Linear Algebra and Optimization DSGA-1014
Fall 2021
CDS at NYU

Lab 1

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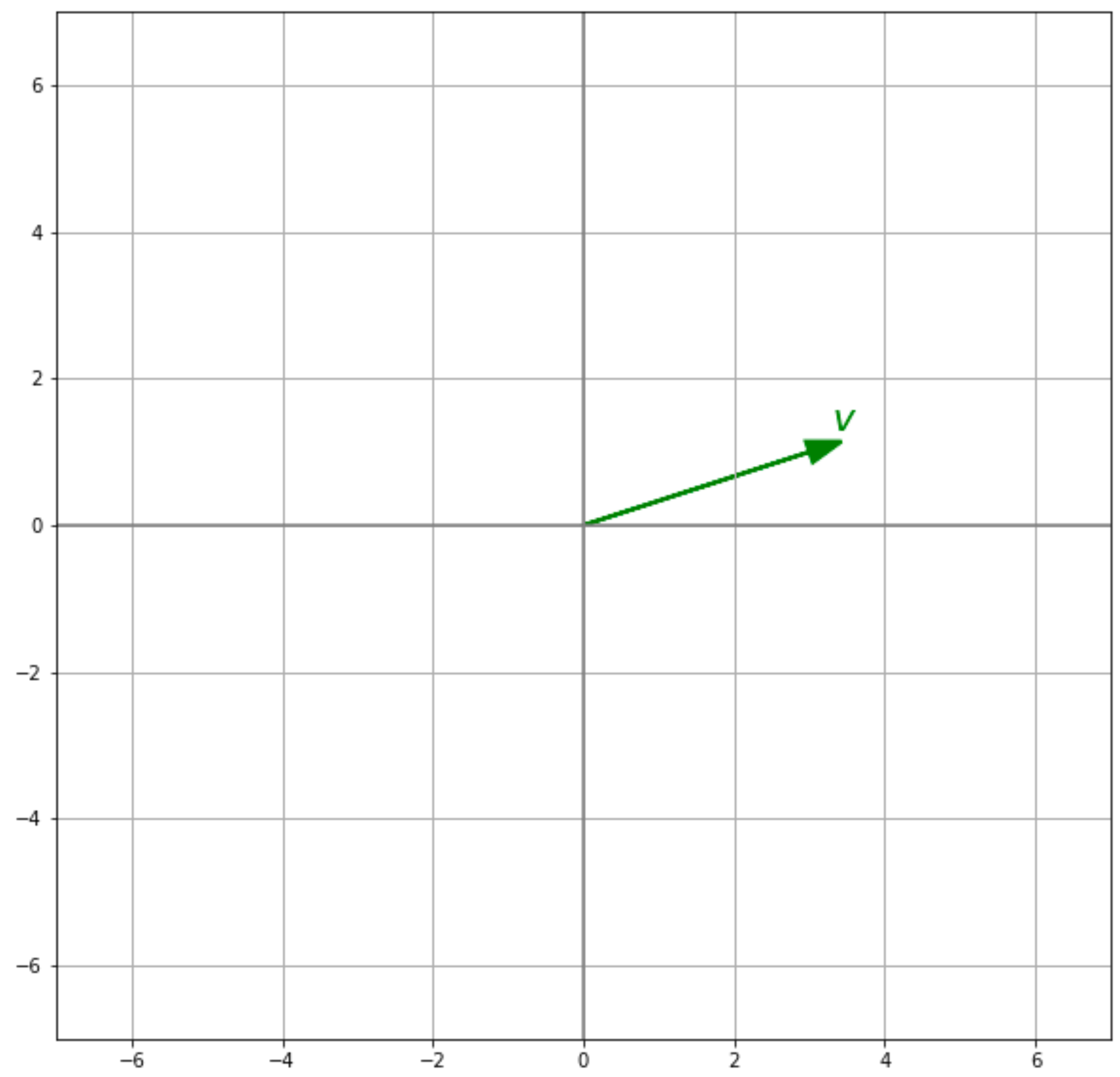
Geometric intuition

Take these two vectors in $\mathbb{R}^2$:

