Logic Gates

CLASS 9

HW 6.2-solution (Continued):

10) DeMorgan's law:

$$(x + y)' = x'\,y'$$
$$(x\,y)' = x' + y' \quad \forall\, x, y \in B$$

Proof

By duality we need to prove only the first equality: $(x + y)' = x'y'$

We use the same reasoning as for the previous property, 9).

That is, we use the axiom that defines the complement, namely 5):

In the first equality, which is the element and which do we have to prove is the complement?

Element = $x + y$, and its complement = $x'y'$.

Substitute in 5). We need to prove:

$$x + y + x'\,y' = 1$$
$$(x + y) \cdot (x'\,y') = 0$$

HW 7.1 - assigned: If not done yet, continue by proving that the two equalities above hold!

$x + y + x' y' = 1$

$(x + y) \cdot (x' y') = 0$

$(x + y) \cdot 1 + x' y' =$ 2 Identity

$= (x + y) \cdot (x + x') + x'y' =$

$= xx + xx' + yx + yx' + x'y' =$ 7 Idempotant

$= (x + yx) + (xx') + (yx' + x'y') =$ 12 Absorbtion, 5 Complement,

$= x + 0 + x'(y + y') =$ 5 Complement, 2 Identity

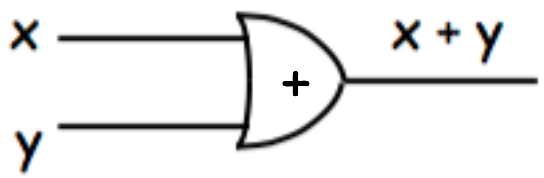
$= x + x' \cdot 1 = x + x' = 1$

$(x + y) \cdot (x' y') =$

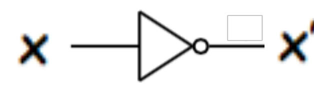
$= xx'y + yx'y' =$ 5 Complement

$= 0 \cdot y + 0 \cdot x' = 0$ 8 Identity, Annulment

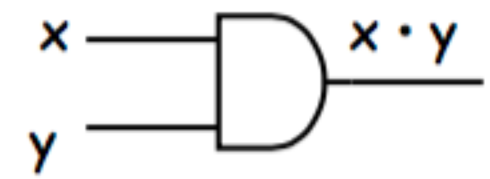
Gates



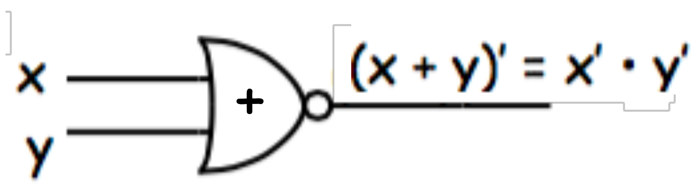
OR



Inverter



AND



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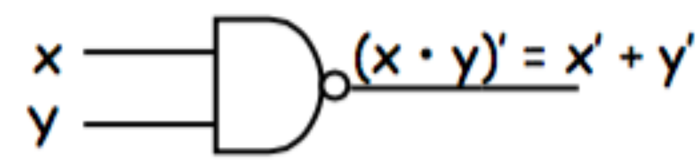

