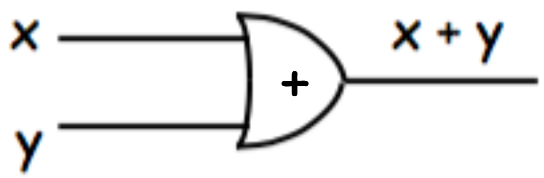
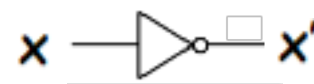
CL=CSCI 160

CLASS 9

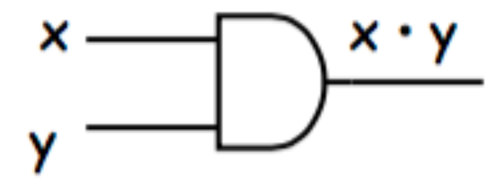
Gates



OR



Inverter



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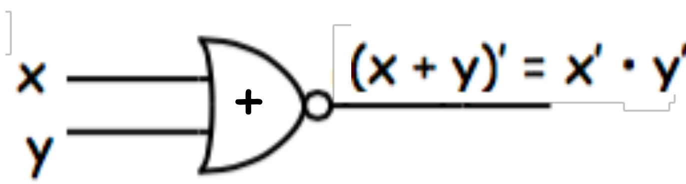
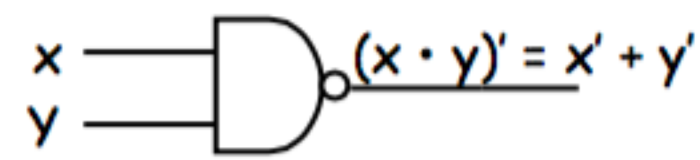


NOR



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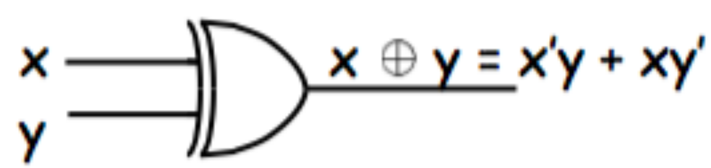


