CL=CSCI 160

CLASS 3

GIVE ANSWERS IN THE SPACES INDICATED BELOW. JUSTIFY YOUR ANSWERS! DO WELL!!!

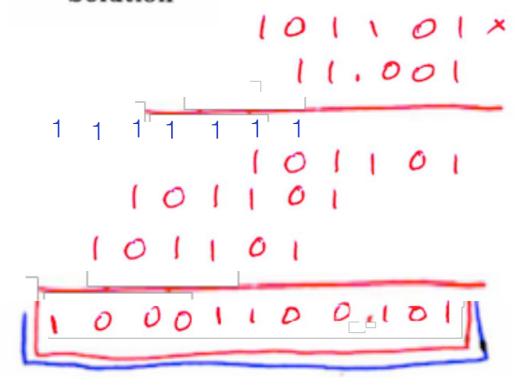
1. Consider the following binary numbers:

$$M = 101101$$

$$N = 11.001$$

Using 3-bit precision compute in binary:

 MN Solution

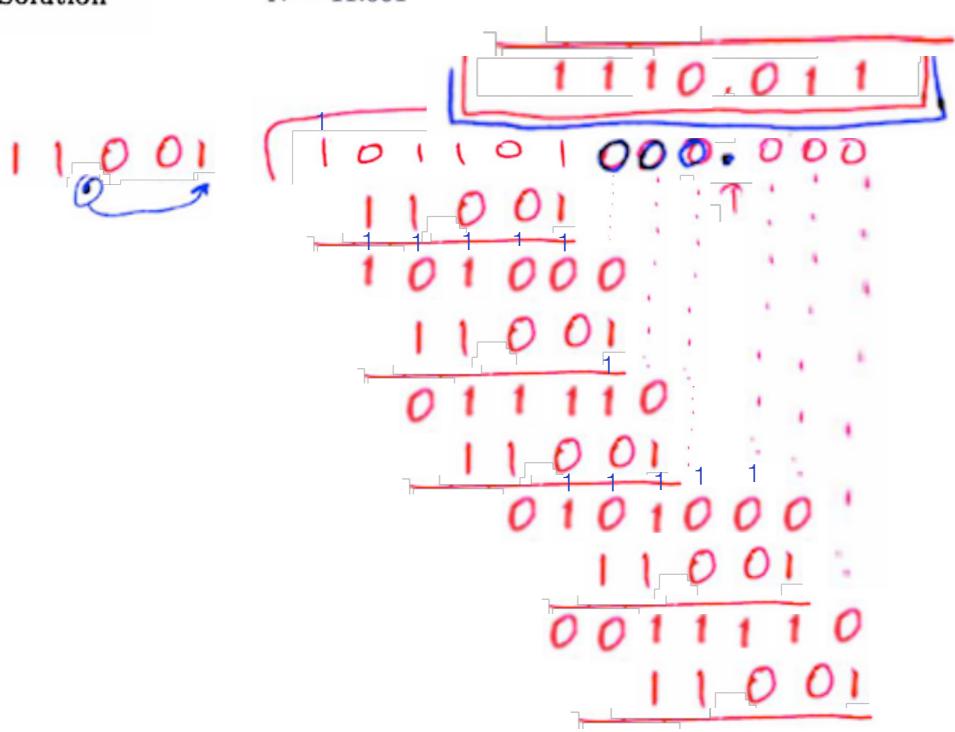


2) M/N Solution

$$M = 101101$$

 $N = 11.001$

3-bit precision



2. We have the following equality:

$$24_r + 18_r = 40_r$$

where all the numbers are written in the same base r, as indicated. Determine r, and clearly encircle the result.

Solution

Convert to decimal:

$$2r + 4 + r + 8 = 4r \quad \longleftrightarrow \quad \boxed{r = 12}$$

Proof

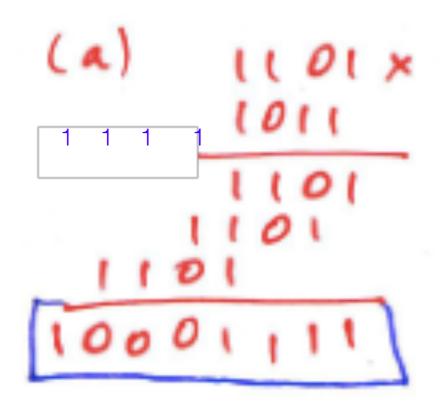
Prove: $24_{12} + 18_{12} = 40_{12}$, that is:

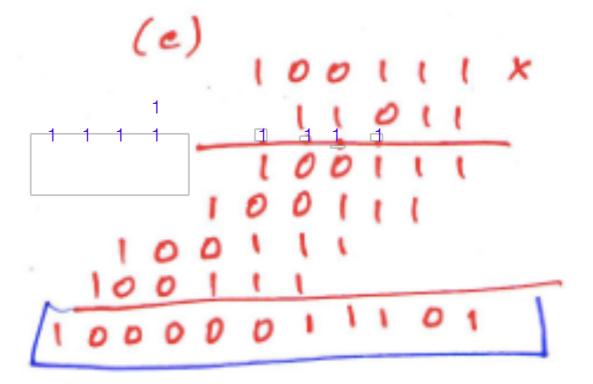
2x12 + 4 + 1x12 + 8 = 4x12, which is equivalent to:

True!

