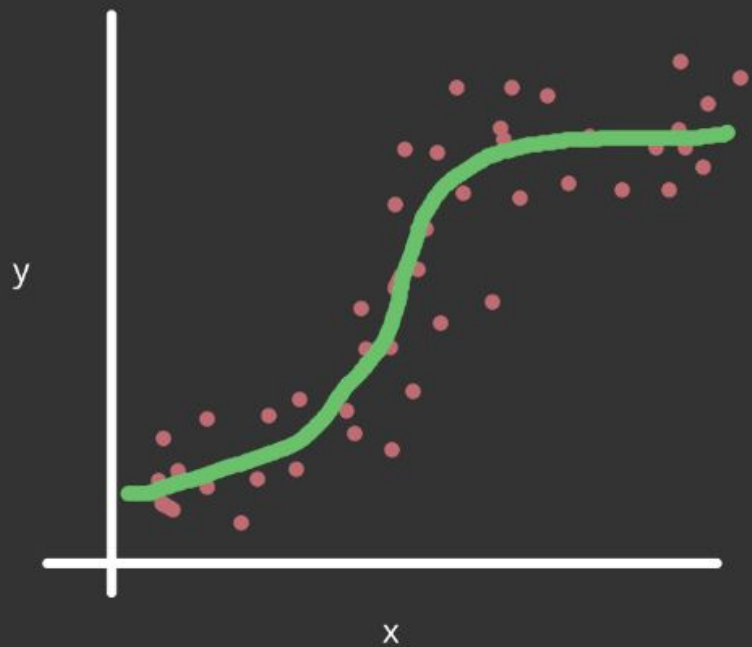


# L05

## Classification

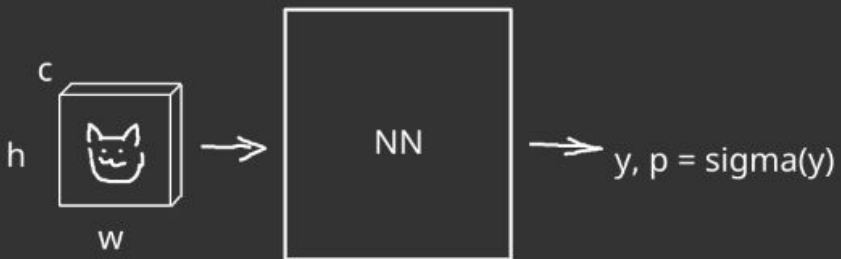
# Regression



# Binary Classification



Cat or Dog?



Sigmoid is activation. But what is the loss? MSE?

$$\sigma(y) = \frac{1}{1 + e^{-y}}$$

$$\frac{d\sigma}{dy} = \sigma(1 - \sigma)$$

$$MSE = (\sigma(y) - t)^2$$

$$MSE' = 2(\sigma(y) - t)\sigma' = 2(\sigma(y) - t)\sigma(1 - \sigma)$$

Sigmoid is activation. But what is the loss? MSE?

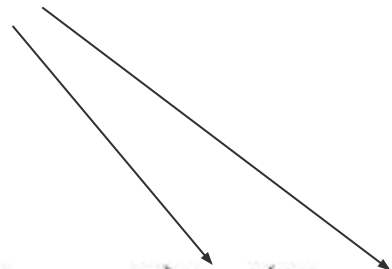
$$\sigma(y) = \frac{1}{1 + e^{-y}}$$

$$\frac{d\sigma}{dy} = \sigma(1 - \sigma)$$

$$MSE = (\sigma(y) - t)^2$$

$$MSE' = 2(\sigma(y) - t)\sigma' = 2(\sigma(y) - t)\sigma(1 - \sigma)$$

- independent on error
- can be easily become 0
- lead to NN paralysis



Sigmoid is activation. But what is the loss? BCE?

