

Stack ADT

Today's Plan



Questions?

Stack ADT

Abstract Data Types

Bag

List

Stack

Stack

34

A data structure representing a stack of things

Objects can be **pushed** onto the stack or **popped** from the stack

Stack

A data structure representing a stack of things

Objects can be **pushed** onto the stack or **popped** from the stack



34

Stack

127

34

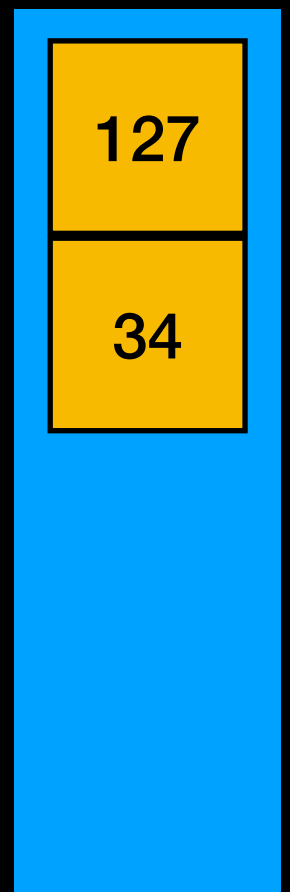
A data structure representing a stack of things

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Stack

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13

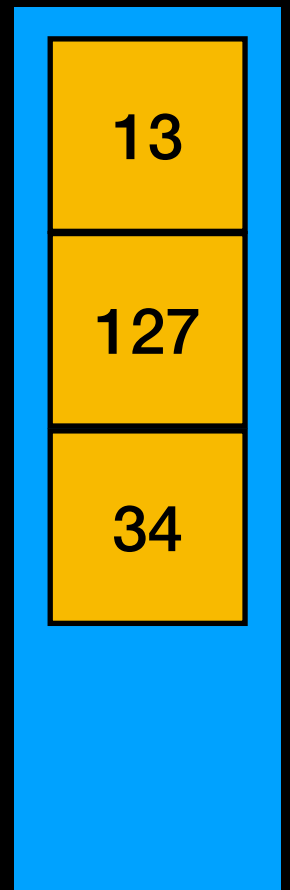
127

34

Stack

A data structure representing a stack of things

Objects can be **pushed** onto the stack or **popped** from the stack



Stack

13

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127

34

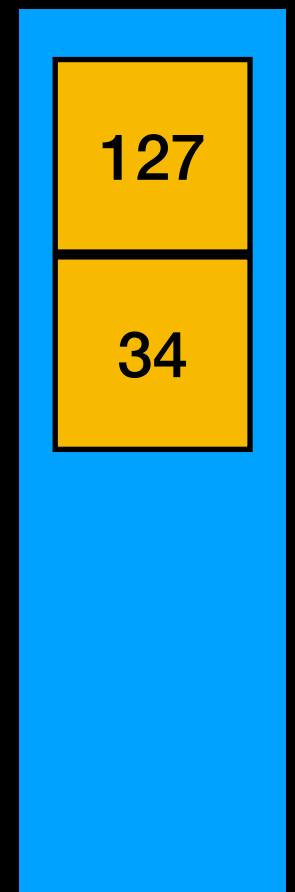
Stack

A data structure representing a stack of things

Objects can be **pushed** onto the stack or **popped** from the stack

LIFO: Last In First Out

Only top of stack is accessible (**top**), no other objects on the stack are visible



Applications

Very simple structure

Many applications:

- program stack

- balancing parenthesis

- evaluating postfix expressions

- backtracking

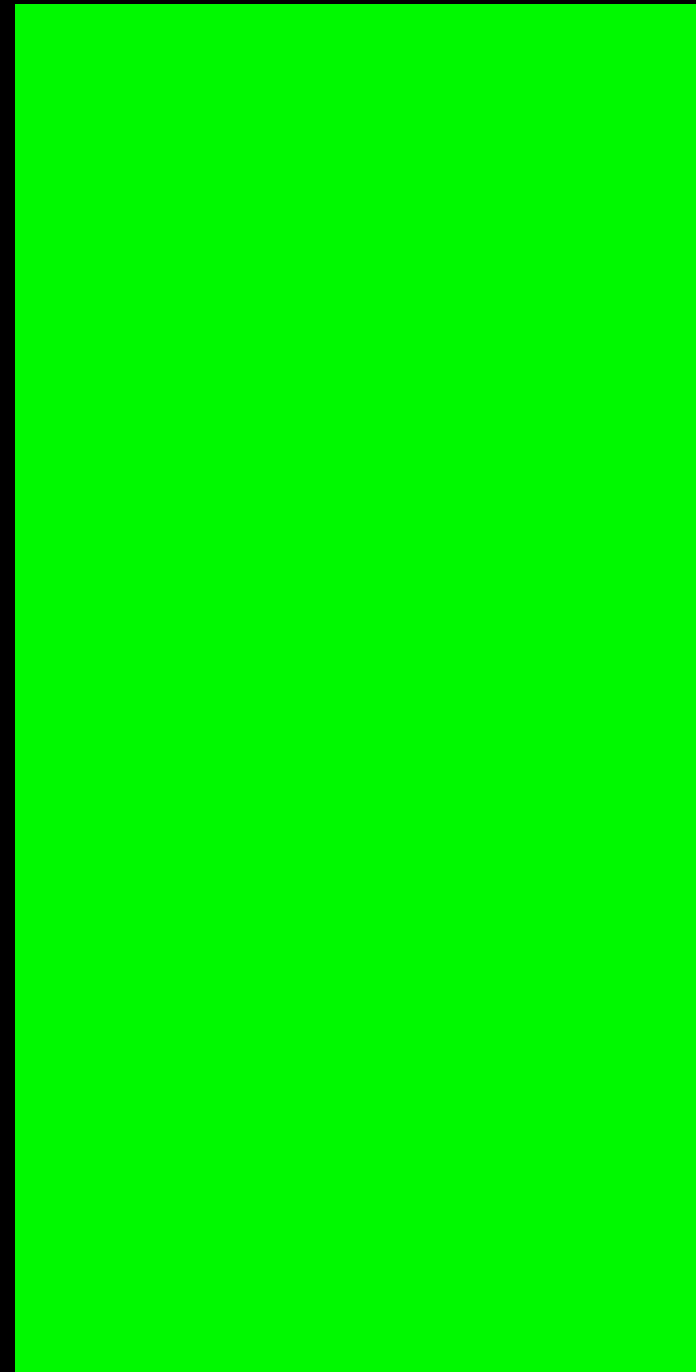
- ... and more

Program Stack

Program Stack

```
1 void f(int x, int y)
2 {
3     int a;
4     // stuff here
5     if(a<13)
6         a = g(a);
7     // stuff here
8 }

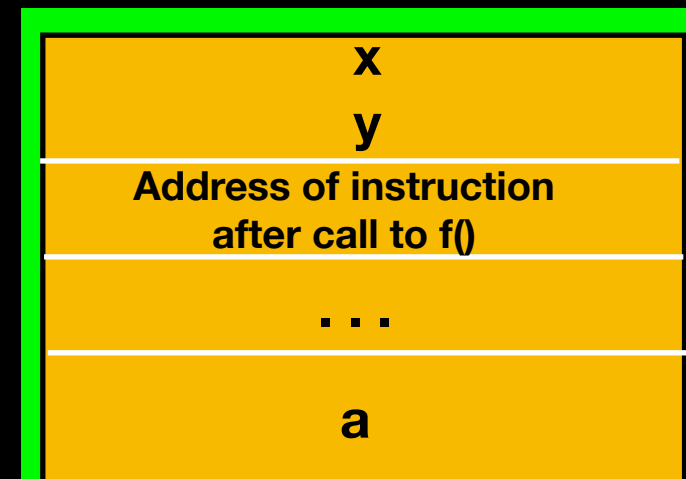
9 int g(int z)
10 {
11     int p ,q;
12     // stuff here
13     return q;
14 }
```



Program Stack

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```

Stack Frame
for f()



```
9 int g(int z)
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```

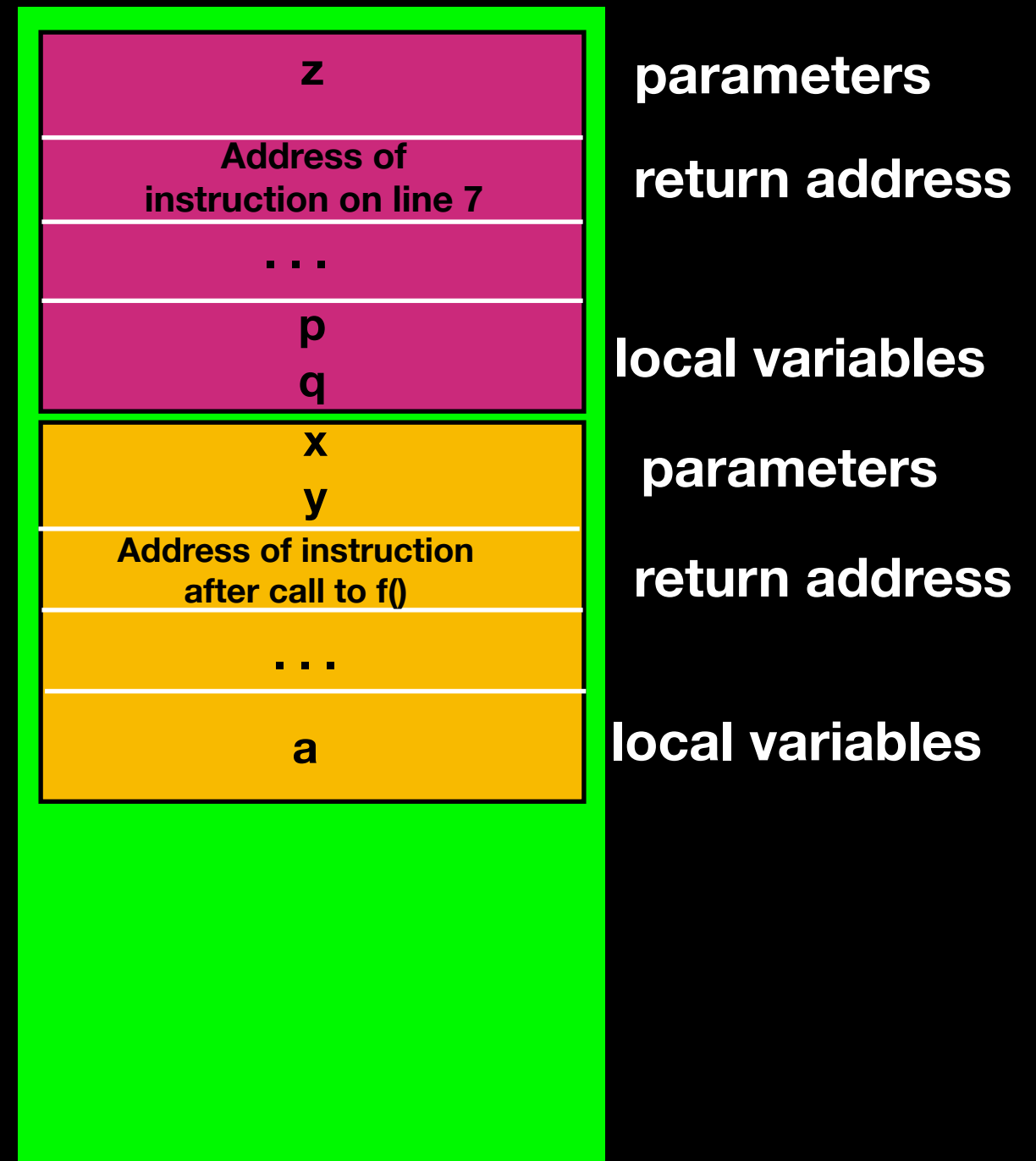
Program Stack

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2 {
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4     // stuff here
5     if(a<13)
6         a = g(a);
7     // stuff here
8 }
```

Stack Frame
for g()

```
9 int g(int z)
10 {
11     int p ,q;
12     // stuff here
13     return q;
14 }
```

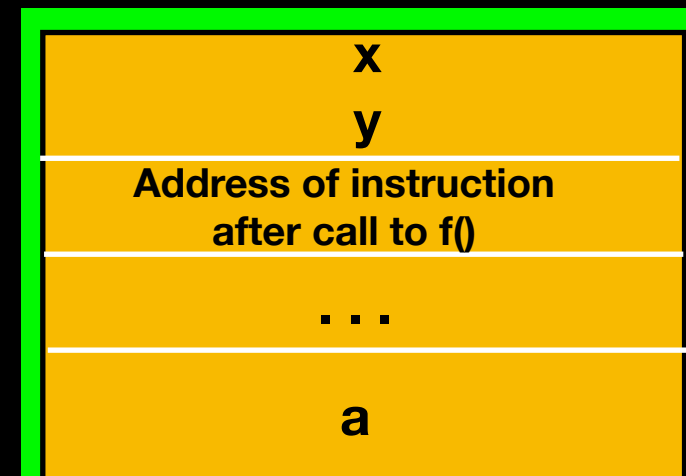
Stack Frame
for f()



Program Stack

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1 void f(int x, int y)
2 {
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5     if(a<13)
6         a = g(a);
7     // stuff here
8 }
```

Stack Frame
for f()



parameters

return address

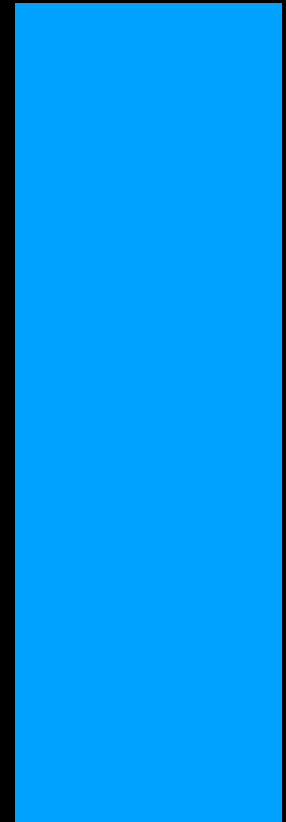

local variables

```
9 int g(int z)
10 {
11     int p ,q;
12     // stuff here
13     return q;
14 }
```

Balancing Parentheses

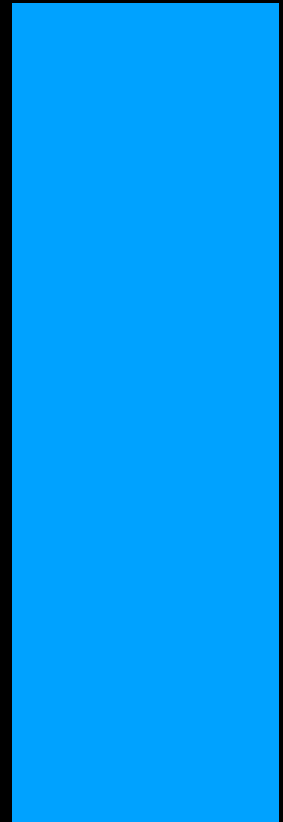

Balancing Parentheses

```
int f(){if(x*(y+z[i])<47){x += y}}
```



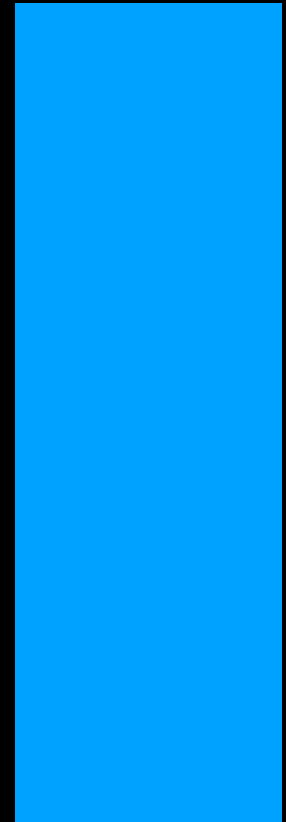

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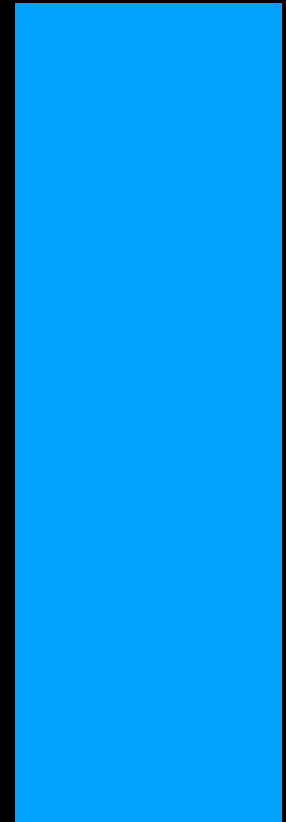

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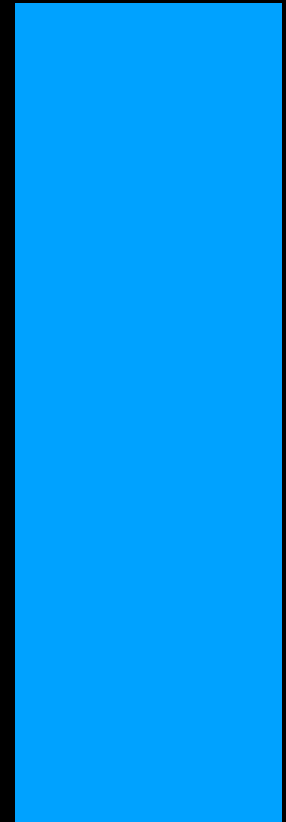

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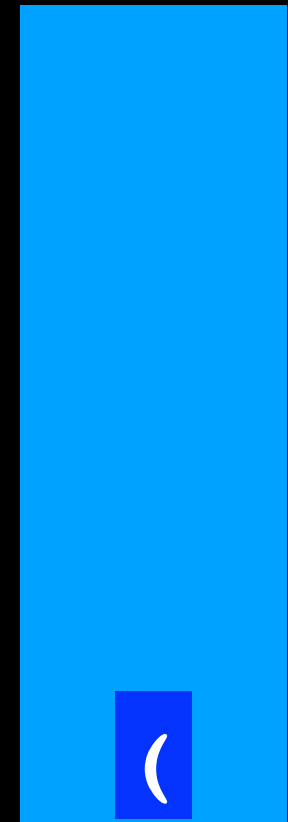
```
int f() {if(x*(y+z[i])<47){x += y}}
```



Balancing Parentheses


```
int f()  
    ↑  
    {if(x*(y+z[i])<47){x += y}}
```

push

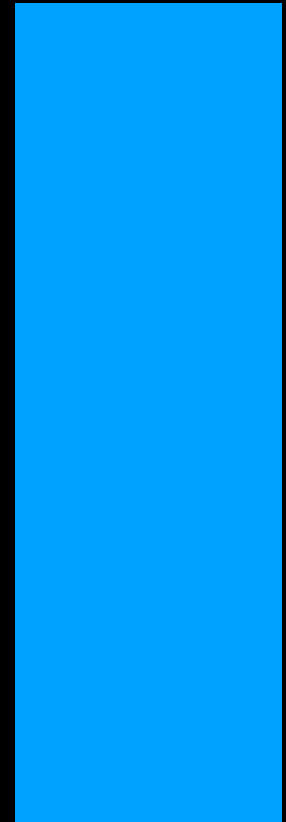


Balancing Parentheses

