CL=CSCI 160

CLASS 17

QUIZ 2.2

I Design a **majority circuit** with three inputs in the following way:

1) **Write the truth table of a function, say $F_3(x,y,z)$,** where the output agrees with the majority of the input, that is $F_3(x,y,z) = 1$ if and only if at least two out of the three variables: x,y,z have the value 1.

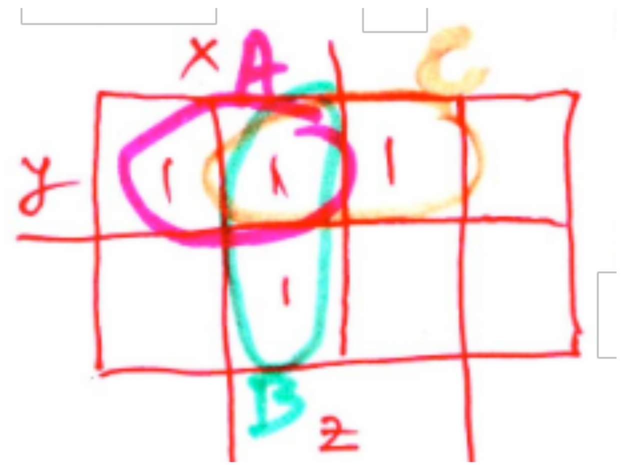
2) **Write the expression of the function $F_3(x,y,z)$** and **minimize** it.

Solution

1)

x	y	z	F_3
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

2) $F_3 = x'yz + xy'z + xyz' + xyz$



All implicants are essential.

$\boxed{F_3 = A + B + C = \quad xy + xz + yz}$

Note: From the expression of the minimal form we can see that we take any 2 out of the 3 variables to form majority

QUIZ 2.2 (cont)

II Next, design a **majority circuit** function, say $F_4(x,y,z,t)$, with four inputs. Handle **ties in a way that is 'optimal'** from the point of view of the minimization of F_4. **Write the truth table of F_4**. Continue this task as HW.

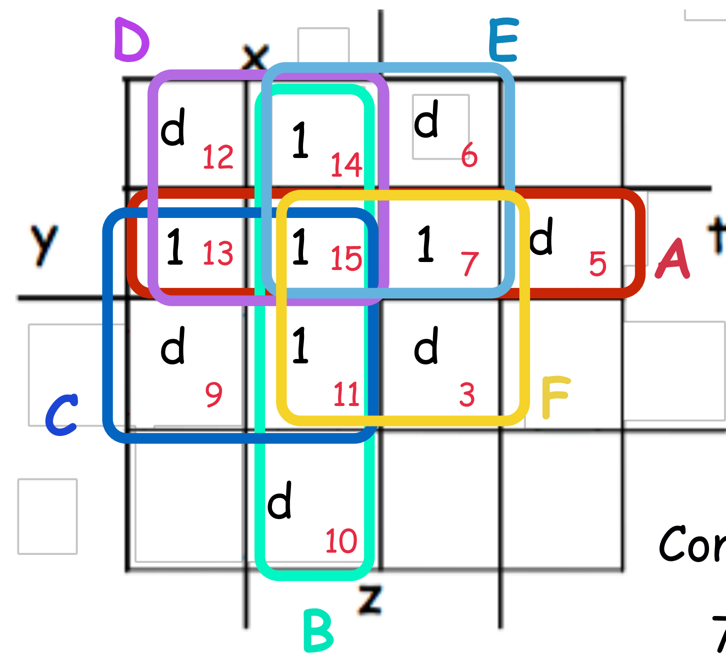
Solution (Q2.2+HW)

What do we do with the ties? **'tie' is not an acceptable value for a function!**

What do we care about?

To minimize F_4, and not who wins in a tie! —> Use d, to our advantage.

x	y	z	t	F_4
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	~~tie~~ d
0	1	0	0	0
0	1	0	1	~~tie~~ d
0	1	1	0	~~tie~~ d
0	1	1	1	1
1	0	0	0	0
1	0	0	1	~~tie~~ d
1	0	1	0	~~tie~~ d
1	0	1	1	1
1	1	0	0	~~tie~~ d
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



Size-4 implicants: A - F

None essential

Consider a 1 that is least covered:

7, 11, 13, 14; say, 7:

7 is covered by A, E, F

Case I: Take A	$F_4 = A + B = yt + xz$
Case II: Take E	$F_4 = E + C = yz + xt$
Case III: Take F	$F_4 = F + D = zt + xy$

<– 3 **Minimal forms**

Note: From the expression of the minimal form we can see that we take any 2 out of the 4 variables and the remaining 2 to form at least a tie.

How would the majority circuit look for 5 variables?

