

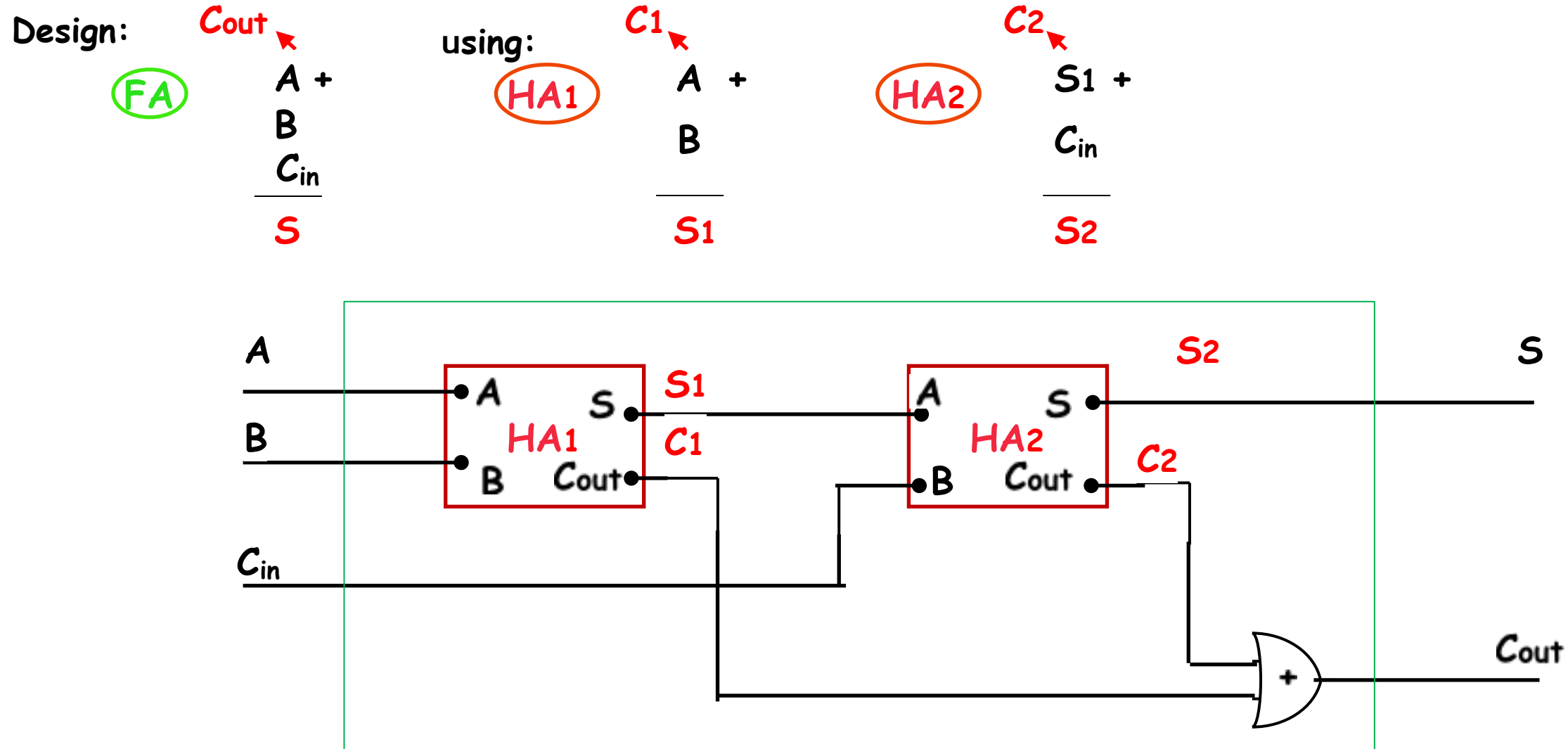
CL=CSCI 160

CLASS 18

HW 17.4

Construct a FA using only HA's and one other gate.