$$\sigma(y) = \frac{1}{1 + e^{-y}} = p$$

$$\frac{d\sigma}{dy} = \sigma(1 - \sigma)$$

$$BCE(p, t) = -t \log(p) - (1 - t) \log(1 - p)$$

Check if there are no  $\log(0)$  or  $\log$  of negative number

Sigmoid is activation. But what is the loss? BCE?

$$\sigma(y) = \frac{1}{1 + e^{-y}} = p \quad \frac{d\sigma}{dy} = \sigma(1 - \sigma)$$
$$BCE(p, t) = -t \log(p) - (1 - t) \log(1 - p)$$

$$\begin{aligned} \frac{d(BCE)}{dy} &= \frac{d(BCE)}{dp} \frac{dp}{dy} = -\frac{t}{\sigma} \sigma' + \frac{1 - t}{1 - \sigma} \sigma' = \\ &= -t(1 - \sigma) + (1 - t)\sigma = \sigma - t \end{aligned}$$



# BCE is Good for

- Probability estimation
- For estimation of any numbers in (0, 1) range

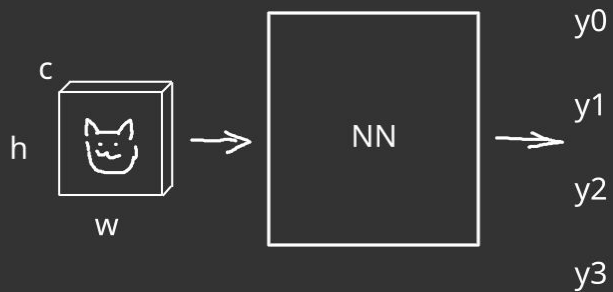




# Multiclass Classification



Cat, Dog, Pig, or Plane?



$$p(y_0) + p(y_1) + p(y_2) + p(y_3) = 1$$

Ref: [link](#)

# Multiclass Classification. Activation Function. Softmax

Must watch: [https://www.youtube.com/watch?v=8ps\\_JEW42xs](https://www.youtube.com/watch?v=8ps_JEW42xs)

$$SM_i(\vec{y}) = \frac{e^{y_i}}{\sum_{j=1}^N e^{y_j}}$$

$$SM_2([2, 3, 5, 0]) = \frac{e^5}{e^2 + e^3 + e^5 + e^0} = 0.84$$

$$SM([2, 3, 5, 0]) = [0.04, 0.11, 0.84, 0.01]$$

## Softmax Properties

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$SM_i(\vec{y}) = \frac{e^{y_i}}{\sum_{j=1}^N e^{y_j}}$$

$$0 < SM < 1$$

$$\sum_i SM_i = 1$$

$$\frac{d(SM_i)}{dy_c} = \frac{-e^{y_i} e^{y_c}}{(\sum_{j=1}^N e^{y_j})^2} = -SM_i \cdot SM_c, c \neq i$$

# Softmax Properties

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$SM_i(\vec{y}) = \frac{e^{y_i}}{\sum_{j=1}^N e^{y_j}}$$

$$\frac{d(SM_i)}{dy_i} = \frac{e^{y_i} \sum_{j=1}^N e^{y_j} - e^{y_i} e^{y_i}}{(\sum_{j=1}^N e^{y_j})(\sum_{j=1}^N e^{y_j})} = SM_i \cdot (1 - SM_i)$$

