Morlet Wavelets at Different Frequencies Morlet Wavelet at 2 Hz 1.0 0.5 0.0 Real Part -0.5 Imaginary Part -1.0Morlet Wavelet at 9 Hz 1.0 Real Part **Imaginary Part** 0.5 0.0 -0.5-1.0 Morlet Wavelet at 16 Hz 1.0 Real Part 0.5 **Imaginary Part** 0.0 -0.5-1.0Morlet Wavelet at 23 Hz 1.0 Real Part 0.5 **Imaginary Part** 0.0 -0.5-1.0Morlet Wavelet at 30 Hz 1.0 Real Part 0.5 **Imaginary Part** 0.0 -0.5-1.0

b) Convolve each wavelet with EEG data from all electrodes and from only the first trial In [18]:

Time (samples)

300

400

500

200

100

0

1.5189717 , -15.421003],

-49.65603

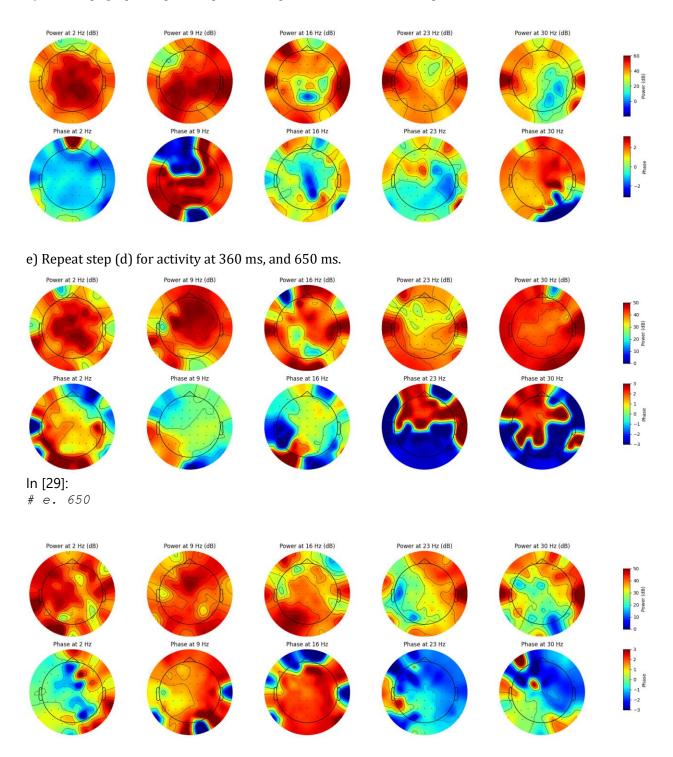
c) Extract power and phase from the result of the complex wavelet convolution and store in a matrix more details of the calculation for part b & c are provided in the code at the end of the report.

[-20.99644 , -7.128301 , 12.264906 , ..., 20.106596 ,

[-27.2869 , -21.165905 , -0.83333369 , ..., -31.996515 ,

, -38.924156]], dtype=float32)

d) Make topographical plots of power and phase at 180 ms at all frequencies



Power Topographies:

Power at 2 Hz (dB): The entire scalp appears to have elevated power, with the central region showing the most pronounced activity. Power at 9 Hz (dB): There's elevated activity at the central and left lateral regions. Power at 16 Hz (dB): The central region, particularly the fronto-central area, exhibits a pronounced decrease in power, surrounded by areas of higher power. Power at 23 Hz (dB): A mixed pattern with some spots of increased power in the central and left lateral regions and decreased power in the fronto-central region. Power at 30 Hz (dB): A pronounced central region with decreased power and elevated power at the peripheral regions. Phase Topographies:

Phase at 2 Hz: A fairly uniform phase distribution with slight phase reversals at the left and right lateral regions. Phase at 9 Hz: Strong phase reversals are observed in the central and frontal regions. Phase at 16 Hz: The frontal and central regions exhibit pronounced phase reversals. Phase at 23 Hz: There's a strong phase reversal in the centro-parietal region. Phase at 30 Hz: The pattern is more complex, with multiple regions showing phase reversals, particularly in the central, parietal, and frontal regions.

