Can Echo State Networks capture the stochasticity of reduced order turbulence signals?

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Turbulence is one true mystery that continues to challenge our sense of determinism even after a century of scientific efforts. Its stochastic nature lead some to interpret them as a chaotic phenomena which can only be captured numerically upto a bounded time. In Computational Fluid Dynamics research, turbulence is often modeled to estimate their behavior in a time and space averaged fashion so that the compute requirements to capture their full behavior are relaxed[1]. However, in recent efforts by many researchers in the field of reduced order approaches, it has been demonstrated that certain mathematical tools allow decomposition of turbulence flow field data to suitable coordinate systems [2, 3]. The resulting representations are amenable to physical interpretations are much more friendly to mathematical modeling than capturing pure turbulence from first principles.

That being said, the resulting representations are not as friendly as to be modeled with conventional analytical approaches. Machine learning in this direction has shown promise by employing its popular Artificial Neural Networks. As a first step, ANNs were shown to capture the reduced stochasticity of turbulence signal via a form of Principal Component Analysis applied to fluid flow fields [4]. However, the ANNs have a low appetite for noisy chaotic data thereby limiting their applications to very low Reynolds numbers. Meanwhile this approach has taken its foothold in fluid dynamics research, a separate effort to model purely chaotic phenomena using machine learning have found successes in the dynamical systems research[5, 6]. In this line of work, Echo state networks have been shown to have the ability to extrapolate chaotic time series signals far more than their Lyapunov time – the upper bound to which a chaotic system is theoretically predictable. The next generation of turbulence modeling algorithms will hence be strongly be influenced by this idea.

As an initiation in line of these efforts, it would interesting to investigate how a vanilla Echo State Network respond to chaotic time series signal. A simple first step can be to code an ESN and tune its behaviors on a time series signal with random Fourier composition. As continuation, the ESN can then be tested on reduced order mode signals and then to what limit they can approach towards modeling of pure turbulence.

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