

Problem

We construct a flow network to solve this problem. We can have a set of X nodes such that each node x_i represents an actress, and a set of Y nodes such that each y_i represents an actor. We then add two nodes to represent each player, p_0 and p_1 and draw an edge from each of these nodes to each node in X , such that there are two edges going into each node in X . We give each of these nodes a capacity of 1. We then add a node s and draw an edge from it to each player, both with a capacity of 1. In order to ensure that there is only one winner, we construct a gadget node g and then draw an edge from this node to each node in Y , each with a capacity of 1. We then connect g to a sink node t via an edge with a capacity of 1. Finally, since we know who has starred with whom, we can use this information to complete our graph, by drawing edges between all actors in X and Y . We give edges between costars a capacity of 1 and all other edges a capacity of 0. Now we have constructed a graph such that a winner is represented by flow of value 1 from the start to the sink via that player's node. Given a current sequence of actors, we can decide which of the two players can force a win by simulating the path on our graph, and seeing if there is an actor in the required set (whether it is X or Y) that has no unused edge of capacity 1 from itself to a node in the opposite set that we can push flow to. This involves running Ford-Fulkerson, which we can do in polynomial time.