```
int f() {if(x*(y+z[i])<47) {x += y}}
```



pop

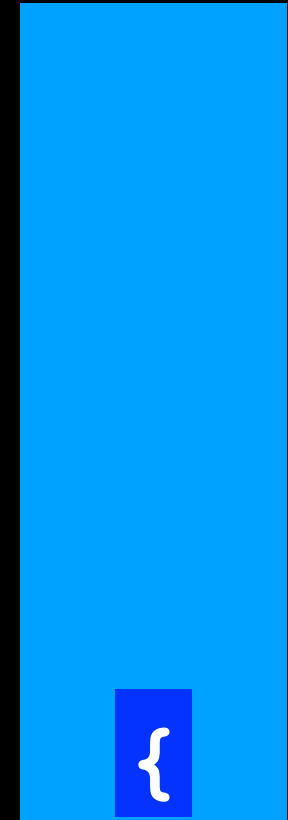


Balancing Parentheses

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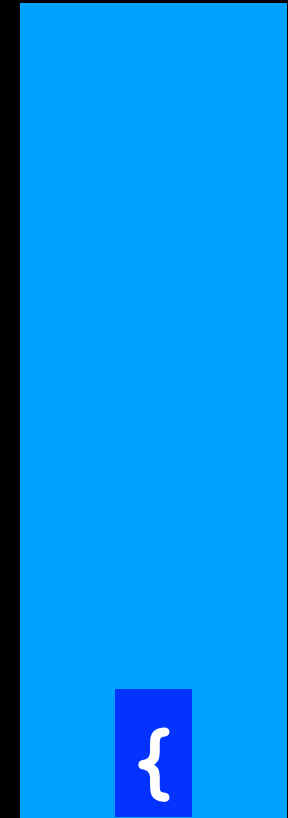



push



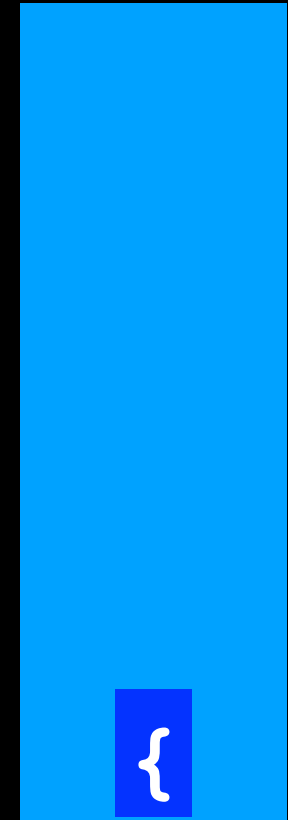

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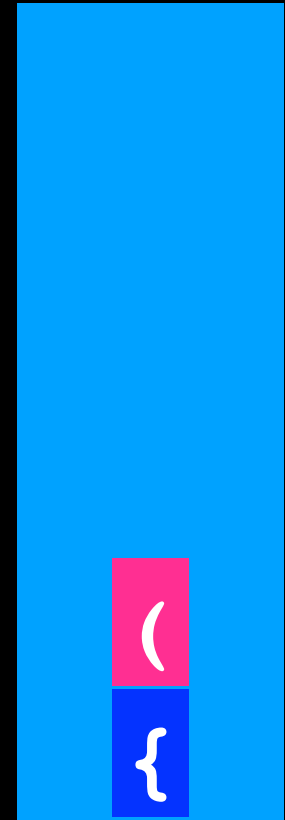
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Balancing Parentheses

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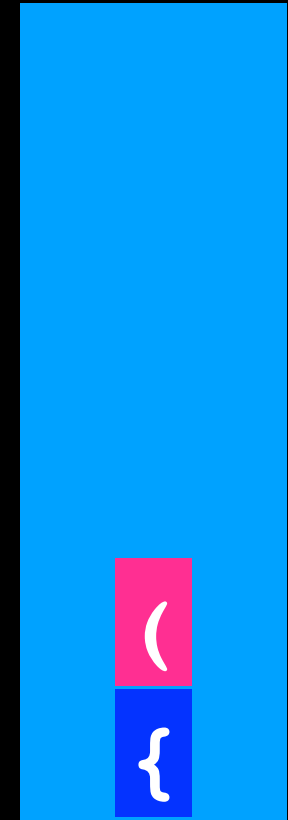
↑



Balancing Parentheses

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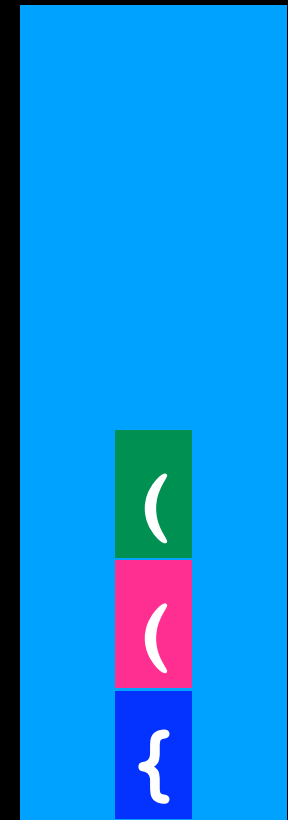


Balancing Parentheses

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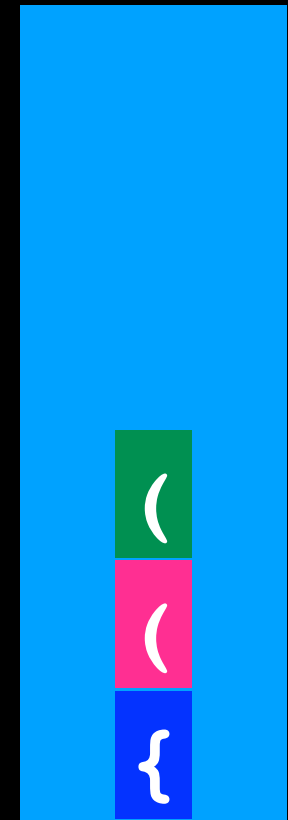
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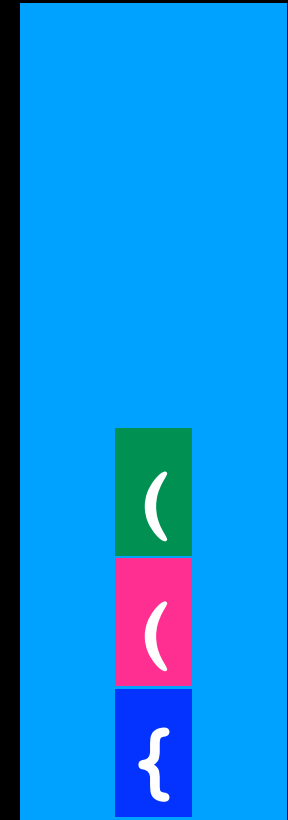
↑



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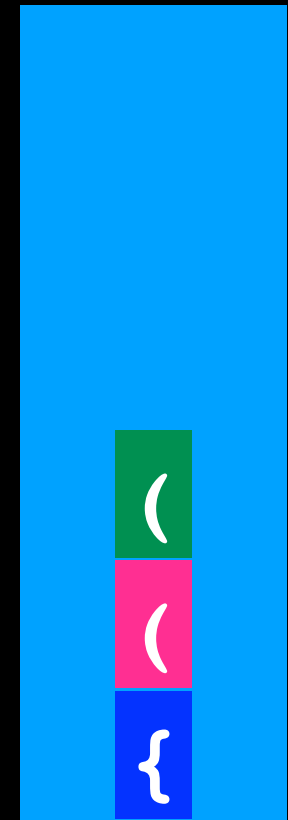
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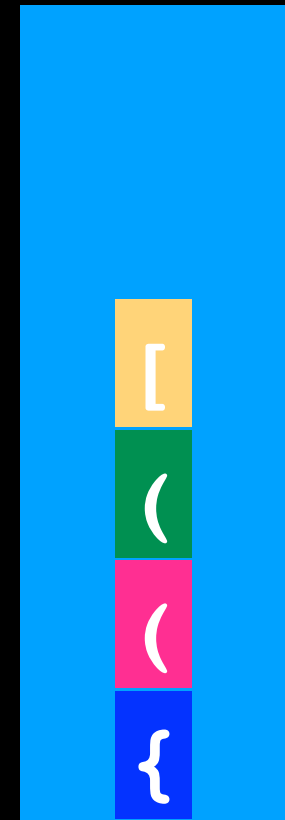


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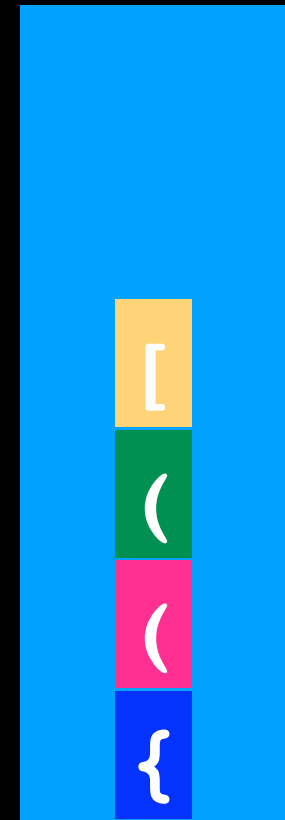
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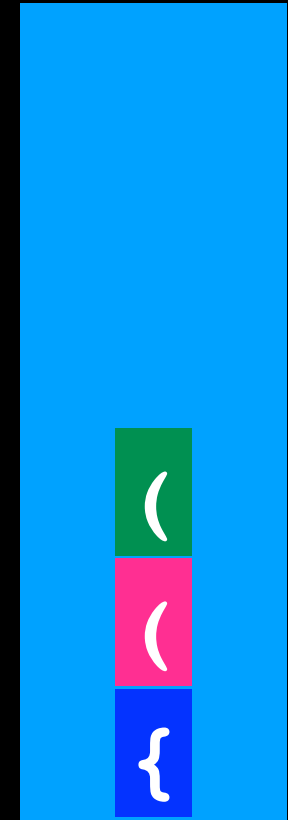
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Balancing Parentheses

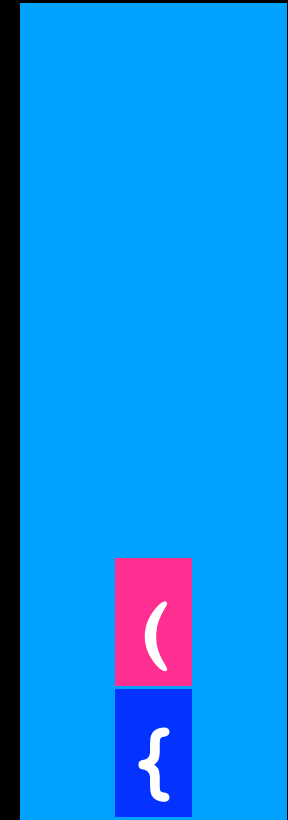
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pop

Balancing Parentheses

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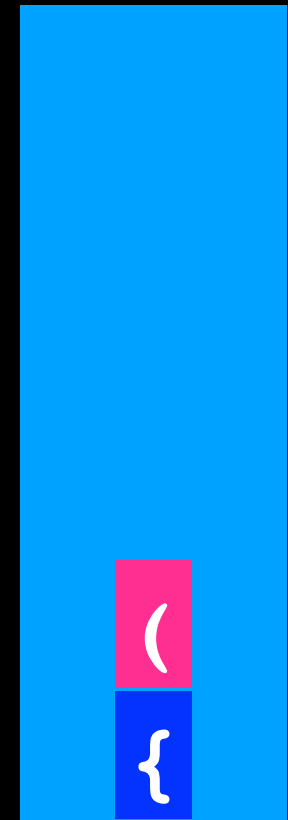


pop

Balancing Parentheses

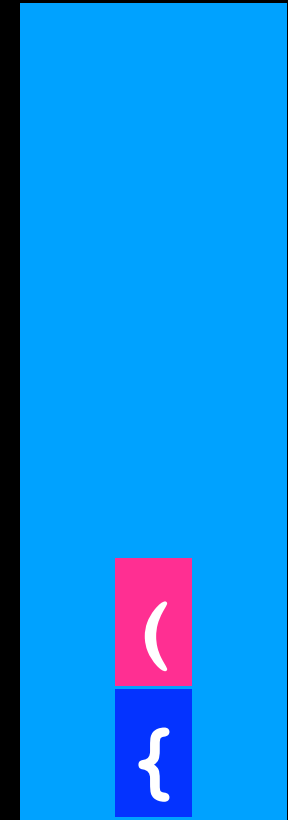
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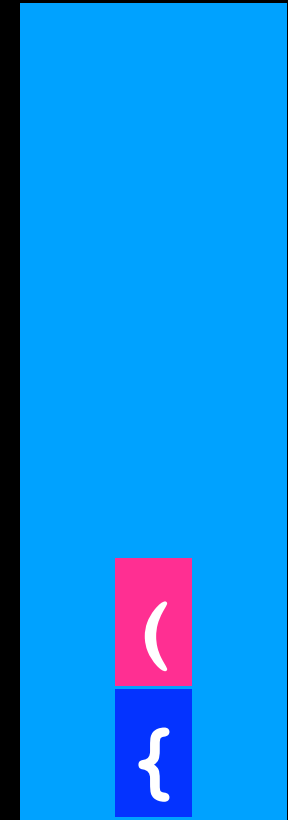
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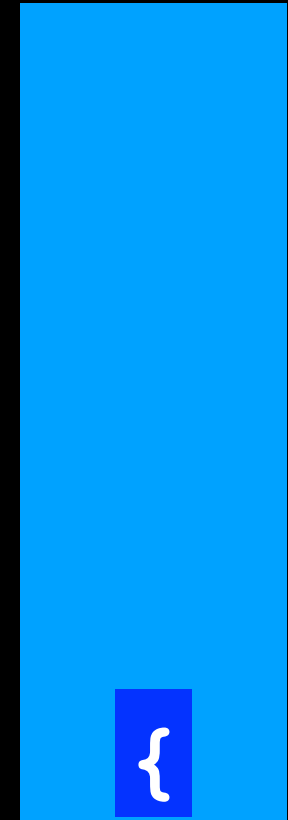
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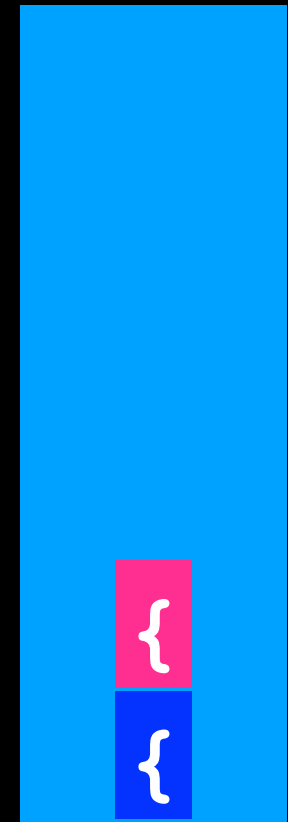
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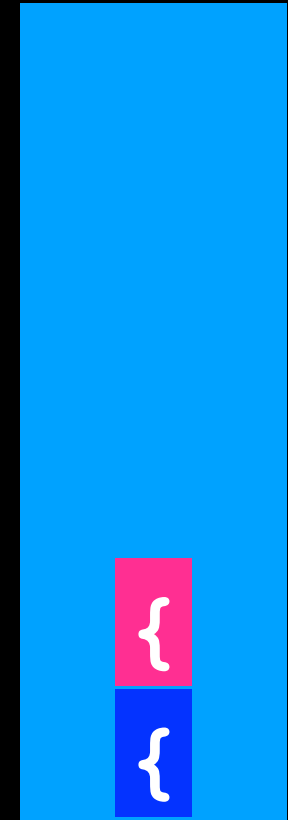


push



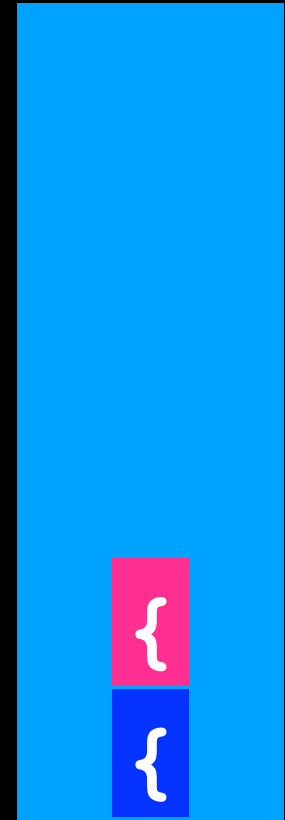
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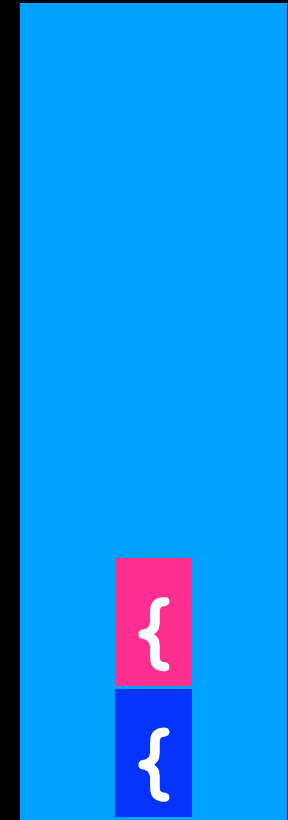
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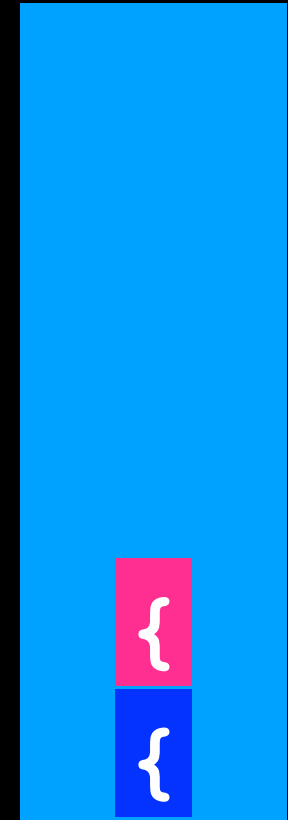
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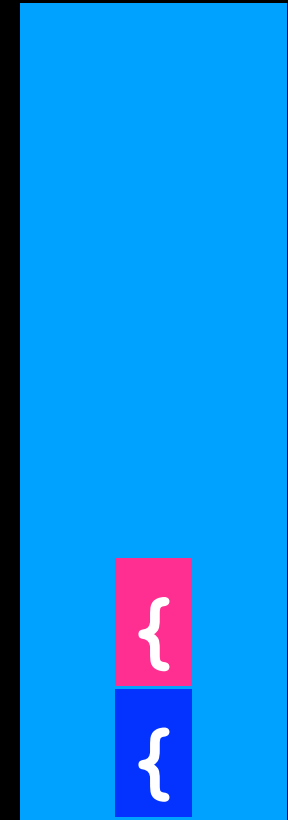
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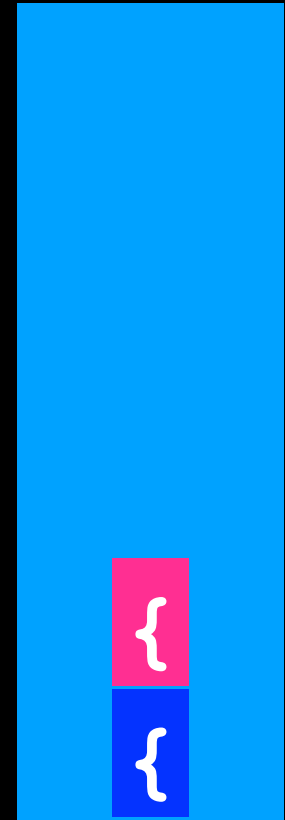
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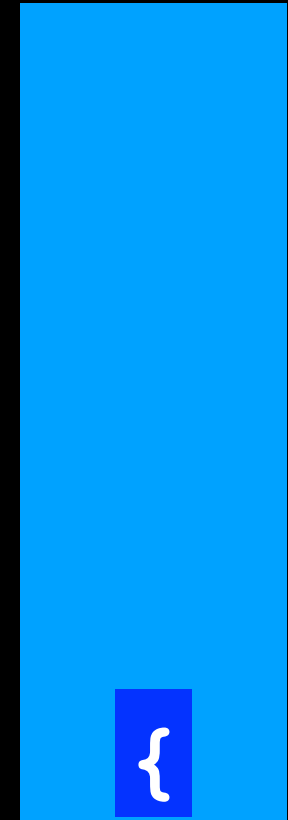
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Balancing Parentheses

```
int f(){if(x*(y+z[i])<47){x += y}}
```



pop

Balancing Parentheses

```
int f(){if(x*(y+z[i])<47){x += y}}
```



Finished reading
Stack is empty
Parentheses are balanced

pop



Balancing Parentheses

```
int f(){if(x*(y+z[i])<47){x += y}
```



Finished reading
Stack not empty
Parentheses **NOT** balanced

A vertical blue rectangle representing a stack. At the bottom of the rectangle, there is a small white box containing an opening curly brace '{', indicating that the stack is not empty.

Balancing Parentheses