Solution



Note: We would think we need to add C_1 and C_2 . Do we? Not if they may not be both =1. Let's see:

Suppose $C_1 = 1 \rightarrow \begin{cases} A=1 \\ \& \\ B=1 \end{cases} \rightarrow S_1 = 0 \rightarrow C_2 = 0$ So: C_1 and C_2 may not be both 1 \rightarrow use OR gate.

We can also prove the diagram is correct by substituting the functions in the diagram:

HA: $S = A'B + AB'$
 $C = AB$

HA1: $S_1 = A'B + AB'$; $C_1 = AB$

HA2: $S_2 = S_1' C_{in} + S_1 C_{in}' = (A'B + AB')' C_{in} + (A'B + AB') C_{in}' = \dots$

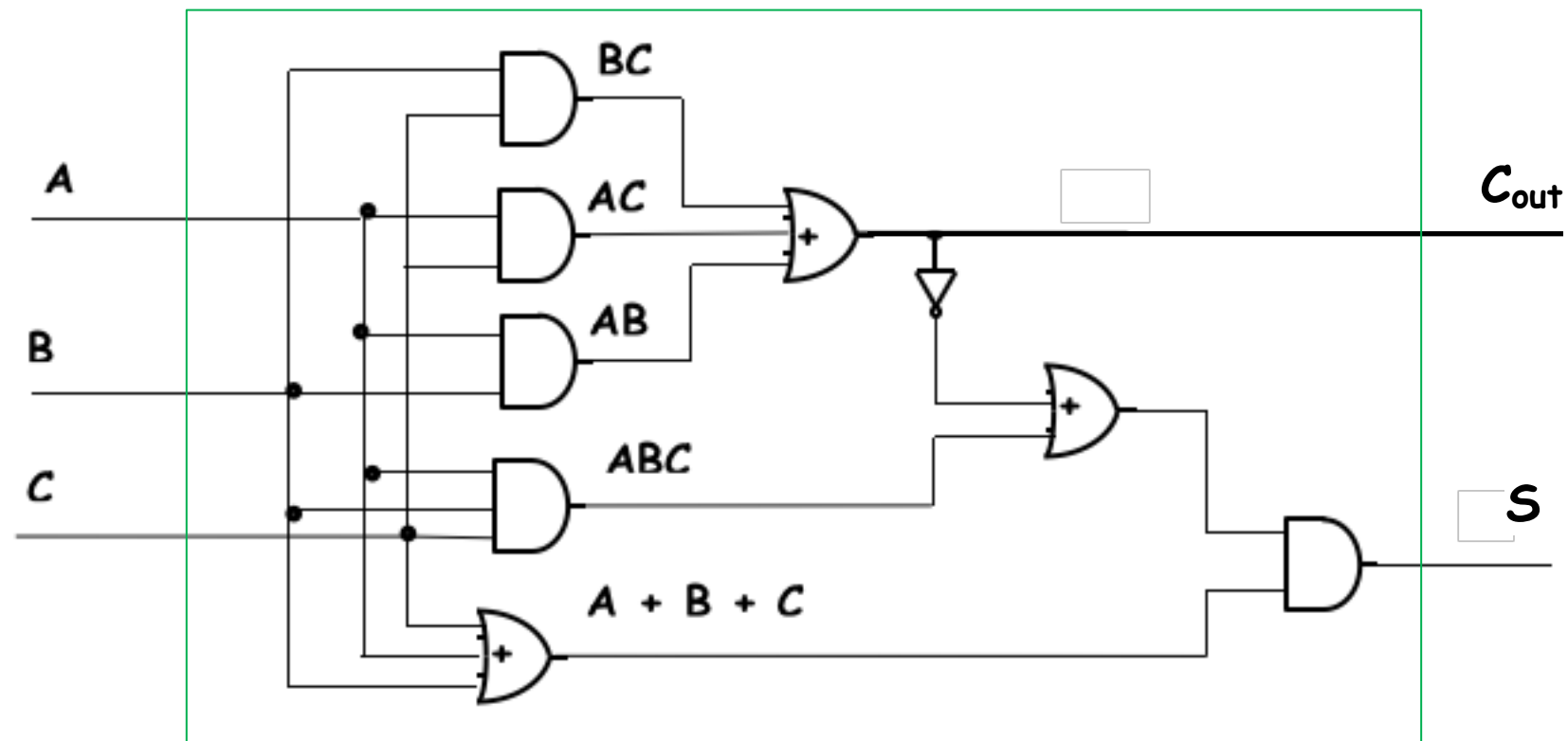
Prove we get:

FA $S = A'B'C + A'BC' + AB'C' + ABC$
 $C = AB + AC + BC$

HW 18.1 - assigned

Continue this proof. Show $S = S_2$ and $C_{out} = C_1 + C_2$

IBM FA:



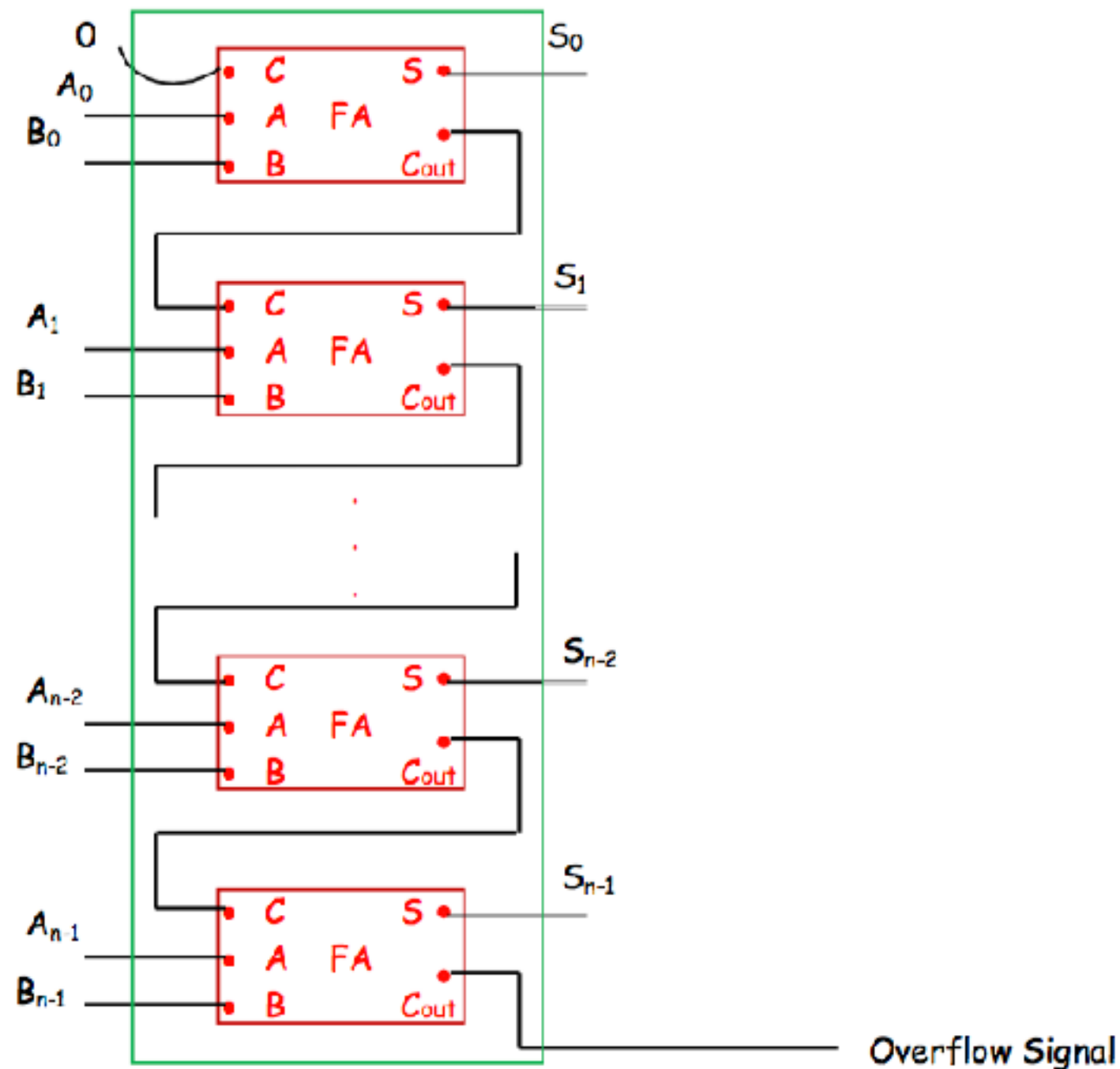
HW 18.2 - assigned

Prove it is indeed a FA, i.e. it creates the functions S , C_{out} of a FA.

Adding multiple digit numbers

Suppose we have two n -digit binary numbers: $A = A_{n-1} A_{n-2} \dots A_1 A_0$ and $B = B_{n-1} B_{n-2} \dots B_1 B_0$

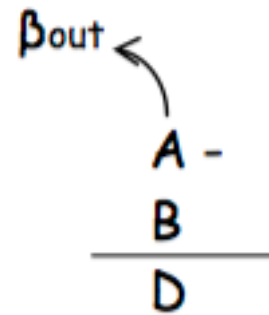
We obtain their sum $S = S_{n-1} S_{n-2} \dots S_1 S_0$ using binary FAs, by adding them bit by bit starting with the lsd's:



Half-Subtractor and Full-Subtractor

HS:

Like HA, it has
2 inputs and 2 outputs.



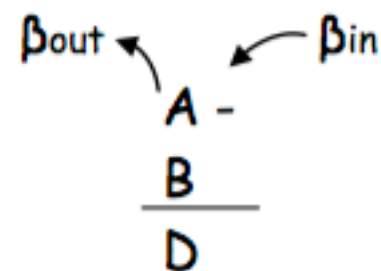
A	B	D	β
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = A'B + AB'$$

$$\beta = A'B$$

FS:

Like FA, it has
3 inputs and 2 outputs.



