PROBABILITY AND STATISTICS TDAB01

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Lecture 1: Probability

PROBABILITY

- **Probability** describes *the chance / risk* that an event will occur

  Example: Coin tossing

  Equal chance for head or tail – probability ½ for each side

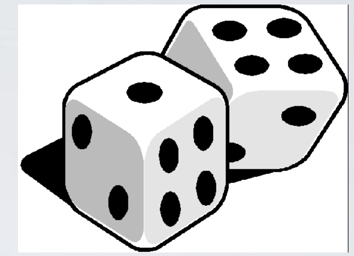
- Probability can be viewed as *relative frequency*:

$$\frac{\text{number of 'successful' outcomes}}{\text{number of trials}}$$

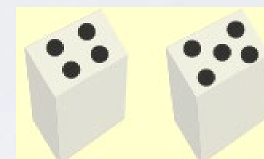
  if number of trials is *very large*

OUTCOMES, EVENTS, SAMPLE SPACE

- **Experiment**: Rolling two dice

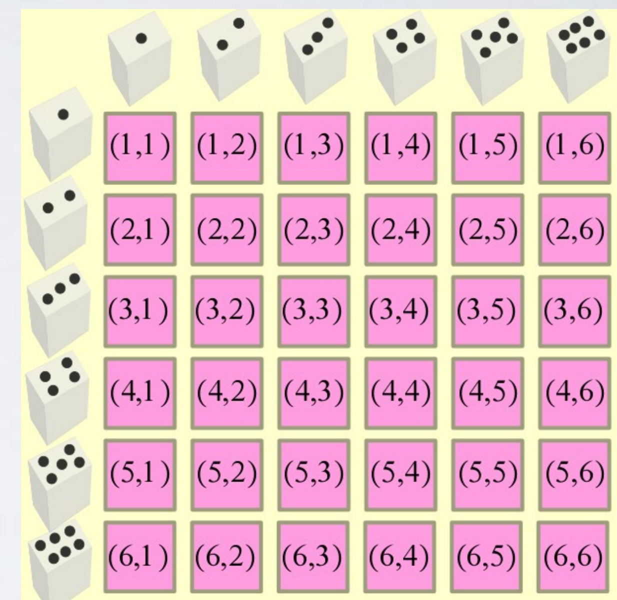


- **Outcome** / elementary result



- **Sample space:** collection of all outcomes

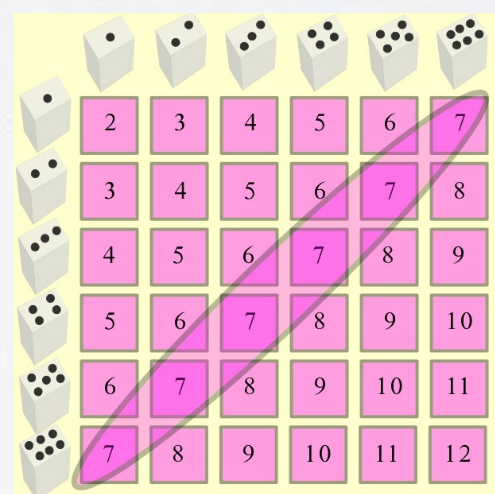
  Notation: Ω or S



- **Event:**. Set of outcomes

  A = {Sum is 7}

  P(A) – Probability of *A*

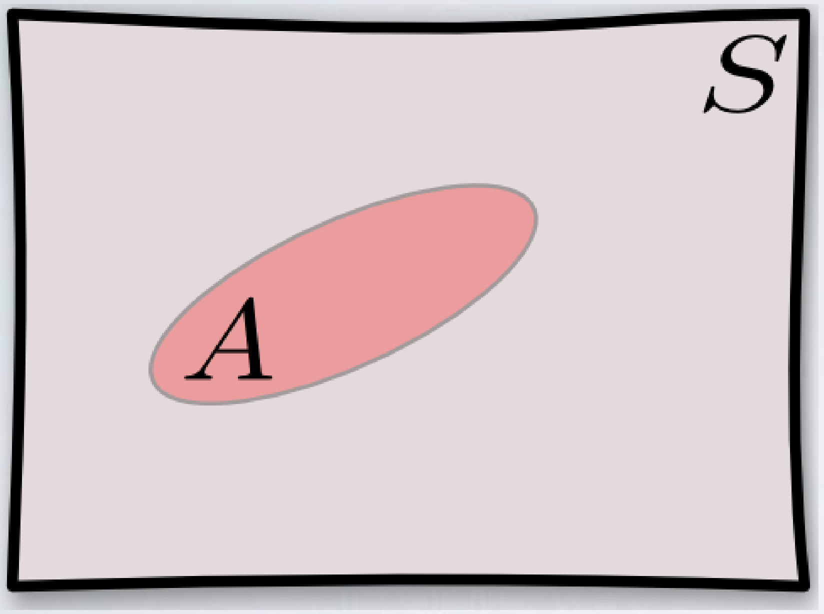


Ø - empty / impossible event

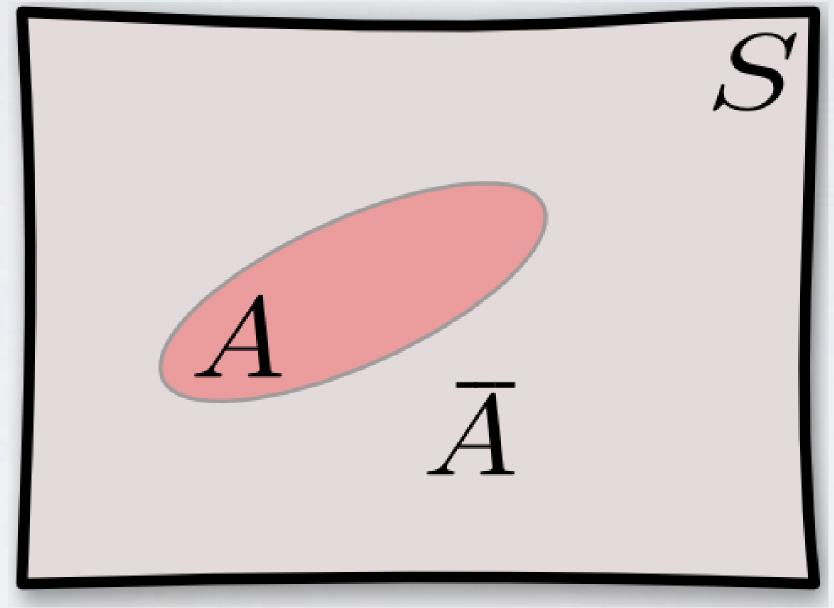
P(Ø)=0 & P(S)=1

SET OPERATIONS

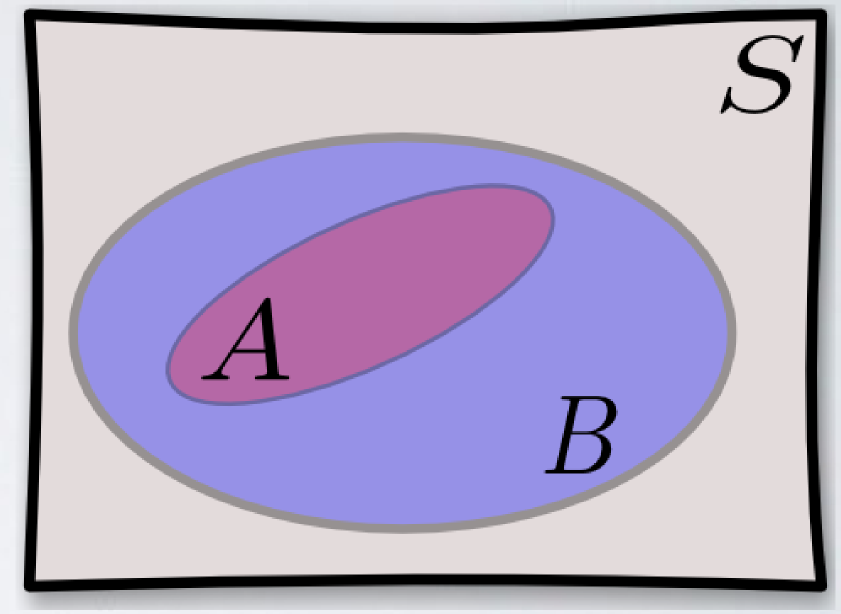
Event *A* included in sample space ***S***