```
for(char ch : st)
{
    if ch is an open parenthesis character
        push it on the stack
    else if ch is a close parenthesis character
        if it matches the top of the stack
            pop the stack
        else
            return unbalanced
    // else it is not a parenthesis
}

if stack is empty
    return balanced
else
    return unbalanced
```

Evaluating Postfix Expressions

Postfix Expressions

Operator applies to the two operands immediately preceding it

Infix:

$2 * (3 + 4)$

$2 * 3 + 4$

Postfix:

$2\ 3\ 4\ +\ *$

$2\ 3\ *\ 4\ +$

Evaluating Postfix Expressions

Operator applies to the two operands immediately preceding it

Postfix:

2 3 4 + *

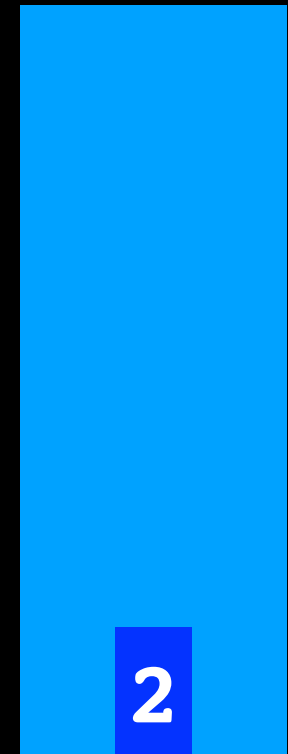
Assumptions / simplifications:

- String is syntactically correct postfix expression
- No unary operators
- No exponentiation operation
- Operands in string are single integer values

Evaluating Postfix Expressions

Postfix:

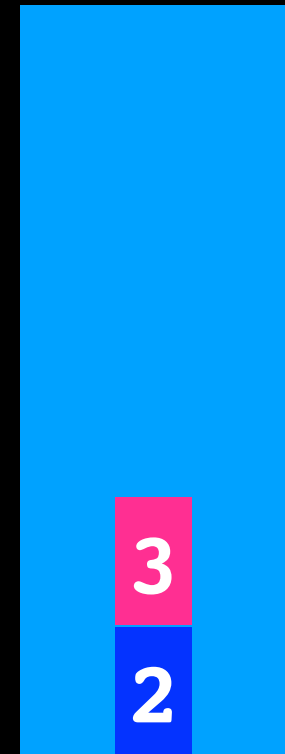
2 3 4 + *



Evaluating Postfix Expressions

Postfix:

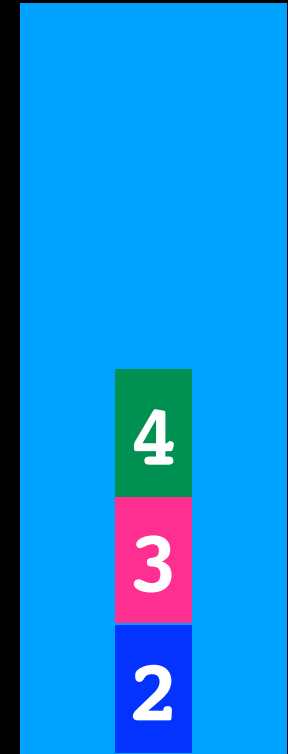

2 3 4 + *



Evaluating Postfix Expressions

Postfix:

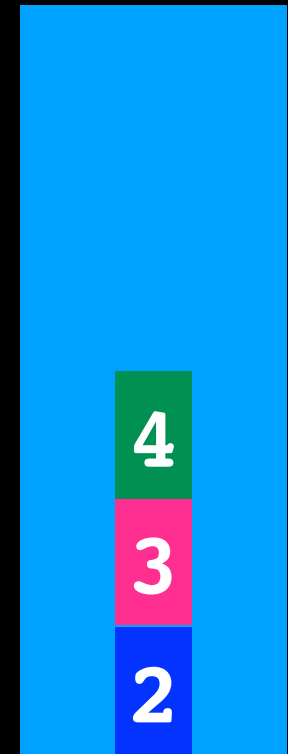

2 3 4 + *



Evaluating Postfix Expressions

Postfix:


2 3 4 + *



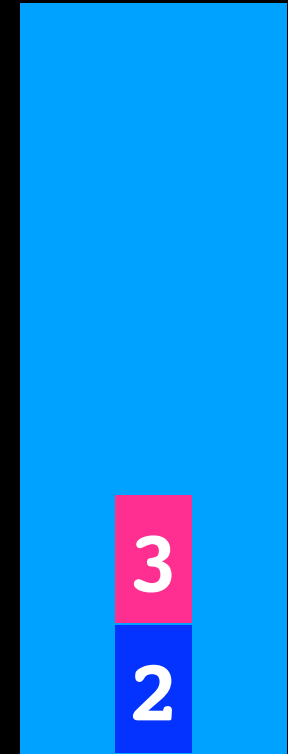
Evaluating Postfix Expressions

Postfix:

2 3 4 + *




4 +



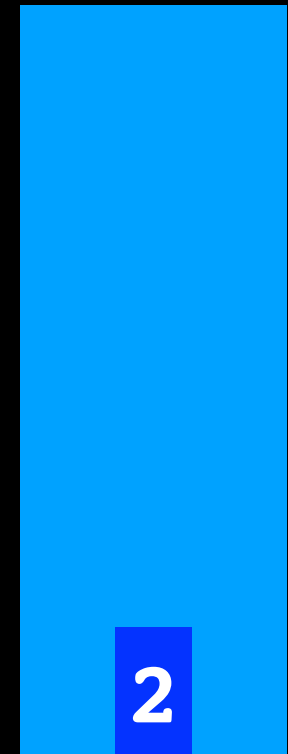
Evaluating Postfix Expressions

Postfix:

2 3 4 + *



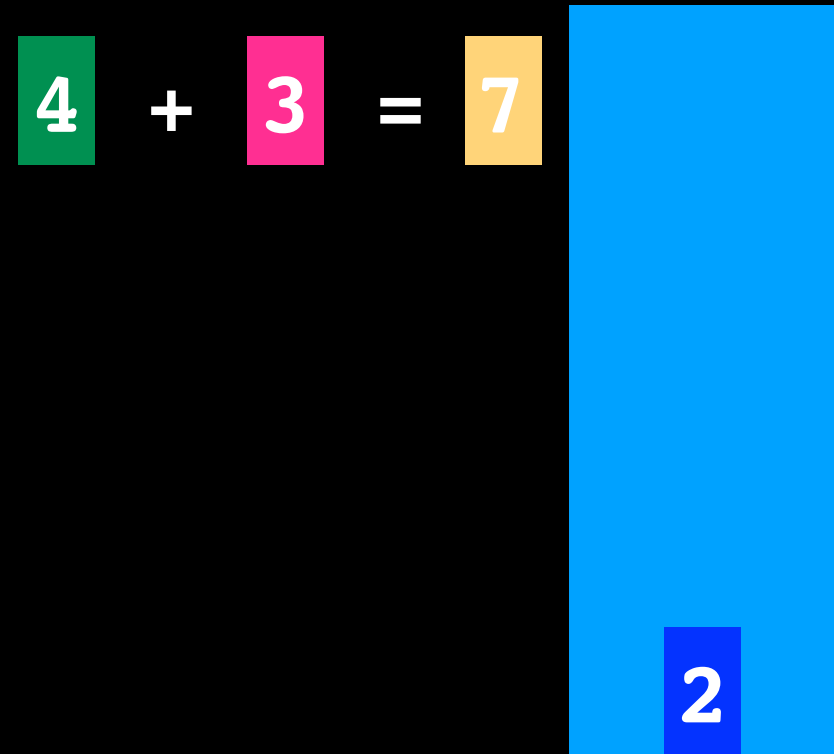

4 + 3



Evaluating Postfix Expressions

Postfix:

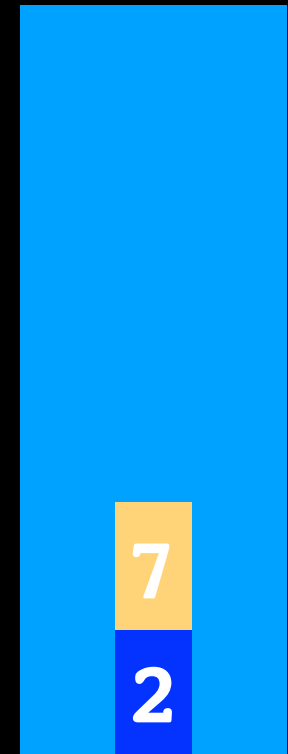

2 3 4 + *



Evaluating Postfix Expressions

Postfix:

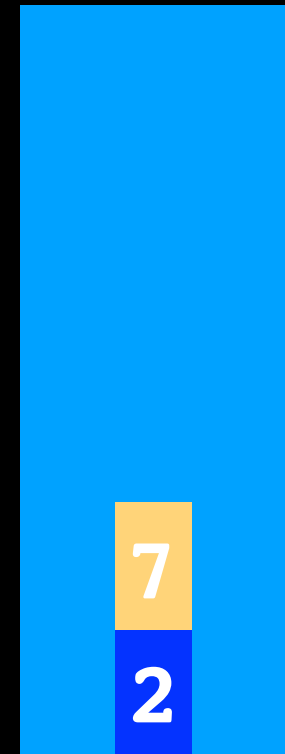
2 3 4 + *



Evaluating Postfix Expressions

Postfix:

2 3 4 + *
↑



Evaluating Postfix Expressions

Postfix:

2 3 4 + *
↑

7

2

Evaluating Postfix Expressions

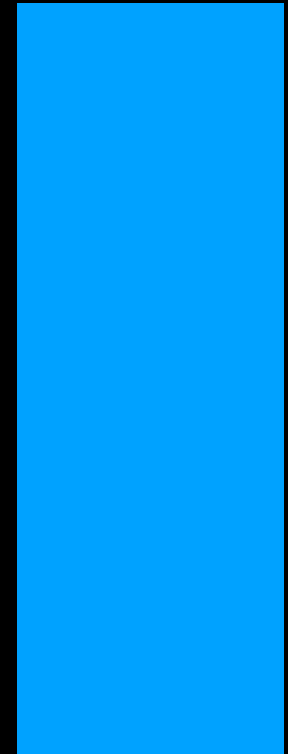
Postfix:

2 3 4 + *
↑

7

*

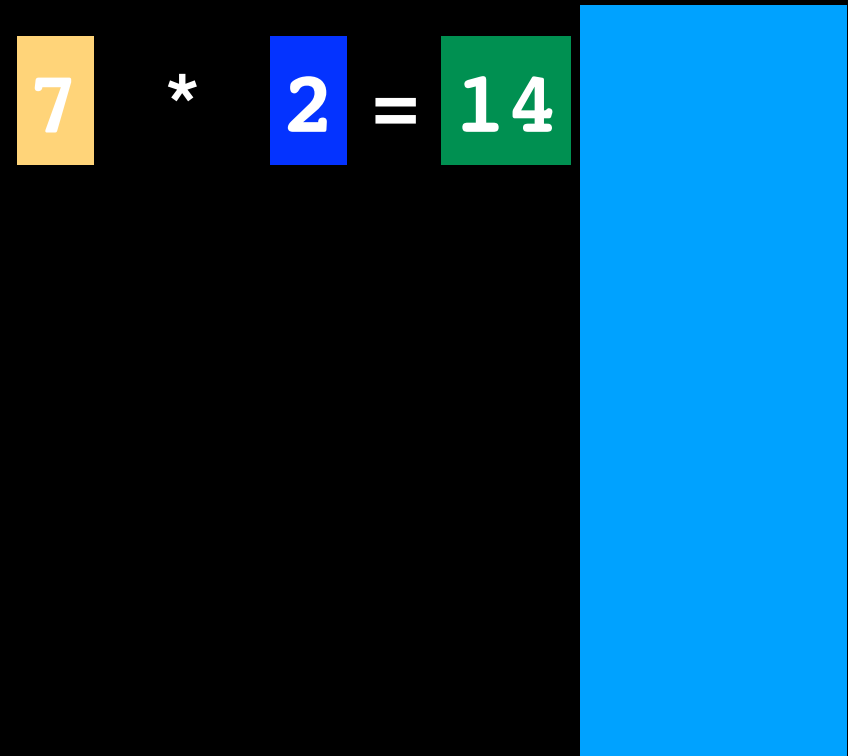
2



Evaluating Postfix Expressions

Postfix:

2 3 4 + *
↑



Evaluating Postfix Expressions

Postfix:

2 3 4 + *



Done reading string
The top of the stack
is the result

14



Evaluating Postfix Expressions

Operator applies to the two operands immediately preceding it

Postfix:

2 3 * 4 +

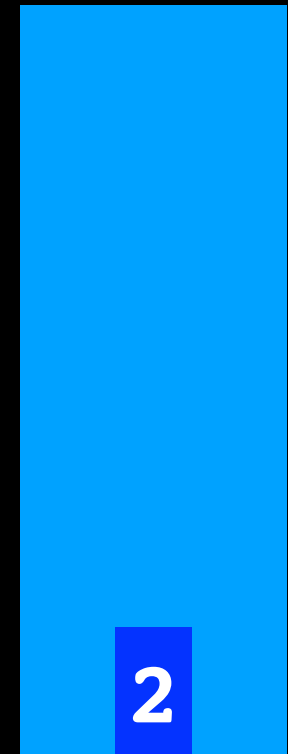
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Evaluating Postfix Expressions

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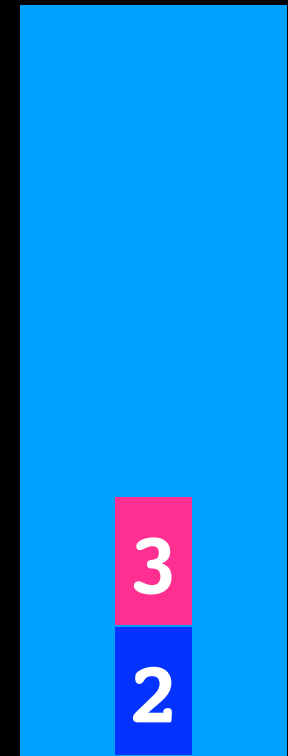
2 3 * 4 +



Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



3

*

2

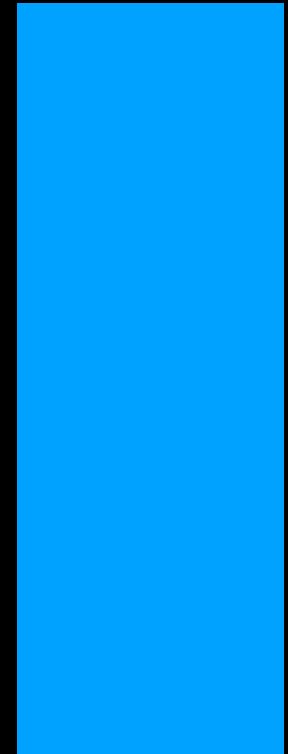
Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



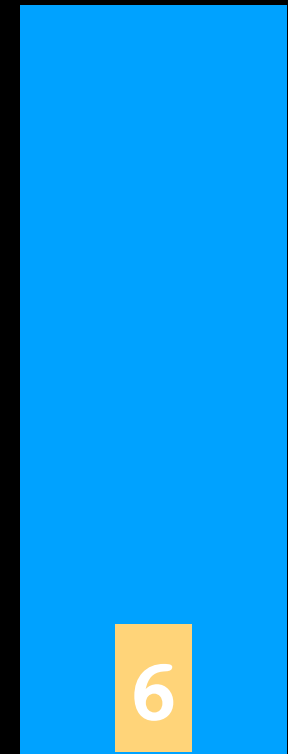
$$\boxed{3} * \boxed{2} = \boxed{6}$$



Evaluating Postfix Expressions

Postfix:

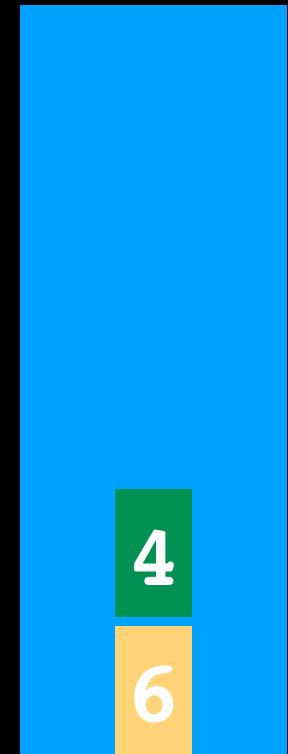
2 3 * 4 +



Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



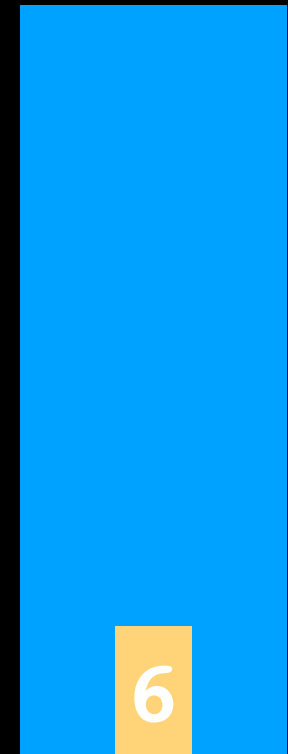
Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



4 +



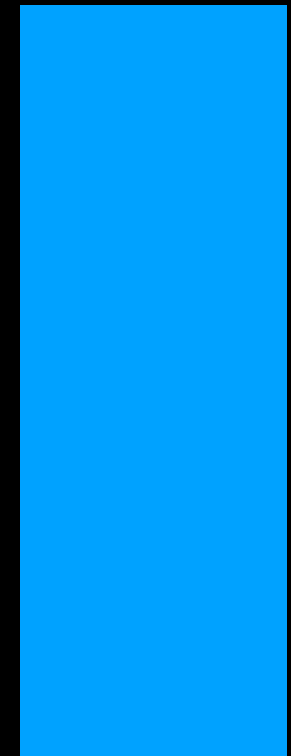
Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



$$4 + 6 = 10$$



Evaluating Postfix Expressions

Postfix:

2 3 * 4 +



Done reading string
The top of the stack
is the result

10



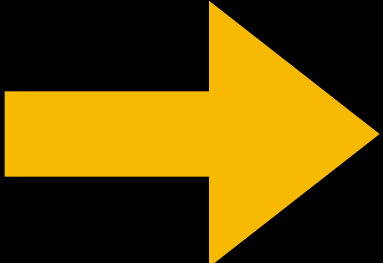
Evaluating Postfix Expressions

