

Solution

To solve this problem, first we want to show that Hitting Set is in NP. We do this by showing that there is some way to verify a proposed solution to this problem in polynomial time. Given a subset H , we can quickly check if it's the right size and if each element in B_i is in H . So Hitting Set is in NP.

Next, Hitting Set looks like a covering problem so we will use Vertex Cover and show that $VertexCover <_p HittingSet$. To do this, we will transform an instance of Vertex Cover (G, k) into a version of Hitting Set $((A, B_1, \dots, B_n), k)$. So graph G is some vertex set and some edge set. To translate this to hitting set, we can imagine the vertices of G as our set A , and each edge as representing a subset B_i such that each two vertices connected by an edge is a subset B_i of A . With this construction, we can see that a subset of A that hits every subset B_i is exactly a subset of vertices that touches every edge. Therefore, we have a hitting set of size k precisely when we have a vertex cover of size k . Additionally, if we do not have a vertex cover of size k then we do not have a hitting set of size k .