Other Logic Operations

CLASS 8

**GIVE ANSWERS IN THE SPACED INDICATED BELOW.
BE CAREFUL WITH, AND JUSTIFY YOUR ANSWERS!
DO WELL!!!**

Simplify as much as possible the following Boolean expression:

$x'yz + x'yz' + xy + x'y + xyz + y'z =$

Solution

$$x'yz + x'yz' + \underline{xy} + \underline{x'y} + xyz + y'z \overset{(4)}{=} x'yz + x'yz' + (x + x')y + xyz + y'z \overset{(5)}{\underset{(2)}{=}}$$

$$= \underline{x'yz} + \underline{x'yz'} + y + xyz + y'z \overset{(4)}{=} x'y(z + z') + y + xyz + y'z \overset{(5)}{\underset{(2)}{=}}$$

$$= x'y + \underline{y} + \underline{xyz} + y'z \overset{(12)}{=} \underline{x'y} + \underline{y} + y'z \overset{(12)}{=} y + y'z \overset{(4)}{=} (y + y')(y + z) \overset{(5)}{\underset{(2)}{=}} \boxed{y + z}$$

Note: I could have applied (12) sooner, and finished quicker!

HW 7.2

Prove:

$$(x+y)(y+z)(z+x') = (x+y)(z+x')$$

Solution

$$\underbrace{(X+Y)(Y+Z)(Z+X')}_{LS} = \underbrace{(X+Y)(Z+X')}_{RS}$$

Note: (12) Absorption: $X + XY = X$

$$LS \overset{(4)}{\underset{(7)}{=}} (XY + XZ + Y + YZ)(Z + X') \overset{(4)}{\underset{(7)}{=}} \overbrace{XYZ + XZ}^{XZ\ (12)} + \overbrace{YZ + YZ}^{YZ\ (7)} + \underbrace{\underbrace{XX'}_{0\ (5)}Y}_{0\ (8)} + \underbrace{XX'Z}_{0\ (5)(8)} + \overbrace{X'Y + X'YZ}^{X'Y\ (12)} = \boxed{XZ + YZ + X'Y}$$

$$RS = XZ + \underbrace{XX'}_{0\ (5)} + YZ + X'Y \overset{(5)}{\underset{(2)}{=}} \boxed{XZ + YZ + X'Y} = LS$$

HW 7.3

2-2 (a) $\underbrace{\bar{X}\bar{Y} + \bar{X}Y} + XY = \bar{X} + Y$ equality to prove

Solution

$$LS = \bar{X}(\bar{Y} + Y) + XY \overset{(5)}{\underset{(2)}{=}} \bar{X} + XY \overset{(4)}{=} (\underbrace{\bar{X} + X}_{1})(\bar{X} + Y) \overset{(5)}{\underset{(2)}{=}} \bar{X} + Y = RS$$

HW 7.3 (continuation)

2-2 (cont.) b) Prove: $\underline{\bar{A} B} + \bar{B}\,\bar{C} + \underline{A B} + \bar{B} C = 1$ **Solution**

$$(4)\quad \underbrace{(\bar{A} + A)}_{1} B \overset{(5)}{\underset{(2)}{=}} B$$

$$LS \overset{(4)}{\underset{(5)}{=}} B + \bar{B}\,\bar{C} + \bar{B} C \overset{(4)}{=} B + \bar{B}(\bar{C} + C) \overset{(5)}{\underset{(2)}{=}} B + \bar{B} \overset{(5)}{=} 1 = RS$$

c) Prove: $Y + \bar{X} Z + X \bar{Y} = X + Y + Z$ **Solution**

To prove: $LS = \underline{Y} + \bar{X} Z + \underline{X \bar{Y}} = X + Y + Z = RS$

$$LS \overset{(4)}{=} (Y + X)(Y + \bar{Y}) + \bar{X} Z \overset{(5)}{\underset{(2)}{=}} X + Y + \bar{X} Z \overset{(4)}{=} (x + y + \bar{x})(x + y + z) \overset{(5)}{\underset{(8)}{=}} x + y + z = RS$$