# Loss Function for Softmax: Cross Entropy

$$p_c = SM_c(\vec{y})$$

$$CE(p, t) = - \sum_{c=1}^N t_c \log p_c$$

Task: take 2 distributions and calculate  $CE$

# Loss Function for Softmax: Cross Entropy. Is It Good?

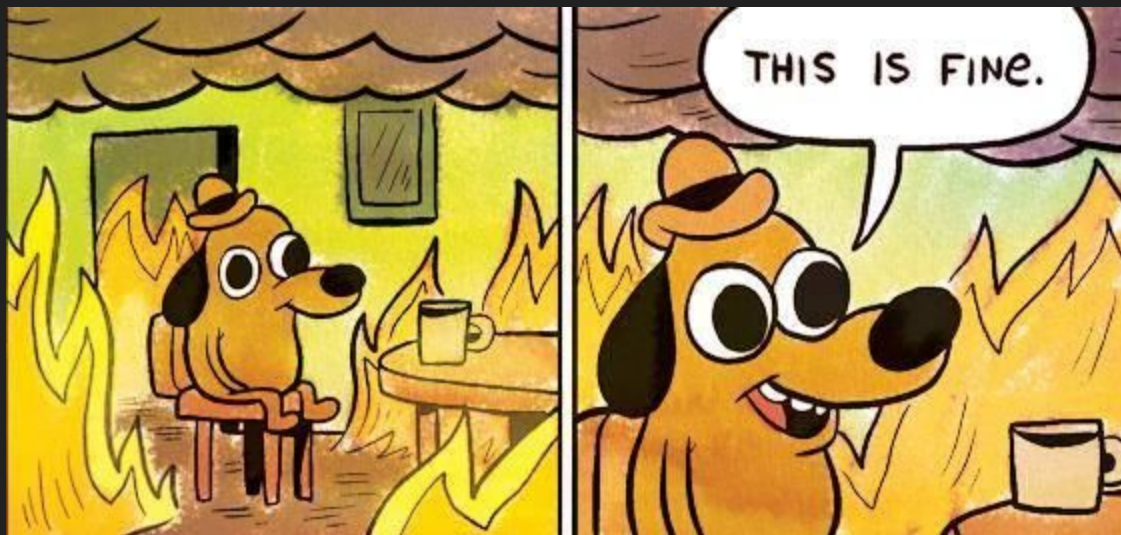
$$p_c = SM_c(\vec{y})$$

$$CE(p, t) = - \sum_{c=1}^N t_c \log p_c$$

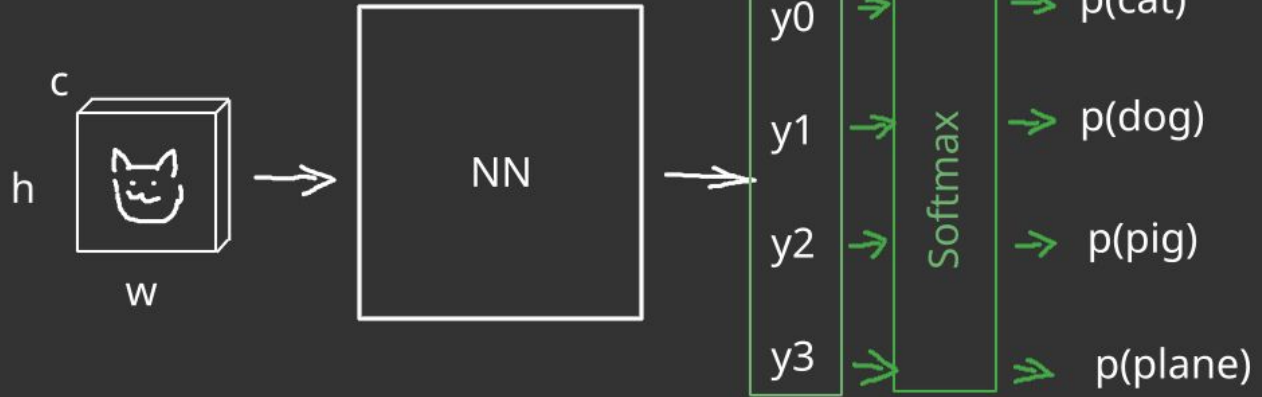
$$\begin{aligned} \boxed{\frac{d(CE)}{dy_i}} &= \frac{- \sum_{c=1}^N t_c \log p_c}{dy_i} = \\ &= - \frac{t_i}{p_i} \frac{d(SM_i)}{dy_i} - \sum_{c \neq i} \frac{t_c}{p_c} \frac{d(SM_c)}{dy_i} = \\ &= - \frac{t_i}{p_i} p_i (1 - p_i) + \sum_{c \neq i} t_c \frac{1}{p_c} p_i p_c = -t_i (1 - p_i) + \sum_{c \neq i} t_c p_i = \boxed{-t_i + p_i} \end{aligned}$$



