

7-Segment Display

CLASS 13

1) Consider the Boolean function

$$f = \Sigma (0, 2, 3, 7, 8, 10, 12, 14, 15)$$

After drawing the K-map, and determining all the prime implicants, find **all minimal forms** of f .

Quiz 2.1

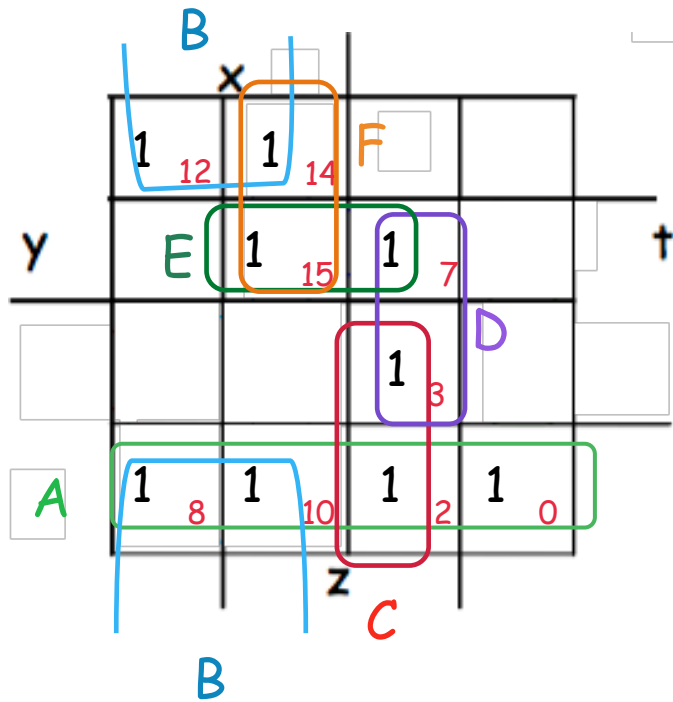
2) Consider the function F , obtained from f by just adding one minterm, the one corresponding to 5, that is,

$$F = \Sigma (0, 2, 3, 5, 7, 8, 10, 12, 14, 15)$$

Just like for the function above, f , find **all minimal forms** of F . Explain **how does the solution change** when the function has just one additional minterm. Encircle and enumerate all the solutions you obtained for f , and for F .

Solution

1)



Essential: $A, B \rightarrow f = A + B + \dots$ We need to cover 3 more 1's.

Take, e.g., 1_3

How can we cover it? We need $\begin{cases} C \\ \text{or} \\ D \end{cases}$

Minimal covers only:

Case 1. C : $f_1 = A + B + C + E \rightarrow$

$$f_1 = y't' + xt' + x'y'z + yzt$$

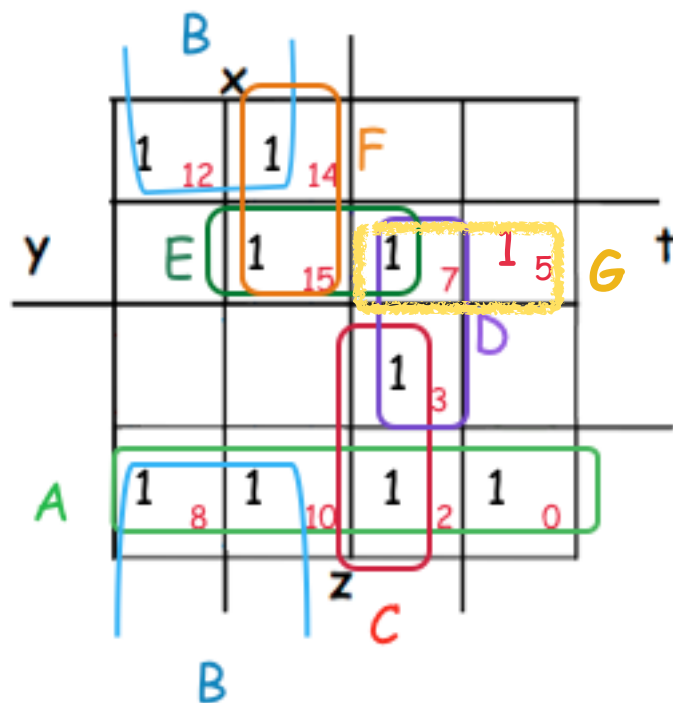
Case 2. D : $f_{2,3} = A + B + D + \begin{cases} E \\ \text{or} \\ F \end{cases}$

$$f_{2,3} = y't' + xt' + x'zt + \begin{cases} yzt \\ xyz \end{cases}$$

2)

Essential: A, B, G We need to cover 2 more 1's: 1_{15} and 1_3 .

