

Homework 3

Due: Thursday 2/14/19 by 12:00pm (noon)

The R library `polynom` lets us easily compute the roots of polynomials. You'll need to install the `polynom` library and load it. I'll ask that you use it a bit in this homework, so here's a little example:

```
library(polynom)

# Create a "polynomial" object for the polynomial
# 1 - 5x + 3x^2 + 2x^3
pol <- polynom(c(1, -5, 3, 2))
# Get the values of x for which 1 - 5x + 3x^2 + 2x^3 = 0
sol <- solve(pol)
```

1. Consider the following $\mathbf{AR}(p)$ models, all with $\sigma_w^2 = 1$.
 - i. $p = 1, \phi_1 = 0.99$
 - ii. $p = 2, \phi_1 = 0.04, \phi_2 = 0.92$
 - iii. $p = 2, \phi_1 = 0.04, \phi_2 = -0.92$
 - iv. $p = 3, \phi_1 = 0.42, \phi_2 = -0.29, \phi_3 = 0.15$
 - (a) For (i)-(iii), find the root of the autoregressive polynomial that is smallest in magnitude by solving $\phi(z) = 0$ for z by hand, without using any special R functions. For (iv), use `polynom` to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is causal.
 - (b) Plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the causal $\mathbf{AR}(p)$ models on a single plot. Include a dotted horizontal line at 0.
 - (c) Based on the plot you made in (b), describe what kinds of values of the autoregressive coefficients ϕ_1, \dots, ϕ_p produce the following behaviors, using at most one sentence for each:
 - $\rho_x(h)$ oscillates between positive and negative values;
 - $\rho_x(h)$ oscillates between larger and smaller but always positive values;
 - $\rho_x(h)$ decays very quickly;
 - $\rho_x(h)$ decays very slowly.
2. Consider the following $\mathbf{MA}(q)$ models, all with $\sigma_w^2 = 1$.
 - i. $q = 1, \theta_1 = 0.99$
 - ii. $q = 2, \theta_1 = 0.04, \theta_2 = 0.92$
 - iii. $q = 2, \theta_1 = 0.04, \theta_2 = -0.92$
 - iv. $q = 3, \theta_1 = 0.42, \theta_2 = -0.29, \theta_3 = 0.15$
 - (a) For (i)-(iii), find the root of the moving average polynomial that is smallest in magnitude by solving $\theta(z) = 0$ for z by hand, without using any special R functions. For (iv), use `polynom` to find the root that is smallest in magnitude. Give the value of this root and indicate whether or not the model is invertible.
 - (b) Plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, 10$ for the invertible $\mathbf{MA}(q)$ models on a single plot. Include a dotted horizontal line at 0.
 - (c) Based on the plot you made in (b), describe what kinds of values of the autoregressive coefficients $\theta_1, \dots, \theta_q$ produce the following behaviors, using at most one sentence for each:
 - The largest lag for which $\rho_x(h)$ is greater than zero for each $\mathbf{MA}(q)$ model.
3. Consider the following $\mathbf{ARMA}(p, q)$ models, all with $\sigma_w^2 = 1$.

- i. $p = 1, q = 1, \phi_1 = 0.99, \theta_1 = 0.99$
 - ii. $p = 1, q = 2, \phi_1 = 0.99, \theta_1 = 0.04, \theta_2 = -0.92$
 - iii. $p = 2, q = 2, \phi_1 = 0.04, \phi_2 = 0.92, \theta_1 = 0.04, \theta_2 = 0.92$
 - iv. $p = 2, q = 2, \phi_1 = 0.04, \phi_2 = -0.92, \theta_1 = 0.04, \theta_2 = 0.92$
 - v. $p = 3, q = 3, \phi_1 = 0.42, \phi_2 = -0.29, \phi_3 = 0.15, \theta_1 = 0.42, \theta_2 = -0.29, \theta_3 = 0.15$
- (a) Indicate whether or not each **ARMA**(p, q) model is causal, and indicate whether or not each **ARMA**(p, q) model is invertible.
- (b) Plot the autocorrelation function $\rho_x(h)$ for $h = 0, \dots, h$ for the causal and invertible **ARMA**(p, q) models on a single plot. Include a dotted horizontal line at 0.
- (c) Based on the plot you made in (b), describe what kinds of values of the autoregressive and moving average coefficients ϕ_1, \dots, ϕ_p and $\theta_1, \dots, \theta_q$ produce the following behaviors, using at most one sentence for each:
- Quickly decaying correlations for small h and slowly decaying correlations for large h ;
 - Periodic/seasonal/cyclical behavior.