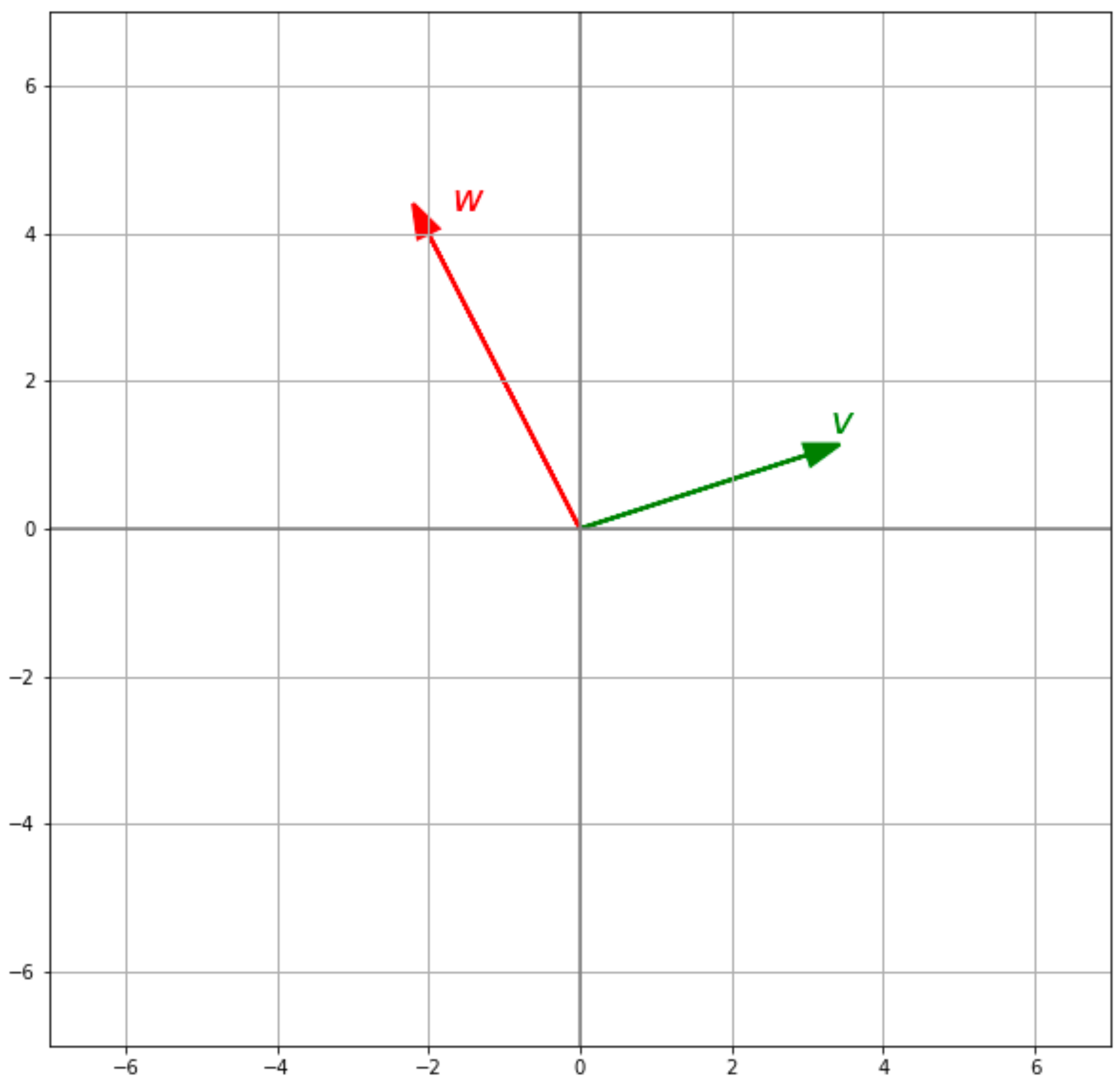
$v = (3,1)$ and $w = (-2,4)$

Describe the following sets geometrically. Which are subspaces of $\mathbb{R}^2$?

