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Project 2 - Studying Networks with Eigenvectors and Eigenvalues

Introduction:
This project will study the network of interactions of members of the Korean pop group "NCT," whose concept involves dividing the large group into subgroups and rotating the members to perform different songs. We will see how the Fiedler Set of the network's Laplacian matrix will split this network into primarily two groups with some overlap, despite there being more than two subgroups.

Network Description and Laplacian Matrix
In order to simplify the network, it is built from members of the 5 sub-units that existed from 2016-2018. Each node in the network is a member of the group, and there is an edge between members that have been in the same sub-unit together. Determining these connections was based on the membership of 5 large sub-units. (The other 3 sub-units during this time period were not included as they did not add any new connections.) This information can easily be found through official membership lists or analyzing music videos with the ability to recognize the individuals.

The result is the following 18 node network. This network graph was created with the help of the CS Academy Graph Editor (https://csacademy.com/app/graph_editor/)

$$L=$$
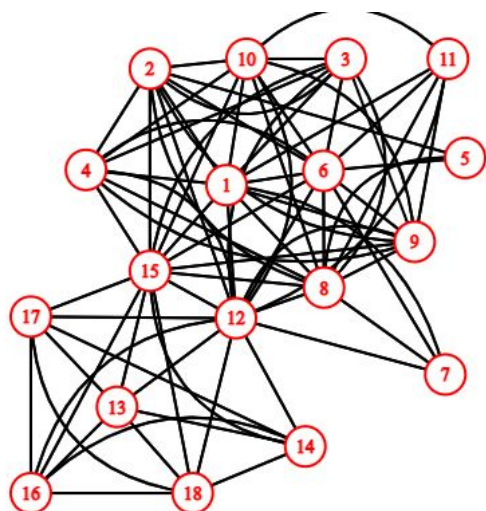
11	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	-1	0	0	-1	0	0	0
-1	10	-1	-1	-1	-1	0	-1	-1	-1	0	-1	0	0	-1	0	0	0
-1	-1	9	-1	0	-1	0	-1	0	-1	0	-1	0	0	-1	0	0	0
-1	-1	-1	9	0	-1	0	-1	-1	-1	0	-1	0	0	-1	0	0	0
0	-1	0	0	3	-1	0	-1	0	0	0	0	0	0	0	0	0	0
-1	-1	-1	-1	-1	12	-1	-1	-1	-1	-1	-1	0	0	-1	0	0	0
-1	0	0	0	0	-1	4	-1	0	0	0	-1	0	0	0	0	0	0
-1	-1	-1	-1	-1	-1	-1	12	-1	-1	-1	-1	0	0	-1	0	0	0
-1	-1	-1	-1	0	-1	0	-1	9	-1	-1	-1	0	0	-1	0	0	0
-1	-1	-1	-1	0	-1	0	-1	-1	10	-1	-1	0	0	-1	0	0	0
-1	0	0	0	0	-1	0	-1	-1	-1	6	-1	0	0	0	0	0	0
-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	16	-1	-1	-1	-1	-1	-1
0	0	0	0	0	0	0	0	0	0	0	-1	6	-1	-1	-1	-1	-1
0	0	0	0	0	0	0	0	0	0	0	-1	-1	6	-1	-1	-1	-1
-1	-1	-1	-1	0	-1	0	-1	-1	-1	0	-1	-1	-1	14	-1	-1	-1
0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	6	-1	-1
0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	6	-1
0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	6

Nodes correspond to the following members: 1- Taeyong; 2- Taeil; 3-Johnny; 4- Yuta, 5- Kun; 6-Doyoung; 7- Ten; 8-Jaehyun 9-WinWin; 10-Jungwoo; 11-Lucas; 12-Mark; 13-Renjun; 14-Jeno, 15-Haechan, 16-Jaemin; 17-Chenle; 18-Jisung

Fiedler Vector, Value, and Set Computation
We can see visually from the network that it is connected, meaning that the multiplicity of the

eigenvalue 0 is 1. (There are no lone nodes that are not connected to another node in the network.) This means the Fiedler value $\lambda_2 > 0$ and we do not need to add any more edges. Using the dcode.fr Eigenvalue calculator (https://www.dcode.fr/matrix-eigenvectors), I found that the smallest eigenvalue greater than 0 is $\lambda_2 = 1.7716$ and the corresponding Fiedler vector (listed as a row vector for space) is:

$\mathbf{v_2}$ = [-0.383519, -0.432518, -0.304217, -0.354181, -1.0771, -0.4453, -0.540029, -0.4453, -0.41112, -0.366571, -0.46852, 0.0707304, 1., 1., 0.157665, 1., 1., 1.,]

By taking only the nodes corresponding to the positive components of $\mathbf{v_2}$, the Fielder set will be F={12, 13, 14, 15, 16, 17, 18}.

Analysis and Discussion:
Interestingly, the Fielder set F corresponds to six members that complete the sub-unit "NCT-Dream." Most of these members do not have other connections outside of this group, which explains how the Fielder set more closely grouped them together. Nodes 12 and 15 represent the two members in this sub-unit that also participated in other groups, which explains why their associated values in $\mathbf{v_2}$ are closer to zero. The graph illustrates these two main groupings as well, with nodes 12 and 15 being like the overlap of a Venn diagram. The color-coded matrix also illustrates these two large groupings with some overlap.

Although I used more than 5 sub-units used to make this network, there were two main large groups which can also be seen illustrated in the graph. It is interesting to note that Nodes 5, 7 and 11 are in neither of these two large groups, so they visually appear to be on the edges of the cluster with significantly fewer edges. Mathematically, they also have the smallest values (a larger negative number) in $\mathbf{v_2}$ and are even farther away from zero. Nodes 7 and 11 connect to node 12, a member whose value is closest to zero, but node 5 has no association with any node in set F, therefore node 5 has the farthest value from zero.

Sources:
For the construction of the dataset, I used the following sources:
1. Vast personal knowledge of the history and discography of NCT
2. List of "temporary" units (https://smtown.fandom.com/wiki/NCT_U)
3. Official Website for lists of two "main" units members (http://nct.smtown.com/Intro)