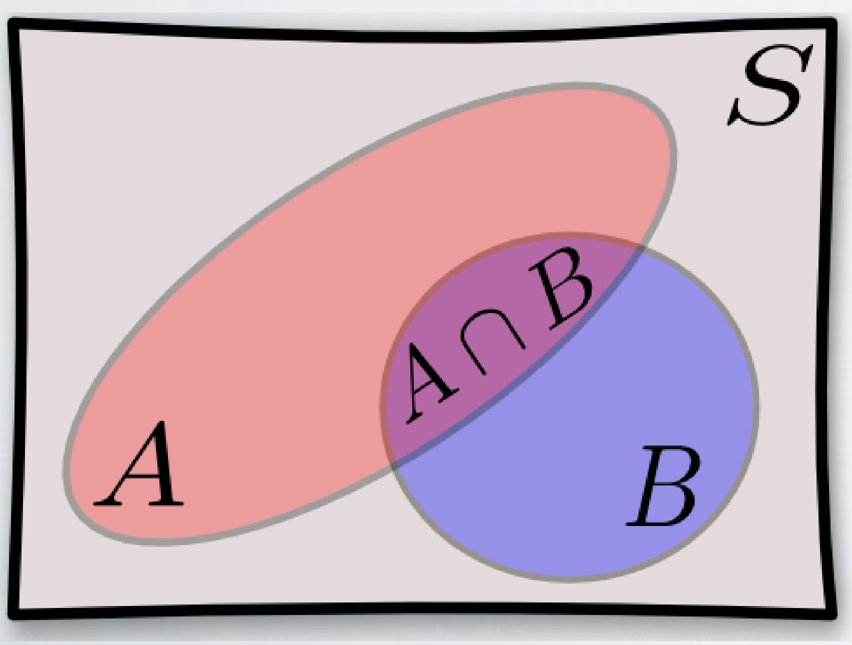
Complement of *A* – „not *A*“
outcomes excluded from *A*



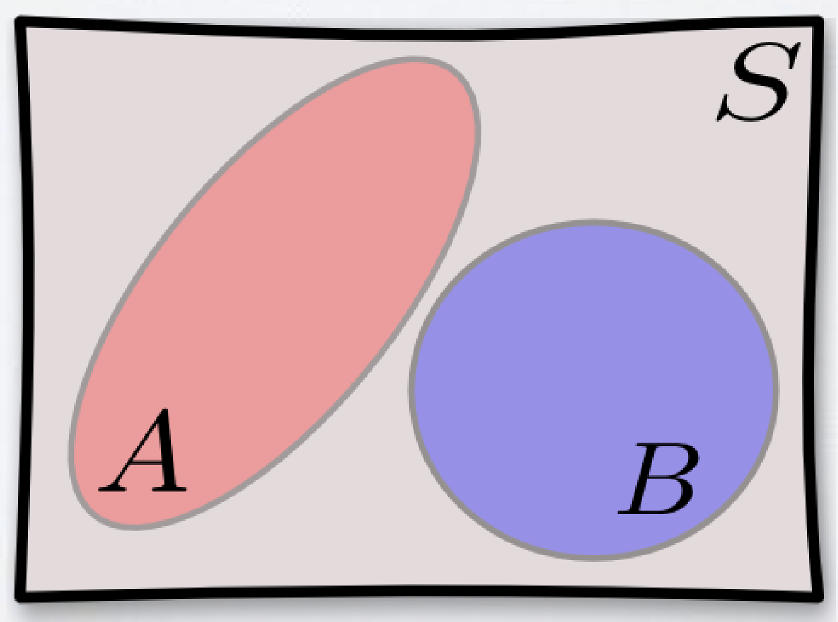
A **subset** of *B*
all elements of *A* are in *B*



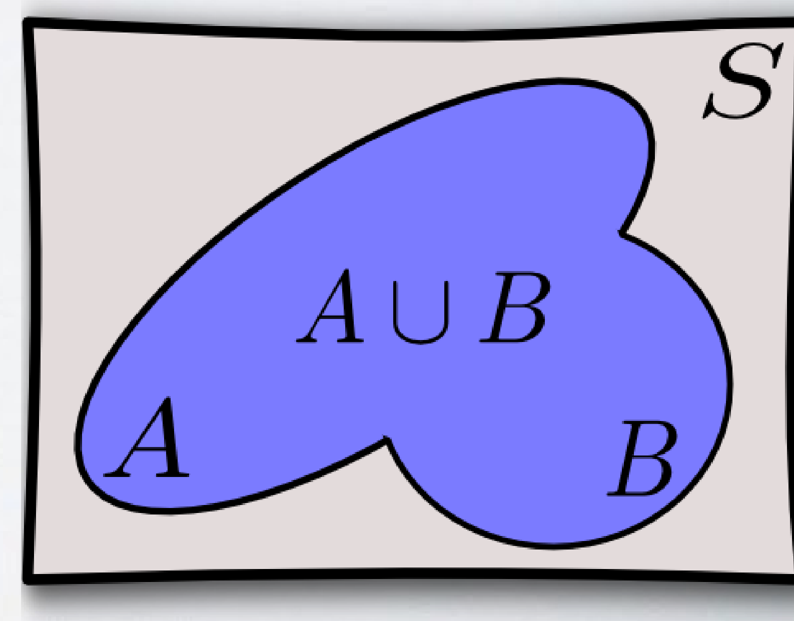
Intersection of *A* and *B* – „*A* and *B*“
all elements that are in both *A* and *B*