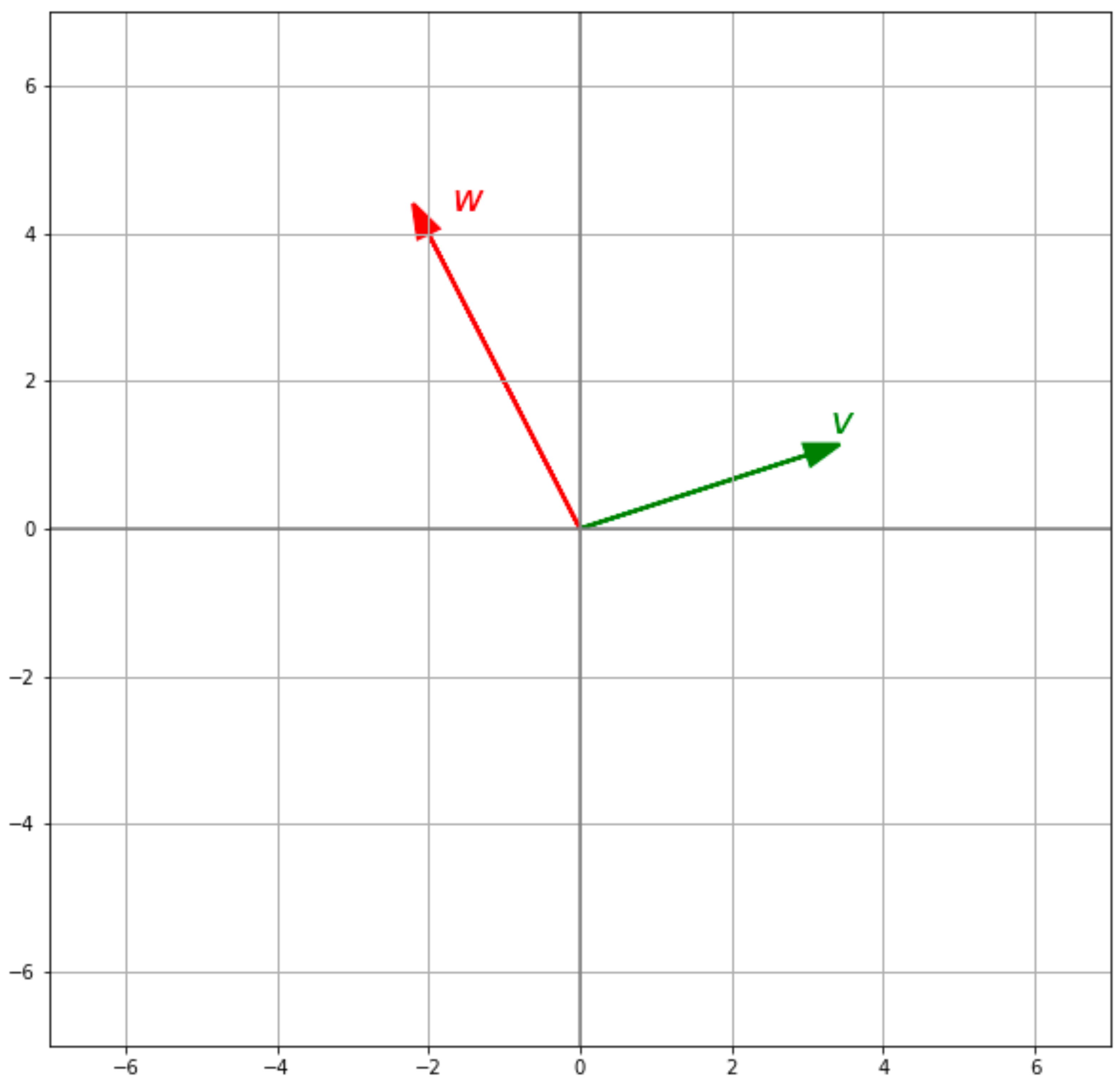
$Span(v)$



$$Span(v) \cup Span(w)$$

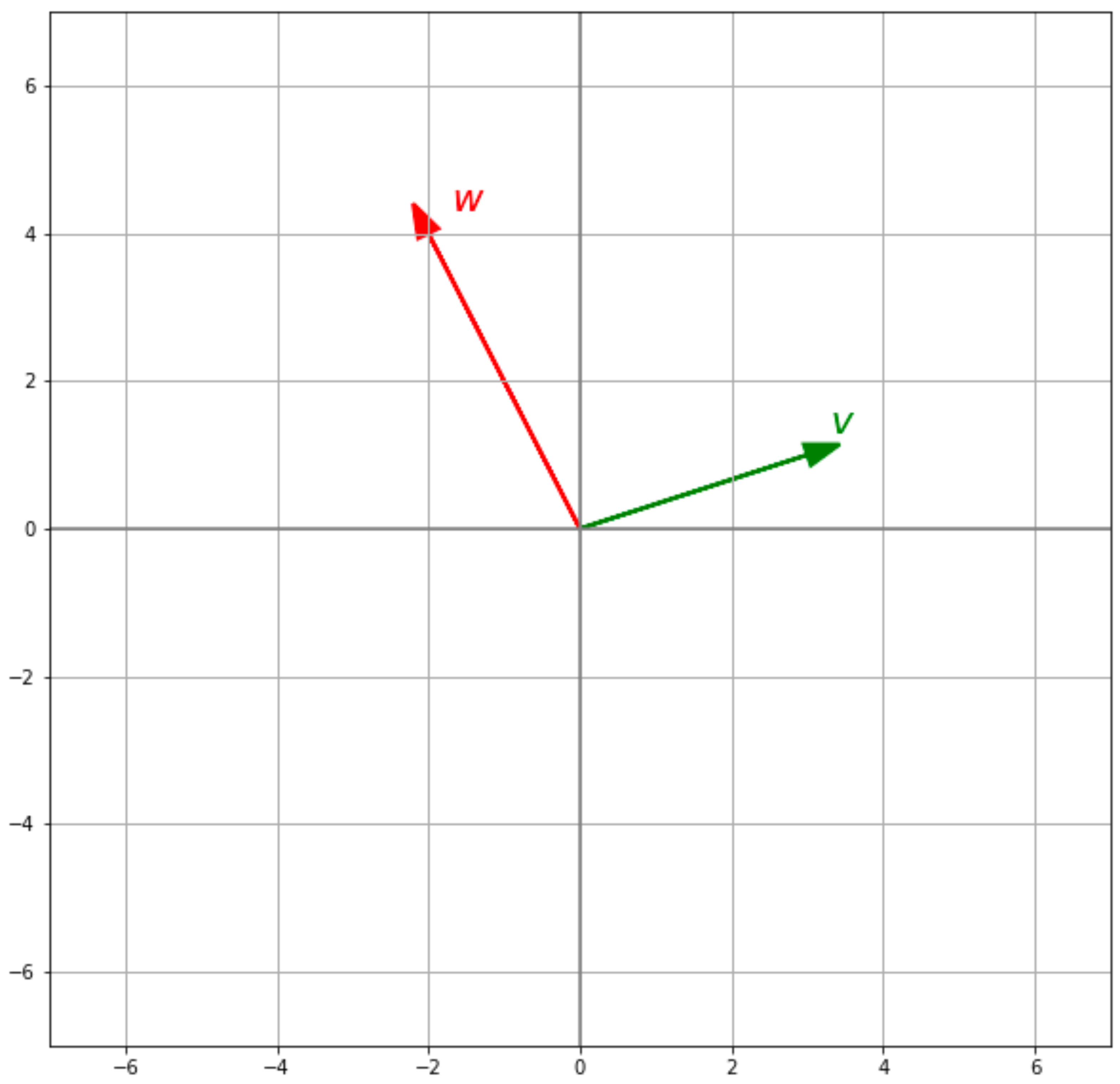


$Span(v) \cap Span(w)$



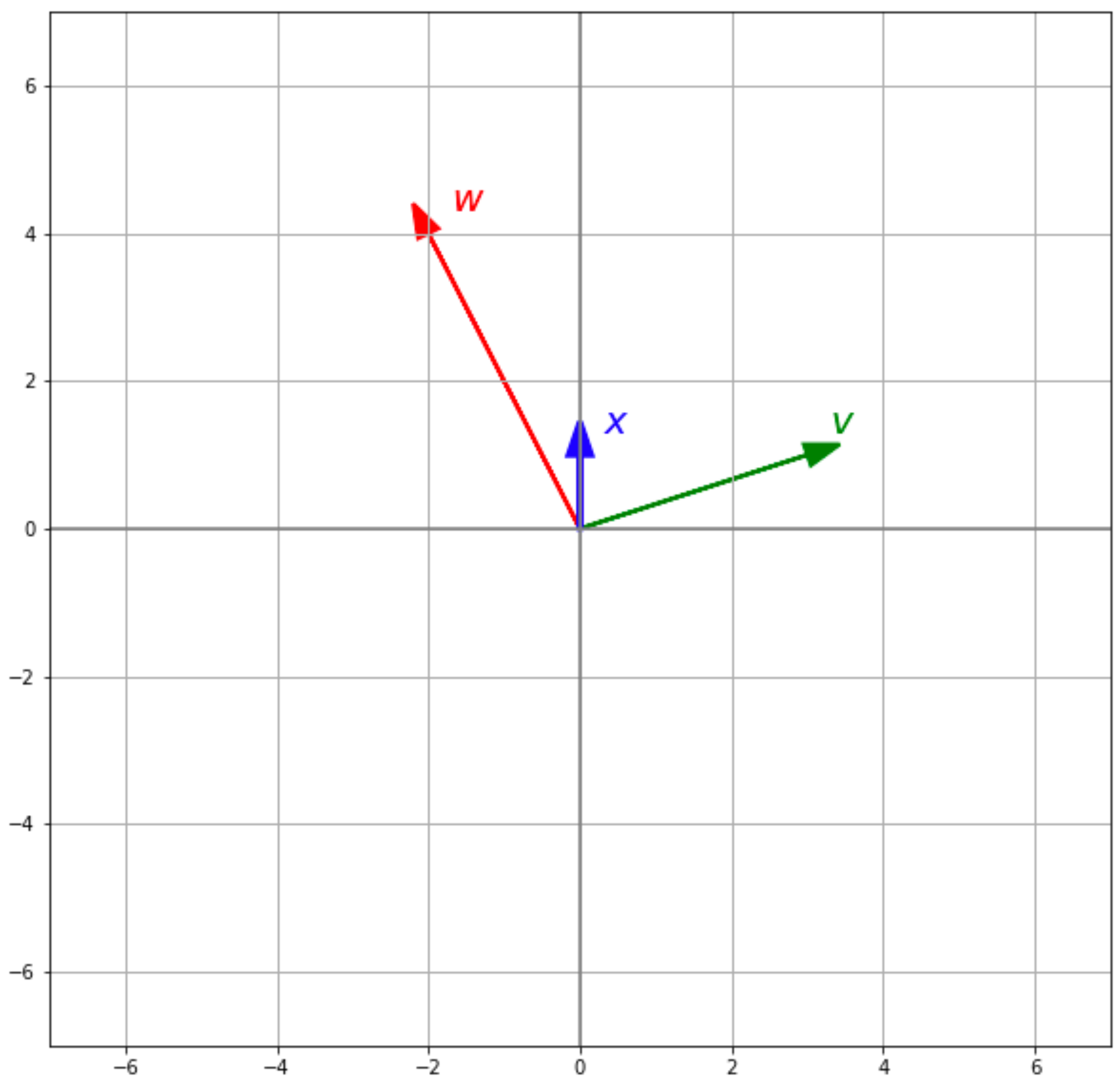
$$Span(v, w)$$

$$\{\alpha v + \beta w : \alpha, \beta \in \mathbb{R}^2\}$$

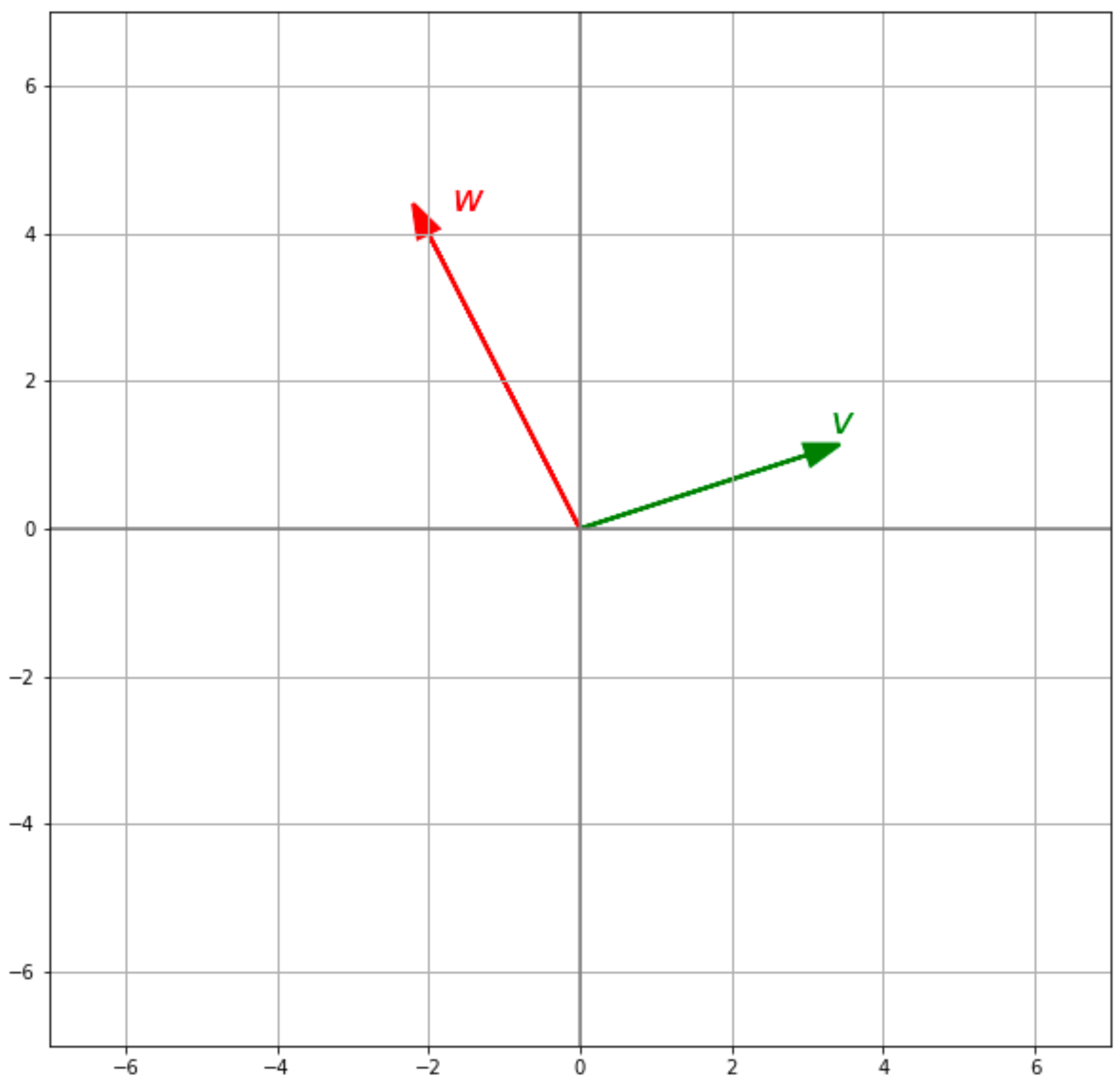


$$Span(v, w, x)$$

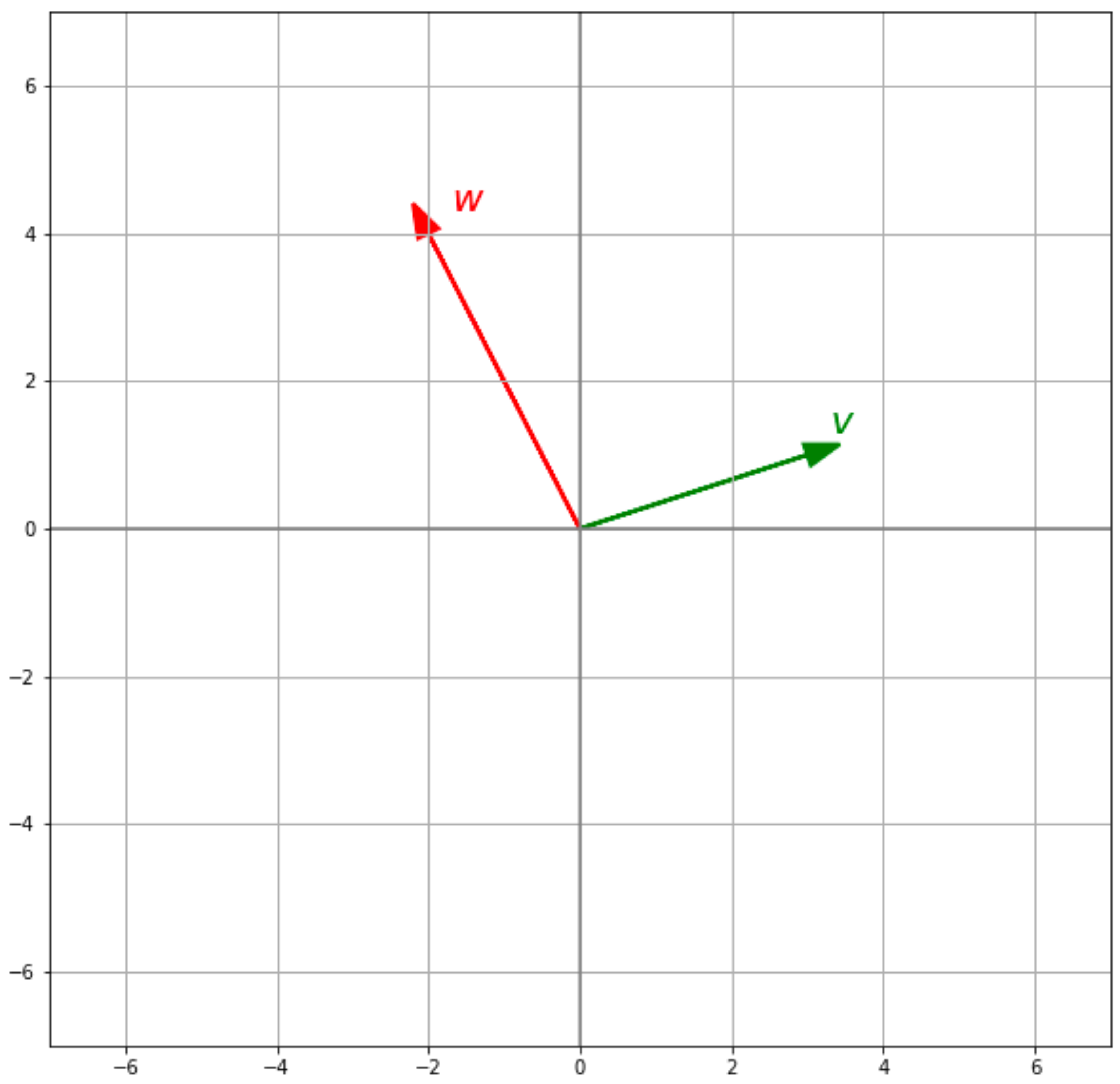
$$x = (0,1)$$



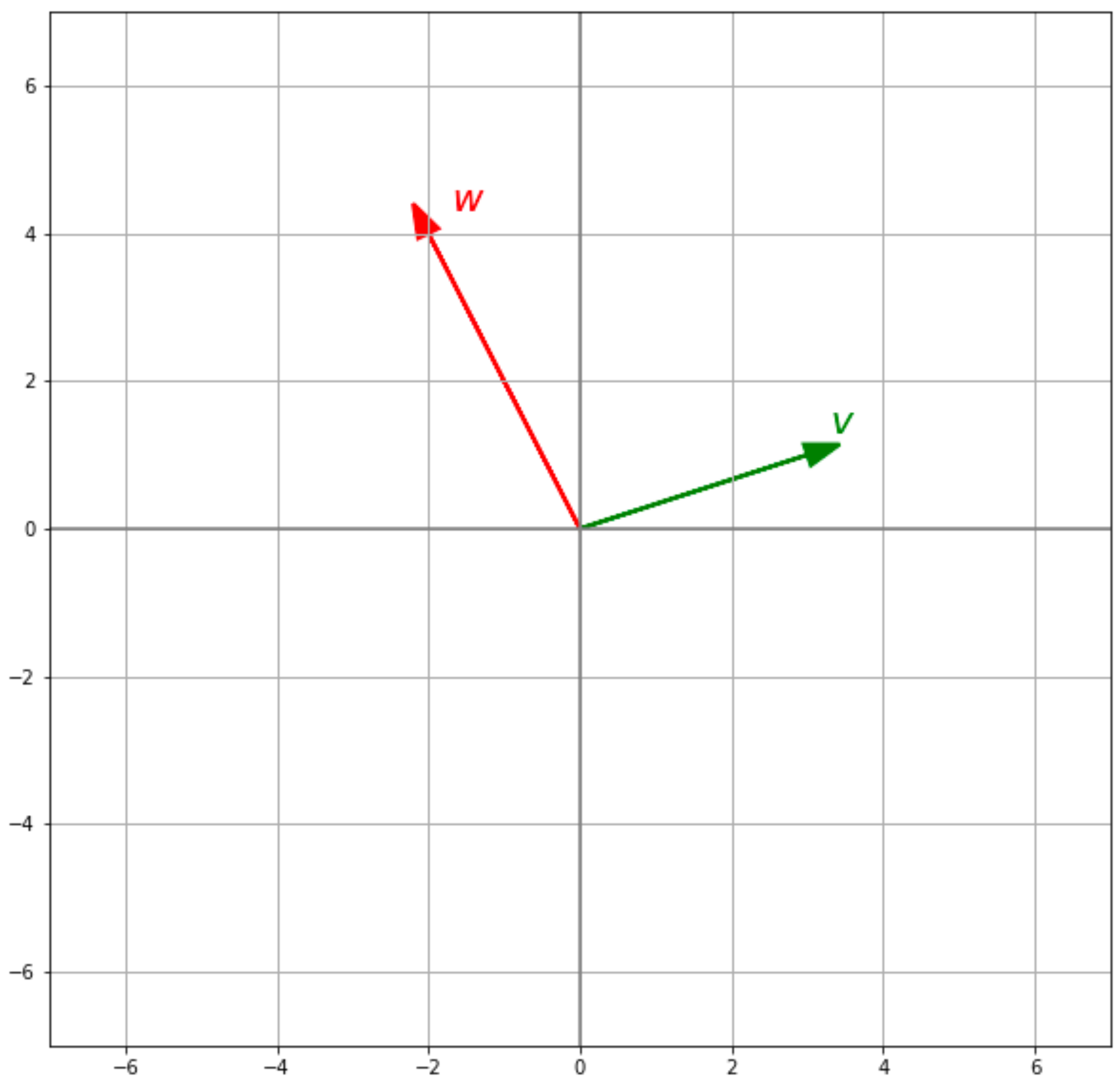
$$\{(1-t)v + tw : t \in \mathbb{R}\}$$



