Solution

Consider this problem with the following rankings:

Name	Preference		
Alice	Bob	Carlos	Don
Bob	Carlos	Alice	Don
Carlos	Alice	Bob	Don
Don	Alice	Carlos	Bob

There is no stable matching in this case. In all three possible matchings, there is an instability.

1. If we pair {Alice, Bob} and {Carlos, Don}, Bob and Carlos prefer each other to their matches

2. If we pair {Alice, Don} and {Bob, Carlos}, then Alice and carlos prefer each other.

3. If we pair {Alice, Carlos} and {Bob, Don}, Bob and Alice prefer each other.

In every possible instance there is an instability, so we have shown that there does not exist a stable matching for all problems of this nature.