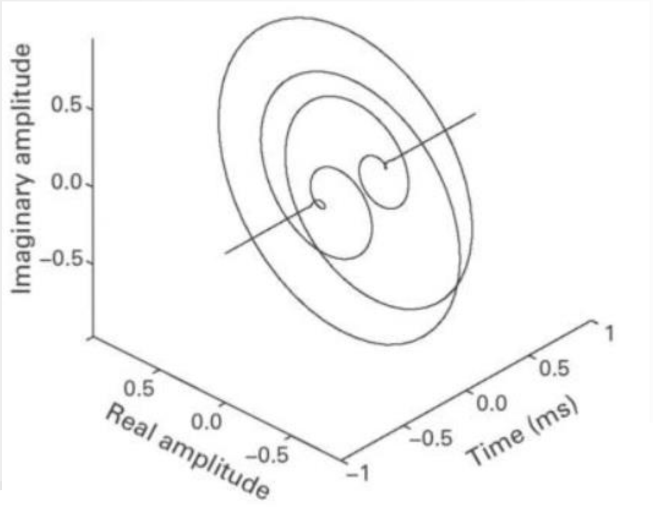
- Convolution with real-time Morlet, acts as a bandpass filter, and does not provide power and phase information.
  - But we need power and phase information too!

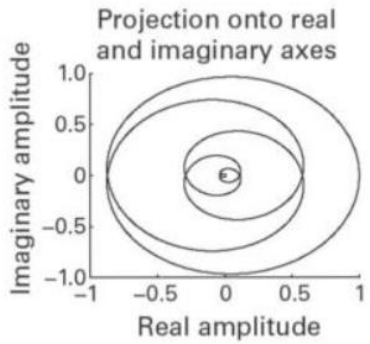
- **Complex Morlet Wavelet (cmw):**

- Can not only band pass filter the data, but also gives power and phase information in time domain.
  - On the other words, complex Morlets are used to estimate the time varying frequency band-specific power and phase.
- A complex Morlet has 3-D: time, real, and imaginary parts

# Time-Frequency Analysis

- ✓ As mentioned before a real-valued wavelet is created by multiplying the sine wave with a Gaussian; a complex Morelet wavelet is created in the same way, except that the sine wave is a complex sine wave.

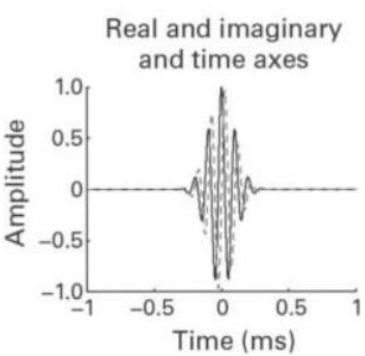



$$cmw = Ae^{-t^2/2s^2} e^{i2\pi ft}$$

$$A = \frac{1}{\left(s\sqrt{\pi}\right)^{1/2}}$$

Where A is a frequency band specific scaling factor.