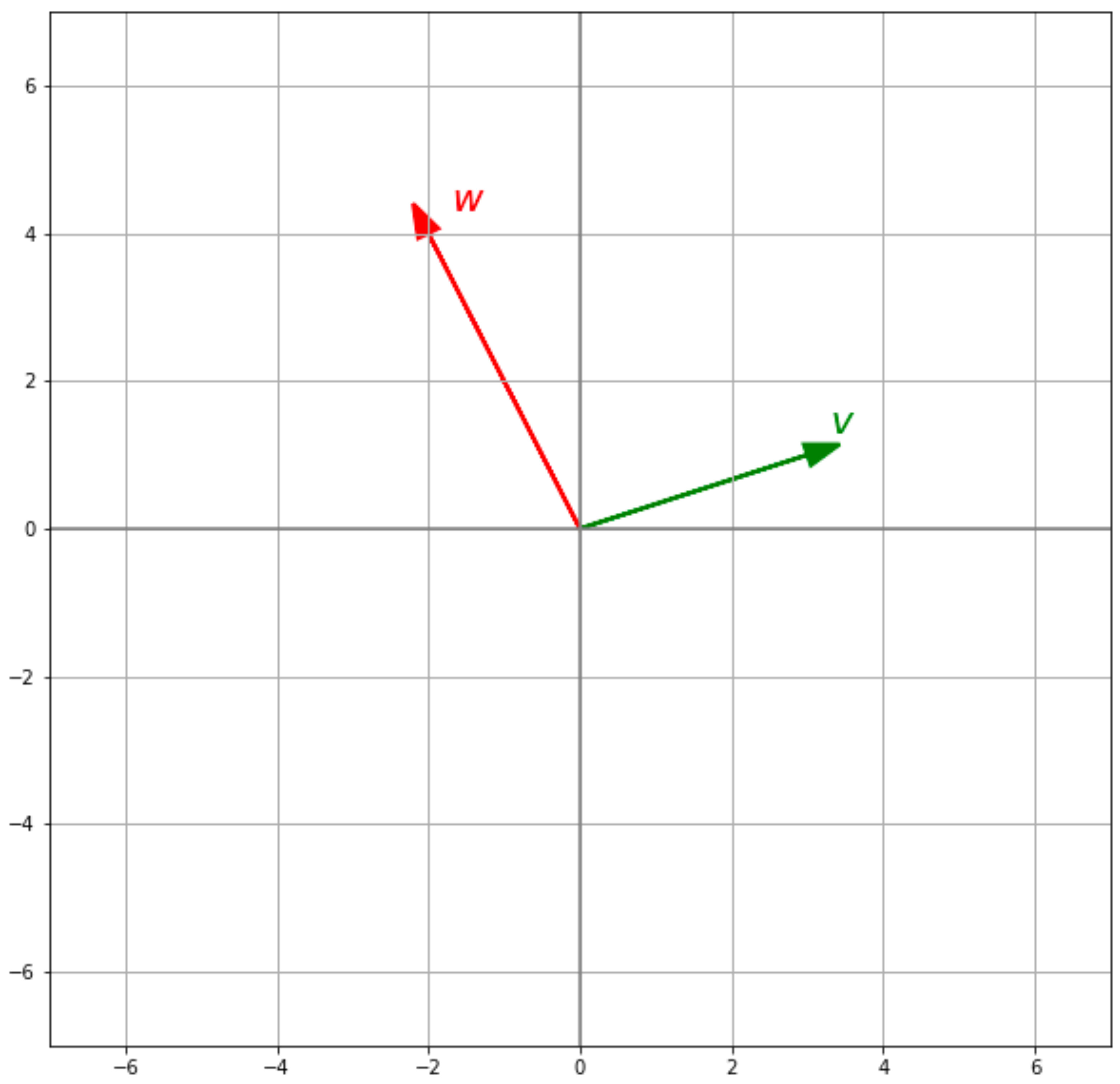
$$\{(1-t)v + tw : t \in \mathbb{R}\}$$



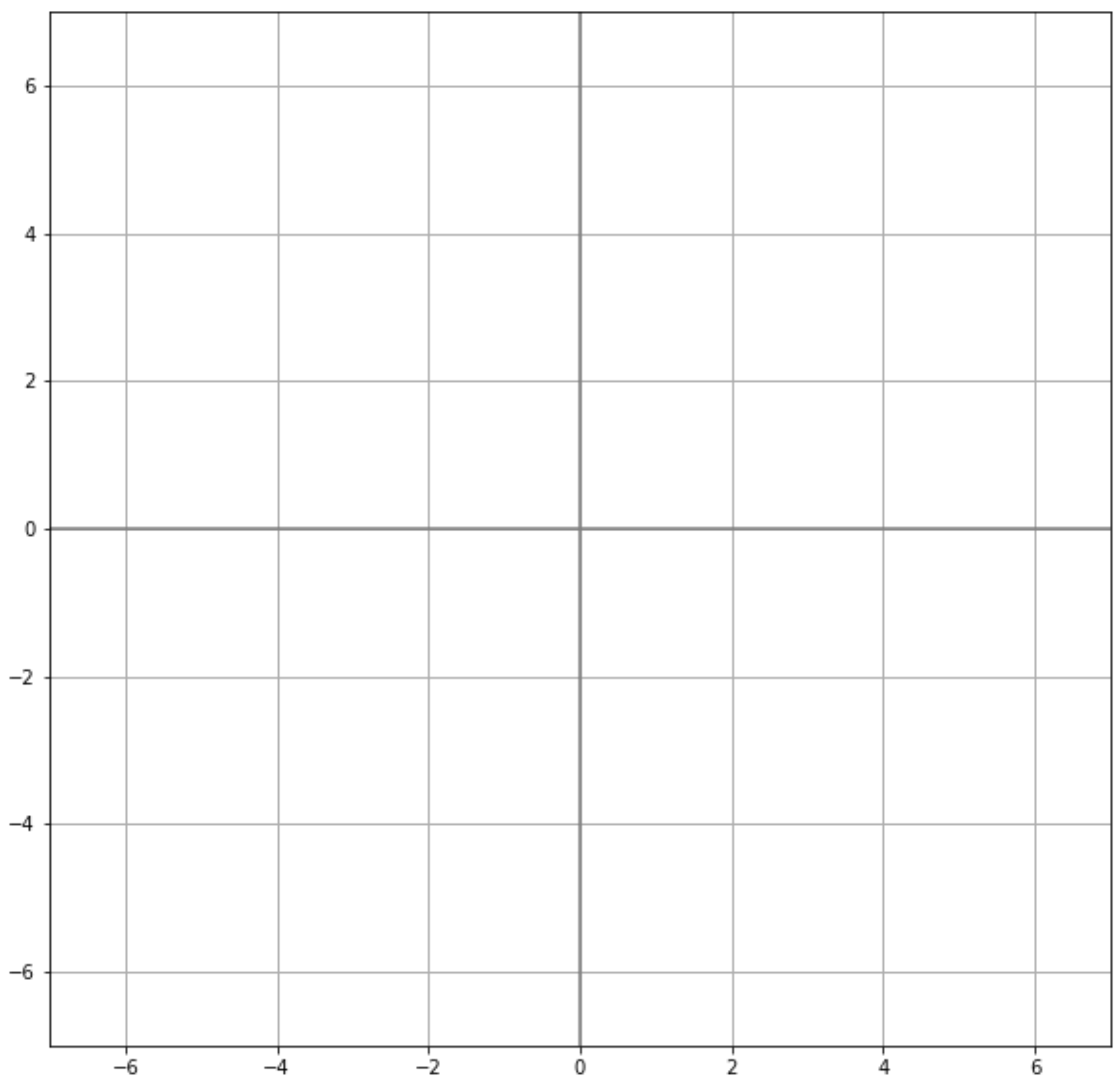
$$\{(1-t)v + tw : t \in [0,1]\}$$



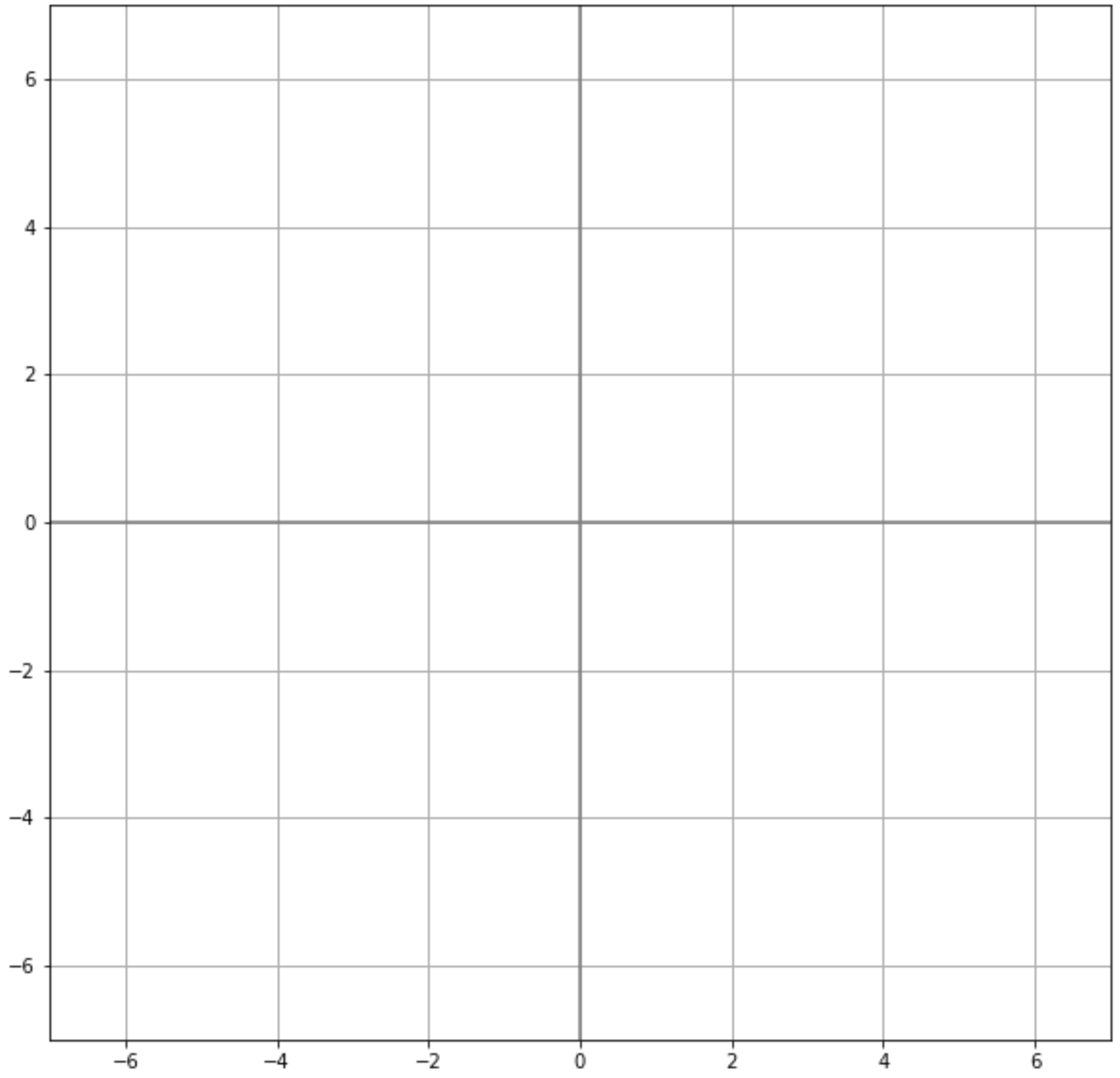
$$\{\alpha v + \beta w : \alpha, \beta \geq 0\}$$



$$\{(a, b) \in \mathbb{R}^2 : a^2 + b^2 = 4\}$$



$$\{(a, b) \in R2 : a2 + b2 \leq 4\}$$



Let $v_1, v_2, v_3, v_4 \in \mathbb{R}^3$ and define $C_1 = \{v_1, v_2\}$ and $C_2 = \{v_3, v_4\}$. If C1 and C2 