

# Homework 6

*Due: Never, To Be Used as Preparation for Exam 2*

## Testing Nonstationarity

For  $n \in \{10, 100, 500\}$  simulate 100 time series according to each of the following models, all of which assume  $w_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_w^2)$ :

- (i)  $x_t = 0.5x_{t-1} + 0.25x_{t-2} + 0.125x_{t-3} + w_t$ ;
- (ii)  $x_t = -10(t/n - 1/2)^2 + w_t$ ;
- (iii)  $x_t = 10(t/n - 1/2)^2 w_t$ ;
- (iv)  $x_t = x_{t-1} + w_t$ ;
- (v)  $x_t = 0.5x_{t-1} + 0.25x_{t-2} + 0.25x_{t-3} + w_t$ ;
- (vi)  $x_t = -x_{t-1} + w_t$ .

In total, you will simulate  $3 \times 6 \times 100$  time series. For each time series, use the `ndiff` function from the `forecast` library to perform an Augmented Dickey-Fuller and a Phillips-Perron test of the null hypothesis that the undifferenced time series is stationary rejects the null. Record whether or not the null is rejected.

- (a) Plot the rejection rates as a function of  $n$  for each model.
- (b) The probability that a test rejects the null when the alternative is true is the **power** of a test. Based on your figure in (a), describe how the power of the tests depend on  $n$ .
- (c) Based on your plot in (a), explain in one sentence how well the tests can detect a non-stationary **AR**(3) process if  $\phi_1$  is not greater than 1 and  $n$  is large.
- (d) Based on your plot in (a), explain in one sentence how well the tests can detect variance nonstationary when  $n$  is large.
- (e) Based on your plot in (a), explain in one sentence how well the tests can detect a non-stationary random walk when  $n$  is large.
- (f) Based on your plot in (a), explain in one sentence how well the tests can detect a nonlinear mean function when  $n$  is large.
- (g) Based on your plot in (a), explain in one sentence how the tests perform when  $x_t$  is a non-stationary **AR**(1) time series with an autoregressive polynomial root that is not equal to 1 and  $n$  is large.
- (h) Based on everything you've observed here, is it sufficient to just apply non-stationarity tests to determine whether or not an observed time series is stationary, or is it also important to look at the data? Answer in at most one sentence.