# Time-Frequency Analysis

## ❑ Reminder:

➢ **Cartesian and Polar Notations and the Complex Plane:**

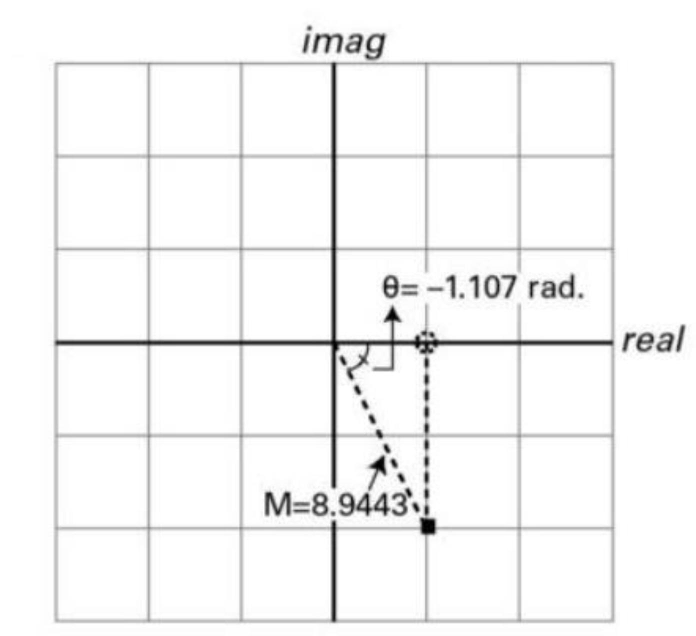
✓ Example: $4 - 8i$

$$M = \sqrt{(real^2 + imag^2)}$$

$$\theta = \arctan\left(imag / real\right)$$

$$real = M\cos\left(\theta\right)$$

$$imag = M\sin\left(\theta\right)$$

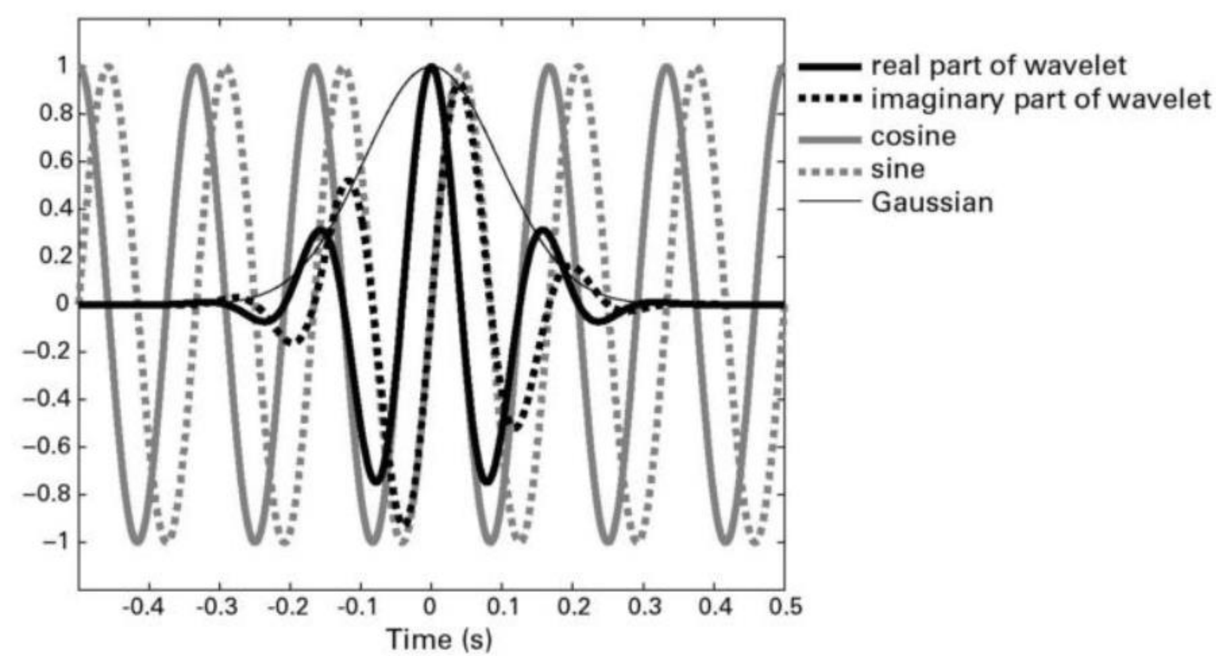



➢ **Euler's Formula:**

$$Me^{i\theta} = M\left[\cos(\theta) + i\sin(\theta)\right]$$

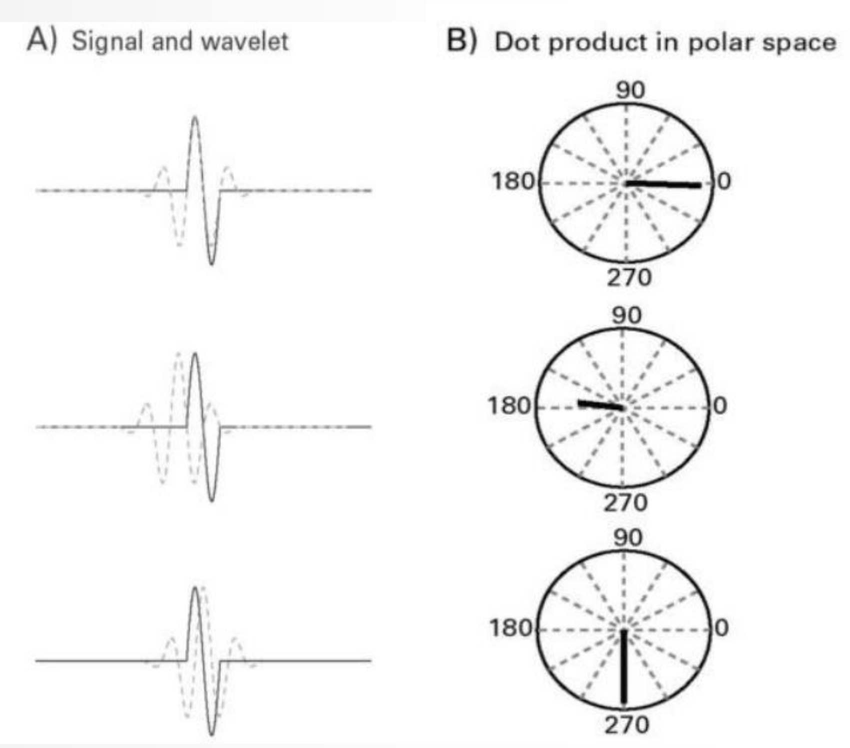
# Time-Frequency Analysis

- **Overlaying of the real and imaginary parts of a complex wavelet along with a cosine and a sine wave.**
