## 3.3 FOUR-VARIABLE K-MAP

The map for Boolean functions of four binary variables (w, x, y, z) is shown in **Fig. 3.8**  $\square$ , which lists the 16 minterms and the squares assigned to each. In **Fig. 3.8(b)**  $\square$ , the map is redrawn to show the relationship between the squares and the four variables. The rows and columns are numbered in a Gray code sequence, with only one digit changing value between two adjacent rows or columns. The minterm corresponding to each square can be obtained from the concatenation of the row number with the column number. For example, the numbers of the third row (11) and the second column (01), when concatenated, give the binary number 1101, the binary equivalent of decimal 13. Thus, the square in the third row and second column represents minterm  $m_{13}$ .

$m_0$	$m_1$	$m_3$	$m_2$			
$m_4$	$m_5$	$m_7$	$m_6$			
$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$			
$m_8$	$m_9$	$m_{11}$	$m_{10}$			
(a)						

\	\ yz	,			y			
W)	rX	00	01	11	10			
		$m_0$	$m_1$	$m_3$	$m_2$			
	00	w'x'y'z'	w'x'y'z	w'x'yz	w'x'yz'			
						,		
		$m_4$	$m_5$	$m_7$	$m_6$			
	01	w'xy'z'	w'xy'z	w'xyz	w'xyz'			
	,					$\int_{X}$		
		$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$	1		
	11	wxy'z'	wxy'z	wxyz	wxyz'			
w t	Į					J		
,,		$m_8$	$m_9$	$m_{11}$	$m_{10}$			
	10	$m_8$ $wx'y'z'$	wx'y'z	wx'yz	wx'yz'			
$\overline{z}$								
(b)								

## FIGURE 3.8

Four-variable map

The map minimization of four-variable Boolean functions is similar to the method used to minimize three-variable functions. Adjacent squares are defined to be squares next to each other (vertically or horizontally, but not diagonally). In addition, the map is considered to lie on a surface with the top and bottom edges, as well as the right and left edges, touching each other to form adjacent squares. For example,  $m_0$  and  $m_2$  form adjacent squares, as do  $m_3$  and  $m_{11}$ . The combination of adjacent squares that is useful during the simplification process is easily determined from inspection of the four-variable map:

One square represents one minterm, giving a term with four literals.

Two adjacent squares represent a term with three literals.

Four adjacent squares represent a term with two literals.

Eight adjacent squares represent a term with one literal.

Sixteen adjacent squares produce a function that is always equal to 1.