$F_5(x_1, x_2, x_3, x_4, x_5) = 1 \iff$ at least 3 out of the 5 variables are 1.

$\binom{5}{3} = \binom{5}{2} = \frac{5 \cdot 4}{1 \cdot 2} = \underline{\underline{10}}$. terms:

$$F_5 = x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_2 x_5 + x_1 x_3 x_4 + x_1 x_3 x_5 + x_1 x_4 x_5 +$$
$$+ x_2 x_3 x_4 + x_2 x_3 x_5 + x_2 x_4 x_5 + x_3 x_4 x_5.$$

We wrote it directly in minimal form

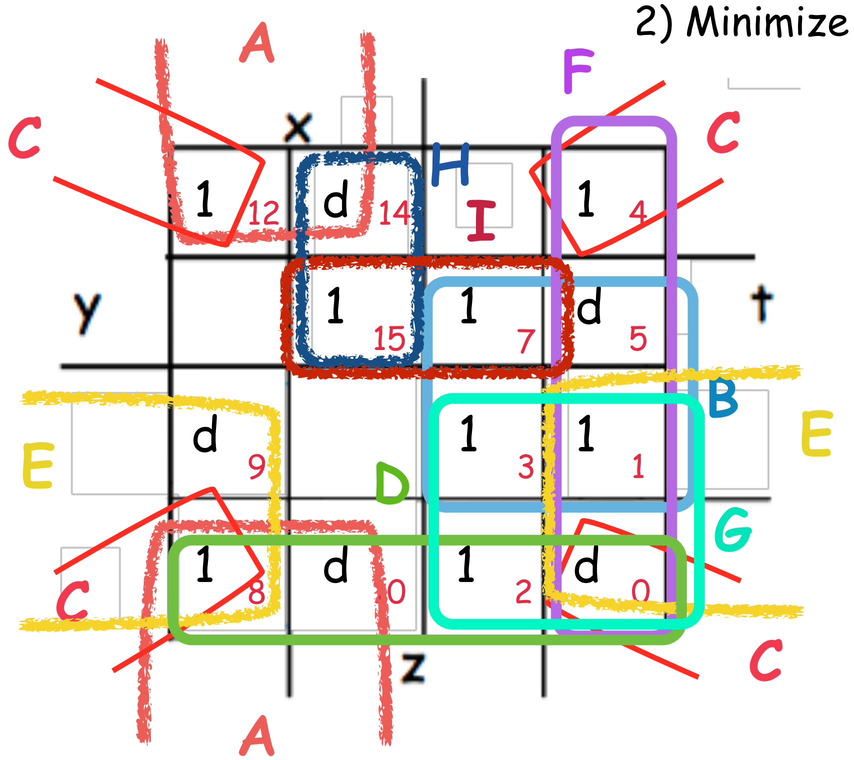
HW 16

Consider the function; we applied the tabulation method on:

$f = \Sigma\,(1, 2, 3, 4, 7, 8, 12, 15) + d\,\Sigma\,(0, 5, 9, 10, 14))$

1) Draw the K-map and find all prime implicants, giving them the same labels (letters), A - I, in class, when applying the tabulation method.
2) Minimize f.

Solution



We have from tabulation method (class 16):

-111	(7, 15)	I
111-	(14, 15)	H

Index	Impl. Binary	Impl. Dec.	
	00--	(0, 1, 2, 3)	G
	0-0-	(0, 1, 4, 5)	F
0	-00-	(0, 1, 8, 9)	E
	-0-0	(0, 2, 8, 10)	D
	--00	(0, 4, 8, 12)	C
1	0--1	(1, 3, 5, 7)	B
	1--0	(8, 10, 12, 14)	A

Essential: none

Least covered 1's: 2, 3, 4, 7, 12, 15: all covered by exactly two implicants. Choose one of them: 2, covered by D, G.

Every minimal form will contain D or G. **Case I**: Take D —> 2 and 8 are covered.

To cover the rest we need one size-2 implicant (for 15) and 2 x size-4 implicants:

$f = D + B + C + \left\{ \begin{matrix} H \\ I \end{matrix} \right\}$ or $f = D + G + C + I$ <— NOT Minimal forms

(**note**: it's redundant, as we don't need D; if you don't note it, go to Case II below)

Case II: Take G —> 1,2,3 are covered. 15 again may be covered only by a size-2 implicants: H, I.

However, if we cover 15 by implicant I, then the remaining 1's may be covered by **just one** size-4 implicant.

$f = G + C + I$ <— Minimal form which means **it's the only one!**

$\boxed{f = x'y' + z't' + yzt}$ <— only Minimal form as expression

HW 17.1 - assigned

A.

