

Prime Implicant Table

Half Full Adder

CLASS 15

QUIZ 2.2

I Design a majority circuit with three inputs in the following way:

1) Write the truth table of a function, say $F_3(x,y,z)$, where the output agrees with the majority of the input, that is $F_3(x,y,z) = 1$ if and only if at least two out of the three variables: x,y,z have the value 1.

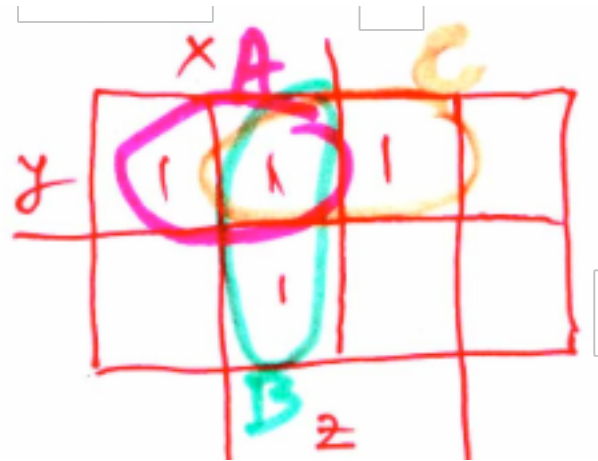
Solution

2) Write the expression of the function $F_3(x,y,z)$ and minimize it.

1)

x	y	z	F_3
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

2) $F_3 = x'yz + xy'z + xyz' + xyz$



All implicants are essential.

$$F_3 = A + B + C = xy + xz + yz$$

Note: From the expression of the minimal form we can see that we take any 2 out of the 3 variables to form majority

QUIZ 2.2 (cont)

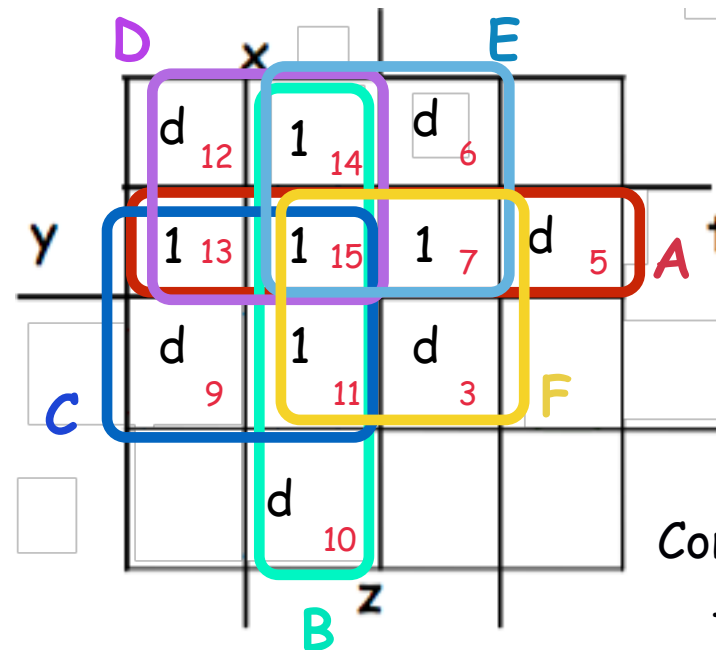
II Next, design a majority circuit function, say $F_4(x,y,z, t)$, with four inputs. Handle ties in a way that is 'optimal' from the point of view of the minimization of F_4 . Write the truth table of F_4 . Continue this task as HW.

Solution (Q2.2+HW)

What do we do with the ties? 'tie' is not an acceptable value for a function!

What do we care about?

To minimize F_4 , and not who wins in a tie! → Use d, to our advantage.



Size-4 implicants: A - F

None essential

Consider a 1 that is least covered:

7, 11, 13, 14; say, 7:

7 is covered by A, E, F

Case I: Take A $F_4 = A + B = yt + xz$

Case II: Take E $F_4 = E + C = yz + xt$

Case III: Take F $F_4 = F + D = zt + xy$

← 3 Minimal forms

Note: From the expression of the minimal form we can see that we take any 2 out of the 4 variables and the remaining 2 to form at least a tie.