- ✓ The real part of the wavelet corresponds to a cosine wave, and the imaginary part corresponds to the sine wave.



# Time-Frequency Analysis

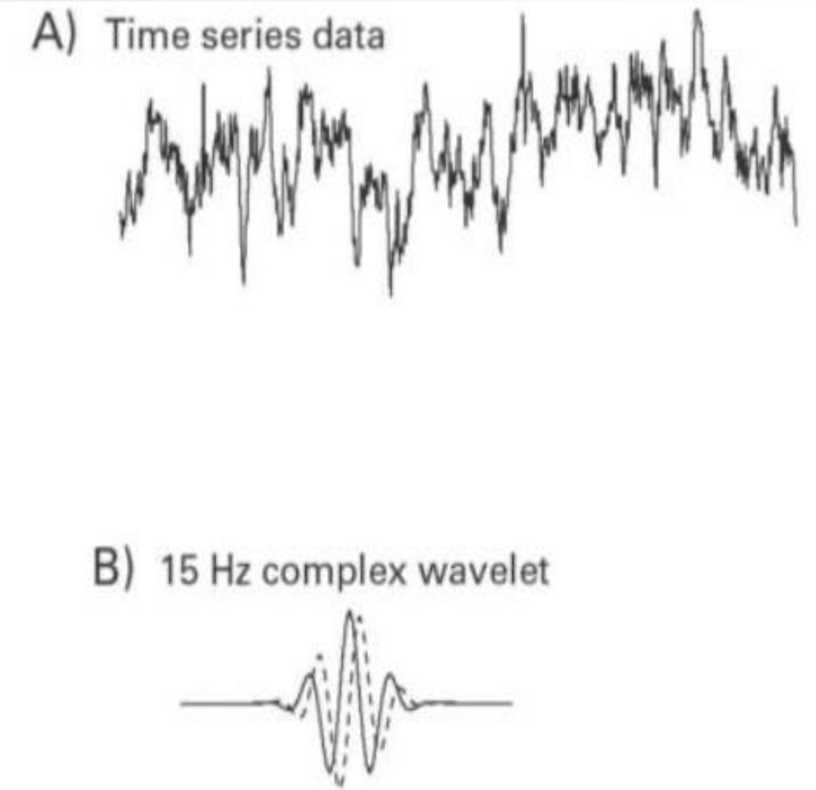
- ✓ The more the wavelet and one-cycle sine wave overlap, the longer the vector is in the complex space, regardless of the phase angle of that vector.
- ✓ The length of the line in the polar space does not depend on the phase relationship between the kernel and the signal. In contrast, the phase relationship between the kernel and the signal is characterized by the angle of the vector.



