$$SSTO = SSR(X_1) + SSE(X_1)$$
$$= SSR(X_1, X_2) + SSE(X_1, X_2)$$

$$SSR(X_2 | X_1) = SSR(X_1, X_2) - SSR(X_1)$$

$$\Rightarrow \underbrace{SSTO}_{\text{total variability in } Y} = \underbrace{SSR(X_1)}_{\text{variability attributable to } X_1} + \underbrace{SSR(X_2 | X_1)}_{\text{residual variability after accounting for } X_1 \text{ that is attributable to } X_2} + \underbrace{SSE(X_1, X_2)}_{\text{remainder after } X_1, X_2 \text{ have been accounted for}}$$

$$SSTO = SSR(X_1) + SSR(X_2 | X_1) + SSR(X_3 | X_2, X_1) + SSE(X_1, X_2, X_3)$$

where $SSR(X_3 | X_1, X_2) = SSR(X_3, X_2, X_1) - SSR(X_1, X_2)$

Decomposition of SSR into extra sums of squares

Given X_1, $SSTO = SSR(X_1) + SSE(X_1)$

Given X_2 as well, $SSTO = SSR(X_1) + SSR(X_2 | X_1) + SSE(X_1, X_2)$

$$SSTO = SSR(X_1, X_2) + SSE(X_1, X_2)$$
$$= SSR(X_2) + SSR(X_1 | X_2) + SSE(X_1, X_2)$$

Recall... in ANOVA conversations previously, we talked about SSTO SSR SSE ... but that was in
MSTO MSR MSE

the context of simple linear regression

In multiple linear regression, an ANOVA table usually contains decompositions of the regression sum of squares into extra sums of squares

$$Y_i = \beta_0 + \sum_{k=1}^{3} \beta_k X_{ik} + \varepsilon_i \quad , \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

ANOVA Table for fitting this model

Sums of Squares	Degrees of Freedom	Mean Squares
$SSR(X_1, X_2, X_3)$	3	$MSR(X_1, X_2, X_3)$
$SSR(X_1)$	1	$MSR(X_1)$
$SSR(X_2 \mid X_1)$	1	$MSR(X_2 \mid X_1)$
$SSR(X_3 \mid X_2, X_1)$	1	$MSR(X_3 \mid X_1, X_2)$
$SSE(X_1, X_2, X_3)$	$n - 4$	$MSE(X_1, X_2, X_3)$

If we add these up, they are $SSR(X_1, X_2, X_3)$, regression sum of squares

usually in order they entered model

Use of Extra Sums of Squares for Testing Hypotheses about β_k

Context: Assume we have $\underset{\sim}{Y}, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_{p-1}$, we're assuming
$Y_i = \beta_0 + \sum_{k=1}^{p-1} X_{ik}\beta_k + \varepsilon_i$, $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

* Testing whether a single $\beta_k = 0$

Null, H_0: $\beta_k = 0$
Alternative, H_a: $\beta_k \neq 0$

we already know how to do a t-test for this, where we compare $t^* = \frac{b_k}{s\{b_k\}}$ to quantiles of a t-distribution with $n-p$ d.f.

Consider $k = 3$
H_0 corresponds to the reduced model,

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \sum_{k=4}^{p-1} \beta_k X_{ik} + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

Fit both models or, make an ANOVA table for the full model and get $SSR(X_3|X_1, X_2, X_4, \ldots, X_{p-1})$ and $SSE(X_1, \ldots, X_{p-1})$

Then we can construct $F^* = \left(\frac{SSR(X_3|X_1, X_2, X_4, \ldots, X_{p-1})}{1}\right) \div \left(\frac{SSE(X_1, \ldots, X_{p-1})}{n-p}\right)$
Under H_0, $F^* \sim F_{1, n-p}$

Use of Extra Sums of Squares for Testing Hypotheses about β_k

Context: Assume we have $Y, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_{p-1}$, we're assuming
$$Y_i = \beta_0 + \sum_{k=1}^{p-1} X_{ik}\beta_k + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

* Testing whether a single $\beta_k = 0$

Null, H_0: $\beta_k = 0$
Alternative, H_a: $\beta_k \neq 0$

Compute
$$F^* = \left(\frac{SSR(X_k \mid X_1, \ldots, X_{k-1}, X_{k+1}, \ldots X_{p-1})}{1}\right) \div \left(\frac{SSE(X_1, \ldots, X_{p-1})}{n-p}\right)$$

Decicision Rule for a level α-test will be:

* If $F^* \leq F(1-\alpha; 1, n-p)$ conclude H_0: $\beta_k = 0$
* If $F^* > F(1-\alpha; 1, n-p)$ conclude H_a: $\beta_k \neq 0$

p-value given by $\Pr(F \geq F^*)$, where $F \sim F_{1,n-p}$

Use of Extra Sums of Squares for Testing Hypotheses about β_k

Context: Assume we have $\underset{\sim}{Y}, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_{p-1}$, we're assuming

$$Y_i = \beta_0 + \sum_{k=1}^{p-1} X_{ik}\beta_k + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

*Testing whether multiple β_k, β_ℓ, $\ell \neq k$ are jointly equal to 0

Null, H_0: $\beta_k = \beta_\ell = 0$
Alternative, H_a: $\beta_k \neq 0$ or $\beta_\ell \neq 0$

Compute

$$F^* = \left(\frac{SSR(X_k, X_\ell \mid X_m, m \neq k, \ell)}{2}\right) \div \left(\frac{SSE(X_1, \ldots, X_{p-1})}{n-p}\right)$$

Decicision Rule for a level α-test will be:

* If $F^* \leq F(1-\alpha; 2, n-p)$ conclude H_0
* If $F^* > F(1-\alpha; 2, n-p)$ conclude H_a

p-value given by $\Pr(F \geq F^*)$, where $F \sim F_{2, n-p}$

Use of Extra Sums of Squares for Testing Hypotheses about

Context: Assume we have $Y, \underset{\sim}{X}_1, \dots, \underset{\sim}{X}_{p-1}$, we're assuming $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$$

*Testing whether multiple β_2, β_3 are jointly equal to 0

Null, H_0: $\beta_2 = \beta_3 = 0$
Alternative, H_a: $\beta_2 \neq 0$ or $\beta_3 \neq 0$

Compute

$$F^* = \left(\frac{SSR(X_2, X_3 \mid X_1)}{2}\right) \div \left(\frac{SSE(X_1, X_2, X_3)}{n-4}\right)$$

Decision Rule for a level α-test will be:

* If $F^* \leq F(1-\alpha; 2, n-4)$ conclude H_0
* If $F^* > F(1-\alpha; 2, n-4)$ conclude H_a

p-value given by $Pr(F \geq F^*)$, where $F \sim F_{2, n-4}$