CL=CSCI 160

CLASS 10

Review **Topics**

(1) Number Representations

- (r - 1)'s, r's complements, algorithm for subtraction, etc
- Find basis x if $(...)_x = ...$

(2) (3) – Boolean algebra:

- Expression manipulation
- Boolean operators

(4) distribution (both)

(12) absorption

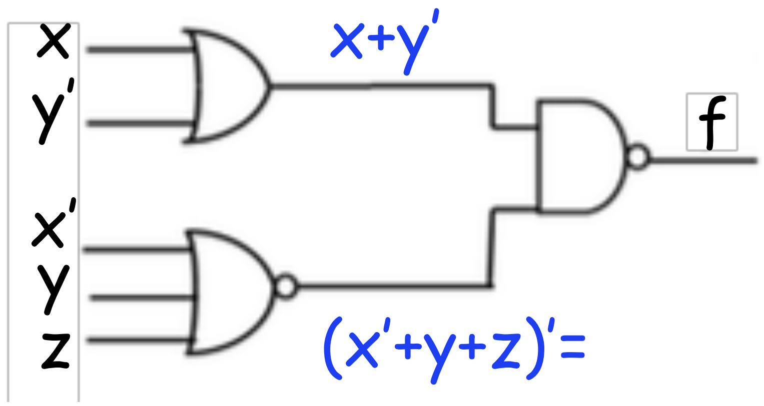
(10) De Morgan

(4) – Circuit – logic diagram (gates)

B. functions; truth table

(4) Gates

Find the truth table + expression of f



$(x'+y+z)'=$

$=xy'z'$

$$f = ((x+y')xy'z')' =$$

$$= (x+y')' + x' + y + z =$$

$$= \underbrace{x'y + x'}_{x'\ (12)} + y + z = \boxed{x' + y + z}$$

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

What can we say about the truth table for f?

When do we have f = 0, and when is f = 1?

It is 0 exactly when x'=0, and y=0, and z=0, that is

for the triple x y z = 1 0 0.

In rest we have f = 1.

(3) Operators

HW 8.1

(a) Prove: Inhibition is not associative

We chose $F_2 = x/y = x\,y'$

$$F_2 \text{ associative} \overset{\text{Det}}{\Leftrightarrow} F_2(F_2(x,y),z) = F_2(x, F_2(y,z)) \quad \forall\, x, y, z$$

$$\Updownarrow$$

$$F_2(xy', z) = F_2(x, yz')$$

$$\Updownarrow$$

$$xy'z' = x(yz')'$$

$$\Updownarrow \; (10)$$

$$xy'z' = x(y'+z) \qquad \forall\, x, y, z$$

$$F_2 \text{ not associative} \overset{\text{Det}}{\Leftrightarrow} \exists\, x, y, z: \quad xy'z' \neq x(y'+z)$$

Make, say, RS = 1, while LS = 0: We need: $x = 1$ and ($y' = 1$ or $z = 1$)

There are multiple such counterexamples.

We only need one, for instance $x = 1$, $z = 1$, and any value for y, say $y = 0$.

We have LS $= 0 \neq 1 =$ RS. We just proved: $\boxed{\textbf{F}_2 \textbf{ is not associative}}$

(b) Prove: Exclusive OR (XOR) is commutative

$F_6(x, y) = x\,y' + x'\,y = x \oplus y$

F_6 commutative $\overset{\text{Det}}{\Leftrightarrow}$ $F_6(x, y) = F_6(y, x) \quad \forall\, x, y$

$\Updownarrow$

$\oplus$ commutative $\overset{\text{Det}}{\Leftrightarrow}$ $x \oplus y = y \oplus x \quad \forall\, x, y$

$\Updownarrow$

$x\,y' + x'\,y = y\,x' + y'\,x \quad \forall\, x, y$

The equality holds for both binary operations + and • are commutative.

