CL=CSCI 160

CLASS 12

Boolean functions - Canonical form vs. standard forms

Independent Switches Problem

Note: $\left.\begin{array}{c} A = 1 \\ \text{and} \\ B = 1 \end{array}\right\} \Longleftrightarrow AB = 1 \qquad \left.\begin{array}{c} A = 1 \\ \text{or} \\ B = 1 \end{array}\right\} \Longleftrightarrow A + B = 1$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let's write the expression of function f, from its truth table.

Sum of minterms representation:

$$f = 1 \Longleftrightarrow \left\{\begin{array}{c} x = 0 \text{ and } y = 0 \text{ and } z = 1 \\ \text{or} \\ x = 0 \text{ and } y = 1 \text{ and } z = 0 \\ \text{or} \\ x = 1 \text{ and } y = 0 \text{ and } z = 0 \\ \text{or} \\ x = 1 \text{ and } y = 1 \text{ and } z = 1 \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{c} x' = 1 \text{ and } y' = 1 \text{ and } z = 1 \\ \text{or} \\ x' = 1 \text{ and } y = 1 \text{ and } z' = 1 \\ \text{or} \\ x = 1 \text{ and } y' = 1 \text{ and } z' = 1 \\ \text{or} \\ x = 1 \text{ and } y = 1 \text{ and } z = 1 \end{array}\right\} \Longleftrightarrow$$

$$\Longleftrightarrow \left\{\begin{array}{c} x'y'z = 1 \\ \text{or} \\ x'yz' = 1 \\ \text{or} \\ xy'z' = 1 \\ \text{or} \\ xyz = 1 \end{array}\right\} \Longleftrightarrow x'y'z + x'yz' + xy'z' + xyz = 1$$

minterms:

$$\Updownarrow$$

$$\boxed{f = x'y'z + x'yz' + xy'z' + xyz}$$

This is called the **canonical sum of products** form.

The canonical form is unique for a given function, and it can be used to compare functions/expressions.

Most times, the canonical form may be reduced, simplified.

Any sum of products, not necessarily containing only minterms, is called **standard**.

Similarly, we have a dual form, called the **canonical product of sums** form; however in this course we will only deal with the sum of minterms representation, as the other is handled similarly.

Example (Short Cut): Obtain the canonical form directly from the truth table of an arbitrary function

	x	y	z	f
0	0	0	0	1 ←
1	0	0	1	1 ←
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1 ←
5	1	0	1	1 ←
6	1	1	0	0
7	1	1	1	1 ←

Canonical sum of products form:

$f = x'y'z' + x'y'z + xy'z' + xy'z + xyz$ also a standard form

$$\left.\begin{array}{l} f = x'y' + xy' + xyz \\ f = y' + xyz \\ \quad [= (y' + y)(y' + xz) = y' + xz] \\ f = y' + xz \end{array}\right\} \text{other standard forms for f}$$

Because the canonical sum of products form is unique we express this function above as:

$f = \sum (0, 1, 4, 5, 7)$

Minimization of functions

Time and space trade-offs

Variety of techniques to obtain gate simplification.

However: simplification depends on the **metric** we use:

- The **number of literals** it contains

  = amount of **wiring** needed to implement the function: # inputs: 3-4 usual, > 8,9 very rare.

- The **number of gates**

  = strong correlation with **# components** needed for implementation;

  simplest design to manufacture is the one with fewest gates, not literals.

- **Number of cascaded levels of gates**

  = reducing # logic levels would **reduce** **the overall delay** in the path input --> output;

however an implementation with minimum delay rarely yields an implementation with fewest # literals or gates.

Traditional minimization techniques: reduce delay at expense of adding more gates.

Other methods: **Trade-off between increased circuit delay and reduced gate count**.