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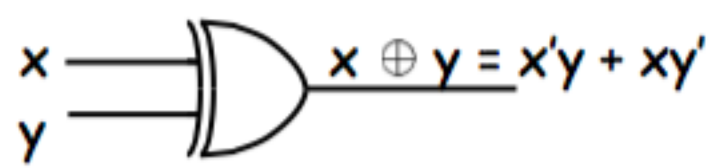
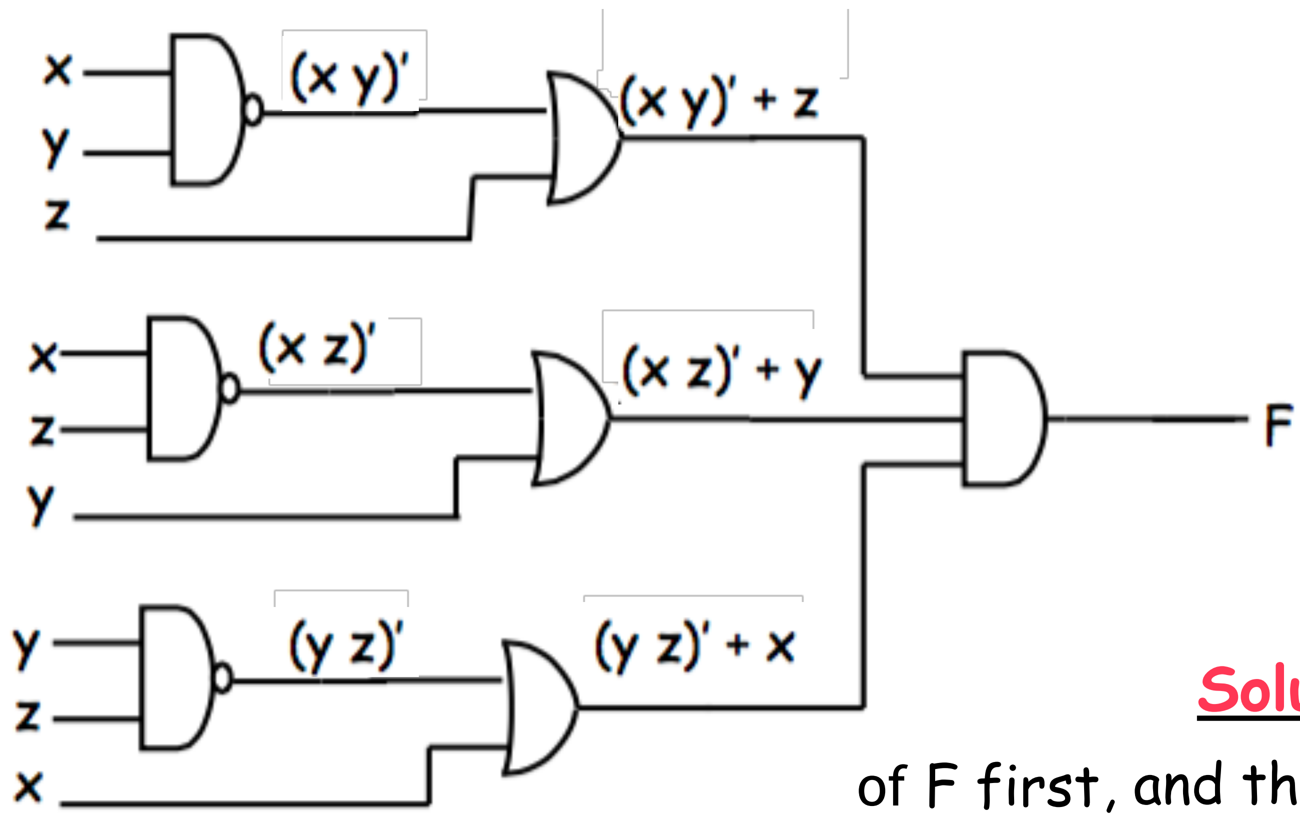


XOR
Exclusive OR

Example – logic diagram



Remember:

AND:

·	0	1
0	0	0
1	0	1

OR:

+	0	1
0	0	1
1	1	1

Problem:

Find the truth table and expression of the function F.

Solution I recommend finding the expression of F first, and then deduce the truth table from the expression.

We have: $(xy)' + z = x' + y' + z$

$(xz)' + y = x' + z' + y$

$(yz)' + x = y' + z' + x$

What is then F?

$$\boxed{F = \underbrace{(x' + y' + z)}\underbrace{(x' + z' + y)}\underbrace{(y' + z' + x)}}$$

What can we say about this product?

It is mostly = 0, like every product.

It is = 0 when any one of its factors = 0

$x' + y' + z = 0$ <--> $x'=0$ and $y'=0$ and $z=0$ <--> $x=1$ and $y=1$ and $z=0$

Similarly we have:

$x' + y + z' = 0$ <--> $x=1$ and $y=0$ and $z=1$

$x + y' + z' = 0$ <--> $x=0$ and $y=1$ and $z=1$

In rest, for all other values of x, y, z, F = 1

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Truth table

HW 8.1

Partial Solution + Hint

(a) Prove: Inhibition is not commutative

Table gives two functions:

$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_4 = x'y$	y/x	Inhibition	y, but not x

Let's choose $F_2 = x/y = x\,y'$

F_2 commutative $\overset{\text{Det}}{\Leftrightarrow}$ $\boxed{F_2(x, y) = F_2(y, x) \quad \forall\, x, y \quad \overset{\text{Det}}{\Leftrightarrow} \quad x/y = y/x}$ $\forall\, x, y$