```
for(char ch : st)
{
    if ch is an operand
        push it on the stack
    else // ch is an operator op
    {
        //evaluate and push the result
        operand2 = pop stack
        operand1 = pop stack
        result = operand1 op operand2
        push result on stack
    }
}
```

Lecture Activity

Describe an algorithm that translates the infix expression below into postfix (you can use drawings to explain):

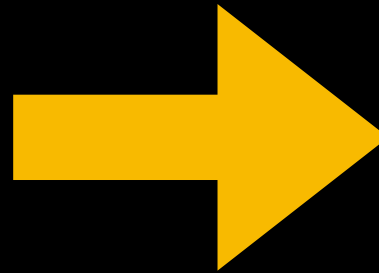
Infix:
 $2 * (3 + 4)$



Postfix:
 $2 3 4 + *$

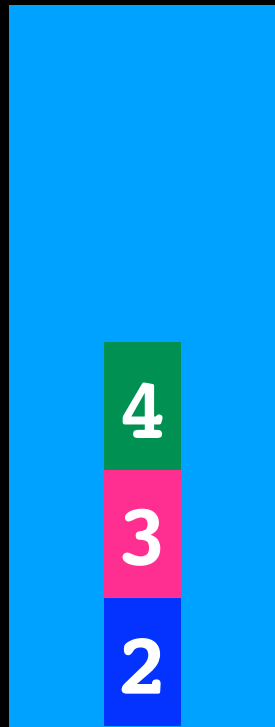
Hint: use 2 stacks, one for operators and parentheses another one for the operands and postfix expression. Once converted use the empty stack to invert the order

Infix:
 $2 * (3 + 4)$

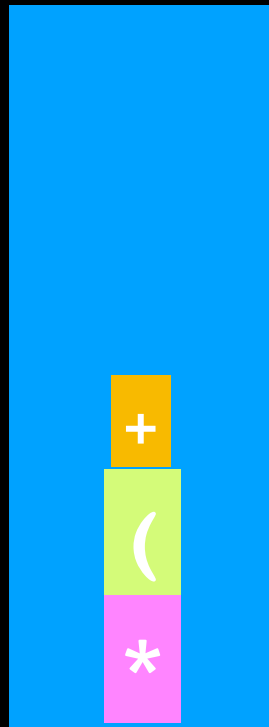


Postfix:
 $2 3 4 + *$

1. Read characters onto corresponding stack until '('

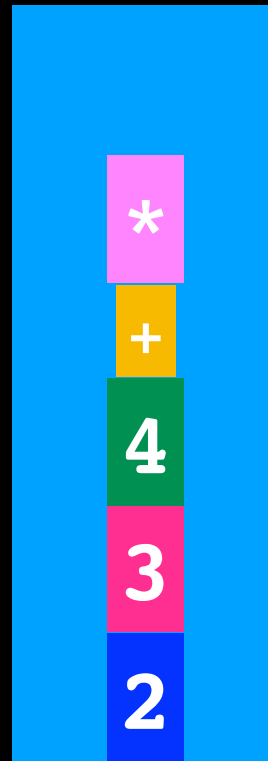


**Postfix
Stack**

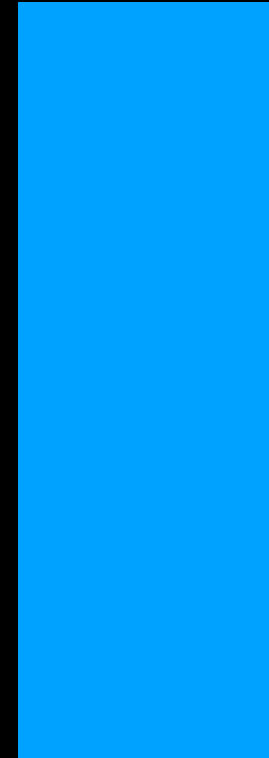


**Operator
Stack**

2. Pop operator stack and push it onto postfix stack ignoring '('



**Postfix
Stack**

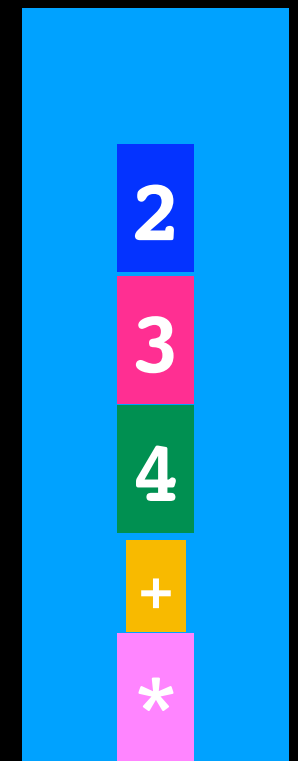


**Operator
Stack**

3. Push everything onto empty stack to invert
Then read pop and print.

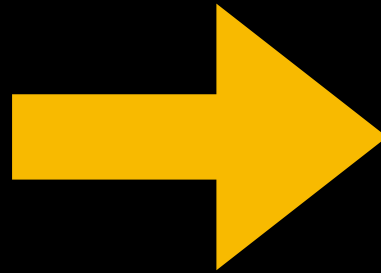


**Postfix
Stack**



**Operator
Stack**

Infix:
 $(2 * 3) + 4$



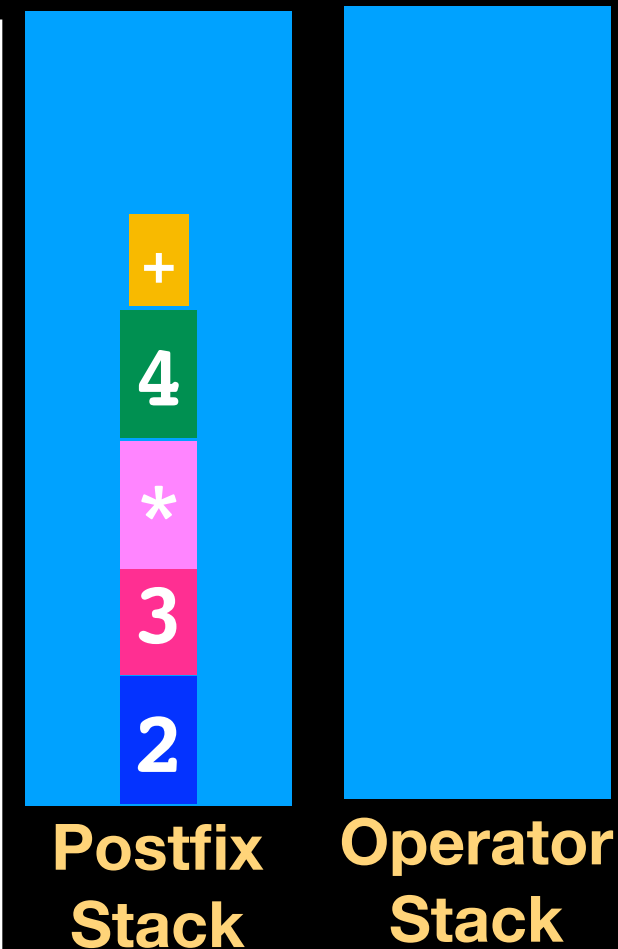
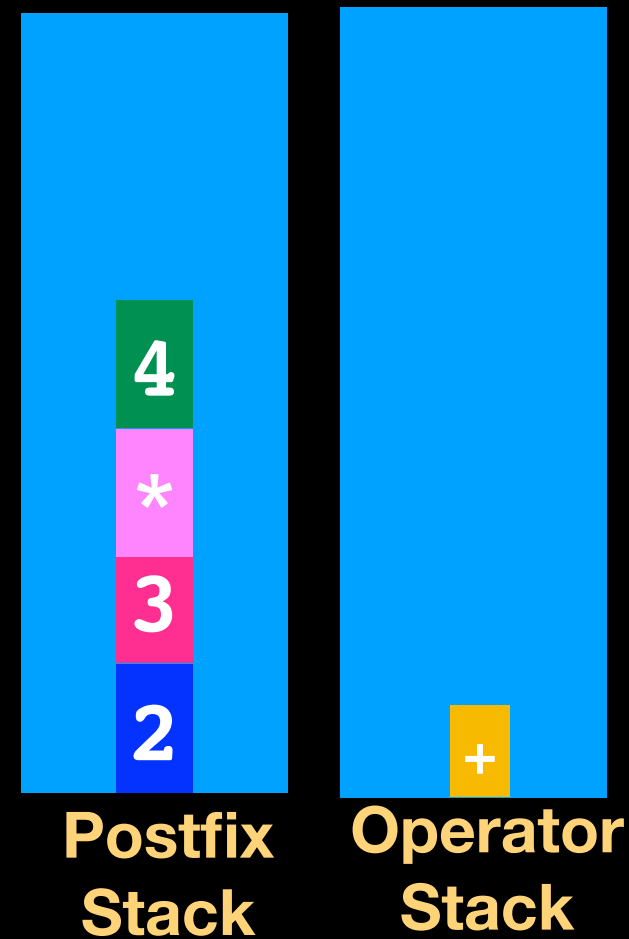
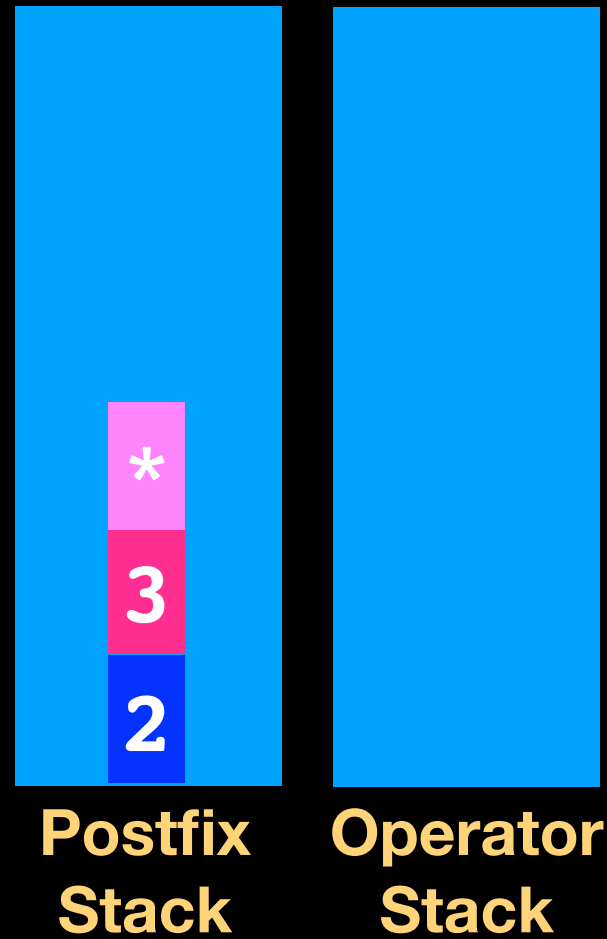
