CL=CSCI 160

CLASS 15

Quiz 2.1

1) Consider the Boolean function

$$f = \Sigma\,(0, 2, 3, 7, 8, 10, 12, 14, 15)$$

After drawing the K-map, and determining all the prime implicants, find **all minimal forms** of f.

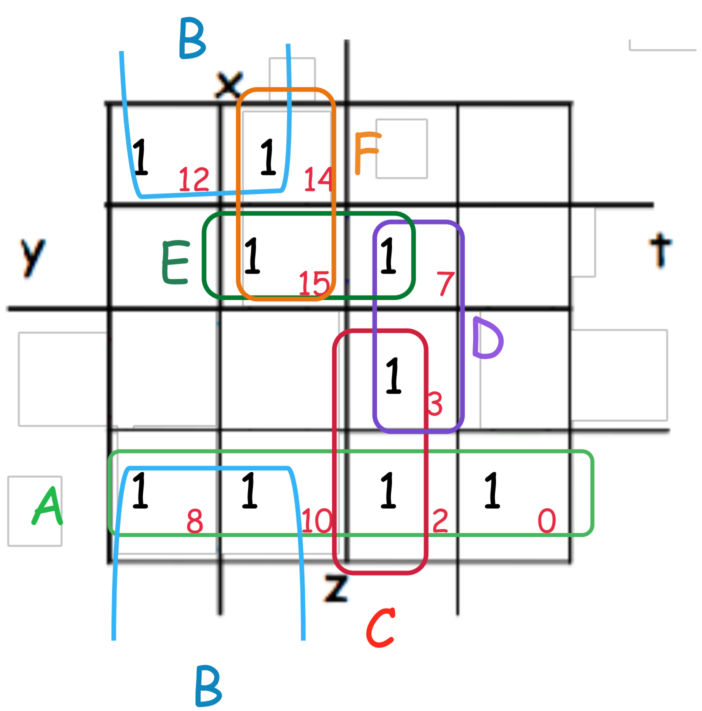
2) Consider the function F, obtained from f by just adding one minterm, the one corresponding to **5**, that is,

$$F = \Sigma\,(0, 2, 3, \mathbf{5}, 7, 8, 10, 12, 14, 15)$$

Just like for the function above, f, find **all minimal forms** of F. Explain **how does the solution change** when the function has just one additional minterm. Encircle and enumerate all the solutions you obtained for f, and for F.

Solution

1)



Essential: A, B ---> f = A + B + ... We need to cover 3 more 1's.

Take, e.g., 1_3. How can we cover it? We need C or D

Minimal covers **only**:

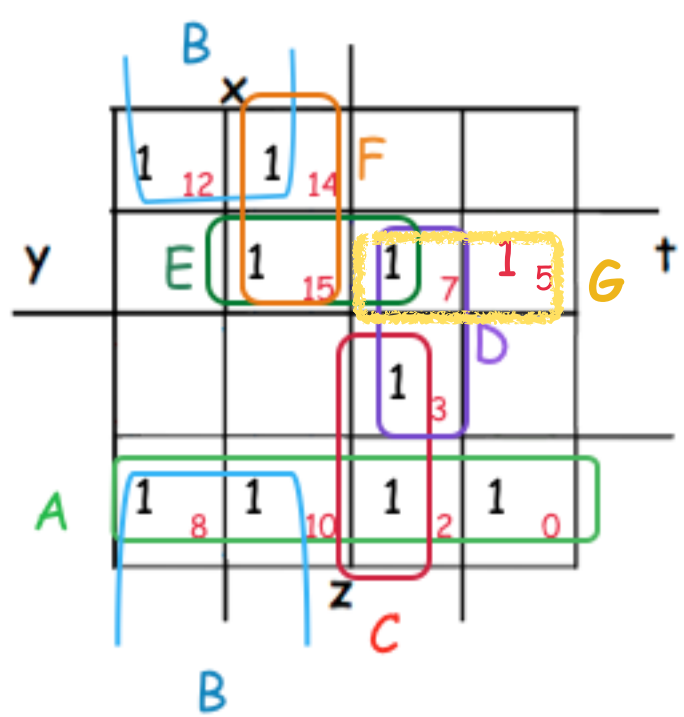
Case 1. C: $f_1 = A + B + C + E$ ---> $f_1 = y't' + xt' + x'y'z + yzt$

Case 2. D: $f_{2,3} = A + B + D +$ E or F $\quad f_{2,3} = y't' + xt' + x'zt + \left\{ \begin{matrix} yzt \\ xyz \end{matrix} \right\}$

2)

Essential: A, B, G We need to cover 2 more 1's: 1_{15} and 1_3.

