Announcements:

* Lectures will be zoomed live
  - During class
  - Recorded and distributed
* Office hours zoom
* Homework TBD] look for announcement
* Exams will be converted to take home

* Start looking for datasets and send them to me
  * has to be time series data
  - ideally data that we can post online
  * equally spaced or can be thought of as equally spaced

Wrapping up ARMA(p,q) Models + Nonstationarity

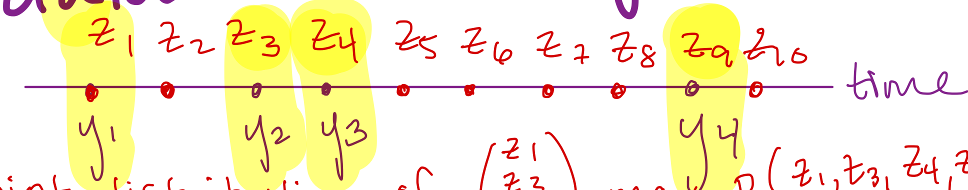
(or z_t)

$$\phi(B)(y_t - \mu) = \Theta(B) w_t,$$

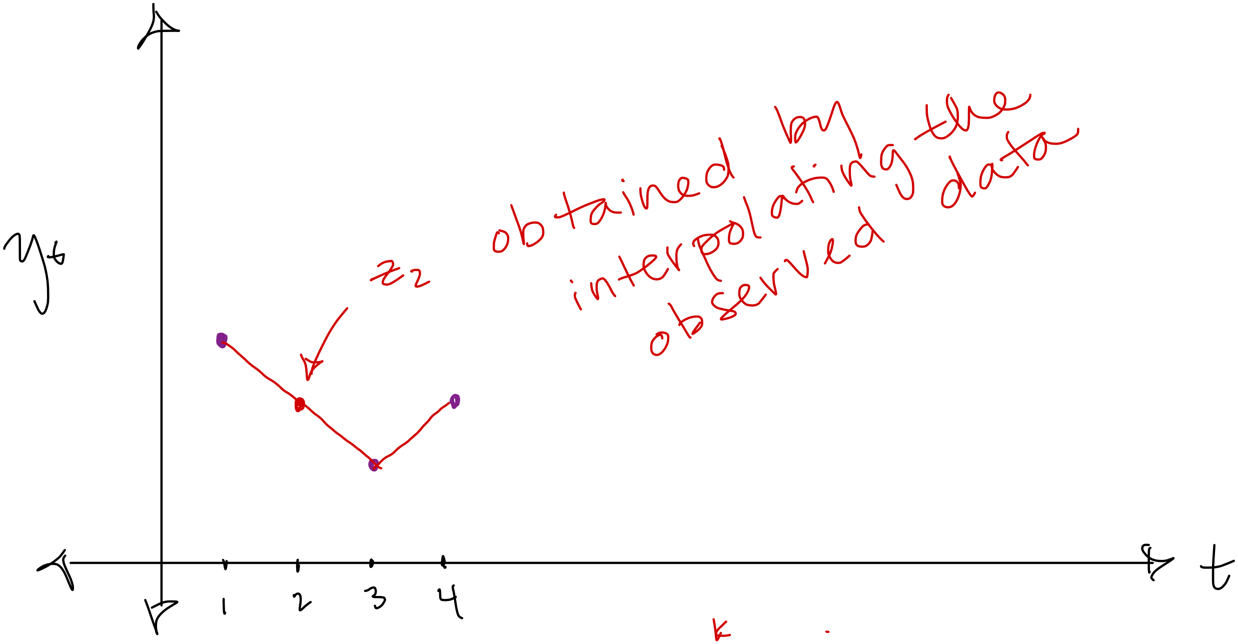
$$w_t \overset{iid}{\sim} N(0, \sigma_w^2)$$

* $\phi_1, \dots, \phi_p$ parameters yield a stationary process, y_t stationary
* model is invertible

What if my data is not equally spaced?