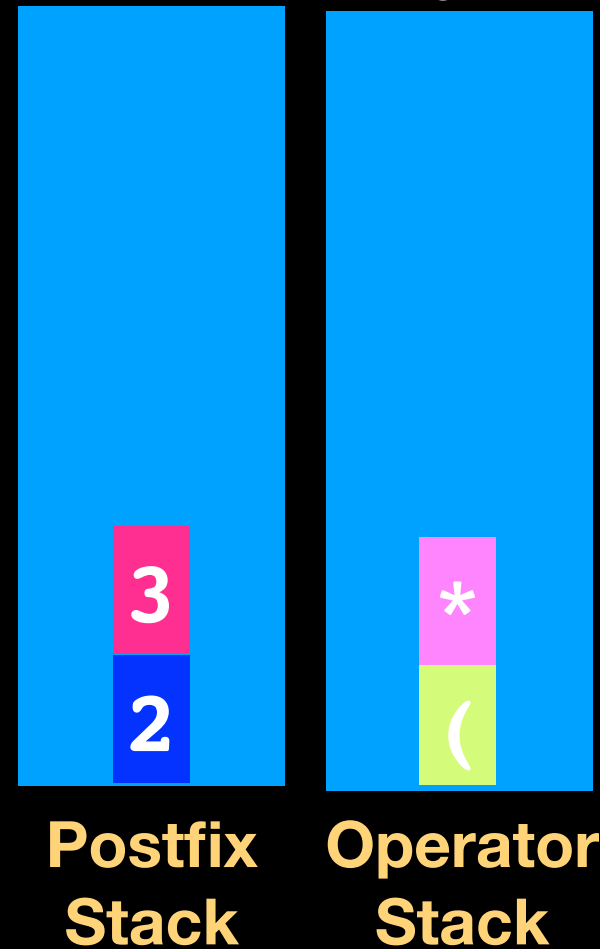
Postfix:
 $2 3 * 4 +$

1. Read characters onto corr. stack until ')' or end of string

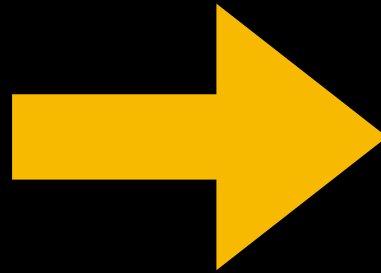
2. If reading a ')', move operators to Postfix Stack until a '(' discard it and continue reading string

3. Keep reading until ')' -> 2. or end of string -> 4.

4. Move operators to Postfix Stack

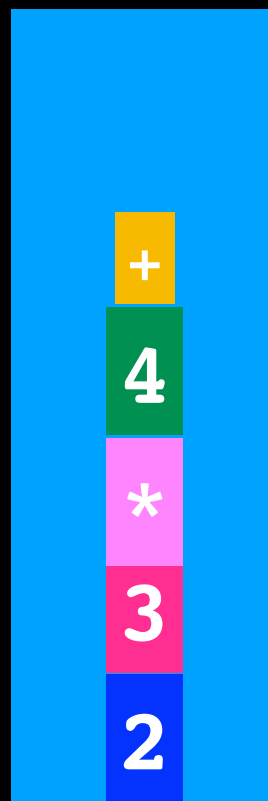


Infix:
 $(2 * 3) + 4$



Postfix:
 $2 3 * 4 +$

4. Move operators to
Postfix Stack



**Postfix
Stack**

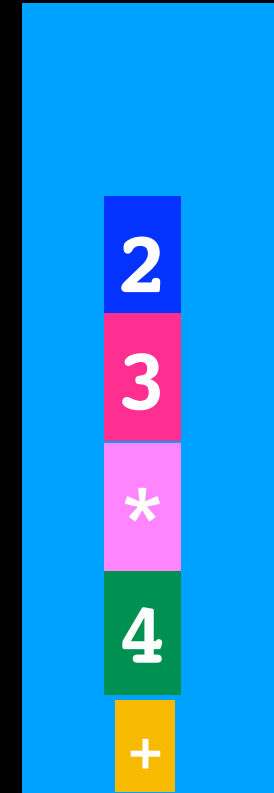


**Operator
Stack**

5. Pop and push onto empty
stack to invert, then print



**Postfix
Stack**

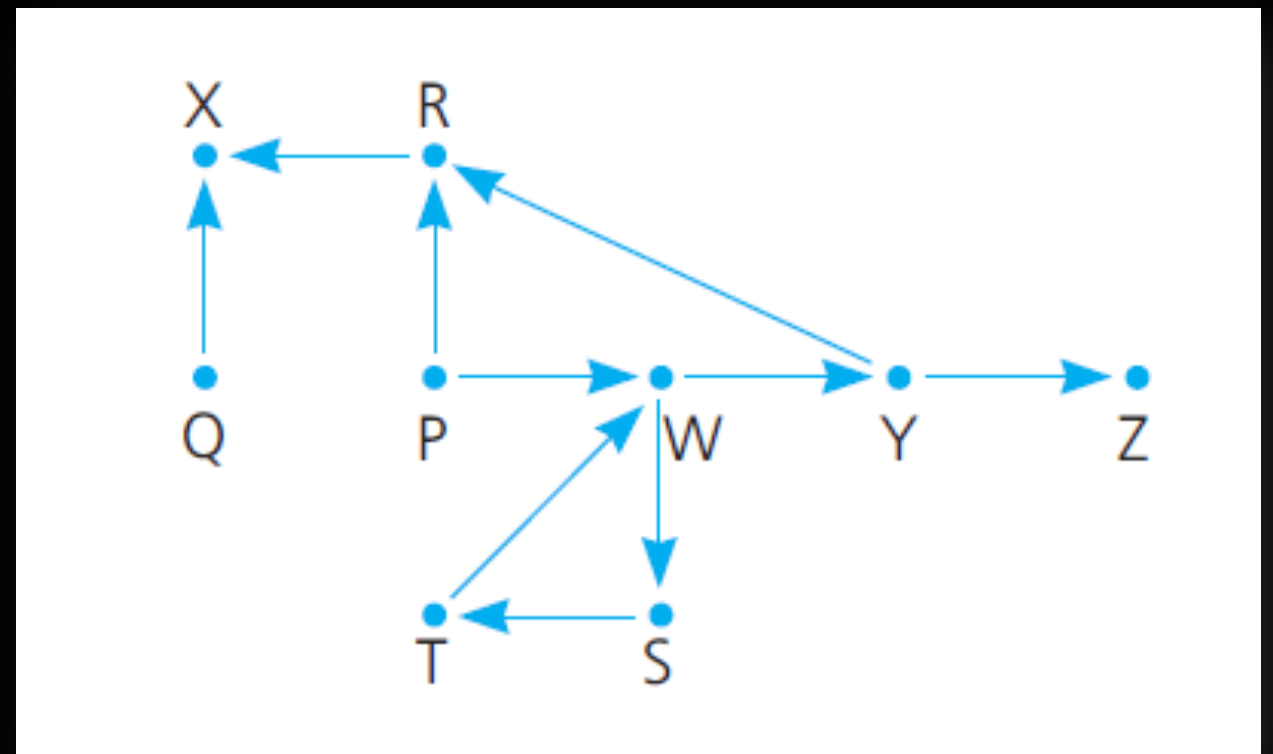


**Operator
Stack**

Search a Flight Map

Fly from Origin to Destination following map

1. Reach destination
2. Reach city with no departing flights (dead end)
3. Go in circles forever



Backtracking

Avoid dead end by backtracking

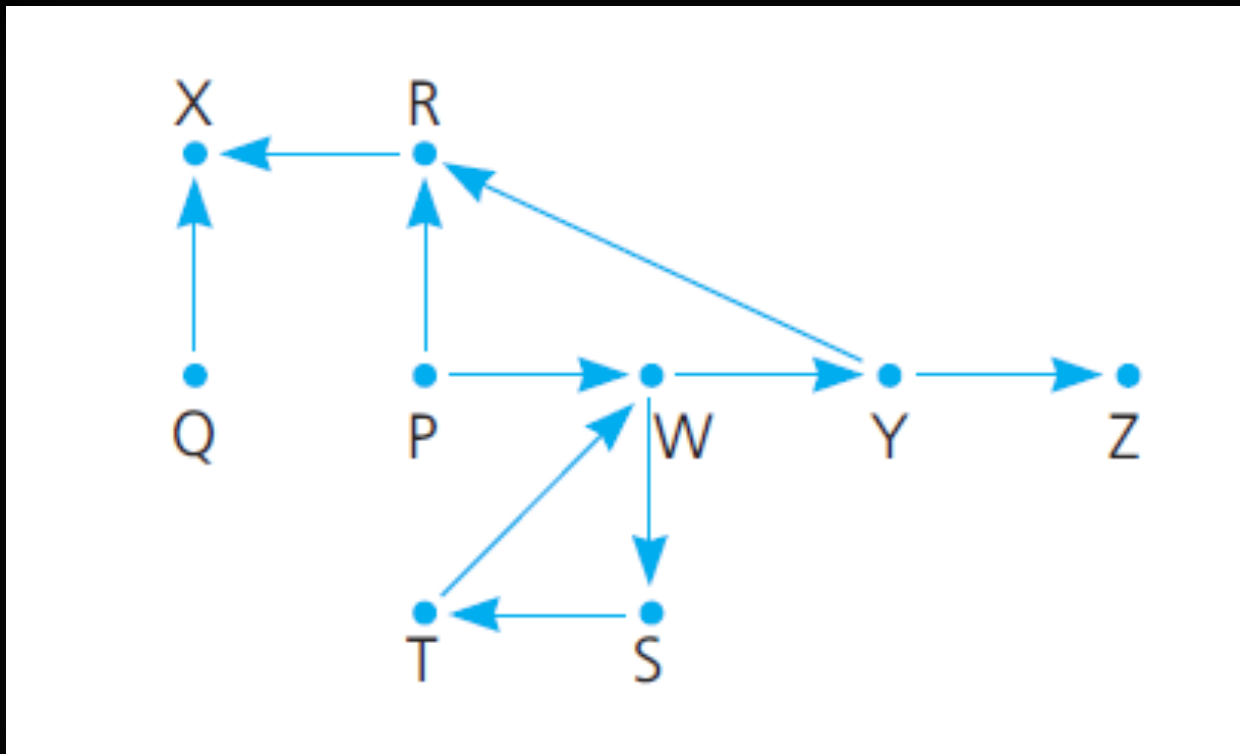
C = visited

C = backtracked

Avoid traveling in circles by marking visited cities

Origin = P , **Destination = Z**

P



Backtracking

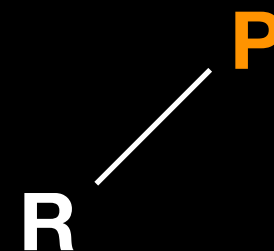
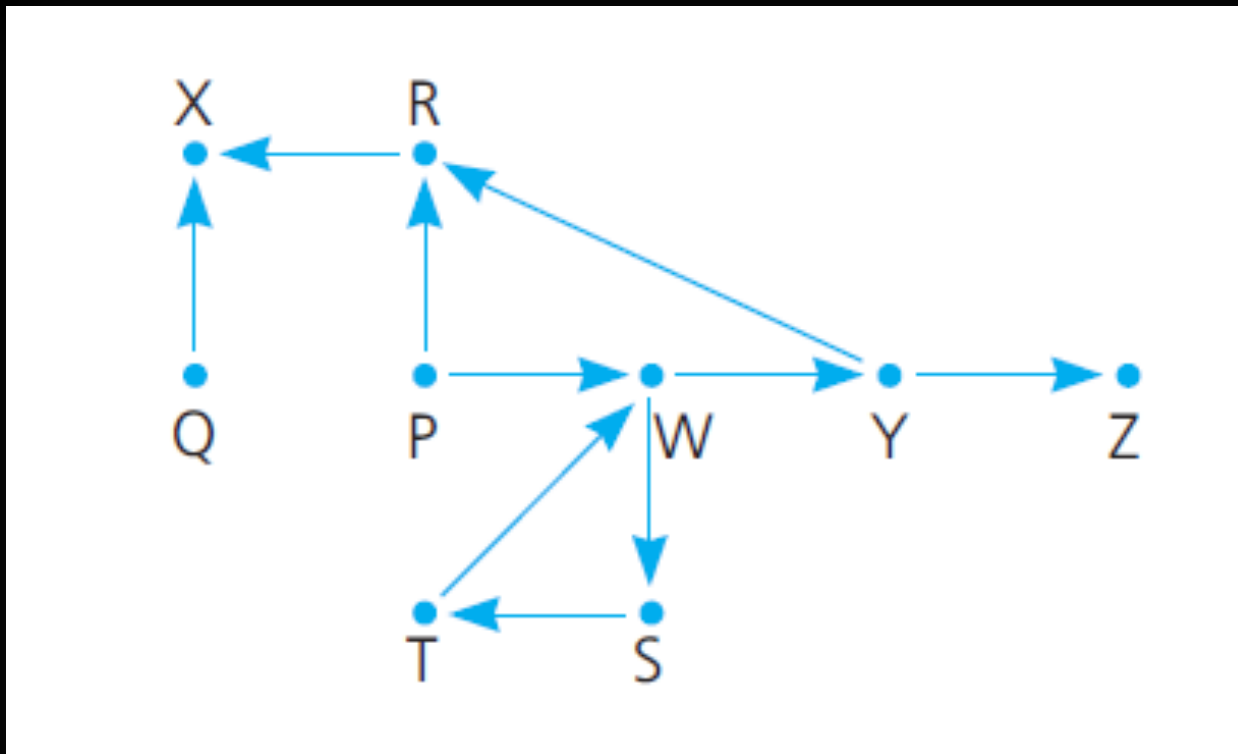
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Backtracking

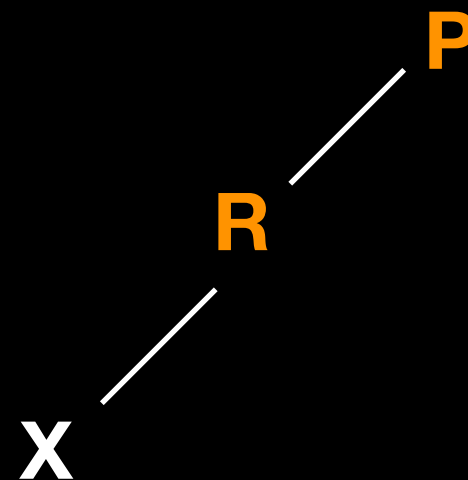
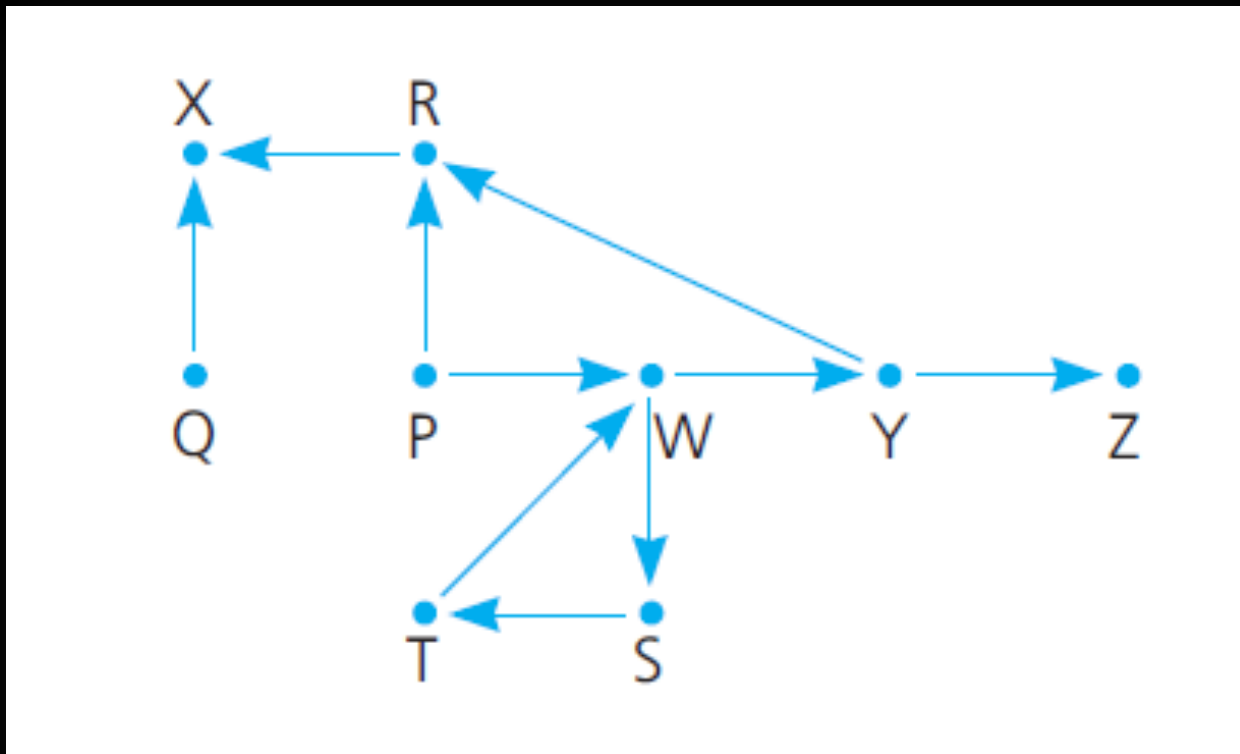
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Backtracking

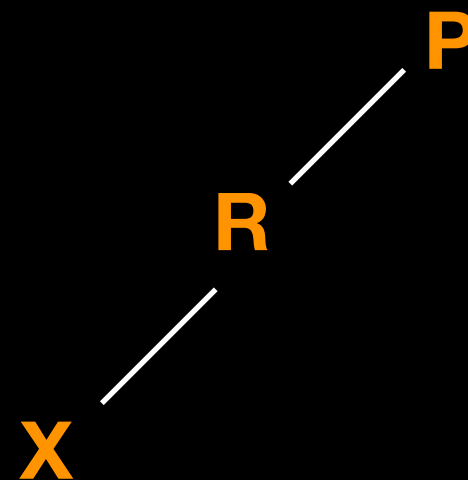
