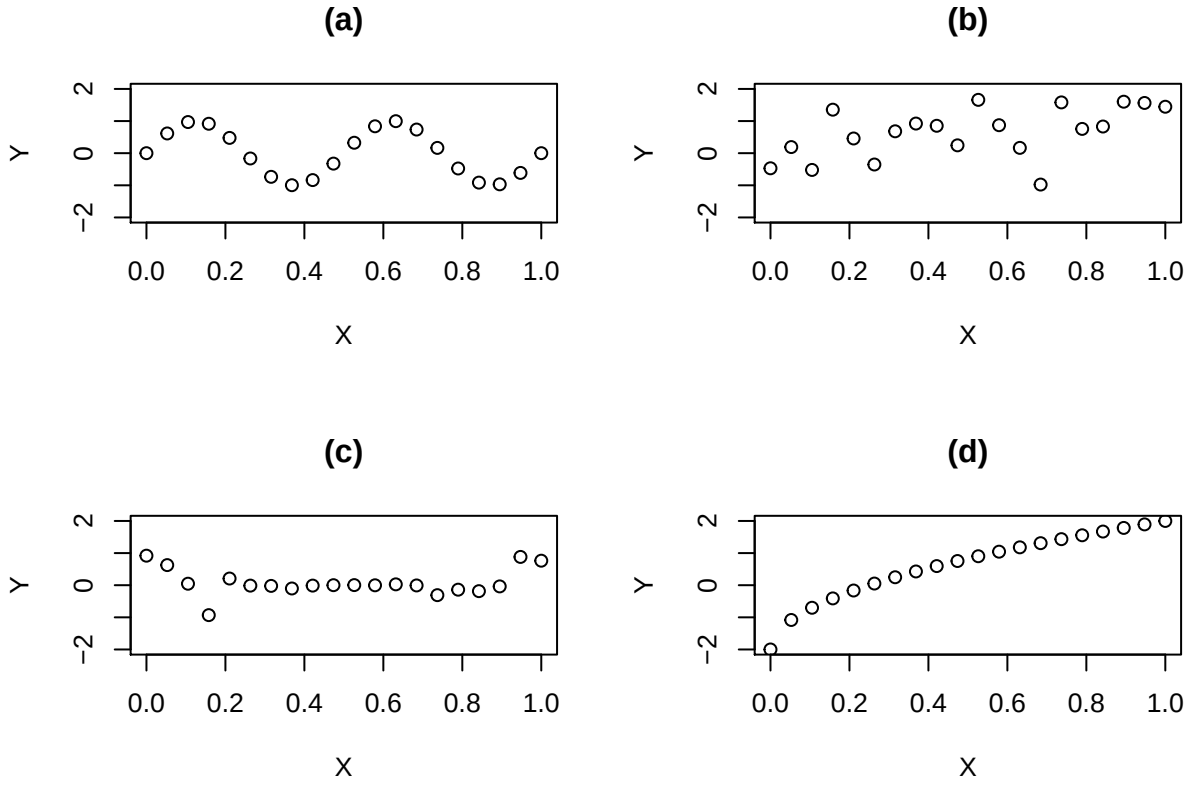
Homework 2

Due: Thursday 2/13/20 by 8:30am

This homework assignment focuses on material covered in Chapter 1 of the textbook.

1. For each of these figures, indicate whether a functional or statistical relationship is depicted between Y and X.



2. Suppose we collected data on the age and shoe size of $n = 3$ people. Let Y_i refer to the i-th subject's shoe size, and X_i refer to subject i's age.

(a) Suppose the first two subjects were the same age, i.e. $X_1 = X_2$, but had different shoe sizes, $Y_1 \neq Y_2$. What feature of the simple linear regression model described in Equation 1.1 of the text is illustrated by this?

(b) Suppose that we only collected data on high school students. If we assume the simple linear regression model described in Equation 1.1 of the text, what is the scope of the model?

3. Suppose Instagram magically knew that every time the number of times user i purchases a product, denoted by Y_i, is related to the number of times the product has been advertised to user i, denoted by X_i, as follows:

$$Y_i = 1 + 2X_i + \epsilon_i$$

where ϵ_i is a random error term with mean $E\{\epsilon_i\} = 0$ and variance $\sigma^2\{\epsilon_i\} = 0.1$; ϵ_i and ϵ_j are uncorrelated so that their covariance is zero (i.e., $\sigma\{\epsilon_i, \epsilon_j\} = 0$ for all $i \neq j$) for $i = 1, \ldots n$.

(a) Which value corresponds to the intercept, β_0? In at most one sentence, interpret it, assuming that the scope of the model includes $X_i = 0$.

(b) Which value corresponds to the intercept, β_1? In at most one sentence, interpret it.

(c) What do you expect the regression function to look like?

(d) Using `R`, make a plot with three panels. Plot the density of the errors ϵ_i for $X_i = 0$, $X_i = 1$, and $X_i = 10$, using a separate panel for each value of X_i. Ensure that the axes are the same across all three plots.

(e) Based on the assumed model and the information provided, can we conclude that the number of times user i purchases a product Y_i is independent of the number of times user j purchases a product Y_j?

(f) Based on the assumed model and the information provided, can we state the exact probability that a single value Y_i will be greater than 4 given that $X_i = 1$?

(g) Simulate $n = 100$ observations from the model, with $X_i = i$. Using `R`, make a scatter plot of the data and overlay the regression function on the scatterplot.

(h) Repeat (g), but instead of assuming that $\sigma^2\{\epsilon_i\} = 0.1$, assume that $\sigma^2\{\epsilon_i\} = 10$. In at most one sentence, describe how increasing $\sigma^2\{\epsilon_i\}$ changes how the regression function relates to the scatter plot.

4. Problem 1.28 from the `.pdf` version of the textbook. Requires use of the `crime` data that has been posted on the Homework page.