They can be covered each independently by 2 size-2 implicants:



$$F_{1-4} = A + B + G + \begin{cases} E \\ \text{or} \\ F \end{cases} + \begin{cases} C \\ \text{or} \\ D \end{cases}$$

$$F_1 = y't' + xt' + x'yt + yzt + x'y'z$$

$$F_2 = y't' + xt' + x'yt + yzt + x'zt$$

$$F_3 = y't' + xt' + x'yt + xyz + x'y'z$$

$$F_4 = y't' + xt' + x'yt + xyz + x'zt$$

Comment: By introducing 1_5 we

create a new essential size-2 implicant, \rightarrow only two more 1's still need to be covered, and they can be covered each in two ways, independently from each other \rightarrow 4 forms total. Write:

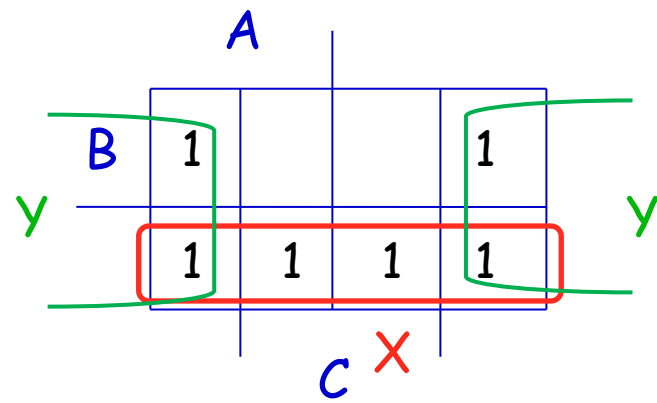
$$F_{1-4} = y't' + xt' + x'yt + \begin{cases} yzt \\ xyz \end{cases} + \begin{cases} x'y'z \\ x'zt \end{cases}$$

Both are OK; 2nd is succinct!