They can be covered each independently by 2 size-2 implicants:



$$F_{1\text{-}4} = A + B + G + \left\{ \begin{matrix} E \\ \text{or} \\ F \end{matrix} \right\} + \left\{ \begin{matrix} C \\ \text{or} \\ D \end{matrix} \right\}$$

$F_1 = y't' + xt' + x'yt + yzt + x'y'z$

$F_2 = y't' + xt' + x'yt + yzt + x'zt$

$F_3 = y't' + xt' + x'yt + xyz + x'y'z$

$F_4 = y't' + xt' + x'yt + xyz + x'zt$

Comment: By introducing 1_5 we create a new essential size-2 implicant, -----> only two more 1's still need to be covered, and they can be covered each in two ways, independently from each other ---> 4 forms total. Write:

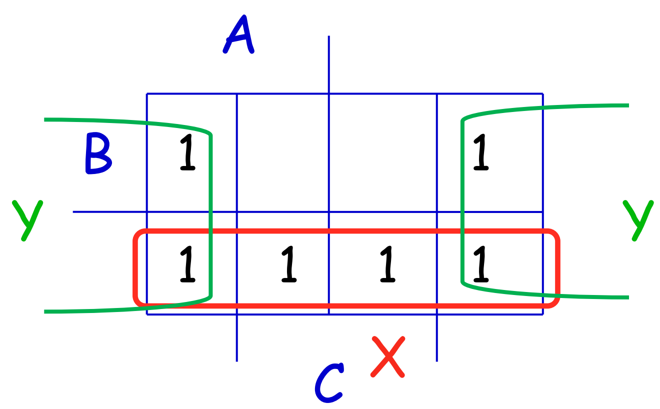
$$F_{1\text{-}4} = y't' + xt' + x'yt + \left\{ \begin{matrix} yzt \\ xyz \end{matrix} \right\} + \left\{ \begin{matrix} x'y'z \\ x'zt \end{matrix} \right\}$$