1–12. Perform the following binary multiplications:

(a) 1101×1011 (b) 0101×1010 (c) 100111×011011





- **1–14.** A limited number system uses base 12. There are at most four integer digits. The weights of the digits are 12^3 , 12^2 , 12, and 1. Special names are given to the weights as follows: 12 = 1 dozen, $12^2 = 1$ gross, and $12^3 = 1$ great gross.
 - (a) How many beverage cans are in 6 great gross + 8 gross + 7 dozen + 4?
 - **(b)** Find the representation in base 12 for 7569_{10} beverage cans.

$$|2^{2}=144 ; |2^{3}=1728$$
(a) $6 \times 1728 + 8 \times 144 + 7 \times 12 + 4 = --$

(b)
$$\frac{7569_{10}}{6912} = 4 \text{ gr. fishs} + 4 \text{ grows} + 6 \text{ dof} + 9 = 6912$$

$$= 657$$

$$= 776$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

$$= 91$$

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1–16. *In each of the following cases, determine the radix r:

(a)
$$(BEE)_r = (2699)_{10}$$
 (b) $(365)_r = (194)_{10}$

(b)
$$(365)_r = (194)_{10}$$

(Note: r is a divisor of 2685: factorize it!) alternative 1



alternative 2 (Use quadratic formula)

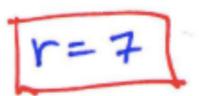
Use formula to solve quadratic equations

to get the result, but the coefficients are large!

Use same formula to solve quadratic

equations to get the result:





(r-1)'s Complement-Representation

r = base Very important: How are our numbers stored?

Definition

$$r = 2$$
 (N is a binary number!)

How did you compute $N_{1c} = 1$'s complement of N? By swapping 0 <-> 1, which is equivalent to:

$$N_{1c} = 1 \dots 1 \cdot 1 \dots 1 - N = 10 \dots 0 \cdot 0 \dots 0 - 0 \cdot 0 \dots 01 - N = 2^n - 2^{-m} - N$$

$$N_{(r-1)c} = r^n - r^{-m} - N$$

Algorithm for subtracting two numbers using only addition and the (r-1)'s complement

To perform M-N do:

- 1) $M + N_{(r-1)c}$
- a) If there is an e. a. c. (= end-around-carry = overflow), then add it to the l. s. d. (= least significant digit) of result from 1). Stop.
 - b) If there is no e. a. c., then the result is negative, and is obtained by taking the (r-1)'s complement of what we obtained at 1); in other words, we compute:

-
$$(M + N(r-1)c)(r-1)c$$

Result from 1)

Stop.

Example in binary + HW assigned

a) e.a.c. -> add it to l.s.d. <u>Alg (M-N):</u> 1) 2) $M + N_{(r-1)c}$ b) no e.a.c. \rightarrow compute -(r-1)'s compl. of 1) **Ex-binary** r = 2 Suppose n = 4, m = 2**2-b) Path:** ii) Swap M <-> N, to get to case 2-b) 2-a) Path: i) Suppose: M: 1101.10 1011.01 M: N: 1011.01 1101.10 N: 1011.01 + 1101.10 + Apply algorithm 1) M: Apply algorithm 1) M: to get M - N: to get M - N: N_{1C} : 0100.10 N_{1C} : 0010.01 e.a.c.= (1)0010.00 1101.10 no e.a.c. ->case2-b) -> -1's compl.: ->case2**-**α)

- 0010.01

as expected

<u>HW</u> (more to follow):

0010.01

Why does this algorithm work?

Why does this algorithm work?

Hint

We know:

$$N = \frac{1}{n} \cdot \frac{1}{m} \cdot \frac{1}$$

1)
$$M + N_{(r-1)c} = M + r^n - r^{-m} - N$$

2) Whether we follow a) or b) depends on the presence of an e. a. c.

What is the magnitude of the e.a.c.? It's an 'overflow':

e.a.c : 10 o = rⁿ On <u>branch a)</u> there is an e. a. c., which means we have:

$$M + r^n - r^{-m} - N \ge r^n$$
 \longleftrightarrow $M - N \ge r^{-m} \longleftrightarrow$ =smallest positive number in our representation e.a.c.

$$\leftarrow \sim M - N > 0$$
 or $M > N$ It also means, that the case when

M - N = 0 will take branch b), which means that 0 will be expressed as -0 by this Alg. Continue justifying the computations in the branches 2-a) and 2-b) as HW.

- 3-A Do the following conversion problems:
 - (a) Convert decimal 34.4375 to binary.
 - (b) Calculate the binary equivalent of 1/3 out to 8 places. Then convert from binary to decimal. How close is the result to 1/3?
 - (c) Convert the binary result in (b) into hexadecimal. Then convert the result to decimal. Is the answer the same?
- 3-B Determine the value of base x if $(211)_x = (152)_8$.
- 3-C Noting that 3² = 9, formulate a simple procedure for converting base-3 numbers directly to base-9. Use the procedure to convert (2110201102220112)₃ to base 9.
- 3-D The solutions to the quadratic equation

$$x^2 - 11x + 22 = 0$$

are
$$x = 3$$
 and $x = 6$.

Determine the base of the numbers in the equation.

3-E Convert the hexadecimal number 68BE to binary, and then convert it from binary to octal and then to base 32.