d) Prove: $\bar{X}\bar{Y} + \bar{Y} Z + X Z + X Y + Y \bar{Z} = \bar{X}\bar{Y} + X Z + Y \bar{Z}$ **Solution 1** Expand both sides, s.t. each term contains all 3 variables:

$$LS \overset{(2)}{\underset{(5)}{=}} \bar{X}\bar{Y}\underbrace{(Z + \bar{Z})}_{1} + \underbrace{(X + \bar{X})}_{1}\bar{Y} Z + X Z \underbrace{(Y + \bar{Y})}_{1} + X Y \underbrace{(Z + \bar{Z})}_{1} + \underbrace{(X + \bar{X})}_{1} Y \bar{Z} \overset{(4)}{=}$$

$$= \bar{X}\bar{Y} Z + \bar{X}\bar{Y}\bar{Z} + X\bar{Y} Z + \cancel{\bar{X}\bar{Y} Z} + X Y Z + \cancel{X\bar{Y} Z} + \cancel{X Y Z} + X Y \bar{Z} + \cancel{X Y \bar{Z}} + \bar{X} Y \bar{Z} \overset{(7)}{=}$$

$$= \bar{X}\bar{Y} Z + \bar{X}\bar{Y}\bar{Z} + X\bar{Y} Z + X Y Z + X Y \bar{Z} + \bar{X} Y \bar{Z}$$

$$RS \overset{(2)}{\underset{(5)}{=}} \bar{X}\bar{Y}(Z + \bar{Z}) + X(Y + \bar{Y}) Z + (X + \bar{X}) Y \bar{Z} \overset{(4)}{=} \bar{X}\bar{Y} Z + \bar{X}\bar{Y}\bar{Z} + X Y Z + X \bar{Y} Z + X Y \bar{Z} + \bar{X} Y \bar{Z} = LS$$

Solution 2 - Hint

d) Prove: $\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

Consider only the terms in LS that are not in RS, and expand **only** those.

$$\bar{y}z + xy = (x+\bar{x})\bar{y}z + xy(z+\bar{z}) = \underbrace{x\bar{y}z}_{\text{absorbed by } xz \in LS} + \overbrace{\bar{x}\bar{y}z}^{\text{absorbed by } \bar{x}\bar{y} \in LS} + \underbrace{xyz}_{\text{by } xz \in LS} + \underbrace{xy\bar{z}}_{\text{absorbed by } y\bar{z} \in LS}$$

Notice how all these terms get absorbed according to (12) by other terms on the LS…

… and they all disappear!

HW 7.3 (continuation)

2-4 +Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C)\cdot(\overline{A} + B)\cdot(B + C) = B \cdot C$$

Hint

I Note that by (5) $A = \overline{B}$, which is equivalent with $B = \overline{A}$

II Multiply out.

2-8 Using DeMorgan's theorem, express the function

$$F = A\overline{B}C + \overline{A}\,\overline{C} + AB$$

(a) with only OR and complement operations
(b) with only AND and complement operations.

Hint Use De Morgan's (10):

for (a):

$$x \cdot y \overset{(9)}{=} ((x \cdot y)')' \overset{(10)}{=} (x' + y')'$$

for (b):

$$x + y = ((x + y)')' = (x' \cdot y')'$$

2-9 **Complement the following expressions:**

a) $A\overline{B} + \overline{A}B$

We perform a) --> the rest still as HW 8.0 (if not done yet)

$$\overline{(A\overline{B} + \overline{A}B)} \overset{(10}{=} \overline{(A\overline{B})} \cdot \overline{(\overline{A}B)} \overset{(10)}{=} (\overline{A} + B)(A + \overline{B}) \overset{(4)}{\underset{(5)}{=}} AB + \overline{A}\,\overline{B}$$

Table

Boolean Expressions for the 16 Functions of Two Variables

Boolean functions	Operator symbol	Name	Comments
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_3 = x$		Transfer	x'
$F_4 = x'y$	y/x	Inhibition	y, but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$(x \oplus y)'$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y, then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x, then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1

HW 8.1 - assigned

By substitution the Boolean expression equivalent of the binary operation as defined in **Table of 16 functions on 2 variables**, show the following:

(a) The inhibition operation is neither commutative nor associative.

(b) The exclusive-OR operation is commutative and associative.