Let's take a break and sip tea



# A lot of math. What about multiclass classification?

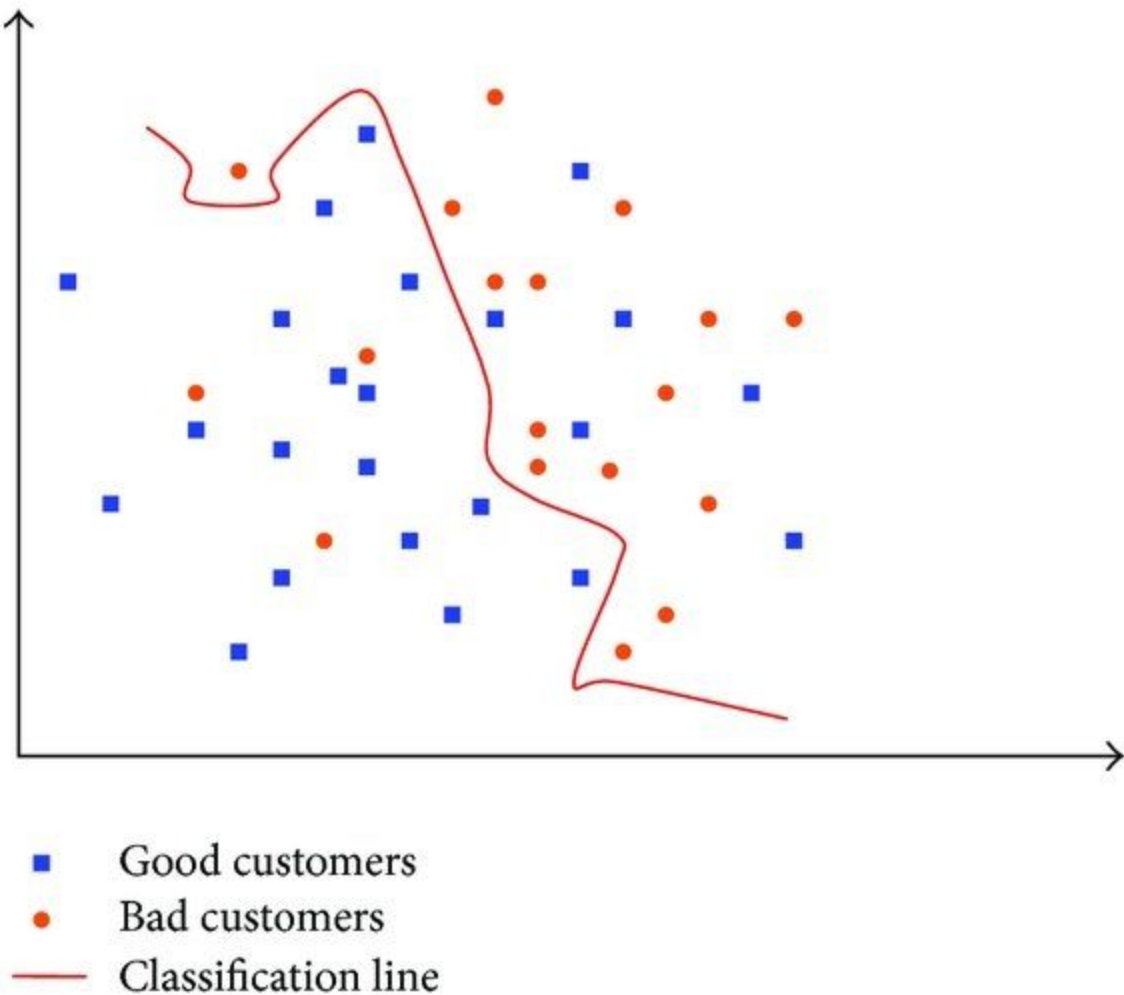


Cat, Dog, Pig, or Plane?