Are there any prominent topographical features in power or in phase?

Yes, the topographical maps show distinct patterns for both power and phase across various scalp regions.

Do these differ for different frequencies?

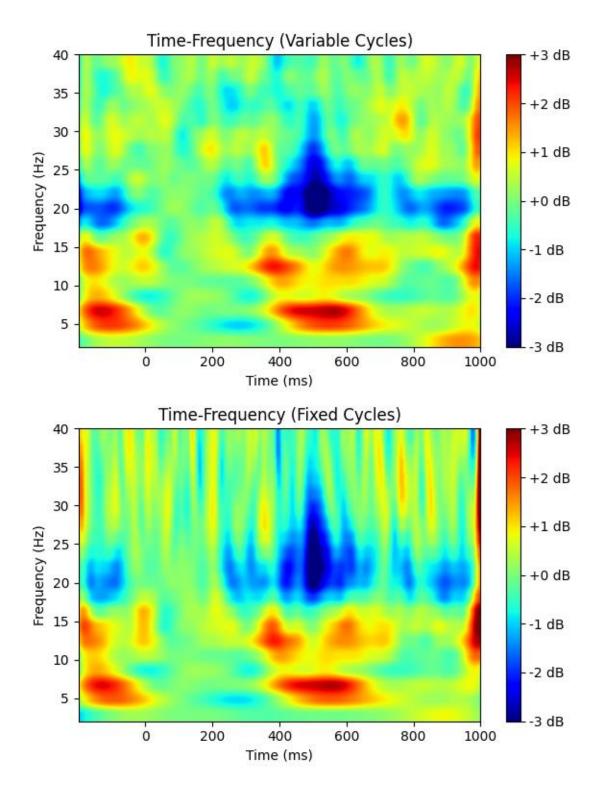
Yes, the topographical patterns vary across different frequencies. For example, at 2 Hz, power is elevated across the entire scalp, while at 30 Hz, central regions show decreased power. Phase distributions also differ, with distinct phase reversals seen at different frequencies.

Do power and phase have similar topographical distributions?

Not entirely. While some regions show overlapping features in power and phase, distinct differences are evident in various frequencies and scalp regions. both power and phase maps exhibit distinct topographical patterns at different frequencies. The power maps tend to have more localized features of high or low power, while the phase maps demonstrate several regions of phase reversals.

Is there any reason to suspect that they might have similar or different topographies?

Yes, power and phase often represent different neural dynamics and processes, which can lead to varied topographical distributions. Power typically captures the amplitude of oscillatory activity, while phase reflects the timing or synchrony of neural events. Given these inherent differences, one would expect them to exhibit distinct topographical patterns.



g) comparison:

Variable cycles provide a more adaptive and arguably intuitive view of the data, especially when interested in a broad range of frequencies. In contrast, fixed cycles offer a consistent view across frequencies, which can be useful when one desires uniform time and frequency resolution.

1. Time-Frequency Distribution:

Variable Cycles:

The time-frequency representation appears smoother, with broader frequency bands and more gradation between areas of activation. A prominent activity is seen around 25 Hz and between 400-600 ms, appearing as a dark blue patch indicating high power. Additional areas of increased power are seen around 10-15 Hz spanning various time points.

Fixed Cycles:

The representation is more striated, with pronounced vertical stripes throughout. This suggests that there's consistent power across multiple frequencies at specific time points. Like the variable cycles, there's a strong activity around 25 Hz and 400-600 ms, but the contour appears sharper. Elevated power around 10-15 Hz is also evident but appears more intermittent due to the vertical striations.

2. Resolution & Precision:

Variable Cycles:

Offers a more adaptable representation where the width of the wavelet (and thus time and frequency resolution) changes depending on the frequency. This allows for a more precise view at lower frequencies and a broader view at higher frequencies.

Fixed Cycles:

The fixed number of cycles across all frequencies can lead to a compromise between time and frequency resolution. This results in the prominent vertical patterns seen in the heat map due to this fixed-width wavelet convolution at all frequencies.

