

# Homework 2

*Due: Thursday 2/7/19 by 12:00pm*

## The AR(1) Model

2. This problem will ask you to work with the autoregressive (AR) model.
  - (a) Describe what R returns when you run `x <- arima.sim(n = 100, list(ar=1), sd = 1)`, and why this occurs.
  - (b) Simulate 1,000 **AR**(1) time series of length  $n = 100$  with  $\sigma_w^2 = 1$  for values of  $\phi_1 = \{-0.5, -0.25, -0.125, 0, 0.125, 0.25, 0.5\}$ . For each value of  $\phi_1$ , compute the percent of simulations in which a level-0.05 test of the null hypothesis that  $\rho(1) = 0$  rejects the null. Plot the percent of simulations in which a test of the null hypothesis rejects the null against  $\phi_1$ .
  - (c) When  $\phi_1 \neq 0$ , the percent of simulations in which a test of the null hypothesis that  $\rho(1) = 0$  rejects the null estimates the **power** of the test. When  $\phi_1 = 0$ , the percent of simulations in which a test of the null hypothesis that  $\rho(1) = 0$  rejects the null estimates the **level** of the test. Is the estimated level 0.05, as we would expect from a level-0.05 test? If not, why not?
  - (d) Describe in at most two sentences how the power of the test relates to the true value  $\phi_1$ . Intuitively, does this make sense?

## The MA(1) Model

2. This problem will ask you to work with the moving average (MA) model.
  - (a) Without using the `arima.sim` function or any other third party function for simulating an MA time series, simulate a length-100 time series  $\mathbf{x}$  according to the MA model:

$$x_t = 0.5w_{t-1} + w_t, w_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$$

- (b) Simulate 1,000 **MA**(1) time series of length  $n = 100$  with  $\sigma_w^2 = 1$  for values of  $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$ . For each value of  $\theta_1$ , compute the percent of simulations in which a test of the null hypothesis that  $\rho(1) = 0$  rejects the null. Plot the percent of simulations in which a test of the null hypothesis rejects the null against  $\theta_1$ .
  - (c) When  $\theta_1 \neq 0$ , the percent of percent of simulations in which a test of the null hypothesis that  $\rho(1) = 0$  rejects the null estimates the **power** of the test. When  $\theta_1 = 0$ , the percent of percent of simulations in which a test of the null hypothesis that  $\rho(1) = 0$  rejects the null estimates the **level** of the test. Is the estimated level 0.05? If not, why not?
  - (d) Describe in at most two sentences how the power of the test relates to the true value  $\theta_1$ . Intuitively, does this make sense?

## Comparing AR and MA Models

3. This problem asks you to compare what you observed in 1. (b)-(d) to what you observed in 2. (b)-(d).
  - (a) Combine the plots from 1. (b) and 2. (b) into a single plot.
  - (b) Compute  $\rho(1)$  under for an **AR**(1) model with  $\phi_1 = \{-0.5, -0.25, -0.125, 0.125, 0.25, 0.5\}$ .
  - (c) Compute  $\rho(1)$  under for an **MA**(1) model with  $\theta_1 = \{-1, -0.268, -0.127, 0, 0.127, 0.268, 1\}$ .
  - (d) In one sentence, interpret what you observe in (a), taking what you find in (b) and (c) into account.