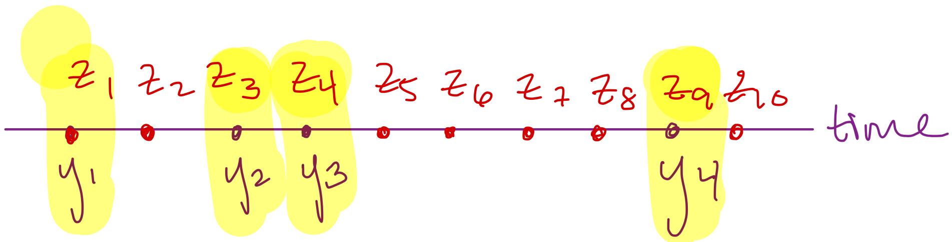


* Figure out joint distribution of $\begin{pmatrix} z_1 \\ z_3 \\ z_4 \\ z_9 \end{pmatrix}$, max $p(z_1, z_3, z_4, z_9)$
* Either interpolate y_1, y_2, y_3, y_4 to get $\tilde{z}$
  - ok idea if n is big and you don't need to add too many obs. to make an equally spaced time series



* Model $y_t = \mu + \sum_{i=1}^{k} t^i \beta_i + \varepsilon_t$
  * Fit model to get $\hat{\beta}$
  * Assume this model holds for z_t,
  * Obtain $z_t = \begin{cases} \hat{\mu} + \sum_{i=1}^{k} \hat{\beta}_i t^i & \text{if } y_t \text{ not observed} \\ y_t & \text{otherwise} \end{cases}$
  * Pick k big enough

$$y_t = \hat{\mu} + \sum \hat{\beta}_i t^i$$



$$\phi(B)(z_t - \mu) = \Theta(B) w_t, \quad w_t \overset{iid}{\sim} N(0, \sigma_w^2)$$

$$\Longrightarrow \underset{\sim}{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_n \end{pmatrix} \sim N\left(\mu \underset{\sim}{1}, \Sigma\right), \quad \sigma_{ij} = \gamma_z(|i-j|)$$

$\uparrow$ depends on $\sigma_w^2, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_p$

$$\Longrightarrow \begin{pmatrix} z_1 \\ z_3 \\ z_4 \\ z_9 \end{pmatrix} \sim N\left(\mu \underset{\sim}{1}, \tilde{\Sigma}\right),$$

$$\tilde{\sigma}_{ij} = \gamma_z(|i-j|)$$

If we use AR(1), $\tilde{\sigma}_{ij} = \dfrac{\phi_1^{|i-j|} \sigma_w^2}{1 - \phi_1^2}$

$$\begin{pmatrix} z_1 \\ z_3 \\ z_4 \\ z_9 \end{pmatrix} \sim N\left(\mu \underset{\sim}{1}, \underset{\sim}{\Sigma}\right) \quad \text{if } AR(1) \ldots$$

$$\underset{\sim}{z} = \begin{pmatrix} z_1 \\ z_3 \\ z_4 \\ z_9 \end{pmatrix} \sim N\left(\mu \underset{\sim}{1}, \left(\frac{\sigma_w^2}{1-\phi_1^2}\right) \begin{pmatrix} 1 & & \text{same as lower} & \\ \phi_1^2 & 1 & & \\ \phi_1^3 & \phi_1 & 1 & \\ \phi_1^8 & \phi_1^6 & \phi_1^5 & 1 \end{pmatrix}\right)$$

works well if

$$\frac{1}{\sqrt{2\pi|\Sigma|}} \exp\left\{-(\underset{\sim}{z} - \mu\underset{\sim}{1})' \Sigma^{-1} (\underset{\sim}{z} - \mu \underset{\sim}{1})\right\}$$

easy to evaluate, but many of our tricks won't work

The z_t's we did not observe

If we rewrite $p(z_1, z_3, z_4, z_9) = \int p(\underset{\sim}{z})\, d\underset{\sim}{z}_0$

can use our estimation tricks