3. Clarity & Interpretation:

Variable Cycles:

The smoother gradient makes it more intuitive to interpret regions of high and low activity and how they evolve.

Fixed Cycles:

The striated pattern can make it slightly harder to pinpoint exact regions of activity. However, the sharp contours can also be beneficial when trying to identify the exact timings of specific events.

```
# a.
def morlet wavelet(frequency, num cycles, sampling rate, duration=2):
    t = np.linspace(-duration/2, duration/2, int(sampling rate * duration),
endpoint=False)
    sine wave = np.exp(2j * np.pi * frequency * t)
    amplitude envelope = np.exp(-t**2 * (np.pi * frequency / num cycles)**2)
    wavelet = sine wave * amplitude envelope
    return wavelet
# Wavelet parameters
frequencies = [2, 9, 16, 23, 30]
n cycles = 4
sampling freq = 256
time window = 2
wavelets = [] # Empty list to store wavelets
# Plotting
fig, axs = plt.subplots(len(frequencies), 1, figsize=(6, 10), sharex=True)
for i, freq in enumerate(frequencies):
    wavelet = morlet_wavelet(freq, n_cycles, sampling_freq, time_window)
    wavelets.append(wavelet) # Append wavelet to the list
    axs[i].plot(np.real(wavelet), label='Real Part')
    axs[i].plot(np.imag(wavelet), label='Imaginary Part', linestyle='--')
    axs[i].set title(f"Morlet Wavelet at {freq} Hz")
    axs[i].legend()
plt.xlabel("Time (samples)")
fig.suptitle('Morlet Wavelets at Different Frequencies', fontsize=16)
plt.tight layout()
plt.show()
b) Convolve each wavelet with EEG data from all electrodes and from only the first trial
                                                                           In [18]:
# b.
# We already have wavelets and their corresponding time vector 't'
convolved data = np.zeros like(First trial eeg) # To store the convolved
data
t = np.linspace(-time window/2, time window/2, int(sampling freq *
time window), endpoint=False)
# Length of the convolved data
len convolved = First trial eeg.shape[1] + len(t) - 1
print('len convolved:', len convolved) #len convolved: 1151
for i, wavelet in enumerate(wavelets):
    # FFT of the wavelet
    wavelet fft = fft(wavelet, n=len convolved)
    # Convolve with each channel
```

```
# FFT of the EEG data
       eeg fft = fft(First trial eeg[ch, :], n=len convolved)
        # Multiply in frequency domain and then IFFT
       conv result = ifft(wavelet fft * eeg fft)
        # Cut the convolved data to original EEG data length
       cut length = len(t) // 2
        convolved data[ch, :] = np.real(conv result[cut length:cut length +
First trial eeg.shape[1]])
# convolved data now contains the result of the convolution of each wavelet
with each channel's data
print('convolved data.shape:', convolved data.shape) # convolved_data.shape:
(64, 640)
len convolved: 1151
convolved data.shape: (64, 640)
                                                                         In [20]:
convolved data
                                                                        Out[20]:
array([[-16.947456 , -23.503107 , -17.0633 , ..., -26.82681
       -30.02092 , -17.705837 ],
       [-20.31652]
                   , -22.39365 , -10.985493 , ..., 0.3493317 ,
        -0.21125636, -0.7934704],
       [-33.983974 , -27.592262 , -4.465775 , ..., -27.515846 ,
       -49.51342 , -43.288696 ],
       [-19.17071 , -7.238967 , 10.825006 , ..., -13.468891 ,
       -30.038723 , -28.459368 ],
       [-20.99644 , -7.128301 , 12.264906 , ..., 20.106596 ,
         1.5189717 , -15.421003 ],
       [-27.2869]
                   , -21.165905 , -0.8333369 , ..., -31.996515 ,
        -49.65603 , -38.924156 ]], dtype=float32)
c) Extract power and phase from the result of the complex wavelet convolution and store in a matrix
                                                                         In [23]:
# C.
# Initialize the 4D array to store power and phase results
results matrix = np.zeros((convolved data.shape[1], len(frequencies),
convolved data.shape[0], 2))
for f_idx, wavelet in enumerate(wavelets):
    wavelet length = len(wavelet)
    # Calculate the length of the convolved data
    n convolution = 640 + wavelet length
    # FFT of the wavelet
    wavelet fft = np.fft.fft(wavelet, n=n convolution)
    for ch idx in range(64):
```

