

Homework 6

Maryclare Griffin

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Testing Nonstationarity

For $n \in \{10, 100, 500\}$ simulate 100 time series according to each of the following models, all of which assume $w_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_w^2)$:

- (i) $x_t = 0.5x_{t-1} + 0.25x_{t-2} + 0.125x_{t-3} + w_t$;
- (ii) $x_t = -10(t/n - 1/2)^2 + w_t$;
- (iii) $x_t = 10(t/n - 1/2)^2 w_t$;
- (iv) $x_t = x_{t-1} + w_t$;
- (v) $x_t = 0.5x_{t-1} + 0.25x_{t-2} + 0.25x_{t-3} + w_t$;
- (vi) $x_t = -x_{t-1} + w_t$.

In total, you will simulate $3 \times 6 \times 100$ time series. For each time series, use the `ndiff` function from the `forecast` library to perform an Augmented Dickey-Fuller and a Phillips-Perron test of the null hypothesis that the undifferenced time series is stationary rejects the null. Record whether or not the null is rejected.

- (a) Plot the rejection rates as a function of n for each model.
- (b) The probability that a test rejects the null when the alternative is true is the **power** of a test. Based on your figure in (a), describe how the power of the tests depend on n .
- (c) Based on your plot in (a), explain in one sentence how well the tests can detect a non-stationary **AR**(3) process if ϕ_1 is not greater than 1 and n is large.
- (d) Based on your plot in (a), explain in one sentence how well the tests can detect variance nonstationary when n is large.
- (e) Based on your plot in (a), explain in one sentence how well the tests can detect a non-stationary random walk when n is large.
- (f) Based on your plot in (a), explain in one sentence how well the tests can detect a nonlinear mean function when n is large.
- (g) Based on your plot in (a), explain in one sentence how the tests perform when x_t is a non-stationary **AR**(1) time series with an autoregressive polynomial root that is not equal to 1 and n is large.
- (h) Based on everything you've observed here, is it sufficient to just apply non-stationarity tests to determine whether or not an observed time series is stationary, or is it also important to look at the data? Answer in at most one sentence.