Homework 2

Due: Thursday 2/7/19 by 12:00pm (noon)

The AR(1) Model

2. This problem will ask you to work with the autoregressive (AR) model.
   (a) Describe what `R` returns when you run `x <- arima.sim(n = 100, list(ar=1), sd = 1)`, and why this occurs.
   (b) Simulate $1,000$ $\mathbf{AR}(1)$ time series of length $n = 100$ with $\sigma_w^2 = 1$ for values of $\phi_1 = \{-0.5, -0.25, -0.125, 0, 0.125, 0.25, 0.5\}$. For each value of ϕ_1, compute the percent of simulations in which a level-0.05 test of the null hypothesis that $\rho(1) = 0$ rejects the null, using $\hat{\rho}(1)$ from class and 3.(h) in Homework 1. Plot the percent of simulations in which a test of the null hypothesis rejects the null against ϕ_1.
   (c) When $\phi_1 \neq 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **power** of the test. When $\phi_1 = 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **level** of the test. Is the estimated level 0.05, as we would expect from a level-0.05 test? If not, why not?
   (d) Describe in at most two sentences how the power of the test relates to the true value ϕ_1. Intuitively, does this make sense?

The MA(1) Model

2. This problem will ask you to work with the moving average (MA) model.
   (a) Without using the `arima.sim` function or any other third party function for simulating an MA time series, simulate a length-100 time series $\boldsymbol{x}$ according to the MA model:

$$x_t = 0.5w_{t-1} + w_t,\ w_t \overset{i.i.d.}{\sim} \mathcal{N}(0, 1)$$

   (b) Using the code you wrote in (a) or `arima.sim`, simulate $1,000$ $\mathbf{MA}(1)$ time series of length $n = 100$ with $\sigma_w^2 = 1$ for values of $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$. For each value of θ_1, compute the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null, using $\hat{\rho}(1)$ from class and 3.(h) in Homework 1. Plot the percent of simulations in which a test of the null hypothesis rejects the null against θ_1.
   (c) When $\theta_1 \neq 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **power** of the test. When $\theta_1 = 0$, the percent of simulations in which a test of the null hypothesis that $\rho(1) = 0$ rejects the null estimates the **level** of the test. Is the estimated level 0.05? If not, why not?
   (d) Describe in at most two sentences how the power of the test relates to the true value θ_1. Intuitively, does this make sense?

Comparing AR(1) and MA(1) Models

3. This problem asks you to compare what you observed in 1. (b)-(d) to what you observed in 2. (b)-(d).
   (a) Combine the plots from 1. (b) and 2. (b) into a single plot.
   (b) Compute the true lag-one autocorrelation $\rho(1)$ under for an $\mathbf{AR}(1)$ model with $\phi_1 = \{-0.5, -0.25, -0.125, 0.125, 0.25, 0.5\}$.
   (c) Compute the true lag-one autocorrelation $\rho(1)$ under for an $\mathbf{MA}(1)$ model with $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$.
   (d) In one sentence, interpret what you observe in (a), taking what you find in (b) and (c) into account.