for ch in range(First trial eeg.shape[0]):

```
# EEG signal for this channel
        eeg signal = First trial eeg[ch idx, :]
        # FFT of the EEG signal
        eeg fft = np.fft.fft(eeg signal, n=n convolution)
        # Convolution in the frequency domain
        conv result fft = wavelet fft * eeg fft
        # Inverse FFT to return to the time domain
        conv result = np.fft.ifft(conv result fft)
        # Correcting the length of the convolved data
        # Ensure integer division and proper trimming
       half wavelet = int(np.ceil((wavelet length - 1) / 2))
        conv result = conv result[half wavelet:-half wavelet or None]
        # Check and adjust the length if necessary
        if conv result.size != 640:
            raise ValueError (f"Unexpected size of convolved result:
{conv result.size}")
        # Calculate power and phase
        power = np.abs(conv result) ** 2
       phase = np.angle(conv result)
        # Store in the matrix
        results matrix[:, f idx, ch idx, 0] = power
        results matrix[:, f idx, ch idx, 1] = phase
print('results matrix.shape:', results matrix.shape)
results matrix.shape: (640, 5, 64, 2)
                                                                         In [24]:
# Shape of the entire results matrix
print("Shape of the results matrix:", results matrix.shape)
# To check the shape of power and phase for a specific frequency and
electrode
f idx = 0 # For example, the first frequency
ch idx = 0 # For example, the first electrode
# Extracting power and phase for this specific frequency and electrode
power example = results matrix[:, f idx, ch idx, 0]
phase example = results matrix[:, f idx, ch idx, 1]
print("Shape of power for frequency index", f idx, "and electrode index",
ch idx, ":", power example.shape)
print("Shape of phase for frequency index", f idx, "and electrode index",
ch idx, ":", phase example.shape)
Shape of the results matrix: (640, 5, 64, 2)
Shape of power for frequency index 0 and electrode index 0: (640,)
Shape of phase for frequency index 0 and electrode index 0: (640,)
```

```
In [27]:
```

```
# d.
# Find the index for 180 ms
time idx = np.argmin(np.abs(times - 180)) #302
# Define frequencies
frequencies = [2, 9, 16, 23, 30] # Adjust according to your frequencies
# Calculate the global minimum and maximum for power (in dB) and phase
power min = 10 * np.log10(results matrix[:, :, :, 0]).min()
power max = 10 * np.log10(results_matrix[:, :, :, 0]).max()
phase min = results matrix[:, :, :, 1].min()
phase_max = results_matrix[:, :, :, 1].max()
# Set up figure
fig, axs = plt.subplots(2, len(frequencies), figsize=(20, 6)) # Adjust the
size as needed
# Loop over frequencies to plot power and phase
for i, freq in enumerate(frequencies):
    # Power (applying 10log10 and normalizing)
    power data = 10 * np.log10(results matrix[time idx, i, :, 0])
    im, = mne.viz.plot topomap(power data, info, axes=axs[0, i],
show=False, cmap='jet')
    axs[0, i].set title(f'Power at {freq} Hz (dB)')
    # Phase (normalizing)
    phase data = results matrix[time idx, i, :, 1]
    im, = mne.viz.plot topomap(phase data, info, axes=axs[1, i],
show=False, cmap='jet')
    axs[1, i].set title(f'Phase at {freq} Hz')
# Adding a common colorbar for power
cax1 = fig.add axes([0.92, 0.55, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=power min, vmax=power max)
cb1 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax1,
orientation='vertical')
cb1.set label('Power (dB)')
# Adding a common colorbar for phase
cax2 = fig.add axes([0.92, 0.15, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=phase min, vmax=phase max)
cb2 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax2,
orientation='vertical')
cb2.set label('Phase')
plt.tight layout(rect=[0, 0, 0.9, 1])
plt.show()
```

```
C:\Users\Maryam\AppData\Local\Temp\ipykernel_3936\1682916526.py:42:
UserWarning: This figure includes Axes that are not compatible with
tight_layout, so results might be incorrect.
  plt.tight layout(rect=[0, 0, 0.9, 1])
```