HW 14.1

$$f = A'B' + AC' + B'C + A'BC'$$

Solution



Essential: X, Y

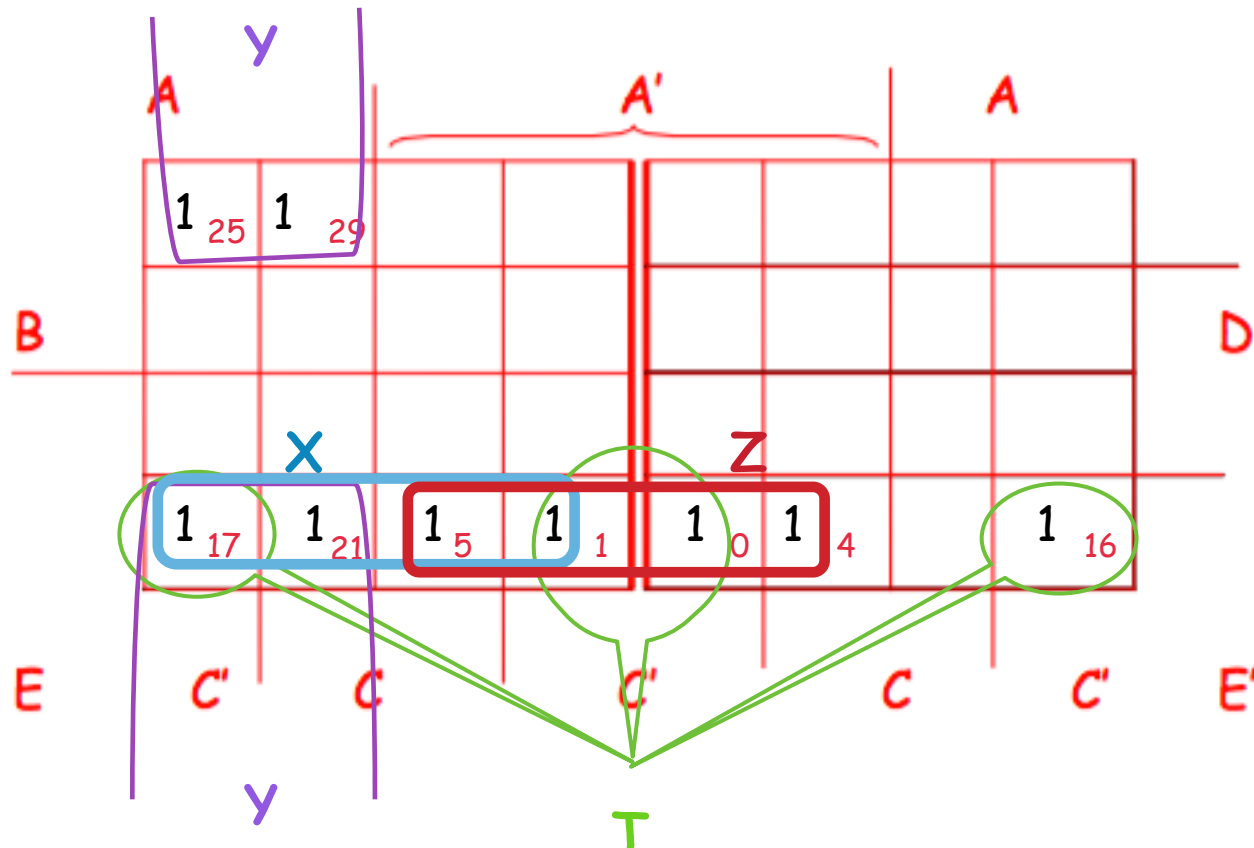
$$f = X + Y = B' + C'$$

HW 14.2

$$f = \Sigma(0, 1, 4, 5, 16, 17, 21, 25, 29)$$

Solution

$$f = \overset{0}{A'B'C'D'E'} + \overset{1}{A'B'C'D'E} + \overset{4}{A'B'CD'E'} + \overset{5}{A'B'CD'E} + \overset{16}{AB'C'D'E'} + \overset{17}{AB'C'D'E} + \overset{21}{AB'CD'E'} + \overset{25}{ABC'D'E} + \overset{29}{ABCD'E}.$$



Question:

Do $1_{16}, 1_{17}, 1_{21}, 1_5$ create an implicant?

Answer:

NO

Try to express the implicant with variables:

The only way: $B'D'$. What's wrong?

$B'D'$ expresses a size-8 implicant: the whole lower row!

Prime implicants: X, Y, Z, T. Essential: Y, Z, T. They cover all 1's.

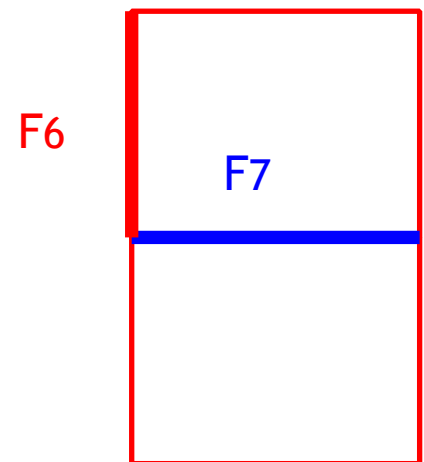
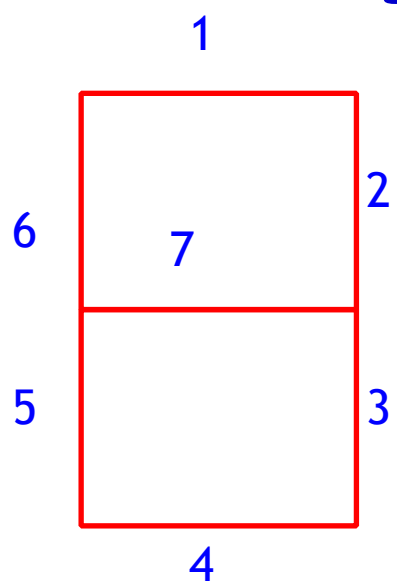
$$f = Y + Z + T = AD'E + A'B'D' + B'C'D'$$

L E D - 7-segment Display of Decimal Digits

Each segment lights up when the digit we want to create **requires** it.