Example

F = a'b'c + a'bc + ab'c + abc'

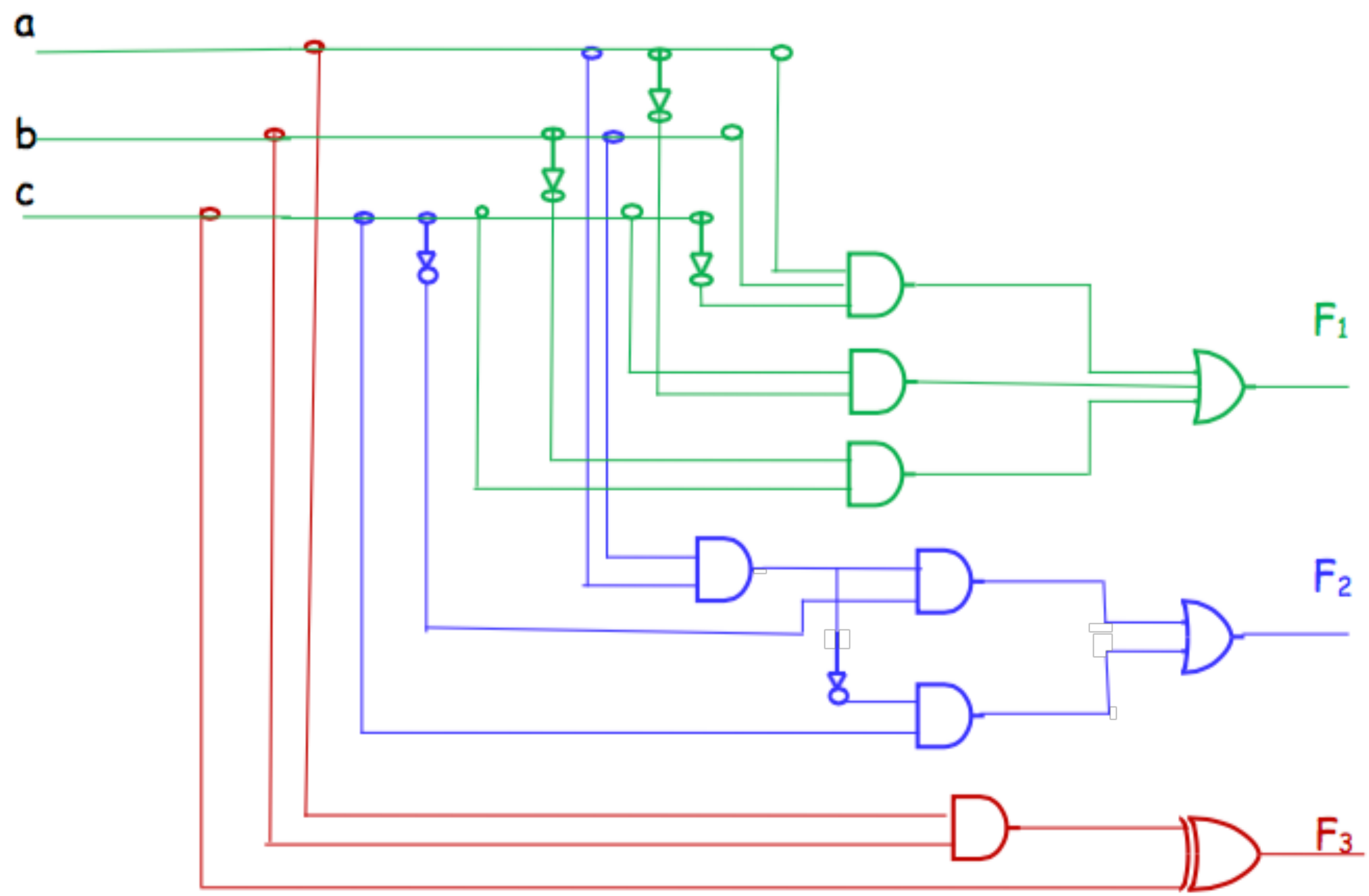
F_1 =abc'+a'c+b'c
- 2-level implementation
- **7 literals** (<12 as the original)

F_2 = tc' + t'c,

set t = ab
- 3-level implementation
- **4 literals**
- longest path: 4 gates (> 3 in F_1)
  --> not as fast as F_1.
- **total # gates in F_2 ≤ in F_1.**

F_3 = (ab)⊕ c
- XOR = complex gate: implement by combining NAND, NOR
- **lowest gate count**, but also worst signal delay, for XOR slow compared to simple AND, OR.

Logical Diagrams (all three functions in one diagram):



HW 12 - assigned

12-A Given the Boolean functions F_1 and F_2,

(a) Show that the Boolean function $E = F_1 + F_2$ contains the sum of the minterms of F_1 and F_2.

(b) Show that the Boolean function $G = F_1 F_2$ contains only the minterms that are common to F_1 and F_2.

12-B Give the truth table of the function:

$$F = xy + xy' + y'z$$