e) Repeat step (d) for activity at 360 ms, and 650 ms.

```
In [28]:
# e. 360ms
# Find the index for 360ms
time idx = np.argmin(np.abs(times - 360))
# Define frequencies
frequencies = [2, 9, 16, 23, 30] # Adjust according to your frequencies
# Calculate the global minimum and maximum for power (in dB) and phase
power min = 10 * np.log10 (results matrix[:, :, :, 0]).min()
power_max = 10 * np.log10(results_matrix[:, :, :, 0]).max()
phase min = results matrix[:, :, :, 1].min()
phase max = results matrix[:, :, :, 1].max()
# Set up figure
fig, axs = plt.subplots(2, len(frequencies), figsize=(20, 6)) # Adjust the
size as needed
# Loop over frequencies to plot power and phase
for i, freq in enumerate(frequencies):
    # Power (applying 10log10 and normalizing)
    power data = 10 * np.log10(results matrix[time idx, i, :, 0])
    im, = mne.viz.plot topomap(power data, info, axes=axs[0, i],
show=False, cmap='jet')
    axs[0, i].set title(f'Power at {freq} Hz (dB)')
    # Phase (normalizing)
    phase data = results matrix[time_idx, i, :, 1]
    im, _ = mne.viz.plot_topomap(phase_data, info, axes=axs[1, i],
show=False, cmap='jet')
    axs[1, i].set title(f'Phase at {freq} Hz')
# Adding a common colorbar for power
cax1 = fig.add axes([0.92, 0.55, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=0, vmax=50)
cb1 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax1,
orientation='vertical')
cb1.set label('Power (dB)')
# Adding a common colorbar for phase
cax2 = fig.add axes([0.92, 0.15, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=-3, vmax=3)
```

```
cb2 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax2,
orientation='vertical')
cb2.set label('Phase')
plt.tight layout(rect=[0, 0, 0.9, 1])
plt.show()
C:\Users\Maryam\AppData\Local\Temp\ipykernel 3936\3697760241.py:42:
UserWarning: This figure includes Axes that are not compatible with
tight layout, so results might be incorrect.
 plt.tight layout(rect=[0, 0, 0.9, 1])
                                                                          In [29]:
# e. 650
# Find the index for 650 ms
time idx = np.argmin(np.abs(times - 650))
# Define frequencies
frequencies = [2, 9, 16, 23, 30] # Adjust according to your frequencies
# Calculate the global minimum and maximum for power (in dB) and phase
power min = 10 * np.log10 (results matrix[:, :, :, 0]).min()
power max = 10 * np.log10 (results matrix[:, :, :, 0]).max()
phase_min = results_matrix[:, :, :, 1].min()
phase max = results matrix[:, :, :, 1].max()
# Set up figure
fig, axs = plt.subplots(2, len(frequencies), figsize=(20, 6)) # Adjust the
size as needed
# Loop over frequencies to plot power and phase
for i, freq in enumerate(frequencies):
    # Power (applying 10log10 and normalizing)
    power data = 10 * np.log10(results matrix[time idx, i, :, 0])
    im, _ = mne.viz.plot_topomap(power_data, info, axes=axs[0, i],
show=False, cmap='jet')
    axs[0, i].set title(f'Power at {freq} Hz (dB)')
    # Phase (normalizing)
   phase data = results matrix[time idx, i, :, 1]
    im, = mne.viz.plot topomap(phase data, info, axes=axs[1, i],
show=False, cmap='jet')
    axs[1, i].set_title(f'Phase at {freq} Hz')
# Adding a common colorbar for power
cax1 = fig.add axes([0.92, 0.55, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=0, vmax=50)
cb1 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax1,
orientation='vertical')
cb1.set label('Power (dB)')
```

```
# Adding a common colorbar for phase
cax2 = fig.add axes([0.92, 0.15, 0.01, 0.3]) # Adjust these values as needed
for positioning