are both linearly independent, what are the possible values for $dim(Span(v_1, v_2, v_3, v_4))$? No proof necessary.

True or False: If $B = \{v_1, \ldots, v_n\}$ is a basis for $\mathbb{R}^n$, and W is a subspace of $\mathbb{R}^n$, then some subset of B is a basis for W.

Suppose $v_1, \ldots, v_m \in \mathbb{R}^n$ are linearly dependent. Prove that if $x \in Span(v_1, v_2, \ldots, v_m)$ then there are infinitely many $\alpha \in \mathbb{R}^m$ with $x = \alpha_1 v_1 + \ldots + \alpha_m v_m$

Let $P_n = \{f(x) = a_0 + a_1x + \ldots + a_nx^n\}$ and define addition and scalar multiplication as follows

Addition. $(f + g)(x) = (a_0 + b_0) + (a_1 + b_1)x + \ldots + (a_n + b_n)x^n$

Multiplication. $(rf)(x) = (ra_0) + (ra_1)x + \ldots + (ra_n)x^n$

Show that P_n is a vector space. What is a basis for this space and what is the dimension?

[Note: f is a function, but f(x) is a real number. That is, once we plug something into our function, we get a real number not a function. For example, if x is a real number, then x^2 is a real number, not a function. This is subtle but extremely important. For example, we know real numbers commute and thus know that f(x) + g(x) = g(x) + f(x). However, we must prove that f + g = g + f]

To prove P_n is a vector space, we need to show it satisfies all the following conditions

(a) $\mathbf{x}+\mathbf{y}=\mathbf{y}+\mathbf{x}$	Commutative law
(b) $(\mathbf{x}+\mathbf{y})+\mathbf{z}=\mathbf{x}+(\mathbf{y}+\mathbf{z})$	Associative law
(c) $\mathbf{x}+\boldsymbol{\theta}=\mathbf{x}$	Additive identity
(d) $\mathbf{x}+(-\mathbf{x})=\boldsymbol{\theta}$	Additive inverse
(e) $(rs)\mathbf{x}=r(s\mathbf{x})$	Associative law
(f) $(r+s)\mathbf{x}=r\mathbf{x}+s\mathbf{x}$	Distributive laws
(g) $r(\mathbf{x}+\mathbf{y})=r\mathbf{x}+r\mathbf{y}$	Distributive laws
(h) $1\mathbf{x}=\mathbf{x}$	Multiplicative identity