Solution

To solve this problem, we first want to show that the Zero Weight Cycle Problem (ZWCP) is in NP. Given a proposed solution to this problem, we can easily verify that it is in fact a simple cycle whose sum of its weights are zero, so we have shown that this problem is in NP.

We can use subset sum to show that the ZWCP is NP complete. Given an instance of subset sum $(S_1, ...S_n, w)$ we can turn this into a graph G using the following construction: We can construct a graph with $n + 1$ nodes, where there is a node 0 and a node for each S_i. We draw edges from nodes S_i to S_j and we give this edge the weight S_i. We draw an edge from each S_i back to 0 and give it the weight S_i, and we draw an edge from node 0 to each S_i and give each of these edges the weight $-w$. Now, a solution to subset sum is represented as a cycle from node 0 to each node S_i in the solution, back to 0. We see that if we have a correct solution to subset sum, this cycle must have a weight 0 because the weight that is $-w$ cancels out the weights that summed up to w in the cycle. On the other hand, we have a 0 weight cycle, then the weights of nodes in the cycle must have summed up to w because we had to use an edge with weight $-w$ to complete the cycle.