Homework 3

Due: Thursday 2/14/19 by 12:00pm (noon)

The `R` library `polynom` lets us easily compute the roots of polynomials. You'll need to install the `polynom` library and load it. I'll ask that you use it a bit in this homework, so here's a little example:

```
library(polynom)

# Create a "polynomial" object for the polynomial
# 1 - 5x + 3x^2 + 2x^3
pol <- polynomial(c(1, -5, 3, 2))
# Get the values of x for which 1 - 5x + 3x^2 + 2x^3 = 0
sol <- solve(pol)
```

You may get complex roots $r = a + bi$. Note that the absolute value of a complex number r is given by $|r| = \sqrt{a^2 + b^2}$.

1. Consider the following $\mathbf{AR}(p)$ models, all with $\sigma_w^2 = 1$.

   i. $p = 1$, $\phi_1 = 0.99$
   ii. $p = 2$, $\phi_1 = 0.04$, $\phi_2 = 0.92$
   iii. $p = 2$, $\phi_1 = 0.04$, $\phi_2 = -0.92$
   iv. $p = 3$, $\phi_1 = 0.42$, $\phi_2 = -0.29$, $\phi_3 = 0.15$

   (a) For (i)-(iii), find the root of the autoregressive polynomial that is smallest in magnitude by solving $\phi(z) = 0$ for z by hand, without using any special `R` functions. For (iv), use `polynom` to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is causal.

   (b) Using `ARMAacf` to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the causal $\mathbf{AR}(p)$ models on a single plot. Include a dotted horizontal line at 0.

   (c) Based on the plot you made in (b), describe what kinds of values of the autoregressive coefficients $\phi_1, \dots, \phi_p$ produce the following behaviors, using at most one sentence for each:

   - $\rho_x(h)$ oscillates between positive and negative values;
   - $\rho_x(h)$ oscillates between larger and smaller but always positive values;
   - $\rho_x(h)$ decays very quickly;
   - $\rho_x(h)$ decays very slowly.

2. Consider the following $\mathbf{MA}(q)$ models, all with $\sigma_w^2 = 1$.

   i. $q = 1$, $\theta_1 = 0.99$
   ii. $q = 2$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
   iii. $q = 2$, $\theta_1 = 0.04$, $\theta_2 = -0.92$
   iv. $q = 3$, $\theta_1 = 0.42$, $\theta_2 = -0.29$, $\theta_3 = 0.15$

   (a) For (i)-(iii), find the root of the moving average polynomial that is smallest in magnitude by solving $\theta(z) = 0$ for z by hand, without using any special `R` functions. For (iv), use `polynom` to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is invertible.

   (b) Using `ARMAacf` to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the invertible $\mathbf{MA}(q)$ models on a single plot. Include a dotted horizontal line at 0.

   (c) Based on the plot you made in (b) and the values you computed for the autocorrelation function $\rho_x(h)$, give the largest lag for which $|\rho_x(h)|$ is greater than zero for each $\mathbf{MA}(q)$ model.

3. Consider the following **ARMA**(p, q) models, all with $\sigma_w^2 = 1$.

i. $p = 1$, $q = 1$, $\phi_1 = 0.99$, $\theta_1 = 0.99$
ii. $p = 1$, $q = 2$, $\phi_1 = 0.99$, $\theta_1 = 0.04$, $\theta_2 = -0.92$
iii. $p = 2$, $q = 2$, $\phi_1 = 0.04$, $\phi_2 = 0.92$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
iv. $p = 2$, $q = 2$, $\phi_1 = 0.04$, $\phi_2 = -0.92$, $\theta_1 = 0.04$, $\theta_2 = 0.92$
v. $p = 3$, $q = 3$, $\phi_1 = 0.42$, $\phi_2 = -0.29$, $\phi_3 = 0.15$, $\theta_1 = 0.42$, $\theta_2 = -0.29$, $\theta_3 = 0.15$

(a) Indicate whether or not each **ARMA**(p, q) model is causal, and indicate whether or not each **ARMA**(p, q) model is invertible.

(b) Using `ARMAacf` to compute $\rho_x(h)$, plot the autocorrelation function $\rho_x(h)$ for $h = 0, \ldots, h$ for the causal and invertible **ARMA**(p, q) models on a single plot. Include a dotted horizontal line at 0.

(c) Based on the plot you made in (b), describe what kinds of values of the autoregressive and moving average coefficients $\phi_1, \ldots, \phi_p$ and $\theta_1, \ldots, \theta_q$ produce the following behaviors, using at most one sentence for each:

- Quickly decaying correlations for small h and slowly decaying correlations for large h;
- Periodic/seasonal/cyclical behavior.