We will focus on the segments, and write one function for each of the 7 segments, e.g. for:

Here it is:



$F_i = 0 \iff$ segment i is off
 $F_i = 1 \iff$ segment i is on

	x	y	z	t	F ₆
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	d
.	1	0	1	1	d
.	1	1	0	0	d
.	1	1	0	1	d
.	1	1	1	0	d
15	1	1	1	1	d

HW
F₇ ←

What should we do with the function for inputs 10-15, which should never occur in our display? Do we care about the values F₆ gets for those inputs? NO

We therefore don't give a value of 0 or 1 for F₆ for those inputs. We will instead use the letter d ('don't care')

These d's, we will use to our advantage when minimizing the function. **NOTE:** The function we create will have to give a value of 0 or 1 for every possible input-occurring or not.

2⁶ possibilities, for d = 0 We minimize 2⁶ functions at once!
or d = 1 We put the d's on the K map, with the 1's.

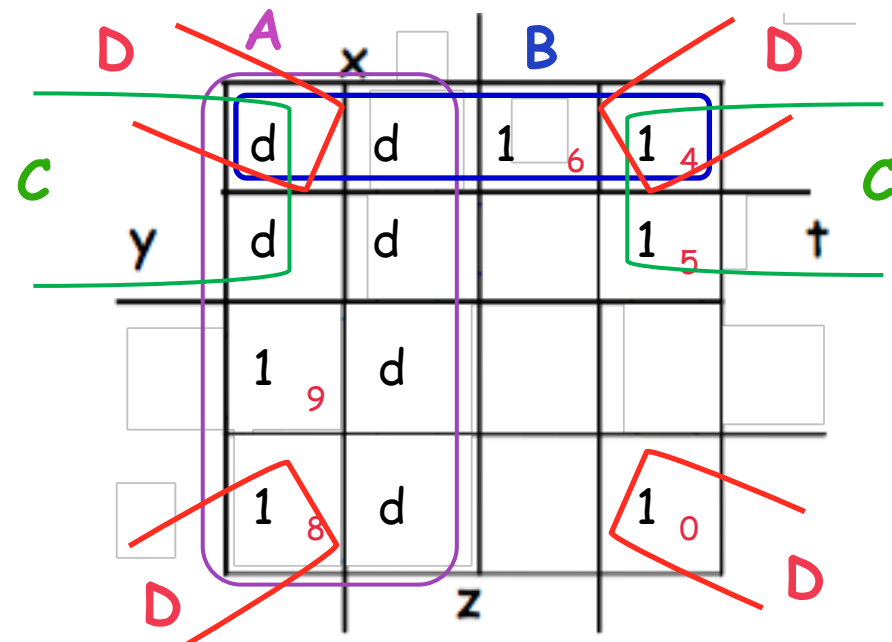
Whether that value will be 0 or 1 will be established so that the function gets the minimal minimal form. It's simpler than it sounds :-)

We use the d's to our advantage:

- 1) when forming implicants, then d = 1, as we want larger implicants- only if they cover at least one 1.
- 2) when performing the covering, we don't have to cover the d's, so d = 0 outside the minimal form.

Let's draw K map + form the prime implicants as a hint for HW 15.1:

From previous page we have:



Prime implicants:

	x	y	z	t	F ₆
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	d
.	1	0	1	1	d
.	1	1	0	0	d
.	1	1	0	1	d
.	1	1	1	0	d
15	1	1	1	1	d

HW 15.1

Finish this by going on to finding all minimal forms for F_6 .

HW 15.2

Find all minimal forms for F_7 .