How would the majority circuit look for 5 variables?

$$F_5(x_1, x_2, x_3, x_4, x_5) = 1 \Leftrightarrow \text{at least 3 out of the 5 variables are 1.}$$

$$\binom{5}{3} = \binom{5}{2} = \frac{5 \cdot 4}{1 \cdot 2} = \underline{10}, \text{ terms:}$$

$$F_5 = x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_2 x_5 + x_1 x_3 x_4 + x_1 x_3 x_5 + x_1 x_4 x_5 + \\ + x_2 x_3 x_4 + x_2 x_3 x_5 + x_2 x_4 x_5 + x_3 x_4 x_5.$$

We wrote it directly in minimal form

HW 16

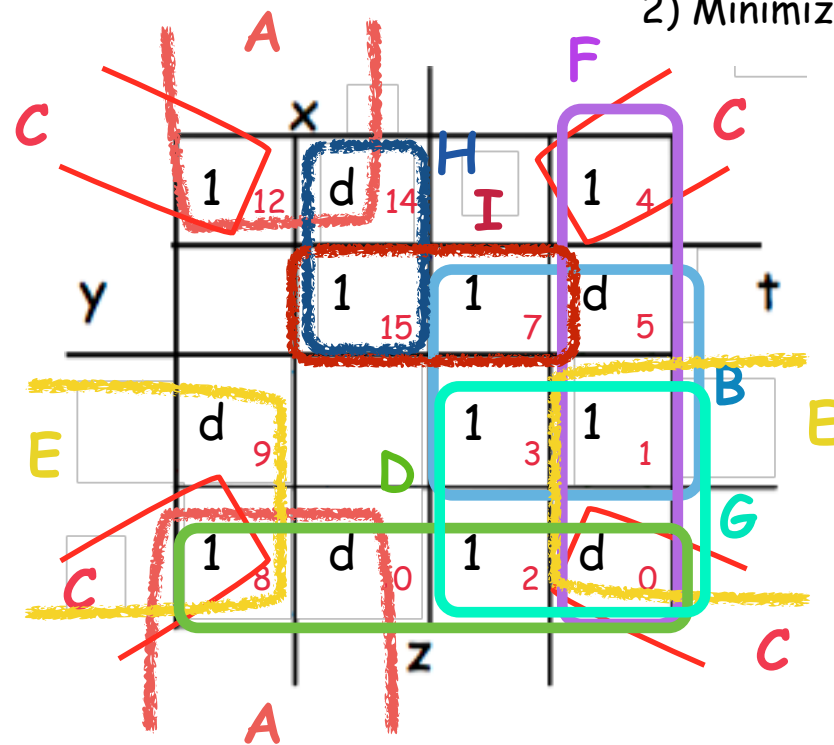
Consider the function; we applied the tabulation method on:

$$f = \Sigma (1, 2, 3, 4, 7, 8, 12, 15) + d \Sigma (0, 5, 9, 10, 14)$$

- 1) Draw the K-map and find all prime implicants, giving them the same labels (letters), A - I, in class, when applying the tabulation method.

- 2) Minimize f.

Solution



We have from tabulation method (class 16):

- 1 1 1	(7, 15)	I
1 1 1 -	(14, 15)	H

Essential: none

Index	Impl. Binary	Impl. Dec.	
0	0 0 - -	(0, 1, 2, 3)	G
	0 - 0 -	(0, 1, 4, 5)	F
	- 0 0 -	(0, 1, 8, 9)	E
	- 0 - 0	(0, 2, 8, 10)	D
	- - 0 0	(0, 4, 8, 12)	C
1	0 - - 1	(1, 3, 5, 7)	B
	1 - - 0	(8, 10, 12, 14)	A

Least covered 1's: 2, 3, 4, 7, 12, 15: all covered by exactly two implicants. Choose one of them: 2, covered by D, G.

Every minimal form will contain D or G. **Case I:** Take D → 2 and 8 are covered.

To cover the rest we need one size-2 implicant (for 15) and 2 x size-4 implicants:

$$f = D + B + C + \begin{cases} H \\ I \end{cases} \quad \text{or} \quad f = D + G + C + I \quad \leftarrow \text{NOT Minimal forms}$$

(note: it's redundant, as we don't need D; if you don't note it, go to Case II below)

Case II: Take G → 1, 2, 3 are covered. 15 again may be covered only by a size-2 implicants: H, I.