Simplify the following Boolean function F together with the don't-care conditions d; then express the simplified function.

(a) $F(x, y, z) = \Sigma(0, 1, 2, 4, 5)$
$d(x, y, z) = \Sigma(3, 6, 7)$

(b) $F(A, B, C, D) = \Sigma(0, 6, 8, 13, 14)$
$d(A, B, C, D) = \Sigma(2, 4, 10)$

B.

A logic circuit implements the following Boolean function:

$$F = A'C + AC'D'$$

It is found that the circuit input combination $A = C = 1$ can never occur. Find a simpler expression for F using the proper don't-care conditions.

Prime Implicant Table [it computes a minimized form]

Consider the function; we applied the tabulation method on:

$f = \Sigma\,(1, 2, 3, 4, 7, 8, 12, 15) + d\,\Sigma\,(0, 5, 9, 10, 14))$

Index	Impl. Binary	Impl. Dec.	
	-111	(7, 15)	I
	111-	(14, 15)	H
0	00--	(0, 1, 2, 3)	G
	0-0-	(0, 1, 4, 5)	F
	-00-	(0, 1, 8, 9)	E
	-0-0	(0, 2, 8, 10)	D
	--00	(0, 4, 8, 12)	C
1	0--1	(1, 3, 5, 7)	B
	1--0	(8, 10, 12, 14)	A

1's (no d's) are the columns; the prime implicants are the rows:

	1	2	3	4	7	8	12	15
~~A~~						x	x	
B	x		x		x			
C				x		x	x	
D		x				x		
E	x					x		
F	x			x				
G	x	x	x					
~~H~~								x
I					x			x

C ⊃ A

I ⊃ H

1⊃3 7⊃15 8 ⊃12

1) No essentials

2) We removed dominated rows A and H

3) We removed dominating columns 1, 7 and 8

Procedure:

1) Find essentials; put them in the minimal form and eliminate from table.

2) Remove: Dominated rows.

3) Remove: Dominating columns.

Repeat steps 1) – 3) until a minimized form is obtained

Note: when a 1 has only one x in its column then the implicant corresponding to that x is **essential**.

Prime Implicant Table

(continued)

	1	2	3	4	7	8	12	15
A						x	x	
B	x		x		x			
C				x		x	x	
D		x				x		
E	x					x		
F	x			x				
G	x	x	x					
H								x
I					x			x

Procedure:

1) Find essentials: put them in the minimal form and eliminate from table.

2) Remove: Dominated rows.

3) Remove: Dominating columns.

Repeat steps 1) – 3) until a minimized form is obtained

We redo the table:

	2	3	4	12	15
B		x			
C			x	x	
D	x				
F			x		
G	x	x			
I					x

1) Essentials: C, I ---> $f = C + I + \ldots$

Remove the columns/rows corresponding to the essential implicants

We redo the table:

	2	3
B		x
D	x	
G	x	x

2) Remove: dominated rows

$$\boxed{f = C + G + I = z't' + x'y' + yzt}$$

Index	Impl. Binary	Impl. Dec.	
	-111	(7, 15)	I
	111-	(14, 15)	H
0	00--	(0, 1, 2, 3)	G
	0-0-	(0, 1, 4, 5)	F
	-00-	(0, 1, 8, 9)	E
	-0-0	(0, 2, 8, 10)	D
	--00	(0, 4, 8, 12)	C

We have the same minimal form as the K-map gave us (HW 16)!

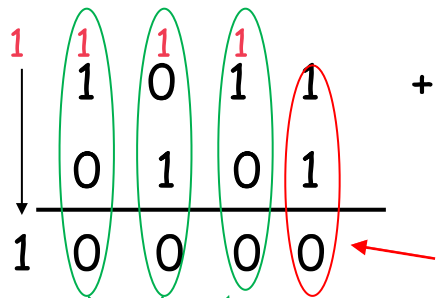
HW 17.2 - assigned

Minimize the function:

$$f = A'B'DE' + E'B'C'D + B'DE' + B'CD + CDE' + BDE'$$

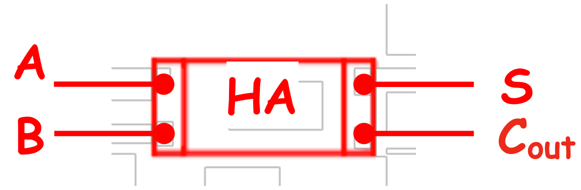
Half-Adder and Full-Adder

Let's add two binary numbers to figure out all steps involved.



