

Recitation 12

<https://cims.nyu.edu/cd2754/>

Fall 2021

Convex functions inequality

Proof the following inequality:

Given convex function f defined on \mathbb{R}^n , show for any x, y in the domain of f

$$f(y) \geq f(x) + \nabla f(x)^T (y - x)$$

Convex functions and convex sets

For $f : \mathbb{R}^n \rightarrow \mathbb{R}$, define the epigraph $\text{epi}(f) \subset \mathbb{R}^{n+1}$ to be the set of all points above the graph of f :

$$\text{epi}(f) := \{(x, t) \in \mathbb{R}^{n+1} \mid t \geq f(x)\}.$$

1. Prove that f is convex if and only if $\text{epi}(f)$ is convex.
2. Prove that if f, g are convex functions, then $h(x) = \max(f(x), g(x))$ is convex.

Questions: Ridge Regression

Let $X \in \mathbb{R}^{n \times d}$, $n > d$, and *not have full rank*. (X is a data matrix)

Recall that the OLS solution is $\hat{x} = (X^T X)^{-1} X^T y$.

1. Since X is not full rank, what does this say about the features?
2. What is the issue with the OLS solution?
3. The ridge regression solution is given by $(X^T X + \lambda I_d)^{-1} X^T y$. How does this fix the issue?
4. Suppose that X has SVD $X = U \Sigma V^T$, and X has singular values $\sigma_1, \dots, \sigma_d$. What are the eigenvalues of $X^T X + \lambda I_d$?
5. How does increasing λ affect the condition number of $(X^T X + \lambda I_d)$?