However, if we cover 15 by implicant I, then the remaining 1's may be covered by just one size-4 implicant.

$$f = G + C + I \quad \leftarrow \text{Minimal form} \quad \text{which means it's the only one!}$$

$$f = x'y' + z't' + yzt$$

← only Minimal form as expression

HW 17.1 - assigned

A.

Simplify the following Boolean function F together with the don't-care conditions d ; then express the simplified function.

(a) $F(x, y, z) = \Sigma (0, 1, 2, 4, 5)$

$d(x, y, z) = \Sigma (3, 6, 7)$

(b) $F(A, B, C, D) = \Sigma (0, 6, 8, 13, 14)$

$d(A, B, C, D) = \Sigma (2, 4, 10)$

B.

A logic circuit implements the following Boolean function:

$$F = A'C + AC'D'$$

It is found that the circuit input combination $A = C = 1$ can never occur. Find a simpler expression for F using the proper don't-care conditions.

Prime Implicant Table [it computes a minimized form]

Consider the function; we applied the tabulation method on:
 $f = \Sigma (1, 2, 3, 4, 7, 8, 12, 15) + d \Sigma (0, 5, 9, 10, 14)$

1's (no d's) are the columns; the prime implicants are the rows:

	1	2	3	4	7	8	12	15
A						x	x	
B	x		x		x			
C				x		x	x	
D		x				x		
E	x					x		
F	x			x				
G	x	x	x					
H								x
I					x			x

$1 \supset 3$ $7 \supset 15$
 $8 \supset 12$

C \supset A

I \supset H

- 1 1 1	(7, 15)	I
1 1 1 -	(14, 15)	H

Index	Impl. Binary	Impl. Dec.	
0	0 0 - -	(0, 1, 2, 3)	G F E D C
	0 - 0 -	(0, 1, 4, 5)	
	- 0 0 -	(0, 1, 8, 9)	
	- 0 - 0	(0, 2, 8, 10)	
	- - 0 0	(0, 4, 8, 12)	
1	0 - - 1	(1, 3, 5, 7)	B A
	1 - - 0	(8, 10, 12, 14)	

Procedure:

1) Find essentials; put them in the minimal form and eliminate from table.

2) Remove: Dominated rows.

3) Remove: Dominating columns.

Repeat steps 1) - 3) until a minimized form is obtained

Note: when a 1 has only one x in its column then the implicant corresponding to that x is essential.

Prime Implicant Table

(continued)

We redo the table:

	1	2	3	4	7	8	12	15
A						x	x	
B	x		x		x			
C				x		x	x	
D		x				x		
E	x					x		
F	x			x				
G	x	x	x					
H								x
I					x			x

Procedure:

- 1) Find essentials; put them in the minimal form and eliminate from table.
 - 2) Remove: Dominated rows.
 - 3) Remove: Dominating columns.
- Repeat steps 1) - 3) until a minimized form is obtained

	2	3	4	12	15
B		x			
C			x	x	
D	x				
F			x		
G	x	x			
I					x

1) Essentials: C, I ----> $f = C + I + \dots$

Remove the columns/rows corresponding to the essential implicants

We redo the table:

	2	3
B		x
D	x	
G	x	x

2) Remove: dominated rows

$$f = C + G + I = z't' + x'y' + yzt$$

	- 1 1 1	(7, 15)	I
	1 1 1 -	(14, 15)	H
Index	Impl. Binary	Impl. Dec.	
0	0 0 - -	(0, 1, 2, 3)	G F E D C
	0 - 0 -	(0, 1, 4, 5)	
	- 0 0 -	(0, 1, 8, 9)	
	- 0 - 0	(0, 2, 8, 10)	
	- - 0 0	(0, 4, 8, 12)	

We have the same minimal form as the K-map gave us (HW 16)!