A	B	β_{in}	D	β_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

HW 18.3 - assigned

Finish and minimize D, β_{out} for FS.

HW 18.4 - assigned

Construct a FS using only HS's and one other gate.

COMPARATOR

We compare two 3-bit binary numbers:

$$A = A_2 A_1 A_0$$

$$B = B_2 B_1 B_0$$

We define the functions:

$$f_{=} = 1 \longleftrightarrow A = B$$

$$f_{=} = \overbrace{(A_2 B_2 + A'_2 B'_2)}^{A_2=1 \& B_2=1} \underbrace{(A_1 B_1 + A'_1 B'_1)}_{A_1=B_1} \underbrace{(A_0 B_0 + A'_0 B'_0)}_{A_0=B_0}$$

$\underbrace{\hspace{10em}}_{A_2=B_2}$

$$f_{<} = 1 \longleftrightarrow A < B$$

$$f_{<} = \underbrace{A'_2 B_2}_{A_2 < B_2} \underbrace{+ (A_2 B_2 + A'_2 B'_2)}_{A_2=B_2} \underbrace{(A'_1 B_1)}_{A_1 < B_1} \underbrace{+ (A_1 B_1 + A'_1 B'_1)}_{A_1=B_1} \underbrace{A'_0 B_0}_{A_0 < B_0}$$

HW 18.5 - assigned

Express the function:

$$f_{>} = 1 \longleftrightarrow A > B$$