Half-adder

Full-adders

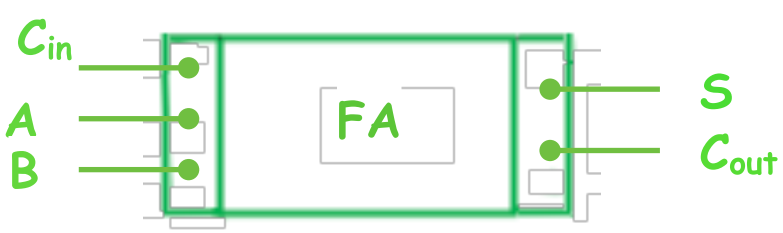


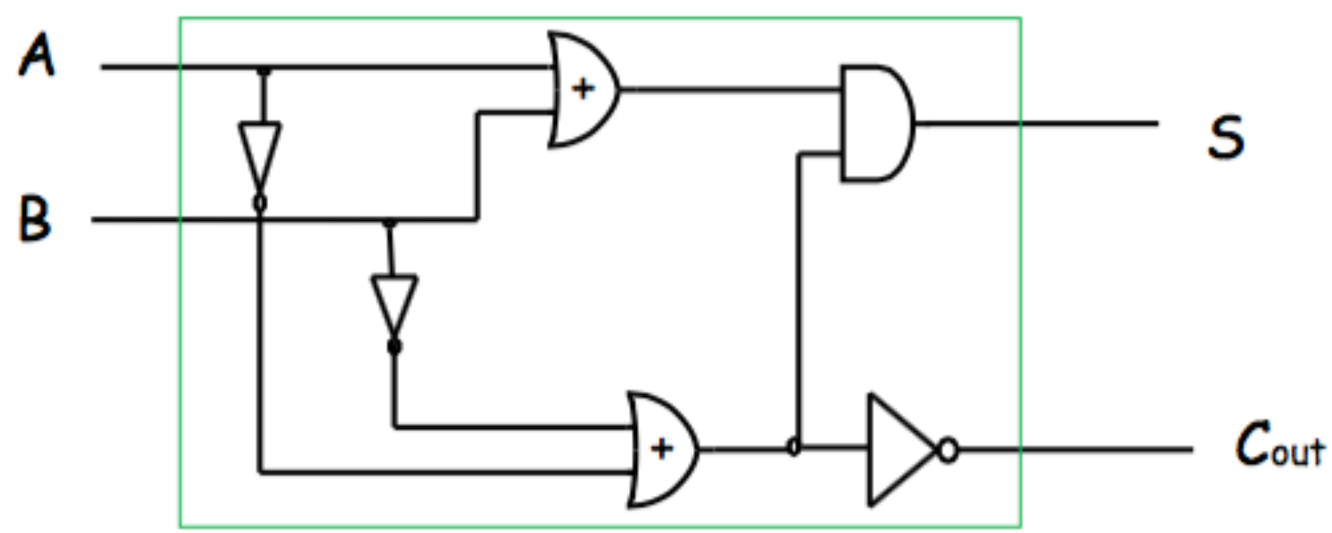
Half-adder:

A	B	S	C_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$S = A'B + AB' = (A' + B')(A + B)$

$C_{out} = A\,B = (A' + B')'$

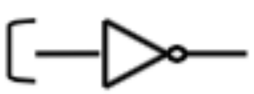




Half-adder with 3 gates

HW 17.3 - assigned

Express H A using only NOR [] gates

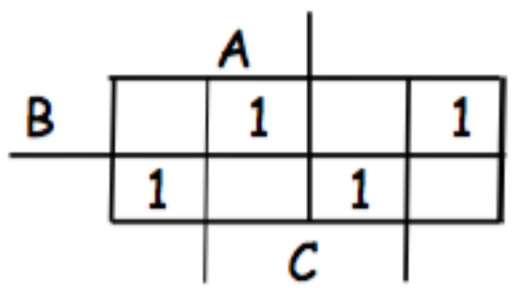
[ is allowed]

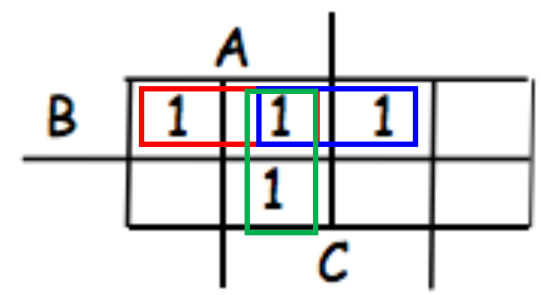
Full-adder

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$S = A'B'C + A'BC' + AB'C' + ABC$

$Cout = A'BC + AB'C + ABC' + ABC$





$S = A'B'C + A'BC' + AB'C' + ABC$

$Cout = AB + AC + BC$

Majority function!

HW 17.4 - assigned: Construct a FA using only HA's and one other gate.