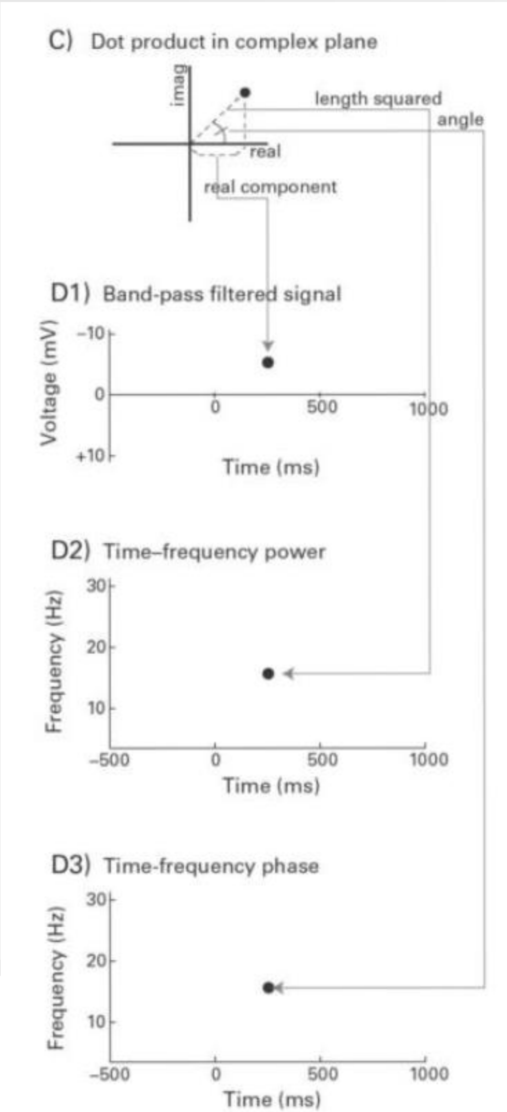

The dot product (one step of the convolution) between a complex Morlet Wavelet (only the real part has only been shown here using a dotted line) and a one-cycle sine wave.

