Half Full Subtractor

CLASS 16

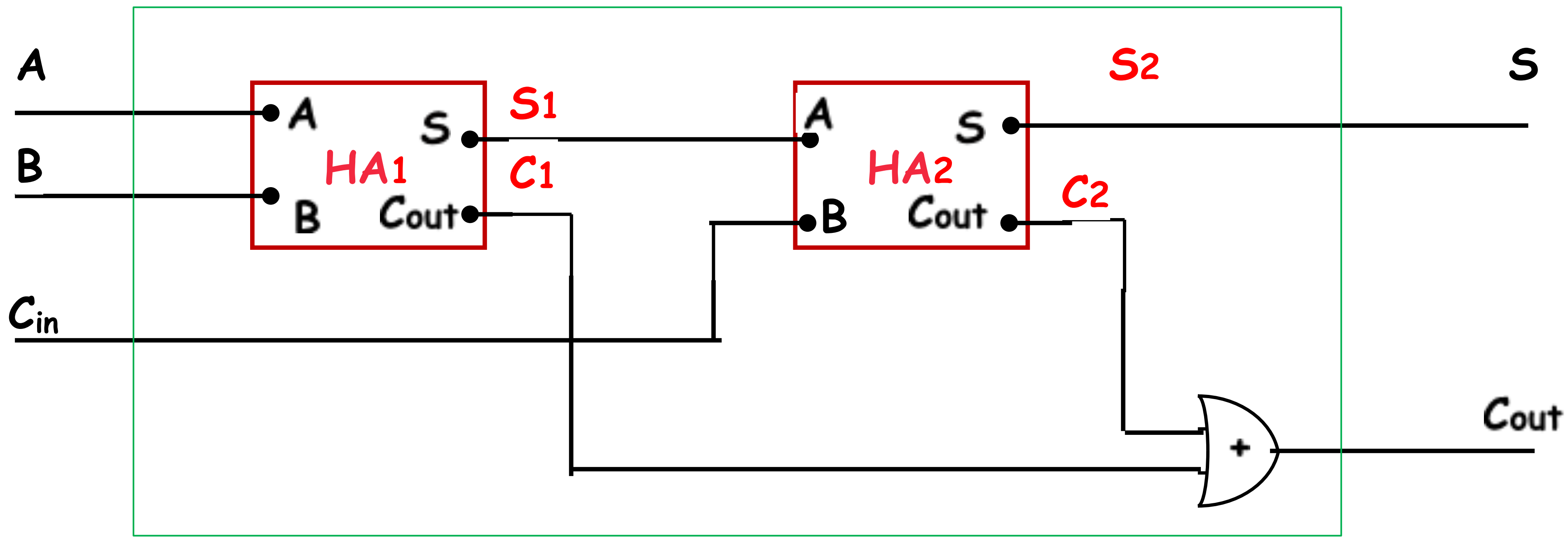
HW 17.4 Construct a FA using only HA's and one other gate.

Solution

Design: FA $C_{out} \leftarrow$ $A + B + C_{in}$ = S

using: HA1 $C_1 \leftarrow$ $A + B$ = S_1

HA2 $C_2 \leftarrow$ $S_1 + C_{in}$ = S_2



Note: We would think we need to add C_1 and C_2. Do we? Not if they may not be both =1. Let's see:

Suppose $C_1 = 1$ ---> $\left\{ \begin{matrix} A = 1 \\ \& \\ B = 1 \end{matrix} \right\}$ ---> $S_1 = 0$ ---> $C_2 = 0$ So: C_1 and C_2 may not be both 1 ---> use OR gate.

We can also prove the diagram is correct by substituting the functions in the diagram:

HA: $S = A'B + AB'$
$C = AB$

HA1: $S_1 = A'B + AB'$; $C_1 = AB$

HA2: $S_2 = S'_1 C_{in} + S_1 C_{in}' = (A'B + AB')' C_{in} + (A'B + AB') C_{in}' = \ldots$