norm = Normalize(vmin=-3, vmax=3)
cb2 = plt.colorbar(ScalarMappable(norm=norm, cmap='jet'), cax=cax2,
orientation='vertical')
cb2.set label('Phase')
plt.tight layout(rect=[0, 0, 0.9, 1])
plt.show()
C:\Users\Maryam\AppData\Local\Temp\ipykernel 3936\619713156.py:42:
UserWarning: This figure includes Axes that are not compatible with
tight layout, so results might be incorrect.
  plt.tight layout(rect=[0, 0, 0.9, 1])
f)
                                                                          In [35]:
#q.
def convolve with wavelet(data, wavelet):
    n signal = data.shape[1]
    n wavelet = len(wavelet)
    n_convolution = n_signal + n_wavelet - 1
    data fft = np.fft.fft(data, n convolution, axis=1)
    wavelet fft = np.fft.fft(wavelet, n convolution)[np.newaxis, :]
    convolution result fft = data fft * wavelet fft
    convolution result = np.fft.ifft(convolution result fft, axis=1)
    # Cut the data
    start = (n \text{ wavelet } -1) // 2
    end = start + n signal
    return convolution result[:, start:end]
# Function to calculate power for variable or fixed cycles
def calculate power(cycles):
    power matrix = []
    for freq, cycle in zip(frequencies, cycles):
        # Extract data for 'FCz'
        data fcz = eeg data[fcz index, :, :] # Assuming trials are on the
second dimension
        # Calculate power for each trial
        power trials = []
        for trial idx in range(data fcz.shape[1]):
            trial 2D = data fcz[:, trial idx].reshape(1, -1) # Making trial
2D for the convolution function
            wavelet = morlet wavelet(freq, cycle, sampling rate)
            convolution = convolve with wavelet(trial 2D, wavelet)
```

```
power = np.abs(convolution)**2
            power trials.append(power[0])
        # Average power across trials
        avg power = np.mean(power trials, axis=0)
        power matrix.append(avg power)
    return np.array(power matrix)
# Baseline correction function
def baseline correction(power matrix, times, baseline range):
   baseline timepoints = np.logical and(times >= baseline range[0], times <=
baseline range[1])
    baseline mean = np.mean(power matrix[:, baseline timepoints], axis=1,
keepdims=True)
    return power matrix / baseline mean # Division for baseline correction
# Constants
baseline range = (-500, -200) # in milliseconds
frequencies = np.arange(2, 41, 2) \# 2 to 40 Hz
cycles = np.linspace(3, 10, len(frequencies)) # Variable cycles from 3 to 10
sampling rate=256
# Find the channel index for 'FCz'
fcz index = raw.ch names.index('Fcz')
# Calculate power for variable and fixed cycles
power variable cycles = calculate power(cycles)
power_fixed_cycles = calculate power([4] * len(frequencies)) # Fixed cycle:
# Apply baseline correction
corrected variable = baseline correction (power variable cycles, times,
baseline range)
corrected fixed = baseline correction(power fixed cycles, times,
baseline range)
# Plotting function
def plot time frequency(data, title):
   plt.figure(figsize=(6, 4))
    plt.imshow(10 * np.log10(data), aspect='auto', cmap='jet',
origin='lower',
               extent=[times[205], times[512], frequencies[0], frequencies[-
111,
               vmin=-3, vmax=3) # Log transform of power; color limits as
specified
    plt.colorbar(format='%+2.0f dB')
   plt.xlabel('Time (ms)')
   plt.ylabel('Frequency (Hz)')
   plt.title(title)
    plt.tight_layout()
   plt.show()
# Plot the results
```

plot_time_frequency(corrected_variable, 'Time-Frequency (Variable Cycles)')
plot_time_frequency(corrected_fixed, 'Time-Frequency (Fixed Cycles)')