F_2 not commutative $\overset{\text{Det}}{\Leftrightarrow}$ $\exists\, x, y$: $F_2(x, y) \neq F_2(y, x)$ $\overset{\text{Det}}{\Leftrightarrow}$ $\exists\, x, y$: $x/y \neq y/x$

In other words we need to find a counterexample to $x/y = y/x$, that is find values in {0, 1} for x, y, such that we have $x/y \neq y/x \overset{\text{Det}}{\Leftrightarrow} xy' \neq yx'$ as every B.A. has 0 and 1 as elements!

Make one side 1 and at the same time the other side 0.

$x = 1$ and $y = 0$ imply:

LS = $1 \cdot 1 = 1$ RS = $0 \cdot 0 = 0$ --> LS ≠ RS --> **F₂ is not commutative**

Done

HW 8.1 - continued Partial Solution + Hint - continued

(b) Prove: Exclusive OR (XOR) is associative Table:

$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both

$F_6(x, y) = x\,y' + x'\,y = x \oplus y$

F_6 associative $\overset{\text{Det}}{\Leftrightarrow}$ $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ $\quad \forall\, x, y, z$

$\overset{\text{Det}}{\Leftrightarrow}$ $F_6(F_6(x,y),z) = F_6(x, F_6(y,z))$ $\quad \forall\, x, y, z$

We need to prove this for all BAs, so we need to use the axioms and properties of a BA.

$LS = (x\,y' + x'\,y) \oplus z = (x\,y' + x'\,y)\,z' + (x\,y' + x'\,y)'\,z =$

$= xy'z' + x'yz' + (x' + y)(x + y')z = xy'z' + x'yz' + x'y'z + xyz$ 5 Complement

$RS = x \oplus (yz' + y'z) = x(yz' + y'z)' + x'(yz' + y'z) = x(y' + z)(y + z') + x'yz' + x'y'z =$

$= xy'z' + xyz + x'yz' + x'y'z$ --> $LS = RS$ --> **F_6 is associative**

Done

HW 7.3 (continuation from class 8)

2-9 b)

$$\overline{((\bar{v}w + x)y + \bar{z})} = (\underbrace{(v+\bar{w})\cdot\bar{x}}_{v\bar{x}+\bar{w}\bar{x}} + \bar{y})z =$$

$$= (v\bar{x} + \bar{w}\bar{x} + \bar{y})z = \boxed{v\bar{x}z + \bar{w}\bar{x}z + \bar{y}z}$$

c)
$$\overline{(wx(\bar{y}z + y\bar{z}) + \bar{w}\bar{x}(\bar{y}+z)(y+\bar{z}))} =$$

$$= (\bar{w} + \bar{x} + (y+\bar{z})(\bar{y}+z))(w + x + y\bar{z} + \bar{y}z) =$$

$$= (\bar{w} + \bar{x} + yz + \bar{y}\bar{z})(w + x + y\bar{z} + \bar{y}z) =$$

$$= \boxed{\bar{w}x + \bar{w}y\bar{z} + \bar{w}\bar{y}z + \bar{x}w + \bar{x}y\bar{z} + \bar{x}\bar{y}z + wyz + xyz + w\bar{y}\bar{z} + x\bar{y}\bar{z}}$$

<u>HW 7.3</u> (continuation)

2-9 d)

$$\overline{(A+\bar{B}+C)(\bar{A}\bar{B}+C)(A+\bar{B}\bar{C})} =$$

$$= \bar{A}B\bar{C} + (A+B)\cdot\bar{C} + \bar{A}\cdot(B+C) \overset{(4)}{=}$$

$$= \bar{A}B\bar{C} + A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C \overset{(12)}{=}$$

$$= A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C$$

What now? Expand and then contract again:

$$= \cancel{AB\bar{C}} + A\bar{B}\bar{C} + AB\bar{C} + \cancel{\bar{A}B\bar{C}} + \cancel{\bar{A}BC} + \bar{A}B\bar{C} +$$

$$+ \bar{A}BC + \bar{A}\bar{B}C = A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C} +$$

$$+ \bar{A}BC + \bar{A}\bar{B}C = \boxed{A\bar{C} + \bar{A}B + \bar{A}C}$$