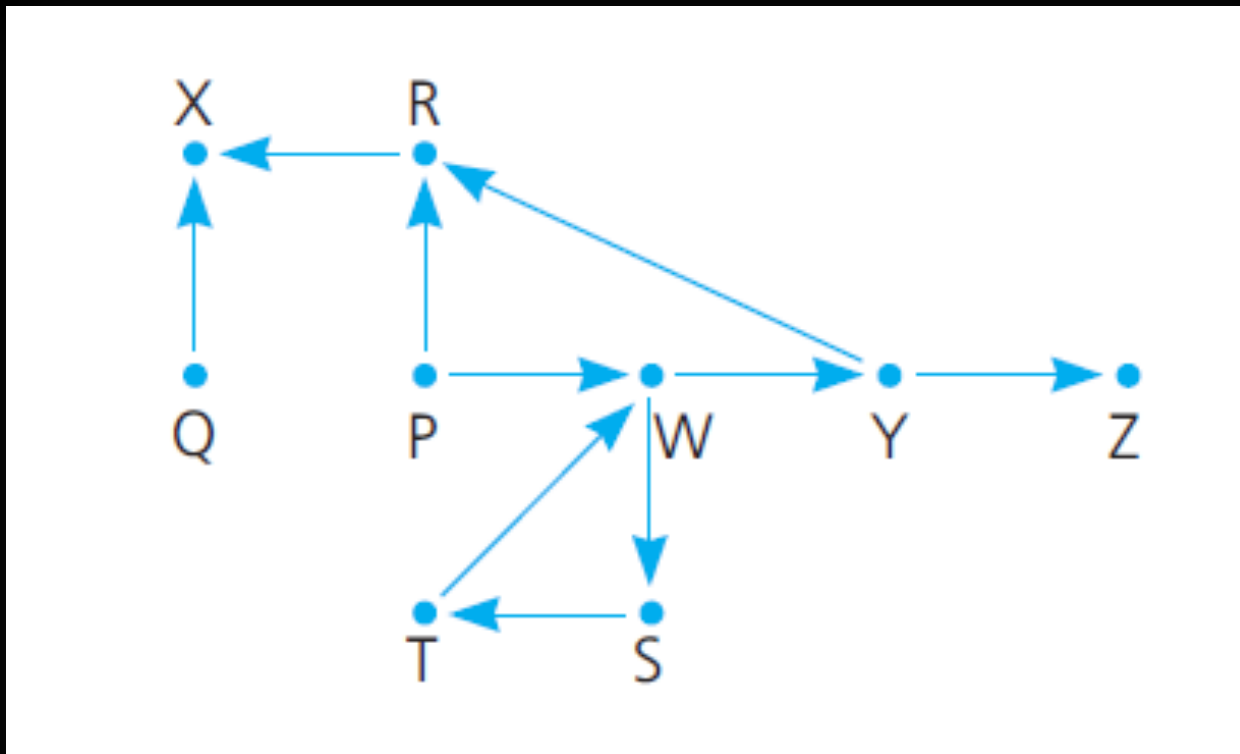
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Backtracking

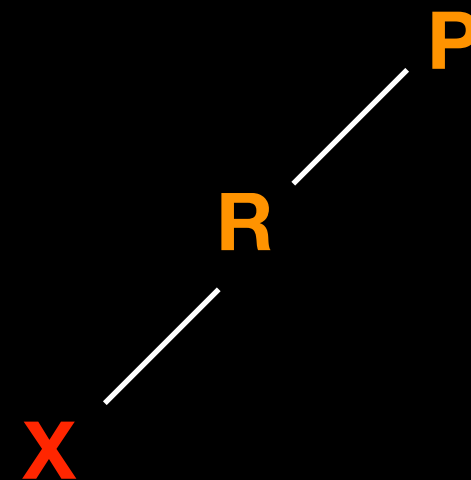
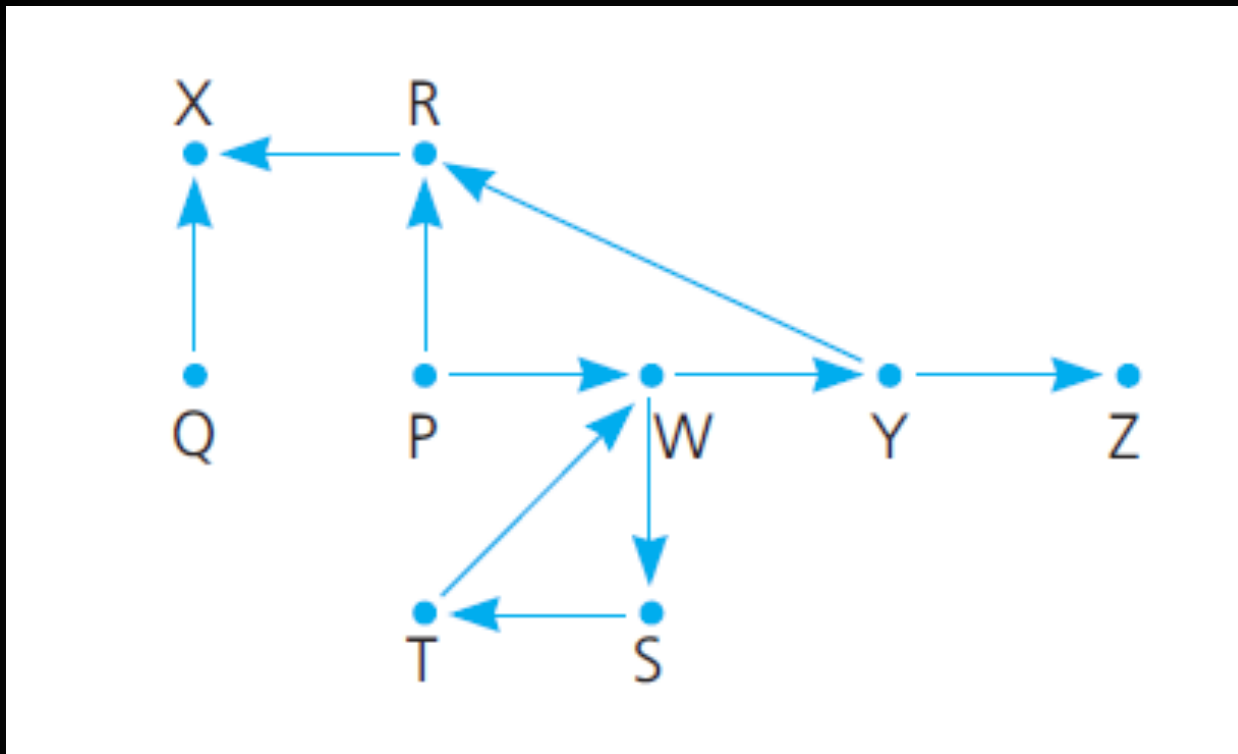
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Backtracking

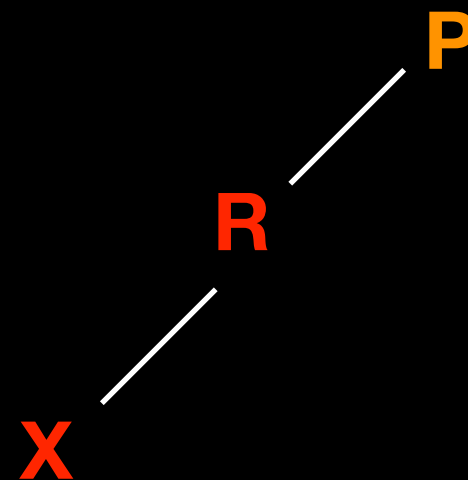
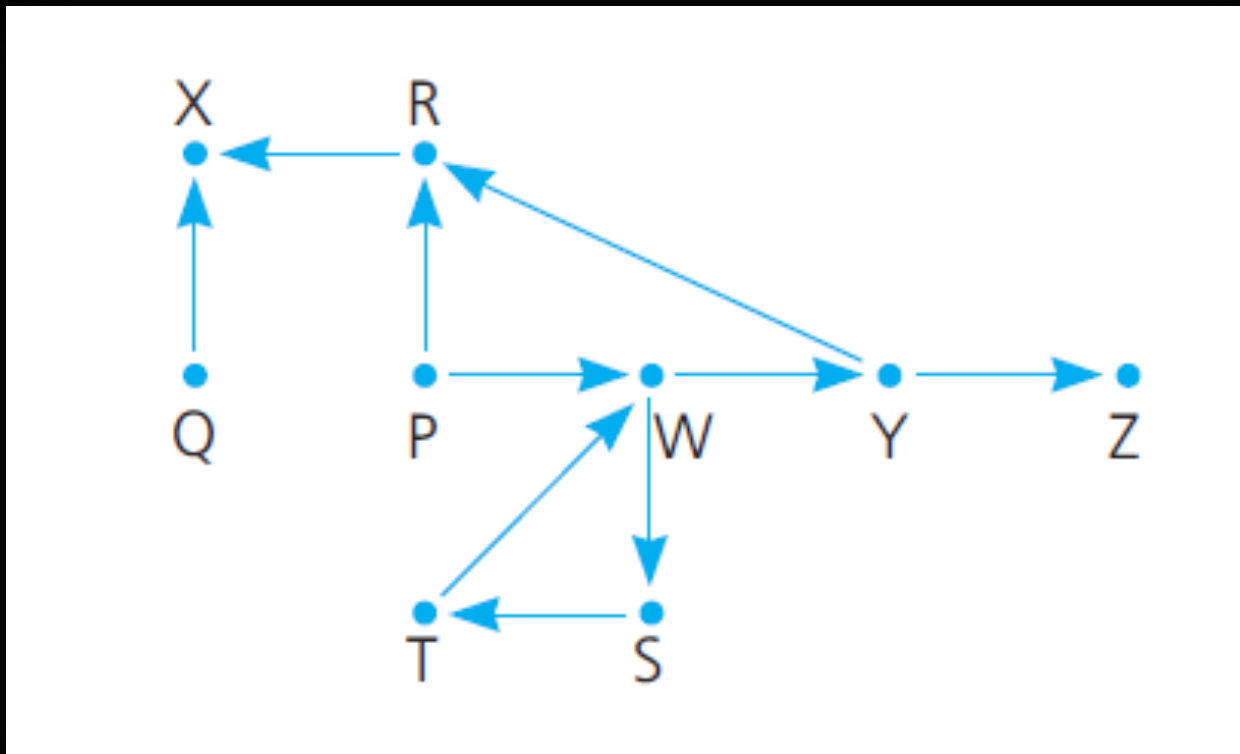
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Backtracking

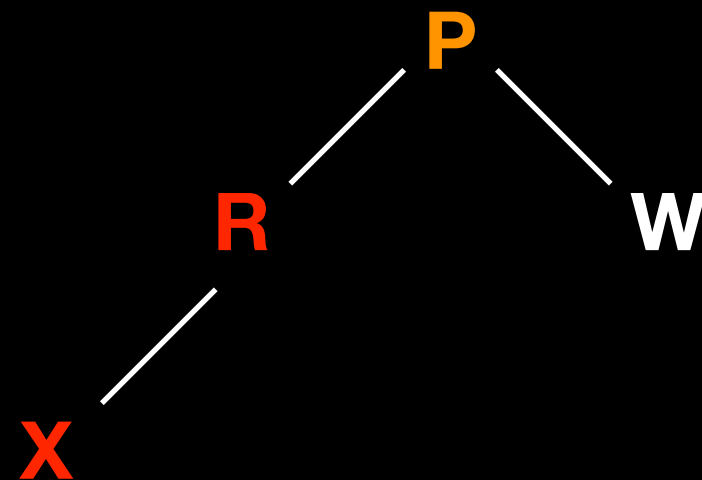
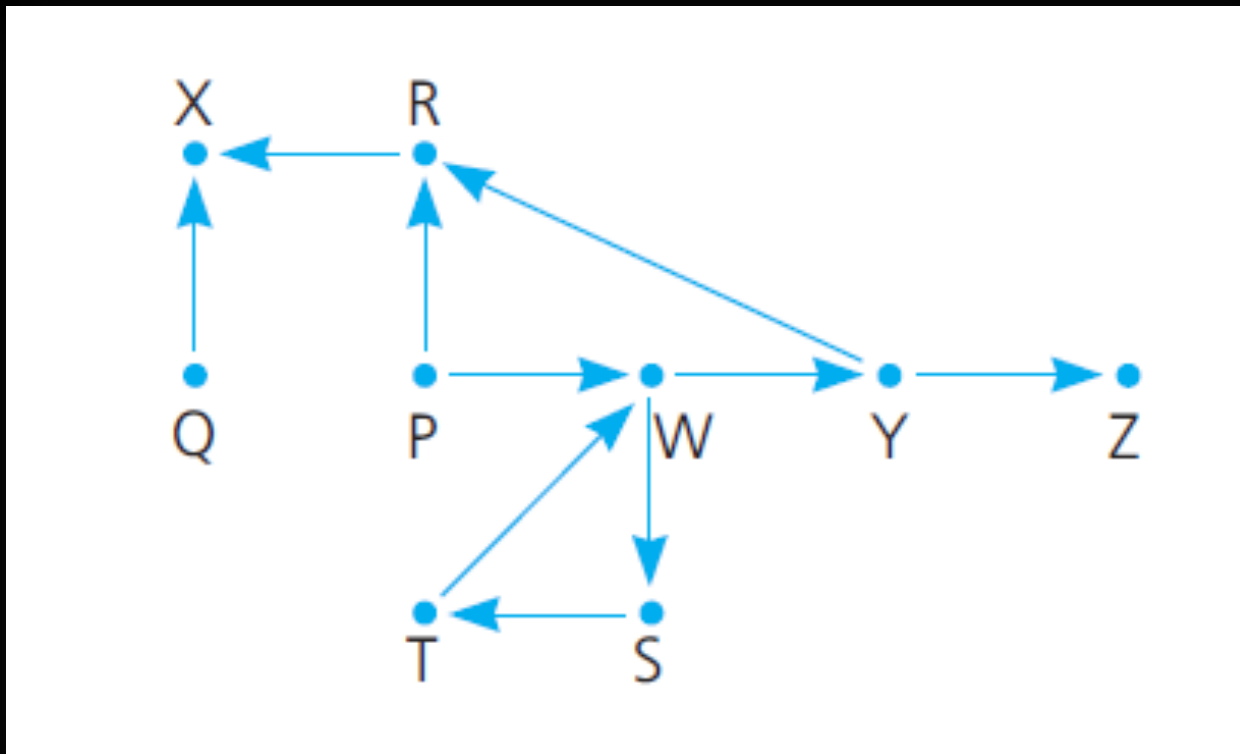
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Backtracking

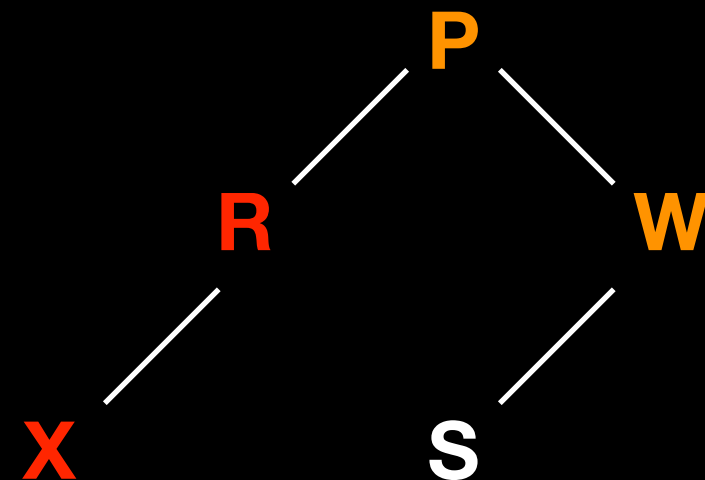
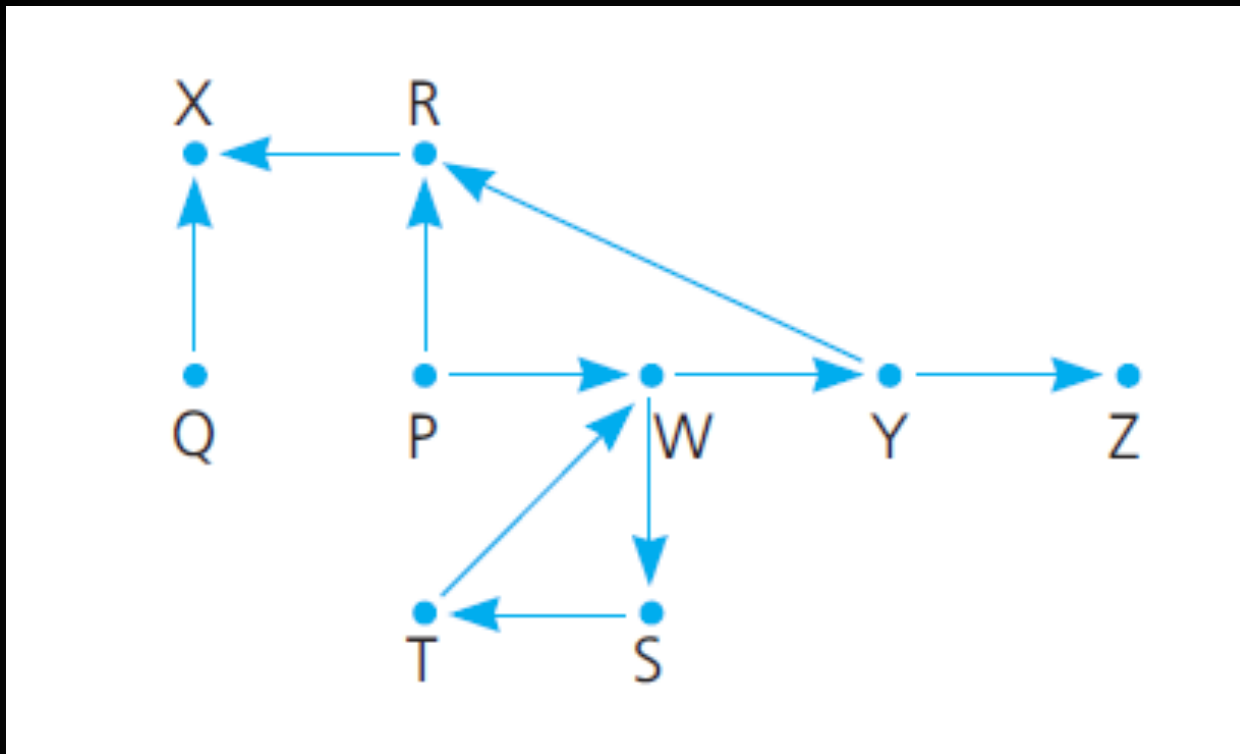
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Backtracking

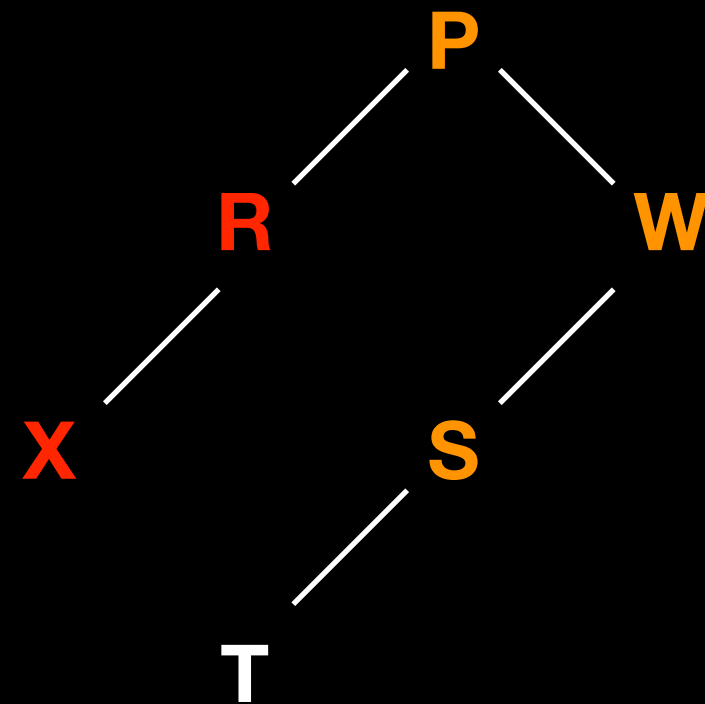
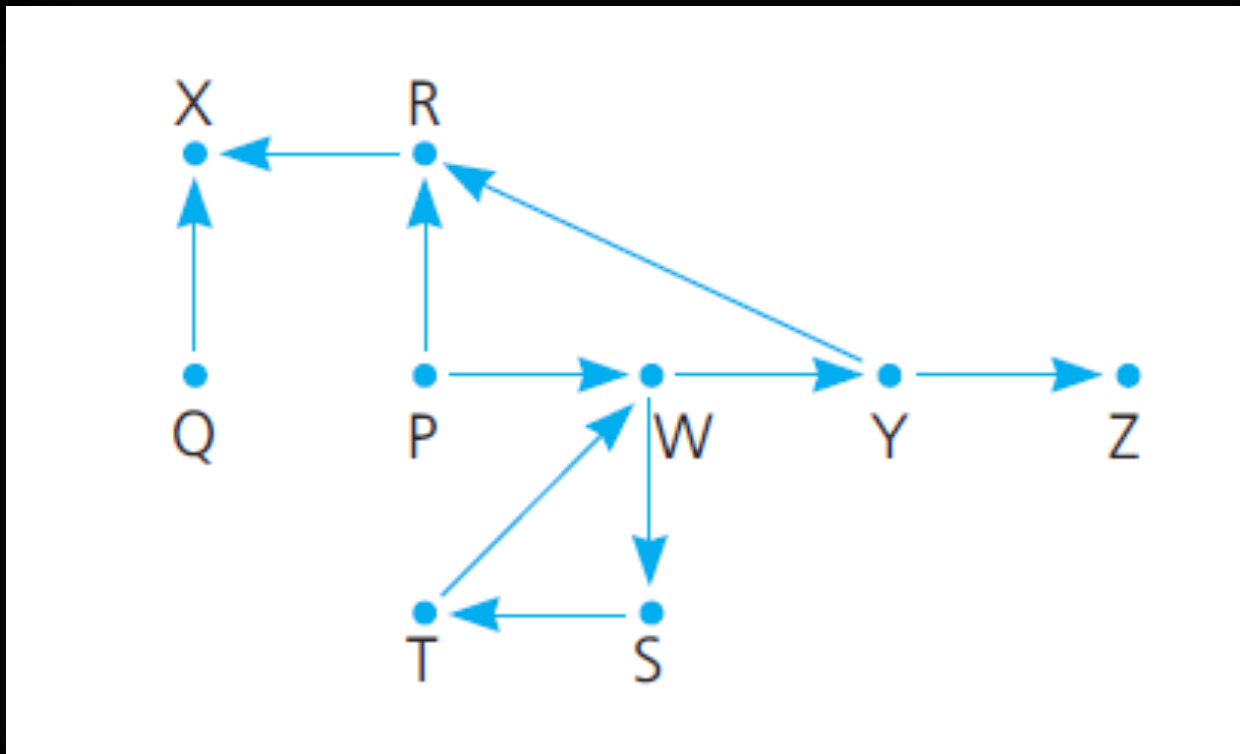
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Backtracking

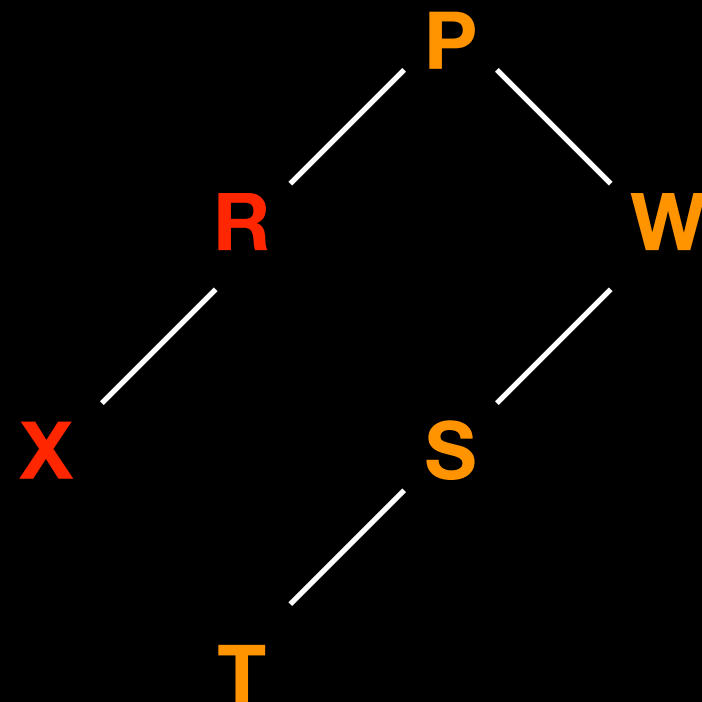
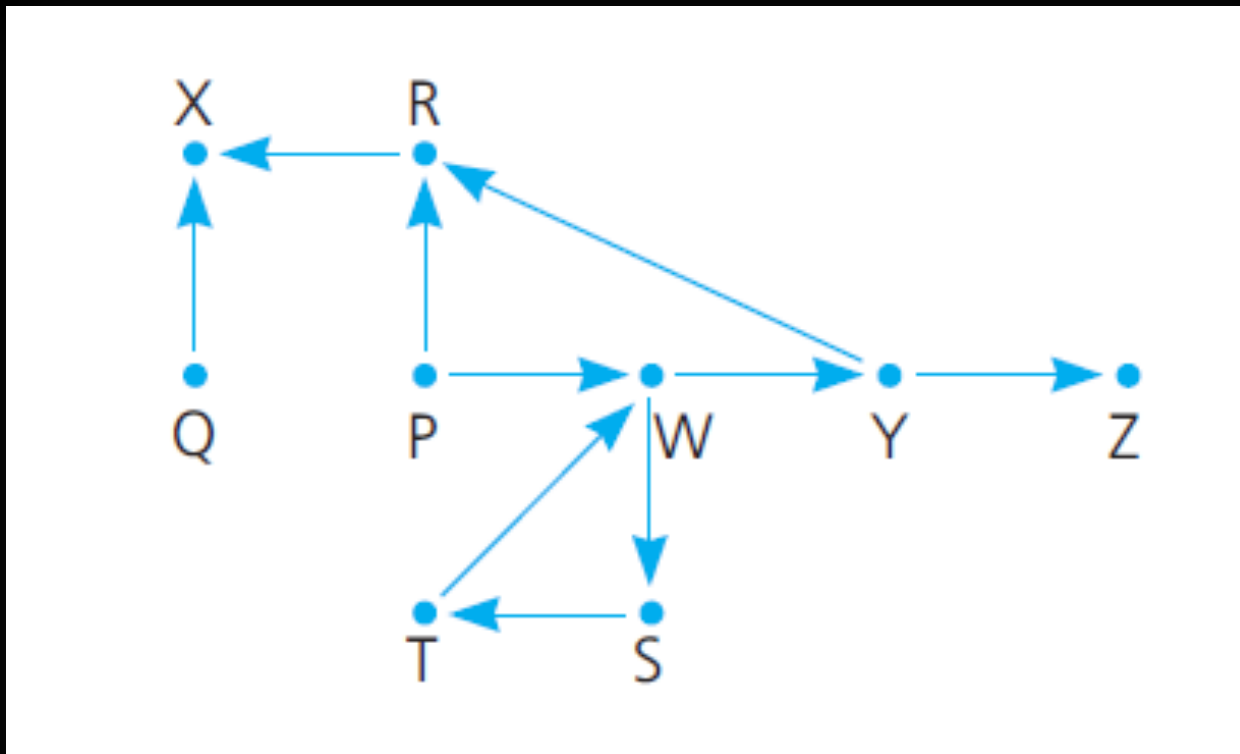
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Backtracking

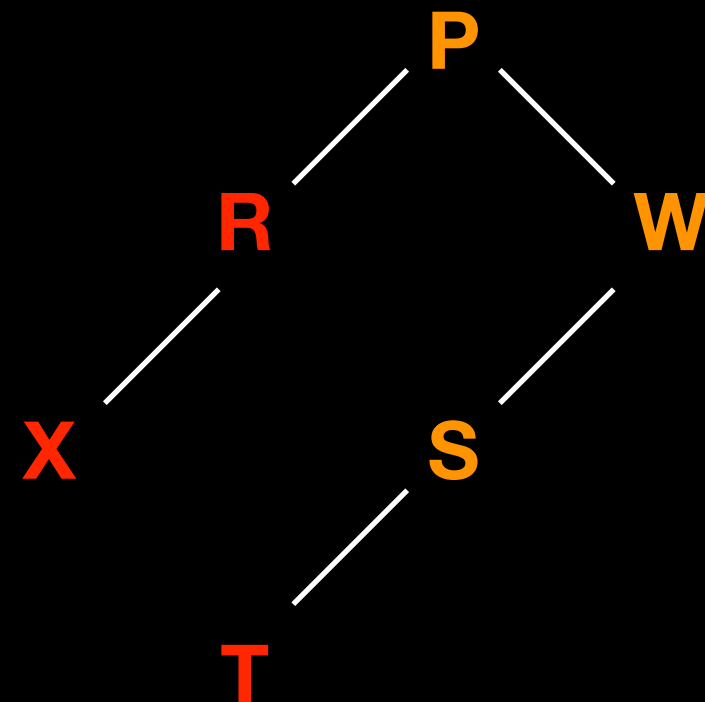
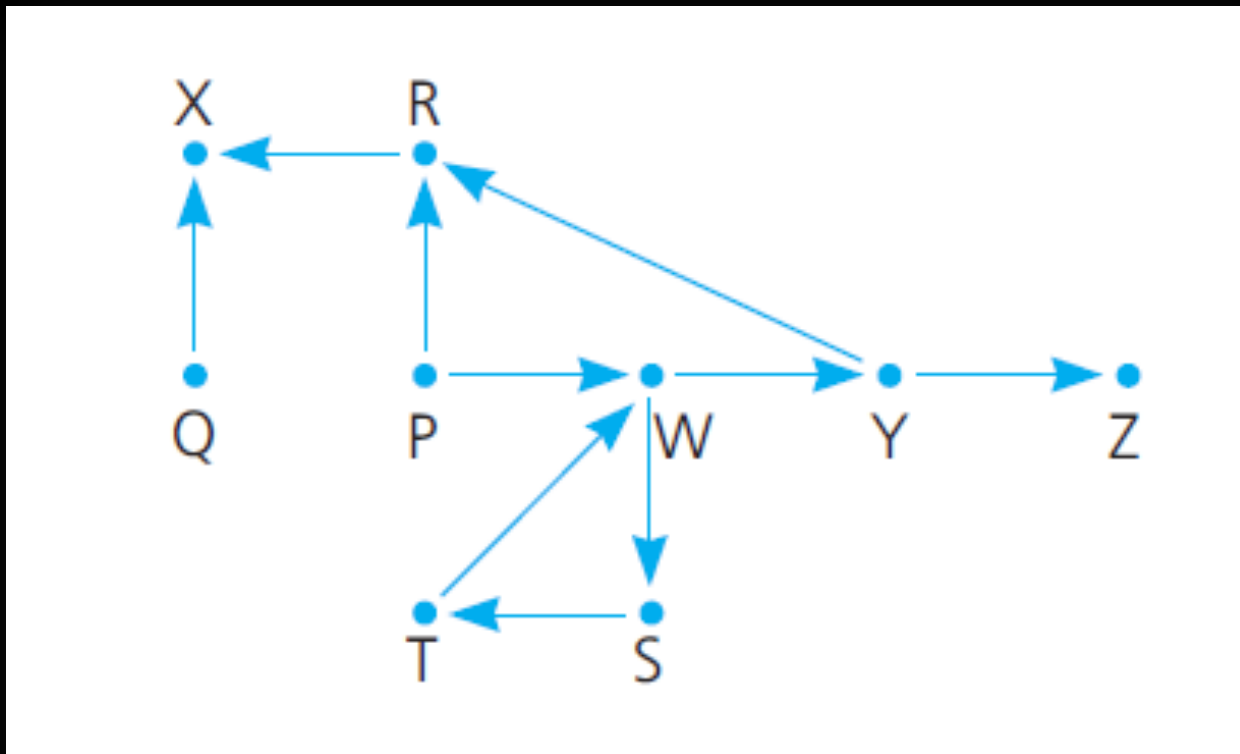
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Backtracking

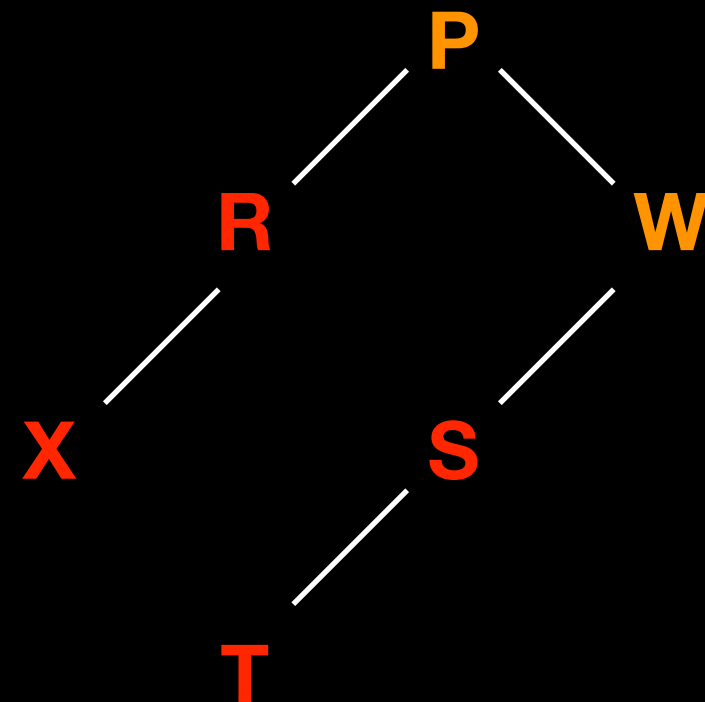
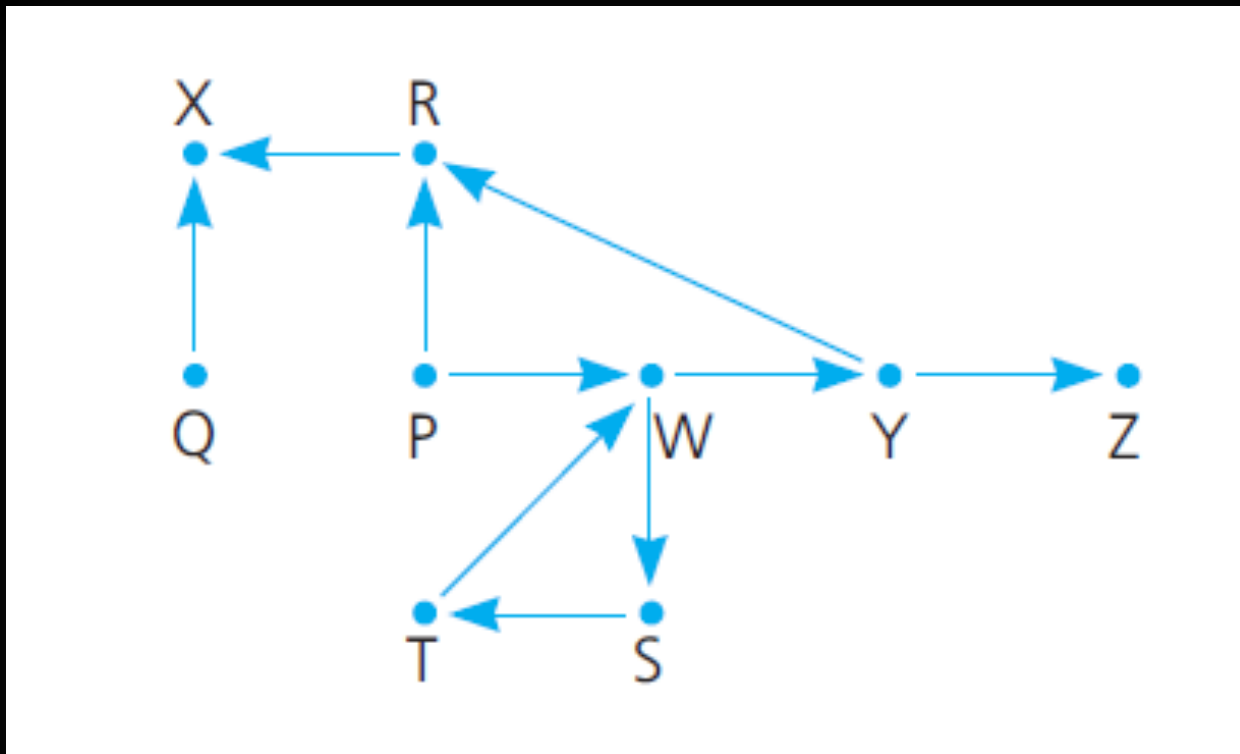
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Backtracking

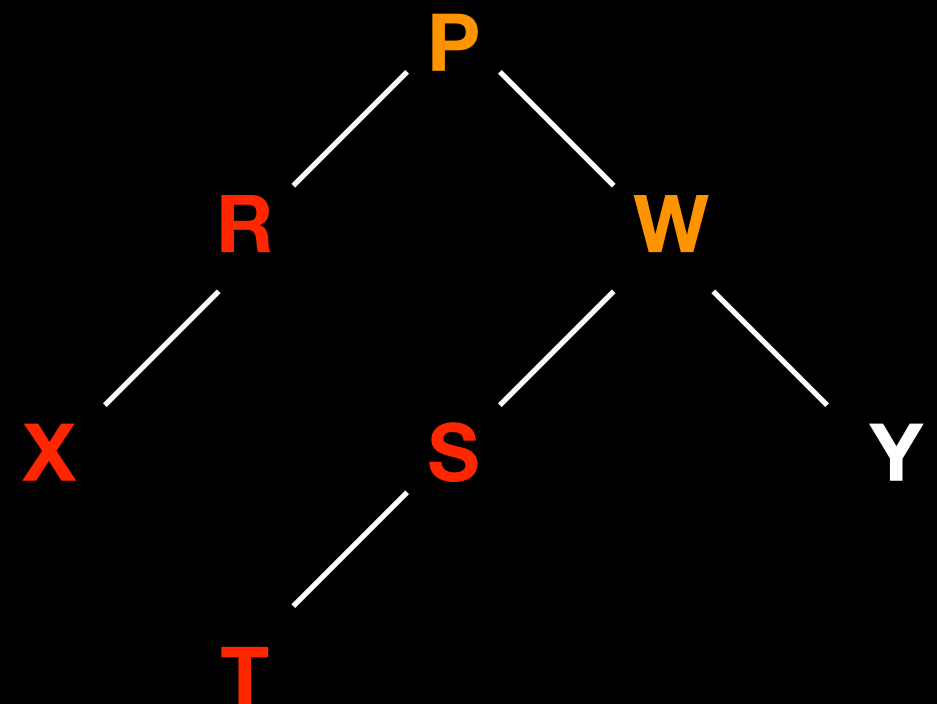
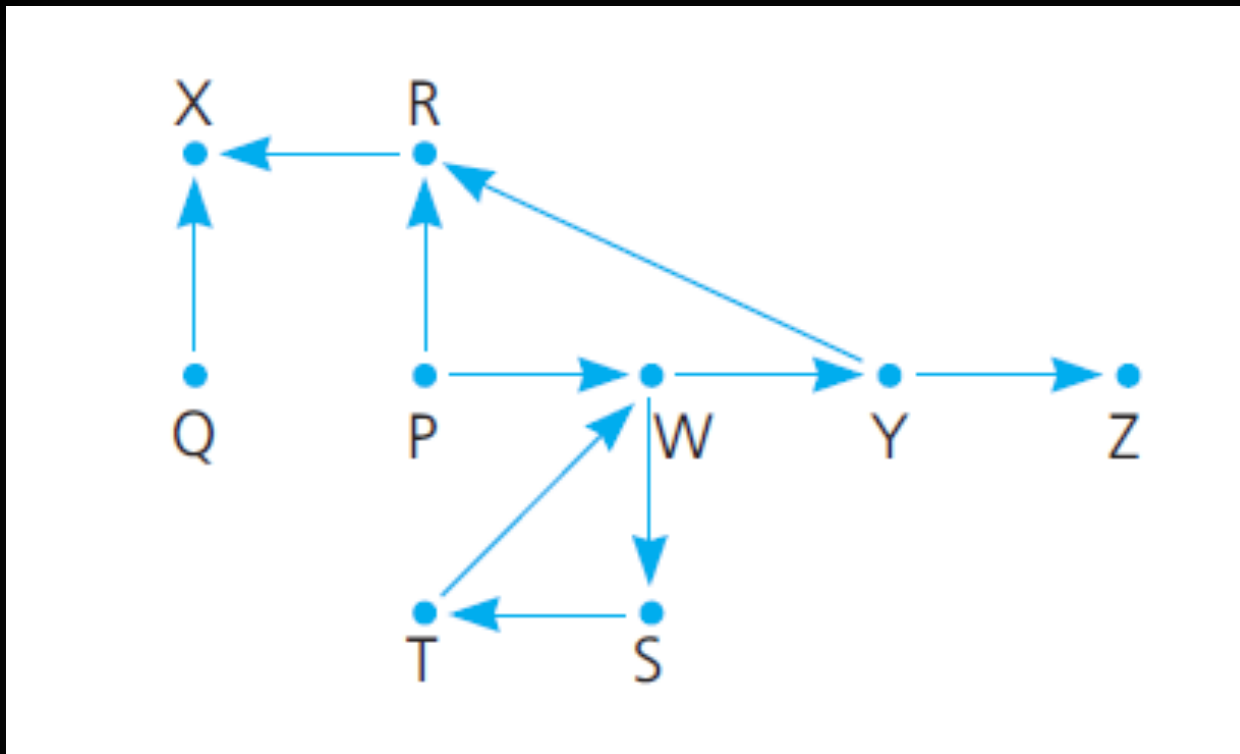
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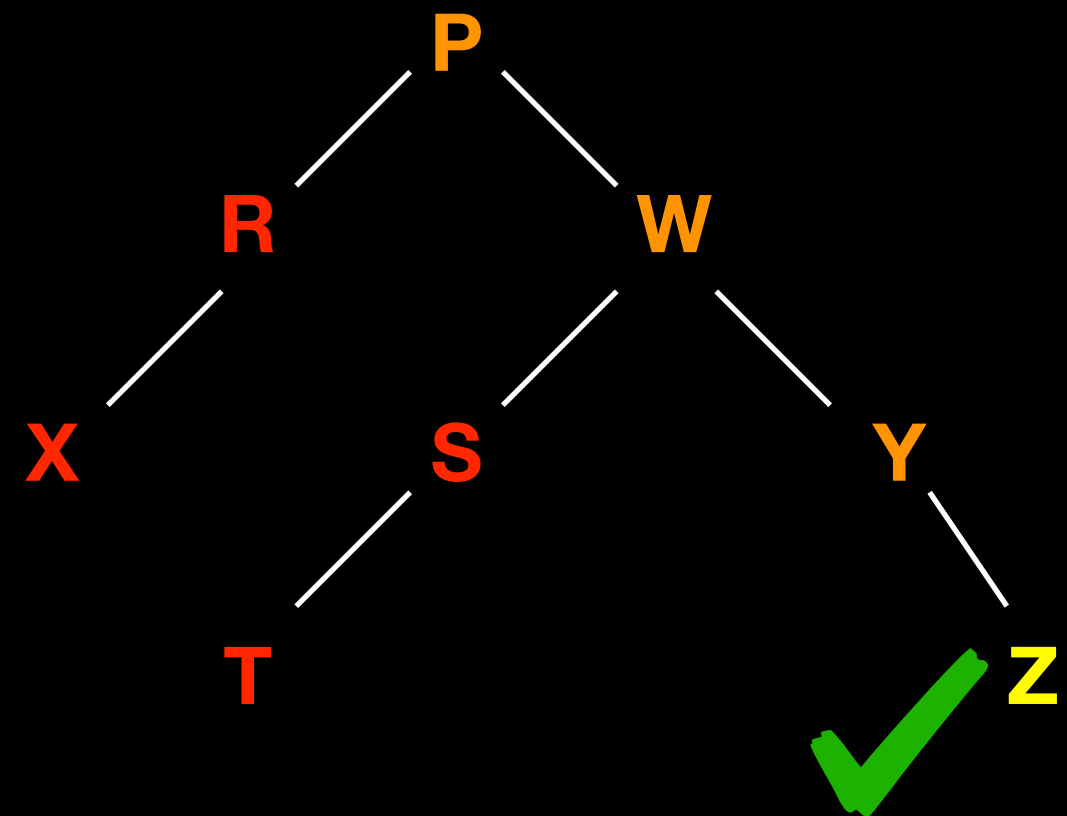