# Time-Frequency Analysis

- **Three piece of information can be extracted from the complex dot product.**



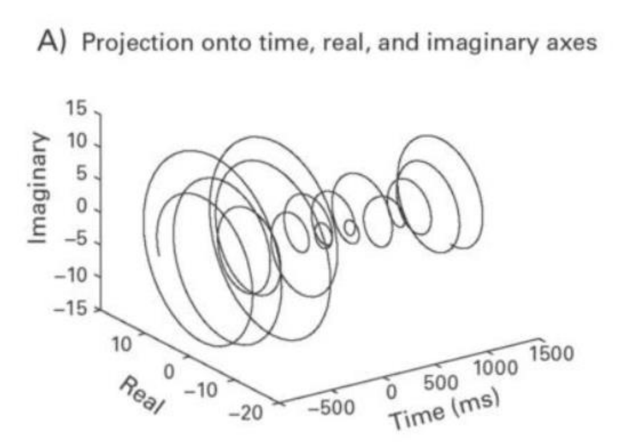
# Time-Frequency Analysis

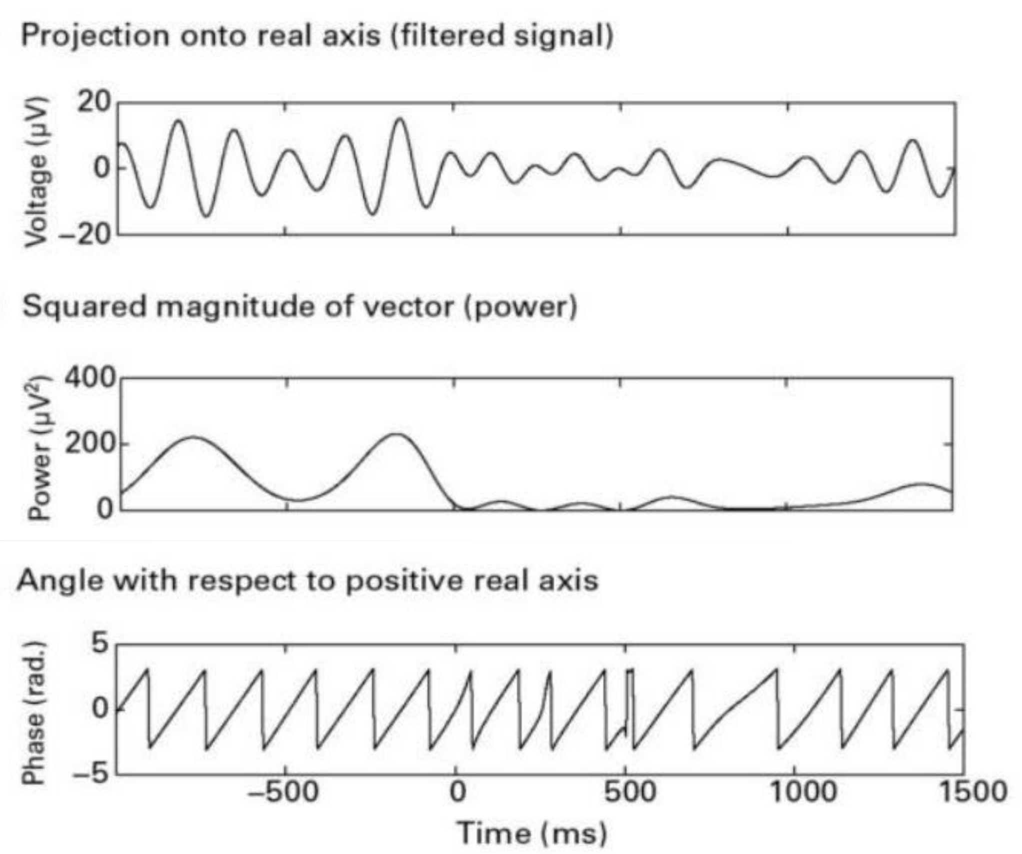



- ✓ The band pass filtered signal which is the projection onto the real axis.
- ✓ The magnitude of the vector which can give us the power information.
  - E.g. if this EEG signal contains a lot of energy at 15 Hz we should get a longer vector.
- ✓ The angle of the vector that can give us the phase angle corresponding to the center of the wavelet.