A and *B* **disjoint**
no common elements / intersection empty



Union of *A* and *B* – „*A* or *B*“
all elements from *A* and / or *B*



RULES OF PROBABILITY

- **Sample space / universal event**: $P(S) = 1$
- **Complement**: $P(\bar{A}) = 1 - P(A)$
- **Union**: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Disjoint events**: $P(A \cup B) = P(A) + P(B)$
- **Independent events**: Occur independently of each other

$$P(A \cap B) = P(A) \cdot P(B)$$

- From **union** to **intersection**

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \overline{A \cap B} = \bar{A} \cup \bar{B}$$

EXAMPLE

H = {hard drive crashes}

A = {first backup crashes}

B = {second backup crashes}

P(*H*)=0.01, ***P***(*A*)=0.02 and ***P***(*B*)=0.02

H, A and B are independent. Probability that file is saved - ?

$$P(\text{file saved}) = 1 - P(\text{ file lost }) = 1 - P(H \cap A \cap B)$$
$$= 1 - P(H) \cdot P(A) \cdot P(B)$$
$$= 1 - 0.01 \cdot 0.02 \cdot 0.02 = 0.999996$$

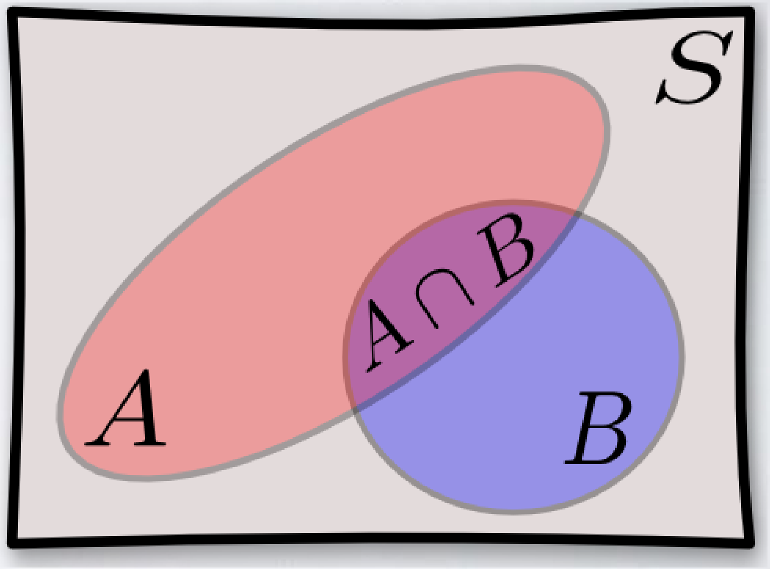
CONDITIONAL PROBABILITY

- Probability that event A occurs given that event B has occurred
- Notation: ***P****(A|B)*
- Definition: $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- Intersection, general case:

$$P(A \cap B) = P(B) \cdot P(A|B)$$

- Independent events:

$$P(A|B) = P(A)$$



BAYES RULE

- Sometimes we know $P(B|A)$, but are interested in $P(A|B)$
- Bayes rule:

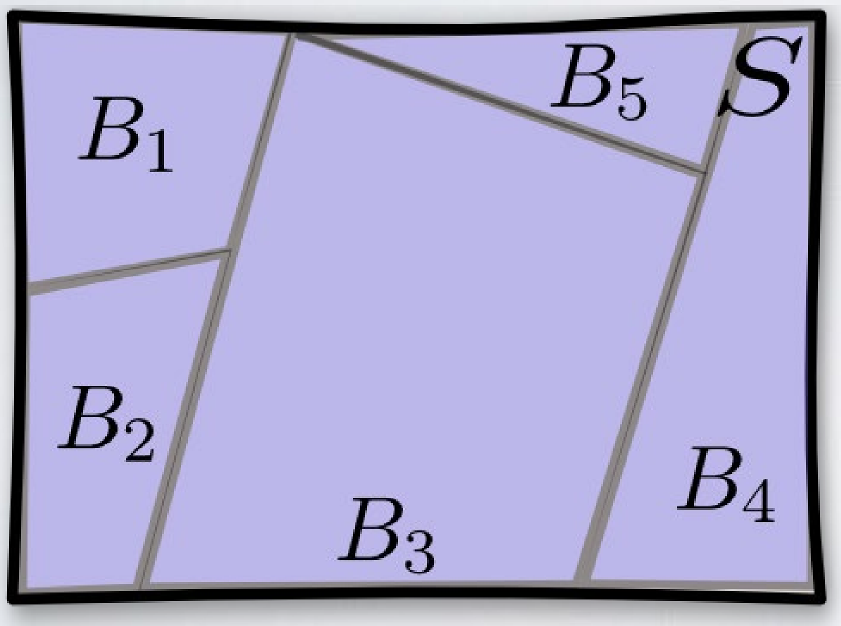
$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

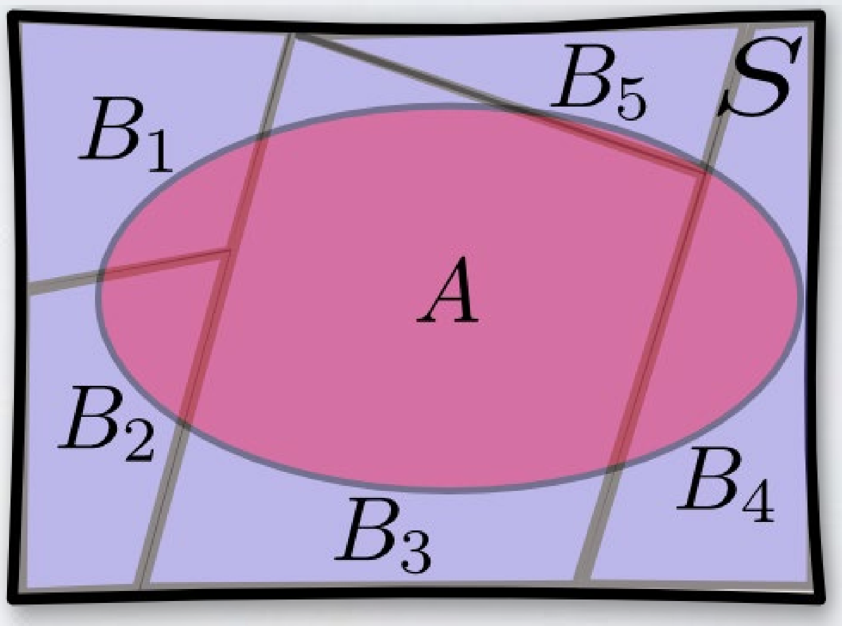
- Example: A = {person has virus}, B = {test positive}
    - $P(A|B) = P$(person has virus | test positive)
    - $P(B|A) = P$(test positive | person has virus)

LAW OF TOTAL PROBABILITY

$B_1, \ldots, B_5$ partition of S:

mutually exclusive/pairwise disjoint and **exhaustive**





$$A = (A \cap B_1) \cup \cdots \cup (A \cap B_5)$$

$$P(A) = P(A \cap B_1) + \ldots + P(A \cap B_5)$$

$$P(A) = P(A|B_1) \cdot P(B_1) + \ldots + P(A|B_5) \cdot P(B_5)$$

BAYES RULE - ALTERNATIVE FORM

- Bayes rule: $$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$
- Law of total probability provides

$$P(B) = P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})$$

- Alternative form of Bayes rule:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

EQUALLY LIKELY OUTCOMES

- Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be sample space with *n* equally likely outcomes: cards, dice, etc.

- *E* – event consisting of *t* equally likely outcomes
- Probability of *E*: t / n or

$$\frac{\text{number of outcomes in E}}{\text{number of outcomes in } \mathbf{\Omega}}$$

- Counting the number of opportunities / outcomes

$\longrightarrow$ Combinatorics

PERMUTATIONS AND COMBINATIONS

Select *k* elements from set of *n* elements

	with replacement	Without replacement
permutation	n^k	$\frac{n!}{(n-k)!}$
combination	$\frac{(k+n-1)!}{k!(n-1)!}$	$\frac{n!}{(n-k)!k!}$

Thank you for attention!