XOR
Exclusive OR

Example – logic diagram

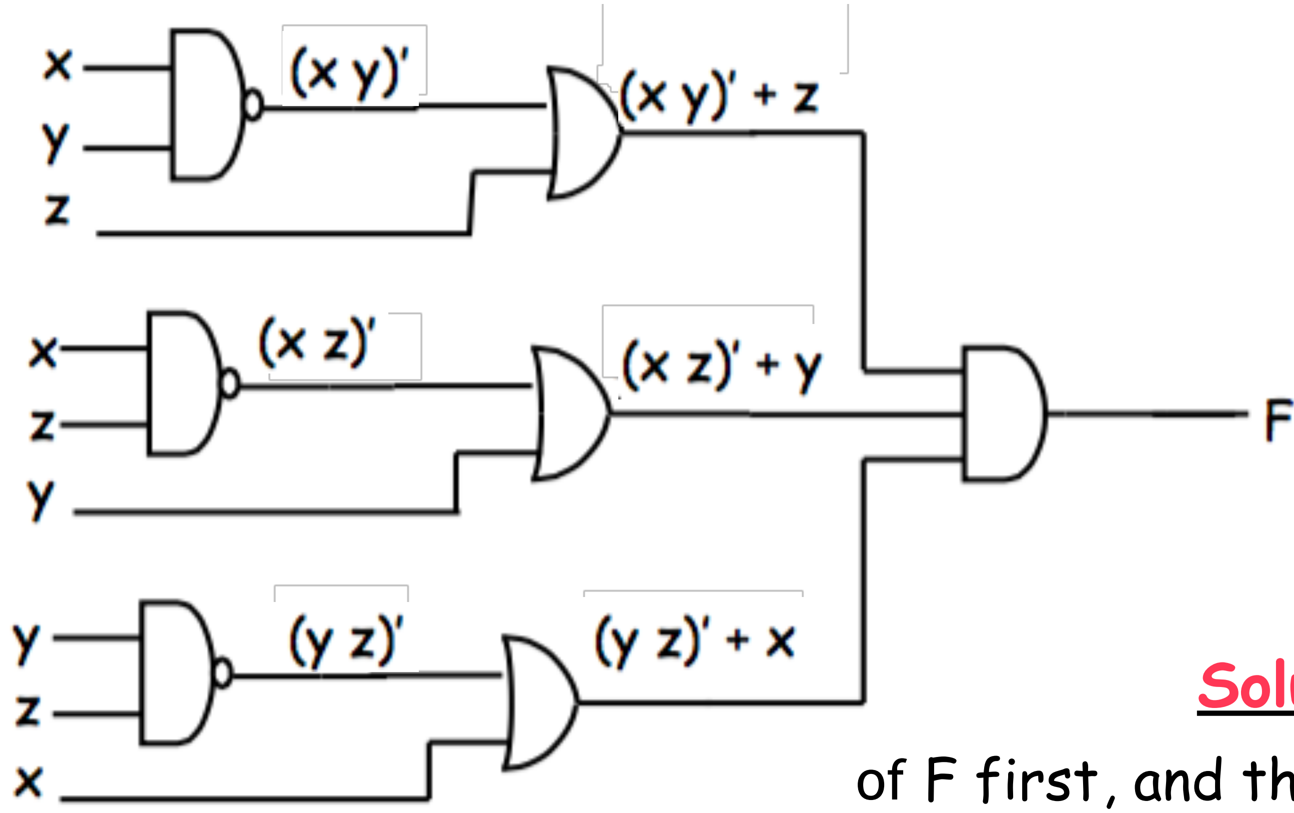
Remember:

AND:

$\cdot$	0	1
0	0	0
1	0	1

OR:

+	0	1
0	0	1
1	1	1



Problem:

Find the truth table and expression of the function F.

Solution I recommend finding the expression of F first, and then deduce the truth table from the expression.

We have: $(xy)' + z = x' + y' + z$

$(xz)' + y = x' + z' + y$

$(yz)' + x = y' + z' + x$

What is then F?

$$\boxed{F = (x' + y' + z)(x' + z' + y)(y' + z' + x)}$$

What can we say about this product?

It is mostly = 0, like every product.

It is = 0 when any one of its factors = 0

$x' + y' + z = 0 \longleftrightarrow x'=0$ and $y'=0$ and $z=0$ $\longleftrightarrow$ $x=1$ and $y=1$ and $z=0$

Similarly we have:

$x' + y + z' = 0 \longleftrightarrow$ $x=1$ and $y=0$ and $z=1$

$x + y' + z' = 0 \longleftrightarrow$ $x=0$ and $y=1$ and $z=1$

In rest, for all other values of x, y, z, F = 1

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Truth table

HW 8.1

Partial Solution + Hint

(a) Prove: Inhibition is not commutative

Table gives two functions:

$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_4 = x'y$	y/x	Inhibition	y, but not x

Let's choose $F_2 = x/y = x\,y'$

F_2 commutative $\overset{\text{Det}}{\Leftrightarrow}$ $\boxed{F_2(x, y) = F_2(y, x) \quad \forall\, x, y \quad \overset{\text{Det}}{\Leftrightarrow} \quad x/y = y/x} \quad \forall\, x, y$

F_2 not commutative $\overset{\text{Det}}{\Leftrightarrow}$ $\exists\, x, y\!: \ F_2(x, y) \neq F_2(y, x) \quad \overset{\text{Det}}{\Leftrightarrow} \quad \exists\, x, y\!: \ x/y \neq y/x$

In other words we need to find a counterexample to $x/y = y/x$, that is find values in {0, 1} for x, y, such that we have $\quad x/y \neq y/x \quad \overset{\text{Det}}{\Leftrightarrow} \quad xy' \neq yx' \quad$ as every B.A. has 0 and 1 as elements!