# Time-Frequency Analysis

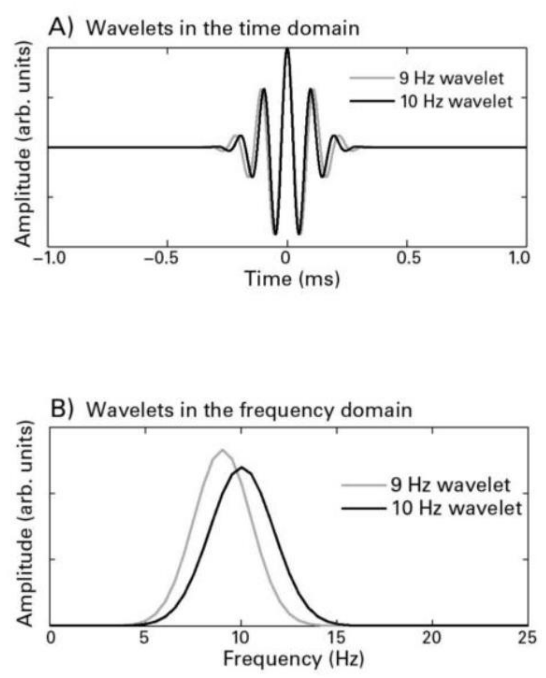
- ✓ After computing the time sliding dot products over time, we can generate the i) power (amplitude squared), ii) phase, and iii) filtered signal (projection into real axis) over time.



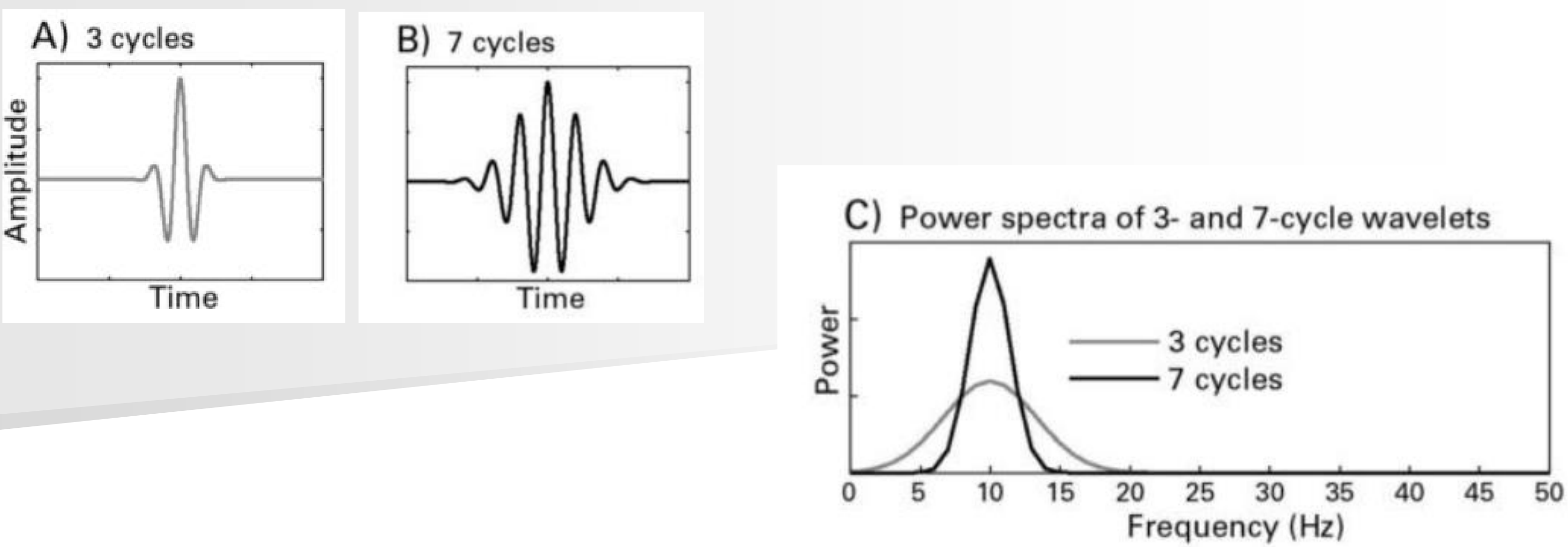
# Time-Frequency Analysis

- ✓ To increase the freedom in post hoc analysis and exploring the best frequency response, using several wavelet frequencies in the range of interest, which are also not close to each other, is suggested (e.g. 6.3 Hz wavelet and 6.6 Hz wavelet are unlikely to produce independent results).

