EVERYTHING HERE IS AN EXAMPLE OF A MISTAKE!! CAN YOU FIND & CORRECT IT ?

① Prove that $S = \{ (x, y) \text{ in } \mathbb{R}^2 \mid x + y = 0 \}$ is a subspace of $\mathbb{R}^2$.

- $u = (1, -1)$ and $v = (2, -2)$ belong to S
- $u + v = (3, -3)$ belong to S
- $\alpha u = (\alpha, -\alpha)$ belong to S

So S is closed under addition and scalar multiplication.

YES it's a subset.

A EXAMPLE IS NOT ENOUGH !!!

② Give the example of a line in $\mathbb{R}^n$.

ANSWER 1: e_1 $(1, 0, 0 \cdots 0)$ it's only vector not a subspace

ANSWER 2: $\mathbb{R}^1 = \mathbb{R}$ $0, 1$ $\underbrace{(0, 0, \cdots 0)}_{n}$ $\underbrace{(1, 0 \cdots 0)}_{n}$

ANSWER 3: $\mathbb{R}^1 = \underbrace{\text{span}(e_1)}_{\text{subspace } \subset \mathbb{R}^n}$

$x_1 \in \mathbb{R}^1$ $(x_1, x_2) \in \mathbb{R}^2$ $(0, 1) \in \mathbb{R}^2$ 3 ...

③ $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$, give a basis of $\ker T$.
$M \to AM$

$\ker T = \{ x \text{ in } \mathbb{R}^n \mid Tx = 0 \}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So let's solve $Tx = 0$ etc ....

$\ker T = \{ X \text{ in } \mathbb{R}^{2\times 2} \mid T(X) = 0 \}$ $\quad T(X) = 0$ if $MX = 0$

(k) Show that $(e_1 \dots e_n)$ is a basis of $\mathbb{R}^n$. What is the dimension of $\mathbb{R}^n$?

Proper demonstration that $(e_1 \dots e_n)$ is linearly indep.

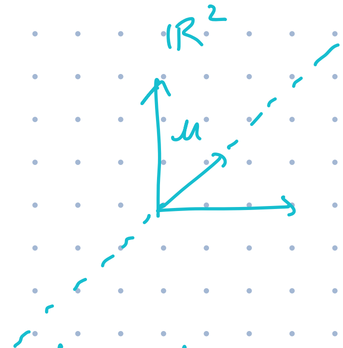
(+) $\mathrm{Span}((e_1, e_2, \dots e_n)) = \mathbb{R}^n$.

$\dim \mathbb{R}^n = n$ — because n elements in the basis.

(*) $(u_1, \dots u_p)$ are p linearly independent vectors in $\mathbb{R}^n$.

Then $\dim(\mathrm{Span}(u_1 \dots u_p)) = p$

and $\mathrm{Span}(u_1 \dots u_p) = \cancel{\mathbb{R}^p}$ $\subset \mathbb{R}^n$



$\dim \mathrm{Span}(u) = 1$

$\mathrm{Span}(u) \neq \mathbb{R}^2$

WHY ARE THESE EXPRESSIONS WRONG. → WHAT'S THE CORRECT WAY

$\mathrm{Im}(A) = Ax$ SET VECTOR

$y \in \mathrm{Im}(A)$

input space

there exist $x \in \mathbb{R}^n$ $y = Ax$

$\mathrm{Im}\,A = \{ Ax \mid x \in \mathbb{R}^n \}$

$x = \ker M$

"x in $\ker M$" "$x \in \ker M$"

$\dim(M)$ for M a matrix

$\dim \mathrm{Im}(M) = \mathrm{rank}\, M$

$\ker M = 0$

$\ker M = \{0\} \Leftrightarrow \dim \ker M = 0$

subspace $S = ax + by + z$

$S = \{ (x, y, z) \text{ in } \mathbb{R}^3 \mid ax + by + z = 0 \}$