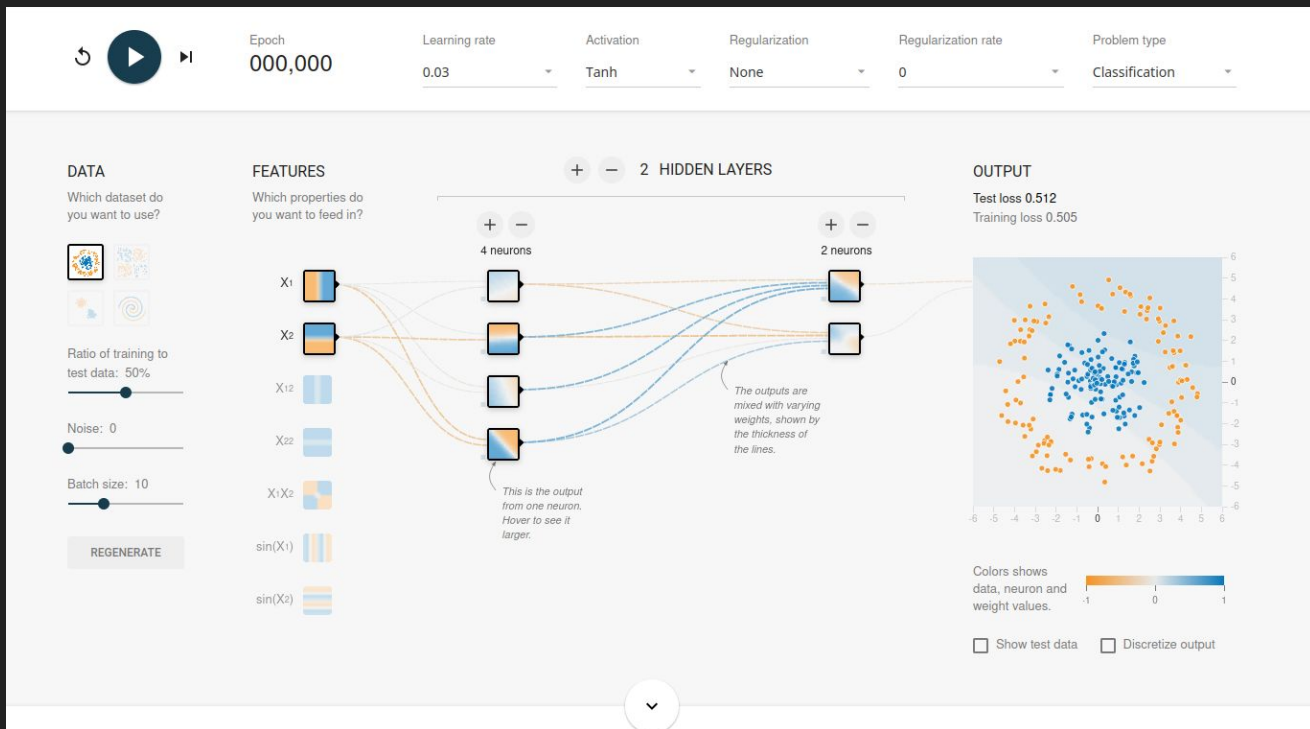
$$p(y_0) + p(y_1) + p(y_2) + p(y_3) = 1$$



# Classification Demo



# Tensorflow Playground



<https://playground.tensorflow.org>

# API

- Datasets:  
<https://pytorch.org/vision/stable/datasets.html>
- Models:  
<https://pytorch.org/vision/stable/models.html>
- Tutorials  
<https://pytorch.org/tutorials/>
- Documentation:  
<https://pytorch.org/docs/stable/index.html>

Trying random stuff for hours instead of reading the documentation



# HW

<https://scikit-learn.org/stable/modules/classes.html#samples-generator>

- `make_blobs()`
- `make_circles()`
- `make_moon()`