Both are OK; 2nd is **succinct!**

HW 14.1

$$f = A'B' + AC' + B'C + A'BC'$$

Solution



Essential: X, Y

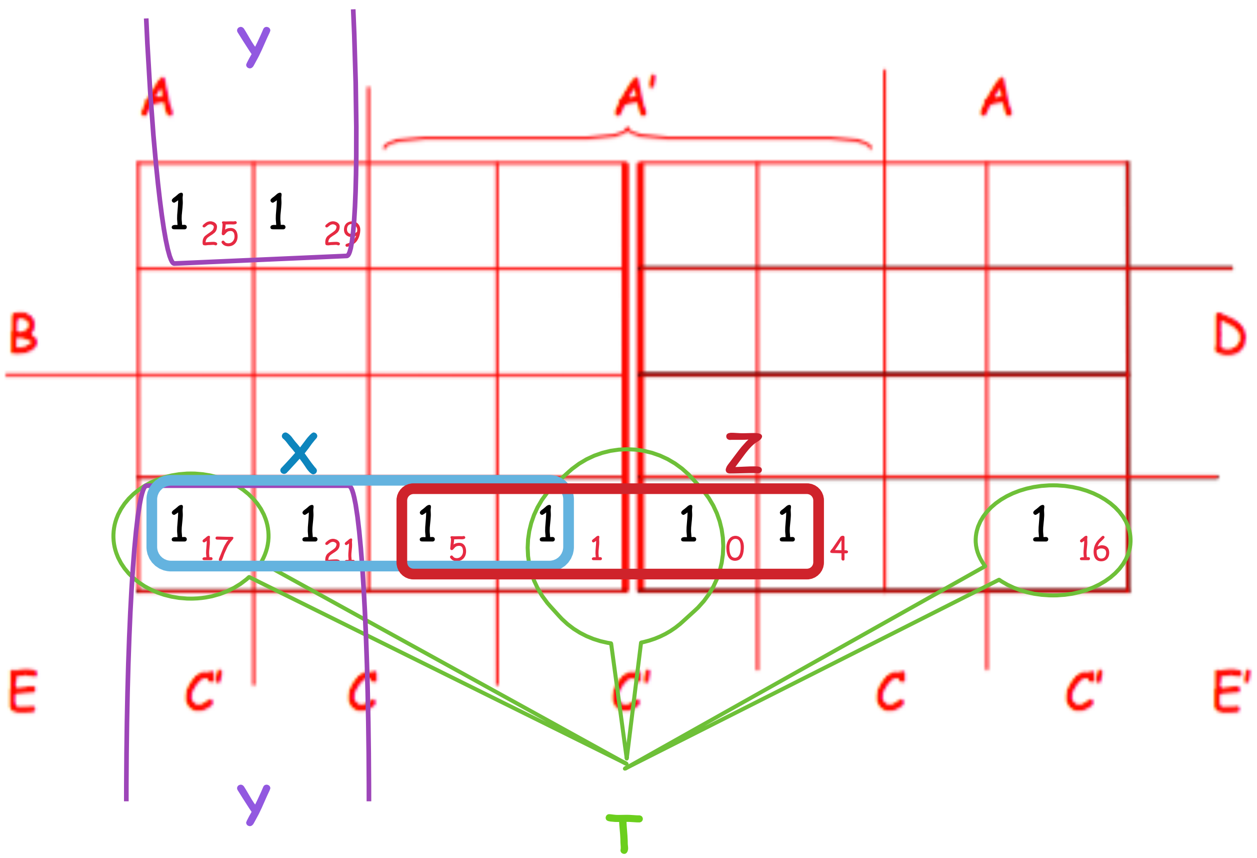
$$\boxed{f = X + Y = B' + C'}$$

HW 14.2

$$f = \Sigma(0, 1, 4, 5, 16, 17, 21, 25, 29)$$

Solution

$$f = \overset{0}{A'B'C'D'E'} + \overset{1}{A'B'C'D'E} + \overset{4}{A'B'CD'E'} + \overset{5}{A'B'CD'E} + \overset{16}{AB'C'D'E'} + \overset{17}{AB'C'D'E} + \overset{21}{AB'CD'E} + \overset{25}{ABC'D'E} + \overset{29}{ABCD'E}.$$



Question:

Do $1_{16}, 1_{17}, 1_{21}, 1_{5}$ create an implicant?

Answer: NO

Try to express the implicant with variables:

The only way: B'D'. What's wrong?

B'D' expresses a size-8 implicant: the whole lower row!

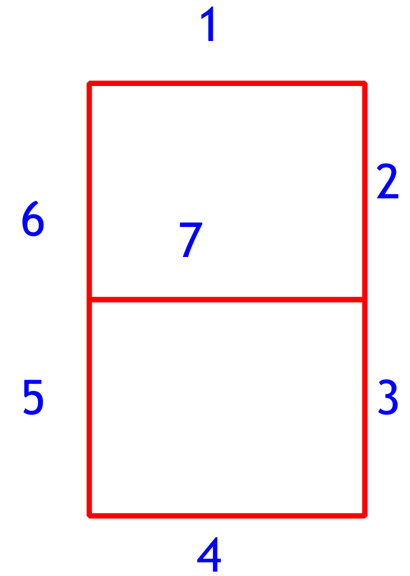
Prime implicants: X, Y, Z, T. Essential: Y, Z, T. They cover all 1's.