We just proved: $\boxed{\textbf{F}_6\textbf{, that is Exclusive OR (XOR) is commutative}}$

(2) Expression manipulation

Basically what we have been doing all along - old HW

2.6)

a) $(xy' + x'y)' \overset{(10)}{=} (x' + y) \cdot (x + y') = \boxed{xy + x'y'}$

b) $((AB' + C)\, D' + E)' \overset{(10)}{=} ((A' + B) \cdot C' + D)\, E' \overset{(4)}{=} \boxed{(A' + B + D)(C' + D)\, E'}$

c) $((x + y' + z)(x' + z')(x + y))' \overset{(10)}{=} \boxed{x' \cdot y \cdot z' + x \cdot z + x' \cdot y'}$

More exercises:

A. $\underbrace{xyz' + x'yz + xyz}_{xy(z+z') = xy} + x'yz' \overset{(7)}{=} \underbrace{xy + yz + x'y}_{y} \overset{(4)}{=} y + yz \overset{(12)}{=} \boxed{y}$

$x'y$

(4) (5) (2)

(5) (2)

$xy\underbrace{(z+z')}_{1} = xy$

$(x+x')yz$

B. $$\underbrace{xyzt + x'yt + xyz't}_{xyt} + yzt \overset{(4)}{\underset{\substack{(5)\\(2)}}{=}} \underbrace{xyt + x'yt}_{yt} + yzt \overset{(4)}{\underset{\substack{(5)\\(2)}}{=}} yt + yzt \overset{(12)}{=} \boxed{yt}$$

C. Prove: $xy + x'z + yz = xy + x'z$

Expand the terms that are in LS \ RS, namely yz

$$\boxed{LS} = xy + x'z + (x + x')yz = xy + x'z + xyz + x'yz \overset{(12)\times 2}{=}$$

$$= xy + x'z = \boxed{RS}$$

(1) Number Representations

Review conversions and operations in any base.

Let's do the r's / (r-1)'s complements and apply the algorithm for subtracting two numbers.

1. r = 16

Compute the (r-1)'s complement of $3ABF09_{hex}$. We have r - 1 = 16 - 1 = 15. Subtract each digit from F=15:

$$(3ABF09_{hex})_{15'C} = \boxed{C540F6}$$

Let's do it via r = 2 (all computations are in binary). How?

$3ABF09_{hex} = 0011\ 1010\ 1011\ 1111\ 0000\ 1001_2 \xrightarrow{\text{1's complement}}$

$\xrightarrow{\text{1's complement}} 1100\ 0101\ 0100\ 0000\ 1111\ 0110_2 = \boxed{C540F6_{hex}}$

2. Let's apply the algorithm for subtracting two numbers

r = 8   n = 4, m = 2   Remember: $N = \overbrace{\qquad\qquad}^{\text{integer part}} \bullet \overbrace{\qquad\qquad}^{\text{fraction part}}$

Alg (M-N): 1) $M + N_{(r-1)c}$ 2) a) e.a.c. -> add it to l.s.d.
b) no e.a.c. -> compute -(r-1)'s compl. of 1)

Let $M_8 = 1671.02$, $N_8 = 2016.12$ Perform M – N using the algorithm using (r-1)'s rep.

1st solution-direct:

1 1
M_8 = 1671.02 +
N_{7c} = 5761.65

7652.67

No e.a.c., branch 2b), so do – 7'c

Result: $\boxed{-0125.10_8}$

2nd solution-convert to binary:

M_2: 001 110 111 001. 000 010
N_2: 010 000 001 110. 001 010

1 1 1 1 1 1 1
M_2: 001 110 111 001. 000 010 +
N_{1C}: 101 111 110 001. 110 101

111 110 101 010. 110 111

No e.a.c., branch 2b), so do – 1'c

-000 001 010 101.001 000 Convert to base 8 to get same result as above.

NOTE

We can do these conversions to other bases and back only if self-complementary:

hex ⟷ binary Yes, for self-complementary

Decimal ⟷ (2421) code
(8, 4, -2, -1) code Yes, for self-complementary

Decimal ⟷̸ BCD No, not self-complementary

see next slide!

Decimal ⟷ (2421) code

(8, 4, -2, -1) code Yes, for self-complementary

Remember:

Decimal ⟷ BCD No, not self-complementary

Table
Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110