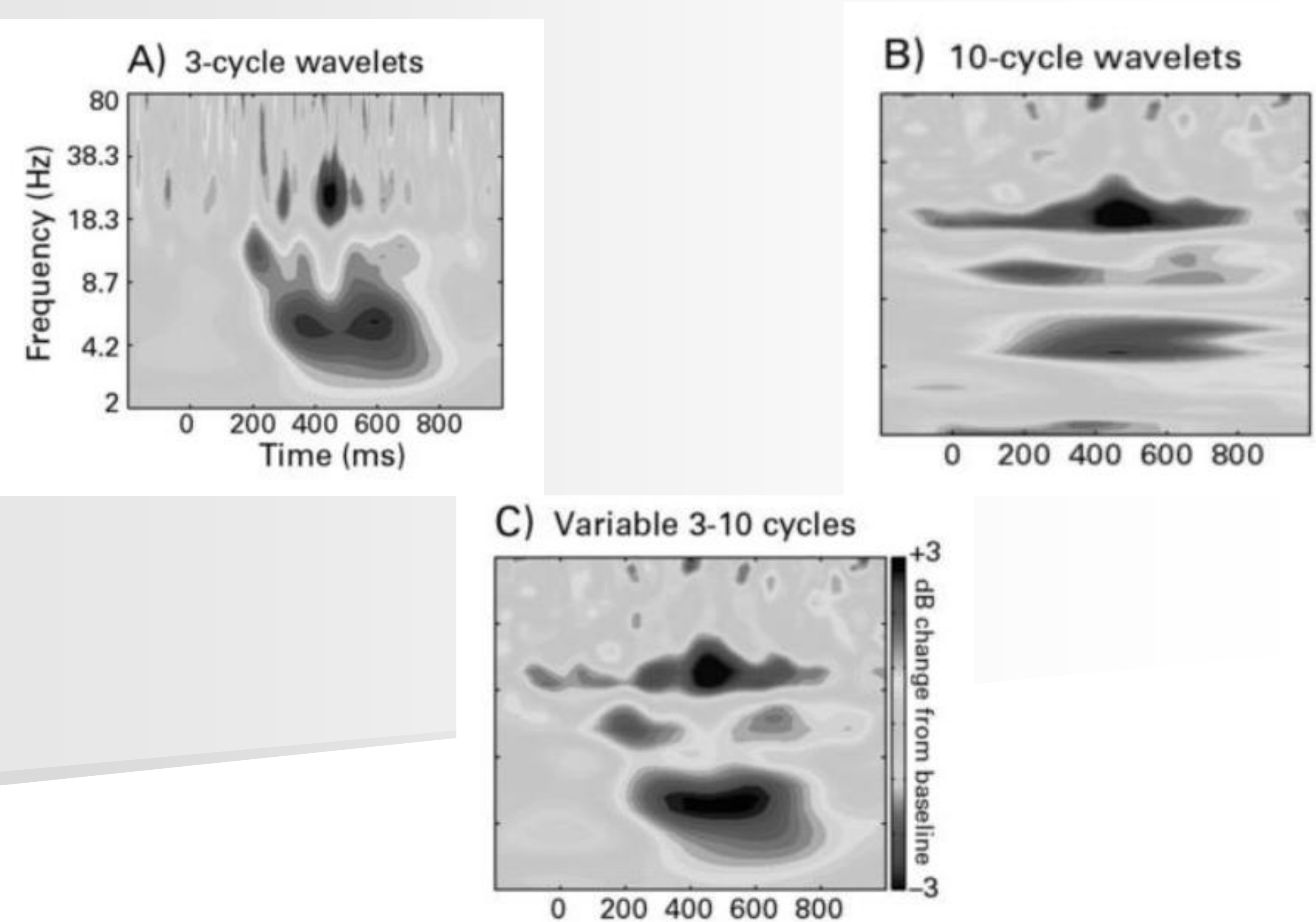


# Time-Frequency Analysis

- **Trade off between temporal and frequency precision:**
  - A larger number of cycles in a Gaussian taper will give us a better frequency precision but at a cost of worse temporal precision.
  - Heisenberg uncertainty principle applied to time-frequency analysis:
    - The more you know about when something happened, the less you know about where (i.e. at which frequency) it happened, and vice versa.