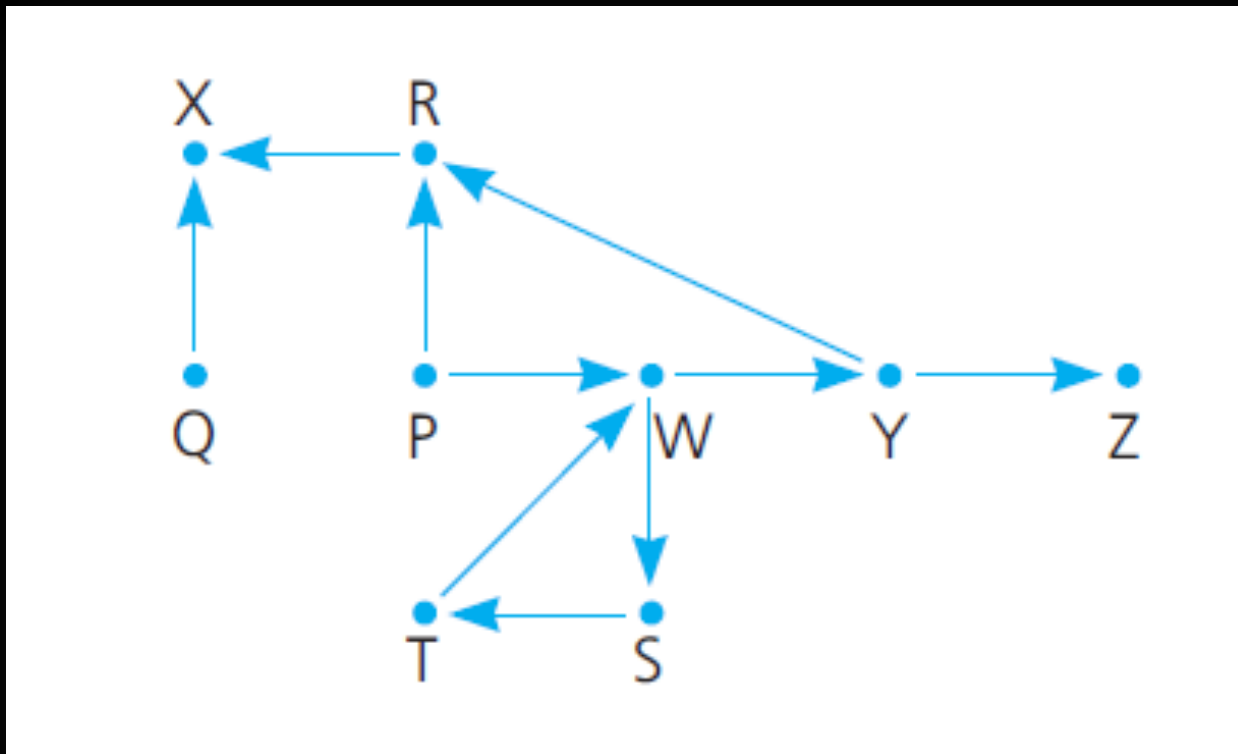
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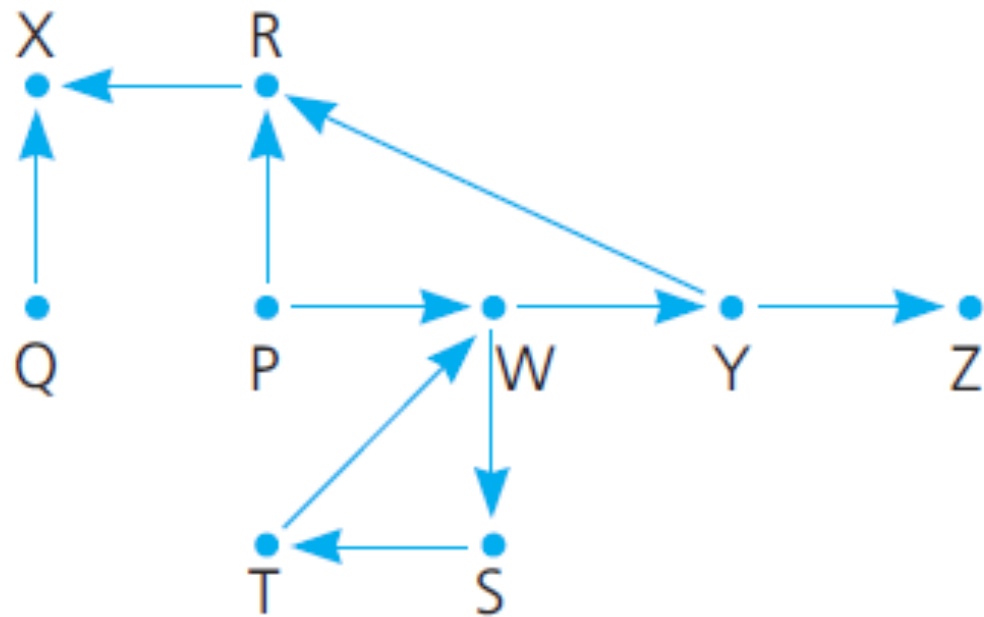
Avoid traveling in circles by marking visited cities

Origin = P , Destination = Z



Backtracking

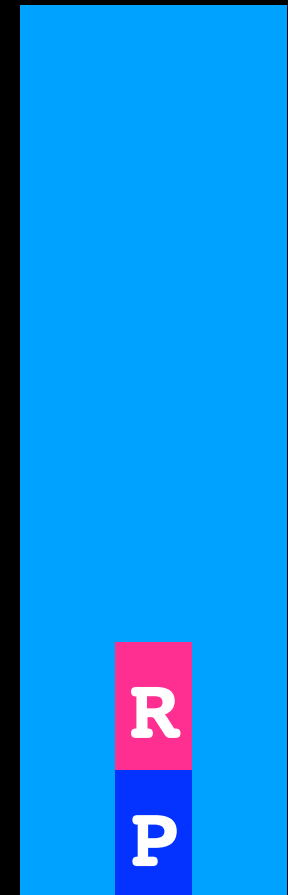
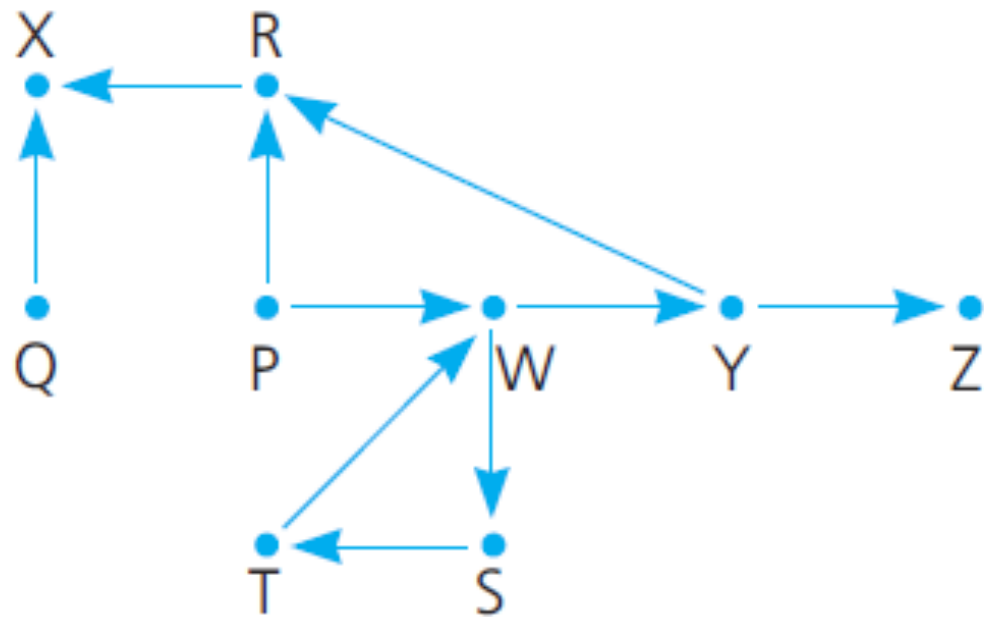
Origin = P , Destination = Z



P

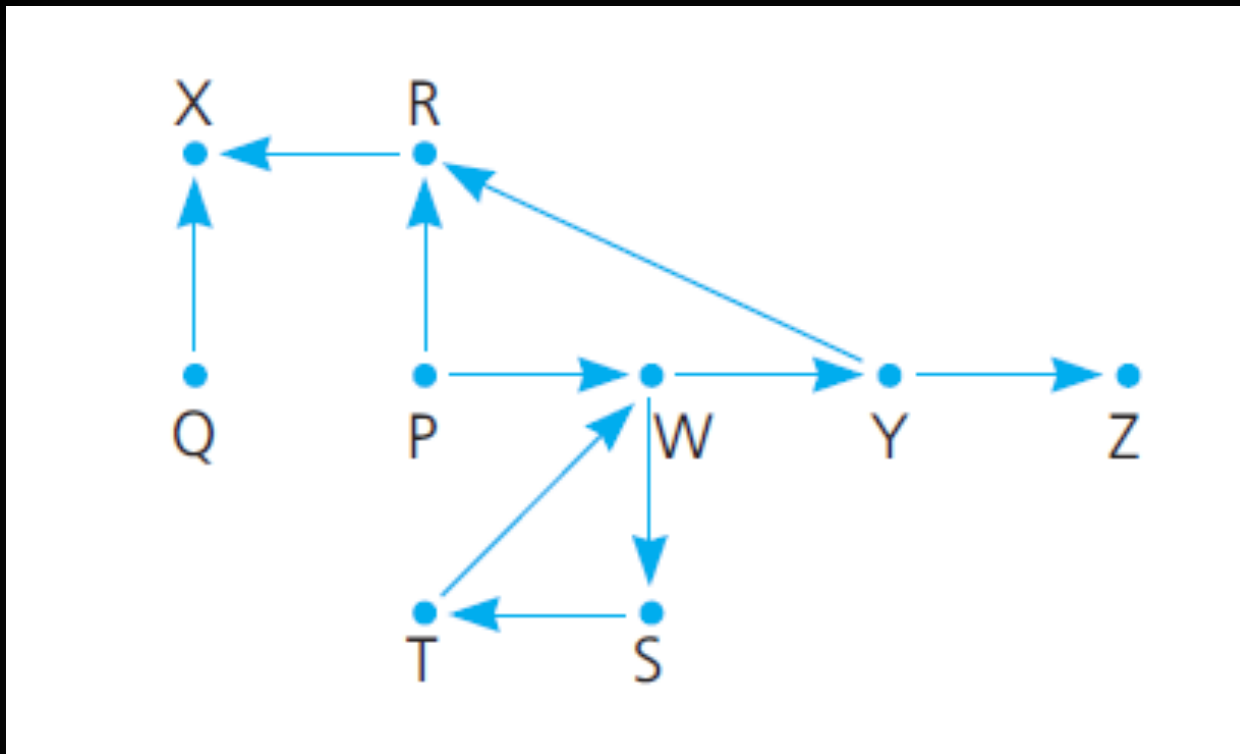
Backtracking

Origin = P , Destination = Z



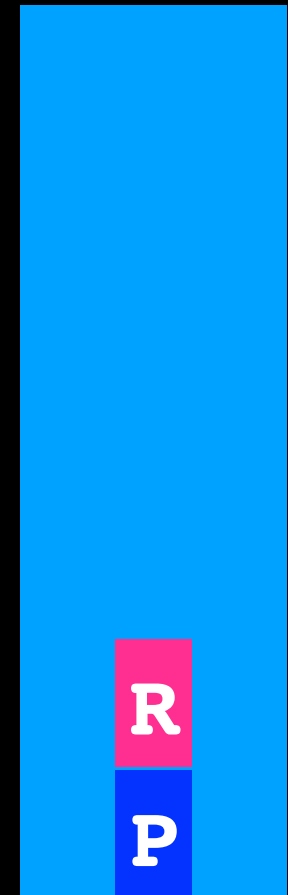
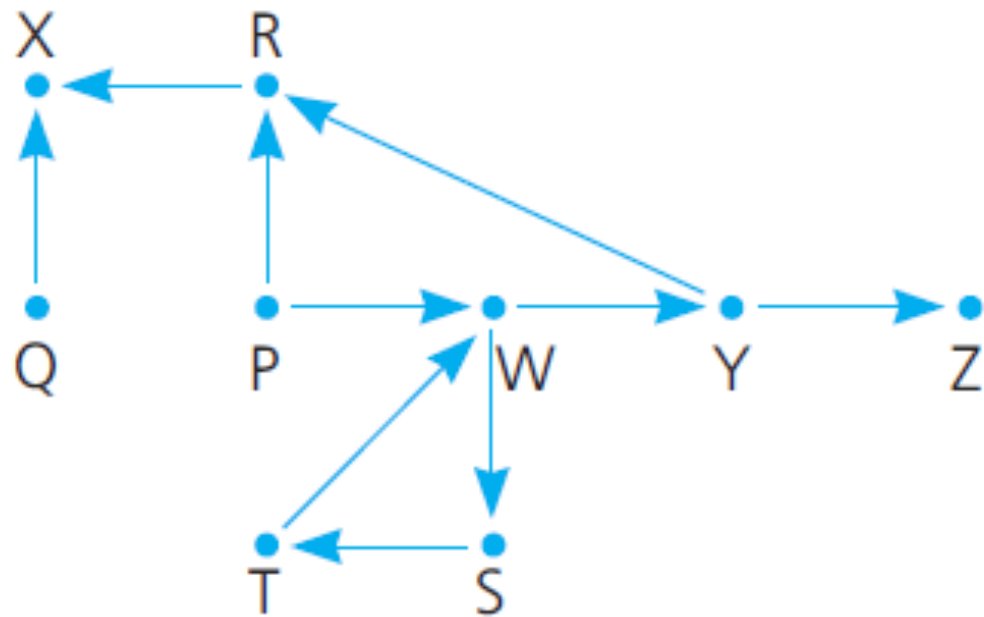
Backtracking

Origin = P , Destination = Z



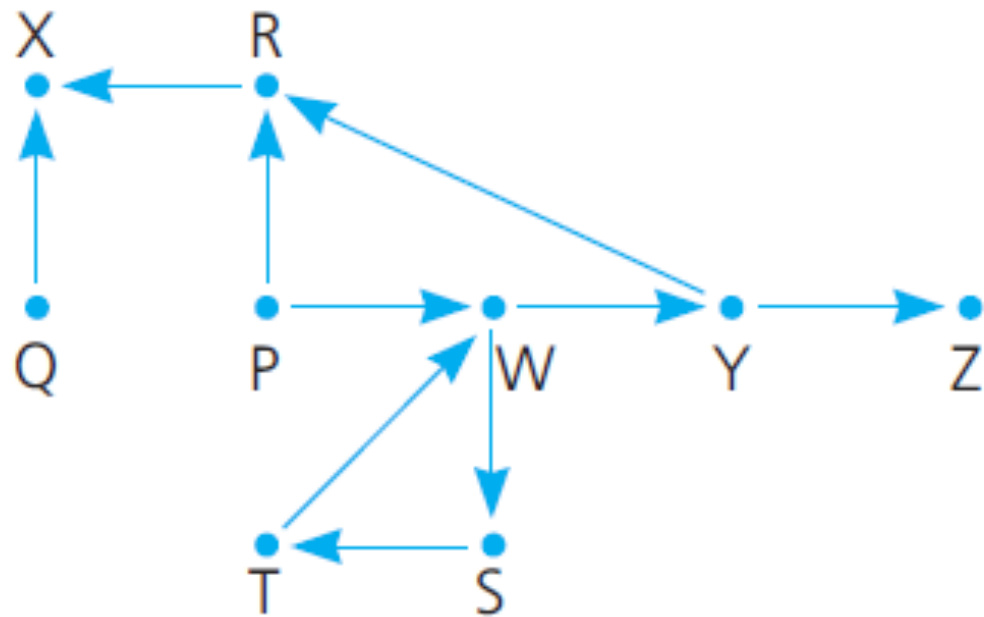
Backtracking

Origin = P , Destination = Z



Backtracking

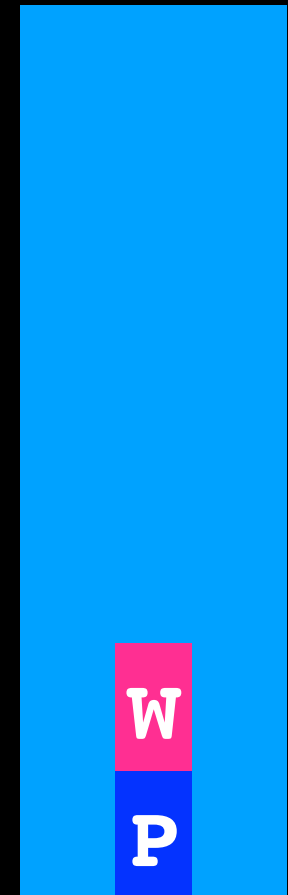
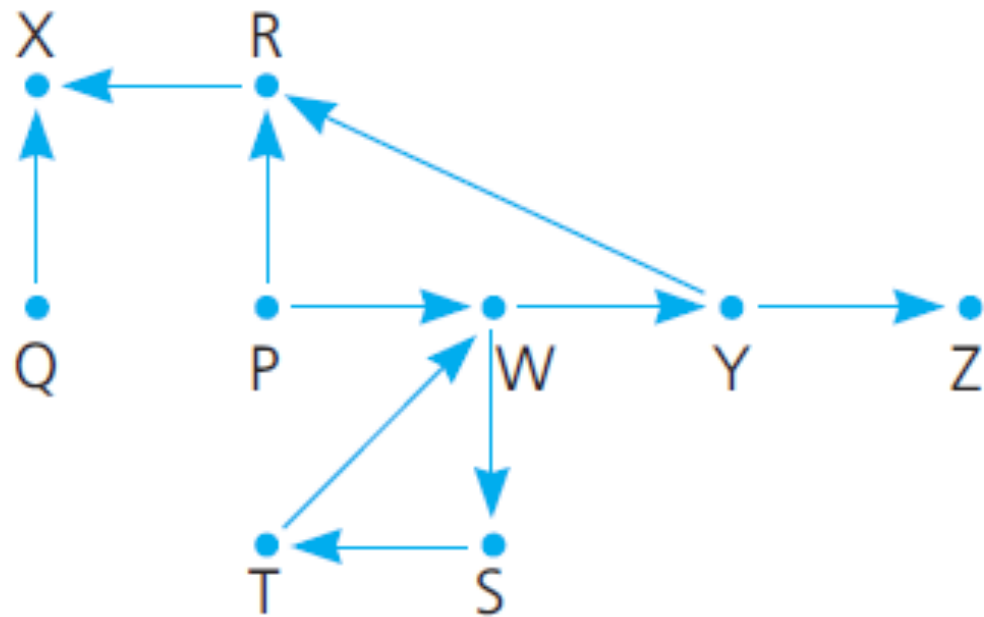
Origin = P , Destination = Z



P

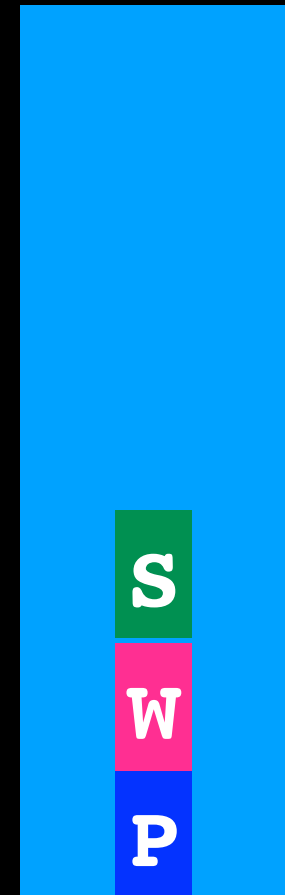
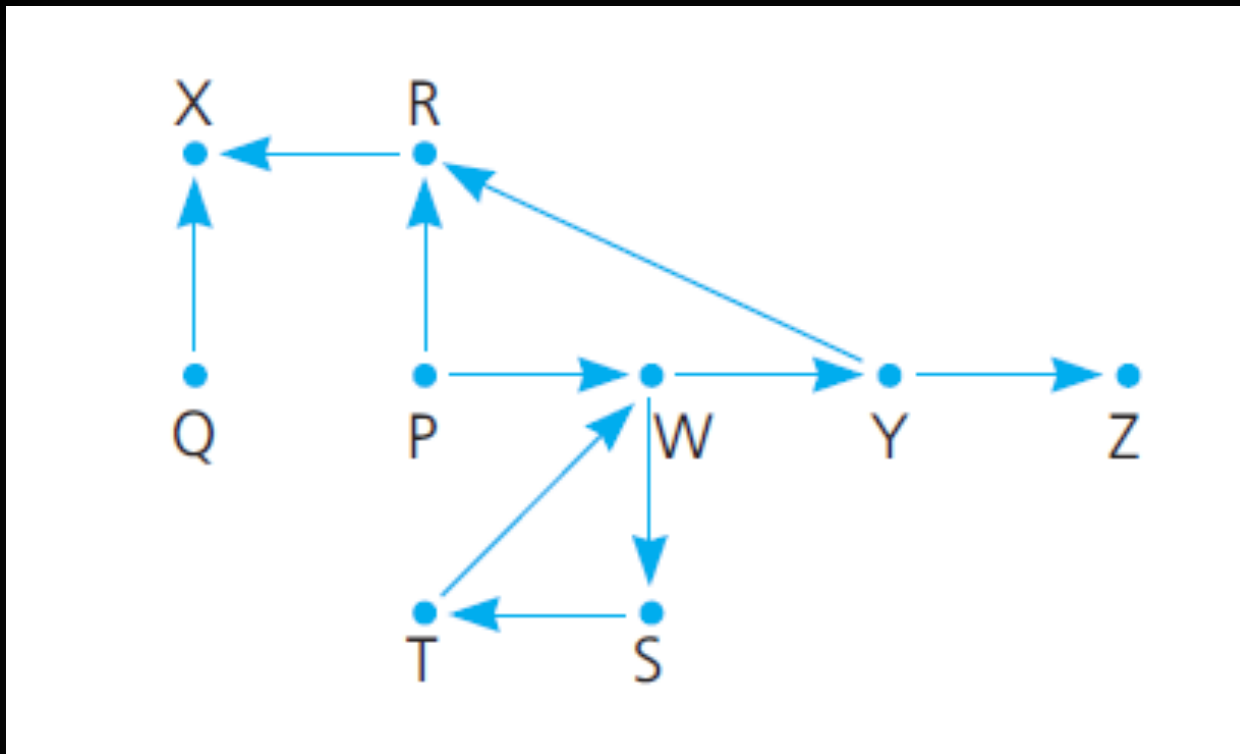
Backtracking

Origin = P , Destination = Z



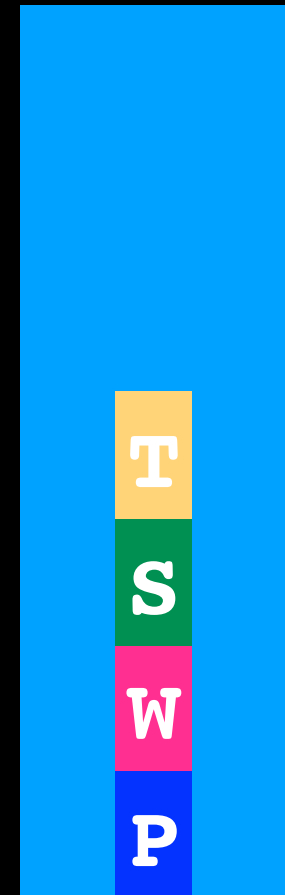
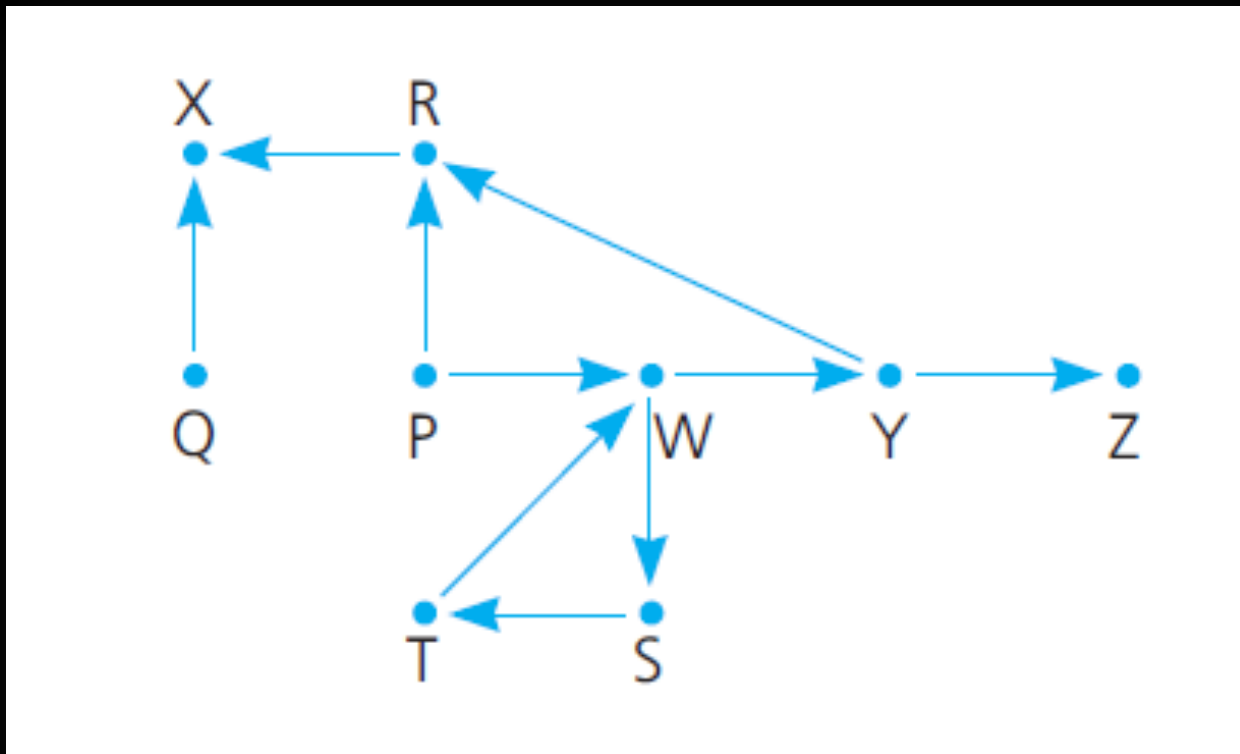
Backtracking

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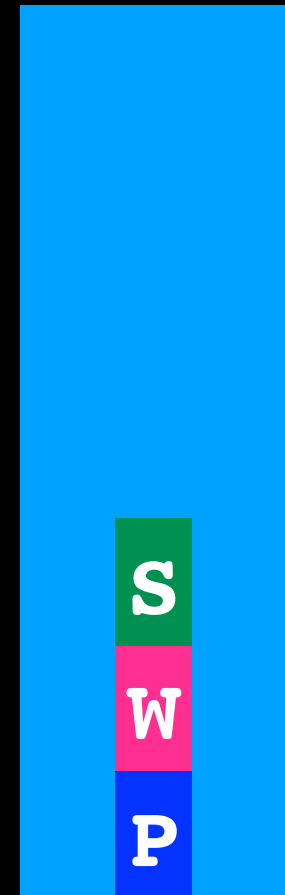
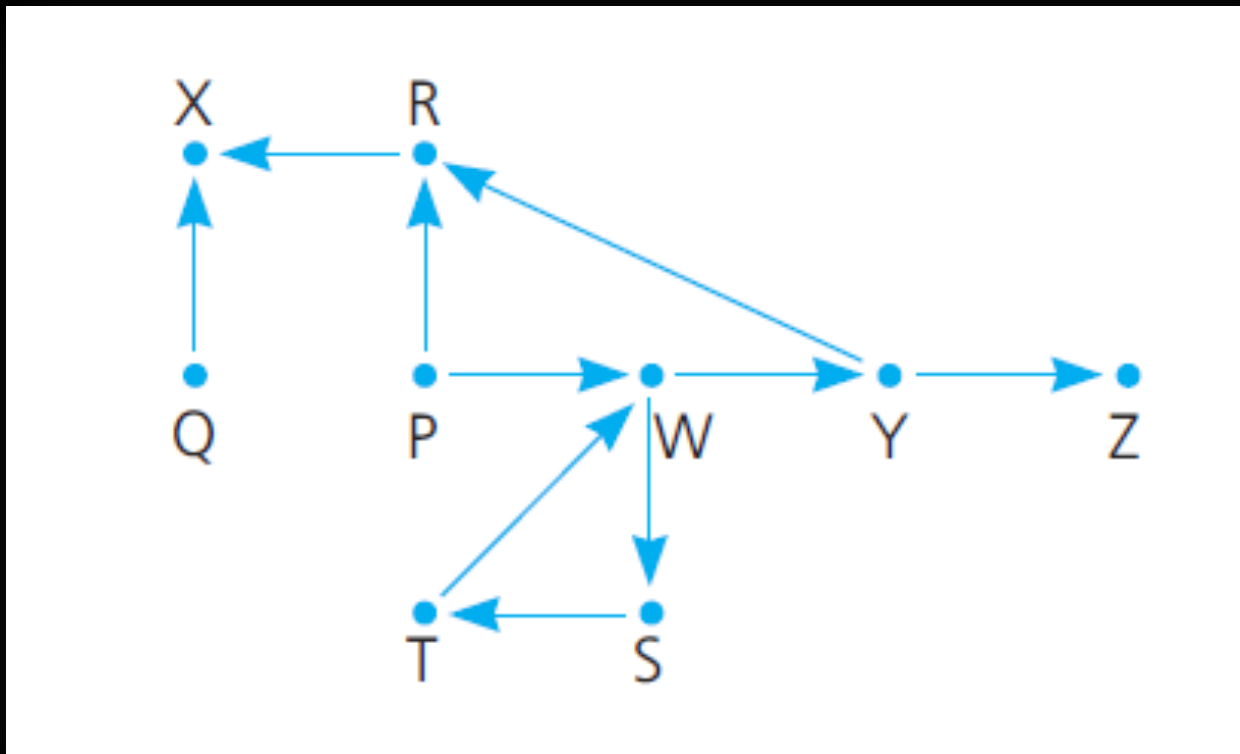
Backtracking

Origin = P , Destination = Z



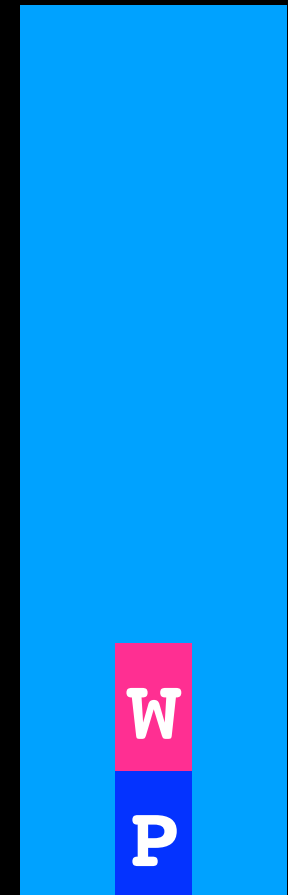
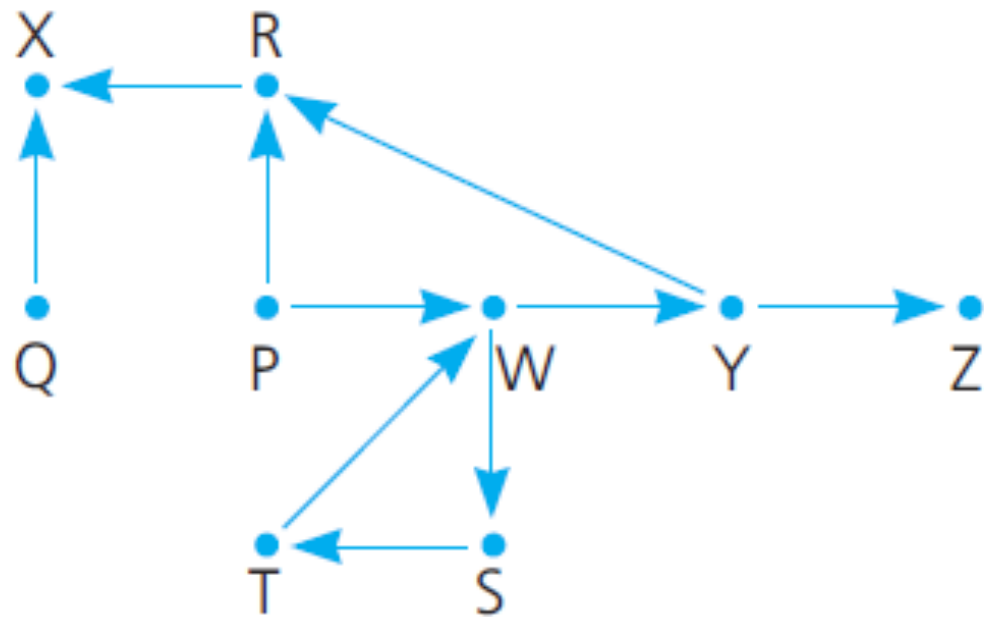
Backtracking

Origin = P , Destination = Z



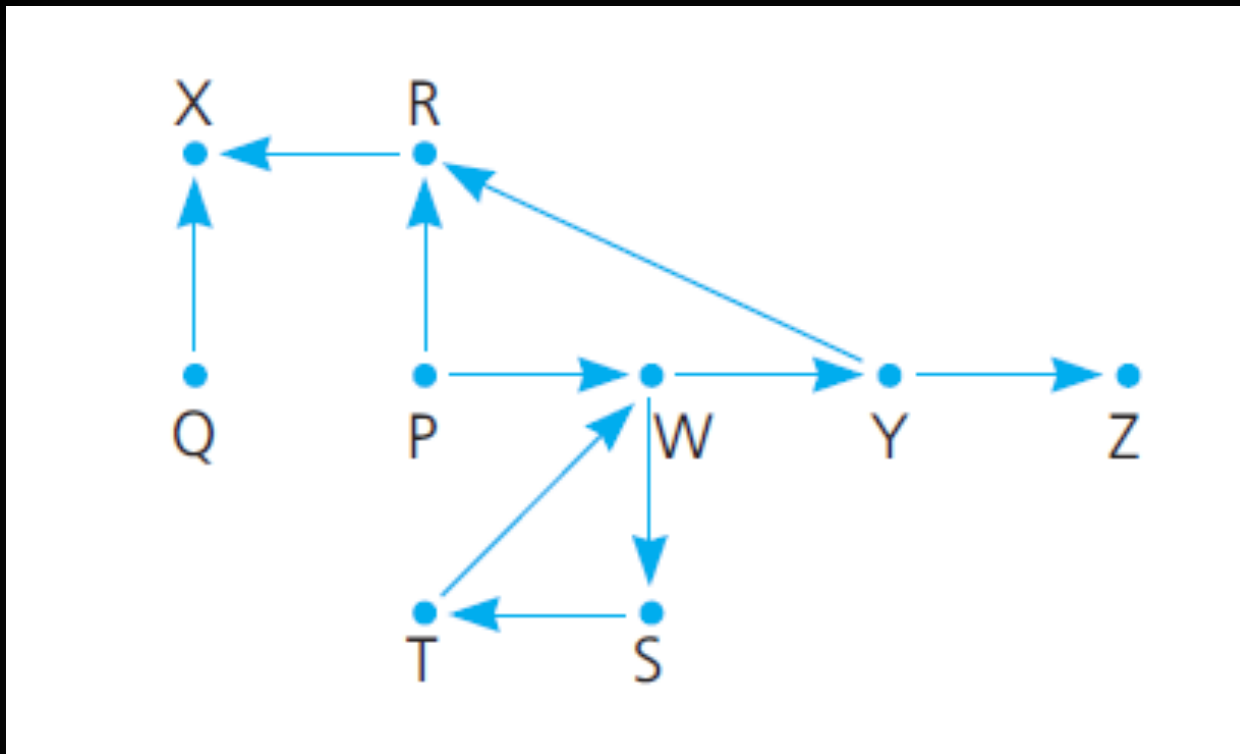
Backtracking

Origin = P , Destination = Z