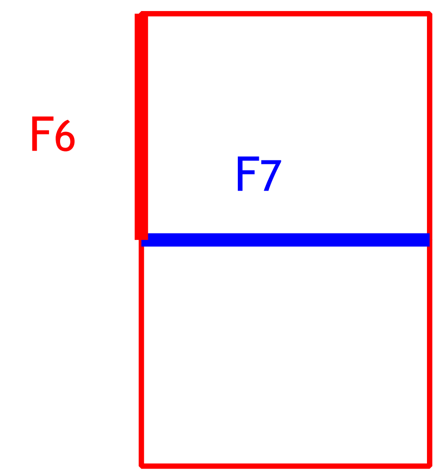
$$\boxed{f = Y + Z + T = AD'E + A'B'D' + B'C'D'}$$

L E D - 7-segment Display of Decimal Digits

Each segment lights up when the digit we want to create **requires it.**

Here it is:





We will focus on the segments, and write one function for each of the 7 segments, e.g. for:

$F_i = 0 \Leftrightarrow$ segment i is off

$F_i = 1 \Leftrightarrow$ segment i is on

	x	y	z	t	F_6
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	d
.	1	0	1	1	d
.	1	1	0	0	d
.	1	1	0	1	d
.	1	1	1	0	d
15	1	1	1	1	d

F_7 HW

What should we do with the function for inputs 10-15, which should never occur in our display? Do we care about the values F_6 gets for those inputs? NO

We therefore don't give a value of 0 or 1 for F_6 for those inputs. We will instead use the letter d ('don't care')

These d's, we will use to our advantage when minimizing the function. **NOTE**: The function we create will have to give a value of 0 or 1 for every possible input-occurring or not.

2^6 possibilities, for d = 0 or d = 1

We minimize 2^6 functions at once!

We put the d's on the K map, with the 1's.

Whether that value will be 0 or 1 will be established so that the function gets the minimal minimal form. It's simpler than it sounds :-)

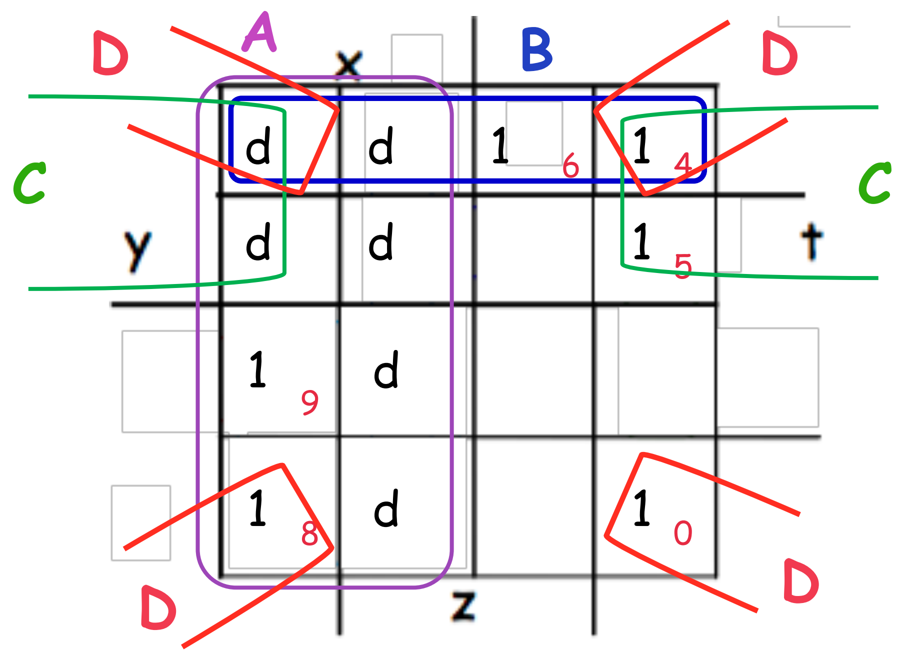
We use the d's to our advantage:

1) when forming implicants, then d = 1, as we want larger implicants- only if they cover at least one 1.

2) when performing the covering, we don't have to cover the d's, so d = 0 outside the minimal form.

Let's draw K map + form the prime implicants as a hint for HW 15.1:

From previous page we have:



Prime implicants:

	x	y	z	t	F_6
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	d
.	1	0	1	1	d
.	1	1	0	0	d
.	1	1	0	1	d
.	1	1	1	0	d
15	1	1	1	1	d

HW 15.1

Finish this by going on to finding all minimal forms for F_6.

HW 15.2

Find all minimal forms for F_7.