# Time-Frequency Analysis



# Time-Frequency Analysis

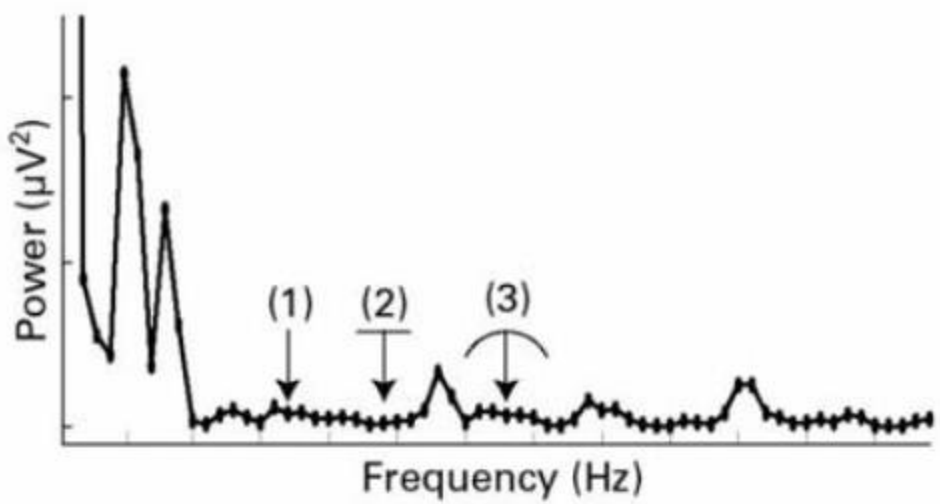
- **Short time FFT:**
- Extracts time-frequency power and phase information
- Overcomes two major limitation of FFT
  - Demonstrates time-varying changes
  - Overcome the stationarity
- Taper the data into overlapping windows (e.g. Hann, hamming, Gaussian)
  - Attenuates the data at the beginning and the end; prevents edge artifact
  - Large buffer zones are not necessary
  - By temporally overlapping the windows, the downside of the tapering the valid EEG at the edges would be compensated
- Get the Fourier transform
  - The number of frequencies returned by FFT is equal to N/2+1 where N is the length of the signal (e.g. If sampling rate is 1000 Hz, we can reconstruct maximum 500 Hz (based on Nyquist rate). If we have 1000ms data we will have 500 frequencies ranges between 0-500. However, if we have 500 ms data, we will have 250 frequencies ranges between 0-500 Hz)

# Time-Frequency Analysis

✓ **Requested frequency:**

Sampling the requested frequency is necessary if the length of the time segment changes as a function of the frequency

Illustration of three different ways to extract the requested frequencies



# Time-Frequency Analysis

- **Time segment length:**
- Trade of between the length of the time segment and the frequency and temporal precision and resolution.
  - The longer the time segment, the better frequency resolution and precisions and vice versa.
- Choice of the length is also related to the lowest frequency we want to analyze; the segment must be long enough to capture one cycle of the activity; e,g, for analyzing 3 Hz activity we need at least 333 ms data segment (preferably 667 ms or 999 ms).
- Adaptive window length (similar to wavelet); lower frequencies longer segments, higher frequency shorter segments.
- **Segments overlap:**
- It improves the temporal precision
- Mitigates the signal loss due to tapering
- Creates smoother time-frequency plots
- No rule but usually 50%-90% for the EEG time-frequency analysis