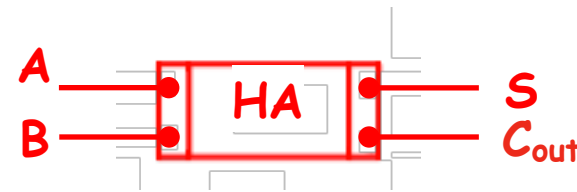
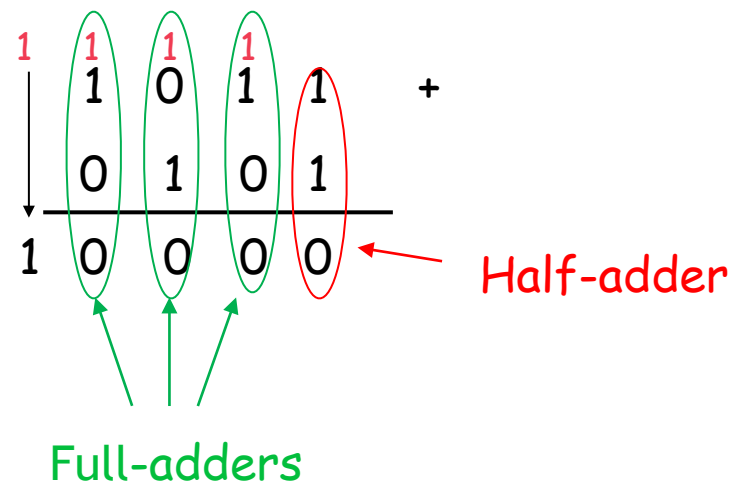
HW 17.2 - assigned

Minimize the function:

$$f = A'B'DE' + E'B'C'D + B'DE' + B'CD + CDE' + BDE'$$

Half-Adder and Full-Adder

Let's add two binary numbers to figure out all steps involved.

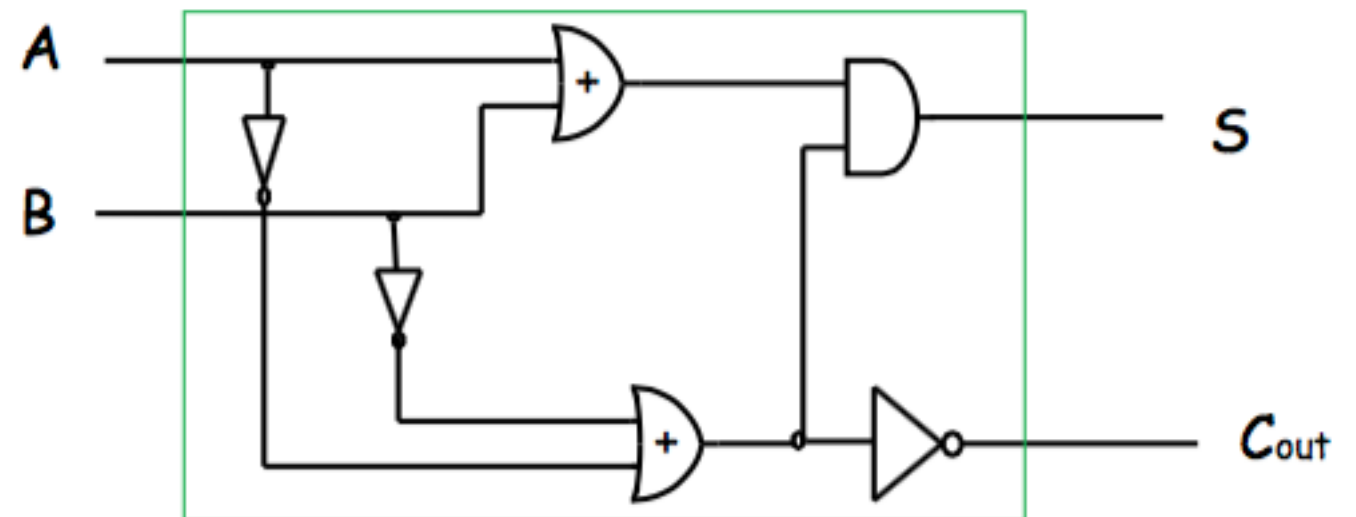


Half-adder:

A	B	S	C _{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1


$$S = A'B + AB' = (A' + B')(A + B)$$


$$C_{out} = AB = (A' + B')'$$



Half-adder with 3 gates

HW 17.3 - assigned

Express H A using only NOR  gates

 is allowed

Full-adder

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = A'B'C + A'BC' + AB'C' + ABC$$

$$C_{out} = A'BC + AB'C + ABC' + ABC$$

	A		
B	1	1	
	1	1	
		C	

$$S = A'B'C + A'BC' + AB'C' + ABC$$

	A		
B	1	1	1
	1		
		C	

$$C_{out} = AB + AC + BC$$

Majority function!

HW 17.4 - assigned: Construct a FA using only HA's and one other gate.