Make one side 1 and at the same time the other side 0.

$x = 1$ and $y = 0$ imply:

$LS = 1 \cdot 1 = 1 \qquad RS = 0 \cdot 0 = 0 \qquad \dashrightarrow LS \neq RS \qquad \dashrightarrow$ **F_2 is not commutative**

Done

(b) Prove: Exclusive OR (XOR) is associative Table:

$F_6 = xy' + x'y$ $\quad x \oplus y \quad$ Exclusive-OR $\quad$ x or y, but not both

$F_6(x, y) = x\,y' + x'\,y = x \oplus y$

F_6 associative $\overset{\text{Det}}{\Leftrightarrow}$ $(x \oplus y) \oplus z = x \oplus (y \oplus z) \quad \forall\, x, y, z$

$\overset{\text{Det}}{\Leftrightarrow}$ $F_6(F_6(x,y),z) = F_6(x, F_6(y,z)) \quad \forall\, x, y, z$

We need to prove this for all BAs, so we need to use the axioms and properties of a BA.

$LS = (x\,y' + x'\,y) \oplus z = (x\,y' + x'\,y)\,z' + (x\,y' + x'\,y)'\,z =$

$= xy'z' + x'yz' + (x' + y)(x + y')z = xy'z' + x'yz' + x'y'z + xyz$

$RS = x \oplus (yz' + y'z) = x(yz' + y'z)' + x'(yz' + y'z) = x(y' + z)(y + z') + x'yz' + x'y'z =$

$= xy'z' + xyz + x'yz' + x'y'z$ $\quad$ --> $\quad LS = RS \quad$ --> $\quad$ **F_6 is associative**

Done

HW 7.3 (continuation from class 8)

2-9 b)

$$\overline{\left((\bar{v}w + x)y + \bar{z}\right)} = \big(\underbrace{(v+\bar{w})\cdot\bar{x}}_{v\bar{x}+\bar{w}\bar{x}} + \bar{y}\big)z =$$

$$= (v\bar{x} + \bar{w}\bar{x} + \bar{y})z = \boxed{v\bar{x}z + \bar{w}\bar{x}z + \bar{y}z}$$

c)

$$\overline{\left(wx(\bar{y}z + y\bar{z}) + \bar{w}\bar{x}(\bar{y}+z)(y+\bar{z})\right)} =$$

$$= \left(\bar{w} + \bar{x} + (y+\bar{z})(\bar{y}+z)\right)(w + x + y\bar{z} + \bar{y}z) =$$

$$= (\bar{w} + \bar{x} + yz + \bar{y}\bar{z})(w + x + y\bar{z} + \bar{y}z) =$$

$$= \boxed{\begin{array}{l} \bar{w}x + \bar{w}y\bar{z} + \bar{w}\bar{y}z + \bar{x}w + \bar{x}y\bar{z} + \bar{x}\bar{y}z + wyz + xyz + \\ + w\bar{y}\bar{z} + x\bar{y}\bar{z} \end{array}}$$

HW 7.3 (continuation)

2-9 d) $\overline{(A+\bar{B}+C)(\bar{A}\bar{B}+C)(A+\bar{B}\bar{C})} =$

$= \bar{A}B\bar{C} + (A+B)\cdot\bar{C} + \bar{A}\cdot(B+C) \overset{(4)}{=}$

$= \bar{A}B\bar{C} + A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C \overset{(12)}{=}$

$= A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C$

What now? Expand and then contract again:

$= A\cancel{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + \bar{A}\cancel{B}\bar{C} + \bar{A}\cancel{B}C + \bar{A}B\bar{C} +$

$+ \bar{A}BC + \bar{A}\bar{B}C = A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C} +$

$+ \bar{A}BC + \bar{A}\bar{B}C = \boxed{A\bar{C} + \bar{A}B + \bar{A}C}$