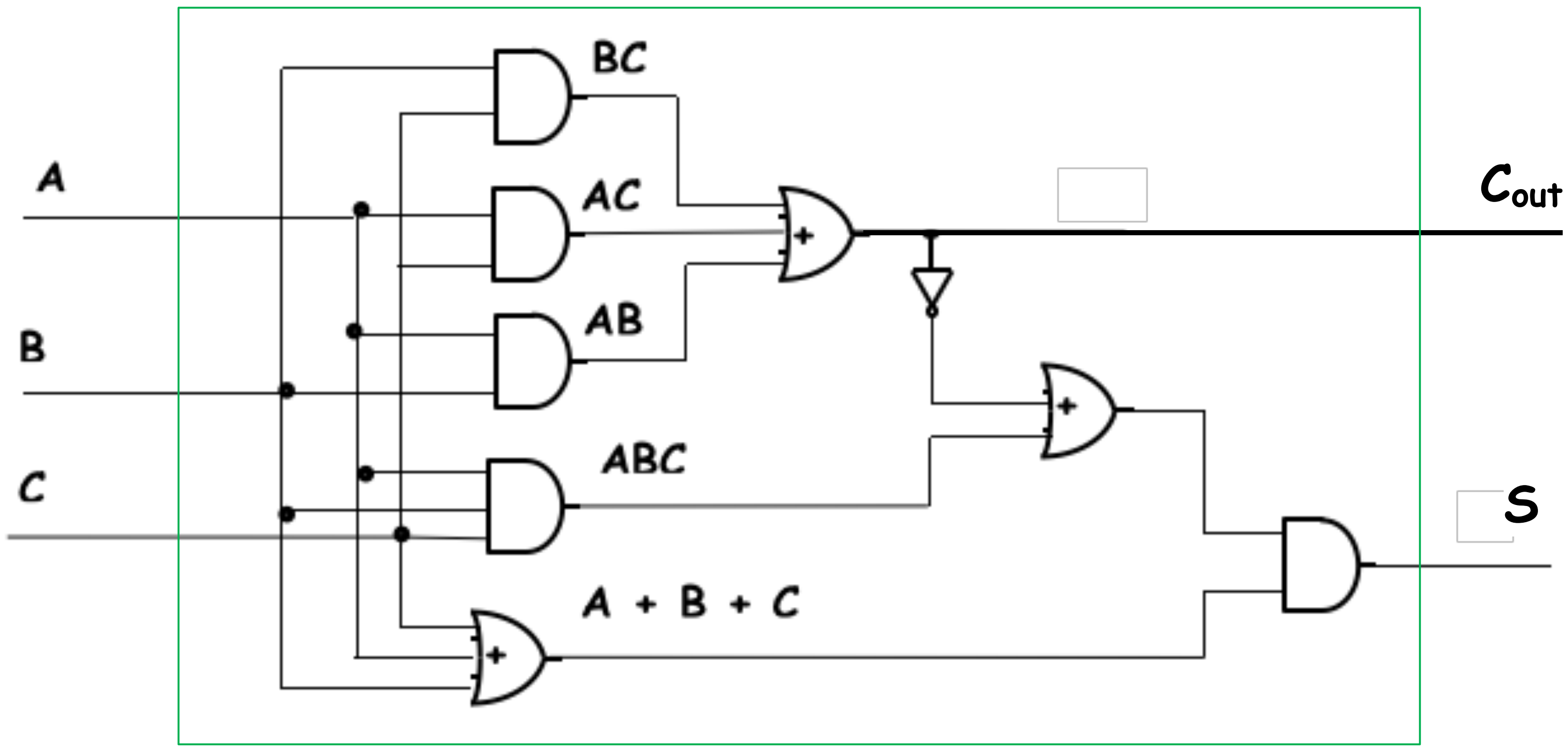
Prove we get:

FA $S = A'B'C + A'BC' + AB'C' + ABC$
$C = AB + AC + BC$

HW 18.1 - assigned

Continue this proof. Show $S = S_2$ and $C_{out} = C_1 + C_2$

IBM FA:



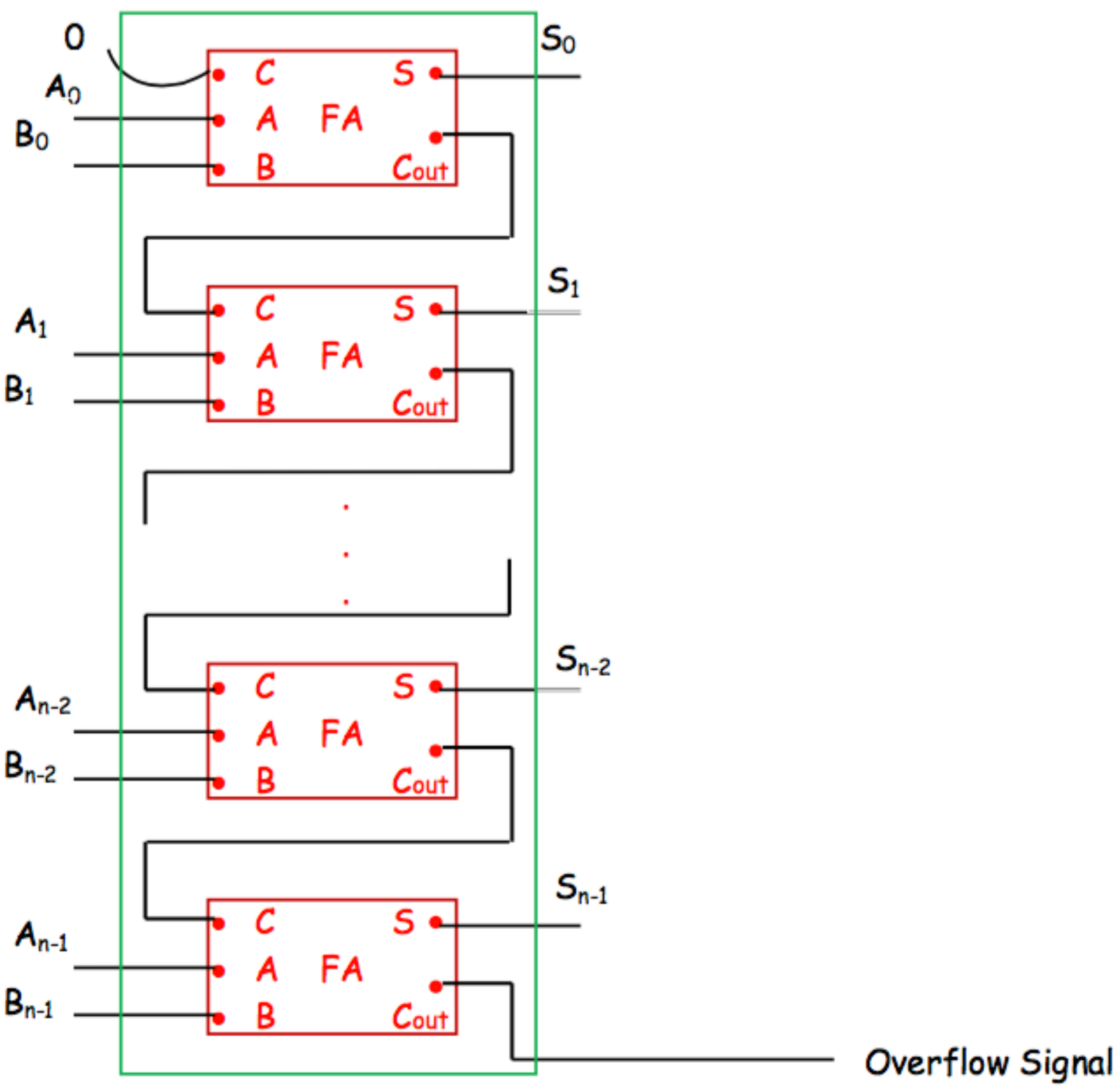
HW 18.2 - assigned

Prove it is indeed a FA, i.e. it creates the functions S, C_{out} of a FA.

Adding multiple digit numbers

Suppose we have two n-digit binary numbers: $A = A_{n-1}\,A_{n-2}\,.\,.\,.\,A_1\,A_0$ **and** $B = B_{n-1}\,B_{n-2}\,.\,.\,.\,B_1\,B_0$

We obtain their sum $S = S_{n-1}\,S_{n-2}\,.\,.\,.\,S_1\,S_0$ using binary FAs, by adding them bit by bit starting with the lsd's:



Half-Subtractor and Full-Subtractor

HS:

Like HA, it has
2 inputs and 2 outputs.

$$\begin{array}{r} \beta_{out} \\ A\ - \\ \underline{B} \\ D \end{array}$$

A	B	D	β
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$D = A'B + AB'$

$\beta = A'B$

FS:

Like FA, it has
3 inputs and 2 outputs.

$$\begin{array}{rl} \beta_{out} & \beta_{in} \\ A\ - & \\ \underline{B} & \\ D & \end{array}$$

A	B	β_{in}	D	β_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

HW 18.3 - assigned

Finish and minimize D, β_{out} for FS.

HW 18.4 - assigned

Construct a FS using only HS's and one other gate.

COMPARATOR

We compare two 3-bit binary numbers: $A = A_2 A_1 A_0$

$B = B_2 B_1 B_0$

We define the functions:

$$\boxed{f_= = 1 \longleftrightarrow A = B}$$

$$f_= = \underbrace{(\overbrace{A_2B_2}^{A_2=1\ \&\ B_2=1} + \underbrace{A'_2B'_2}_{A_2=0\ \&\ B_2=0})}_{A_2 = B_2}\ \underbrace{(A_1B_1 + A'_1B'_1)}_{A_1 = B_1}\ \underbrace{(A_0B_0 + A'_0B'_0)}_{A_0 = B_0}$$

$$\boxed{f_< = 1 \longleftrightarrow A < B}$$

$$f_< = \underbrace{\overbrace{A'_2B_2}^{A_2=0\ \&\ B_2=1}}_{A_2 < B_2} + \underbrace{(A_2B_2 + A'_2B'_2)}_{A_2 = B_2}(\underbrace{A'_1B_1}_{A_1 < B_1} + \underbrace{(A_1B_1 + A'_1B'_1)}_{A_1 = B_1}\ \underbrace{A'_0B_0}_{A_0 < B_0})$$

HW 18.5 - assigned

Express the function: $\boxed{f_> = 1 \longleftrightarrow A > B}$