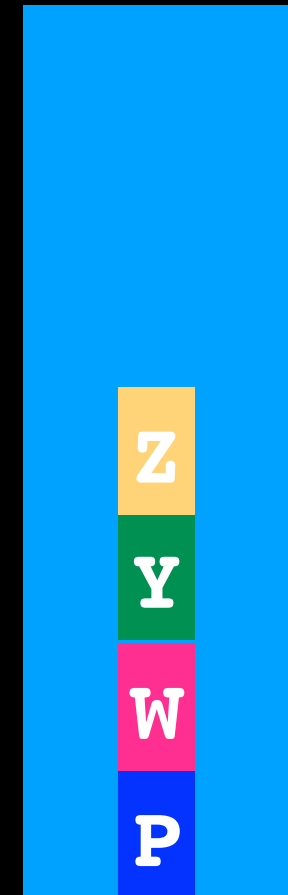
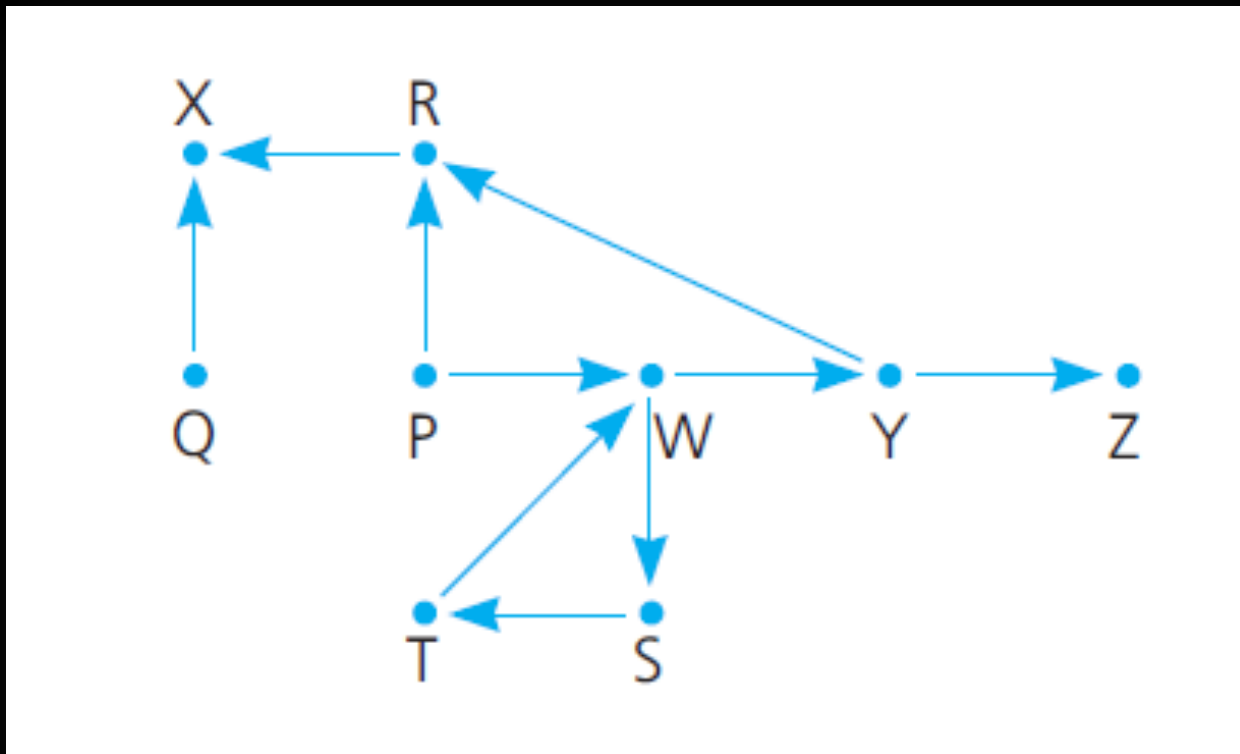
Backtracking

Origin = P , Destination = Z



Backtracking

Origin = P , Destination = Z



Backtracking

```
while(not found flights from origin to destination)
{
    if no flight exists from city on top of stack to
unvisited destination
        pop the stack //BACKTRACK
    else
    {
        select an unvisited city C accessible from city
currently at top of stack
        push C on stack
        mark C as visited
    }
}
```

Program Stack and Recursion

Recursion works because function waiting for result/
return from recursive call are on program stack

Order of execution determined by **stack**

More Applications

Balancing anything!

-html tags (e.g `<p>` matches `</p>`)

Reverse characters in a word or words in a sentence

Undo mechanism for editors or backups

Traversals (graphs / trees)

...

Stack ADT

```
#ifndef STACK_H_
#define STACK_H_

template<typename ItemType>
class Stack
{
public:
    Stack();
    void push(const ItemType& new_entry); //adds an element to top of stack
    void pop(); // removes element from top of stack
    ItemType top() const; // returns a copy of element at top of stack
    int size() const; // returns the number of elements in the stack
    bool isEmpty() const; //returns true if no elements on stack, else false

private:
    //implementation details here

}; //end Stack

#include "Stack.cpp"
#endif // STACK_H_
```