

Homework 2

Due: Thursday 2/7/19 by 12:00pm

The AR(1) Model

2. This problem will ask you to work with the autoregressive (AR) model.
 - (a) Describe what R returns when you run `x <- arima.sim(n = 100, list(ar=1), sd = 1)`, and why this occurs.
 - (b) Simulate 1,000 **AR**(1) time series of length $n = 100$ with $\sigma_w^2 = 1$ for values of $\phi_1 = \{-0.5, -0.25, -0.125, 0, 0.125, 0.25, 0.5\}$. For each value of ϕ_1 , compute the percent of simulations in which a level-0.05 test of the null hypothesis that $\rho(1) = 0$ rejects the null. Plot the percent of simulations in which a test of the null hypothesis rejects the null against ϕ_1 .
 - (c) When $\phi_1 \neq 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **power** of the test. When $\phi_1 = 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **level** of the test. Is the estimated level 0.05, as we would expect from a level-0.05 test? If not, why not?
 - (d) Describe in at most two sentences how the power of the test relates to the true value ϕ_1 . Intuitively, does this make sense?

The MA(1) Model

2. This problem will ask you to work with the moving average (MA) model.
 - (a) Without using the `arima.sim` function or any other third party function for simulating an MA time series, simulate a length-100 time series \mathbf{x} according to the MA model:

$$x_t = 0.5w_{t-1} + w_t, w_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$$

- (b) Using the code you wrote in (a) or `arima.sim`, simulate 1,000 **MA**(1) time series of length $n = 100$ with $\sigma_w^2 = 1$ for values of $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$. For each value of θ_1 , compute the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null. Plot the percent of simulations in which a test of the null hypothesis rejects the null against θ_1 .
 - (c) When $\theta_1 \neq 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **power** of the test. When $\theta_1 = 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **level** of the test. Is the estimated level 0.05? If not, why not?
 - (d) Describe in at most two sentences how the power of the test relates to the true value θ_1 . Intuitively, does this make sense?

Comparing AR and MA Models

3. This problem asks you to compare what you observed in 1. (b)-(d) to what you observed in 2. (b)-(d).
 - (a) Combine the plots from 1. (b) and 2. (b) into a single plot.
 - (b) Compute $\rho(1)$ under for an **AR**(1) model with $\phi_1 = \{-0.5, -0.25, -0.125, 0.125, 0.25, 0.5\}$.
 - (c) Compute $\rho(1)$ under for an **MA**(1) model with $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$.
 - (d) In one sentence, interpret what you observe in (a), taking what you find in (b) and (c) into account.